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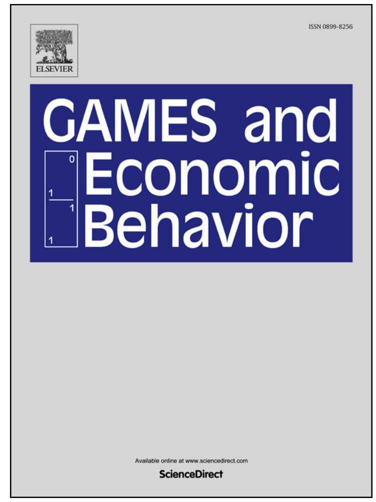


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# Reacting to ambiguous messages: An experimental analysis

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**Abstract**

Ambiguous language is ubiquitous and often deliberate. Recent theoretical work (Bose and Renou, 2014; Kellner and Le Quement, 2018; Beauchêne, J. Li, and M. Li, 2019) has shown how language ambiguation can improve outcomes by mitigating conflict of interest. Our experiment finds a significant effect of language ambiguation on subjects who are proficient at Bayesian updating. For ambiguity averse subjects within this population, a significant part of this effect operates via the channel of subjects' desire to reduce ambiguity. For both ambiguity averse and neutral subjects within this population, an additional behavioral channel is also present. (JEL: C91; D01; D81)

Keywords: Ambiguity aversion; Communication; Persuasion; Laboratory experiment.

## 1. Introduction

Individuals often make use of ambiguous formulations. Such ambiguity often appears deliberate as it could easily be avoided. Governors of the US central bank have been known for their use of cryptic language. In 1995, a speech by Alan Greenspan gave rise to very different headlines, the New York Times

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writing "Doubts voiced by Greenspan on a rate cut" and the Washington Post writing instead "Greenspan hints Fed may cut interest rates" (see Blume and Board, 2014).[1] Other examples include contracts or advertising messages as well as political speech—e.g., former UK Labour Party leader Jeremy Corbyn's stance on Brexit.

How and why senders resort to ambiguous messages remains subject to discussion. Clarifying this requires understanding better how audiences react to such messages. We conduct a lab experiment which focuses on a specific type of ambiguous messages characterized by non-probabilistic uncertainty about the used communication rule. After such a message, a receiver who originally had a unique prior now faces multiple posteriors which he is not willing to compound into a single probability distribution over the unknown state of the world. Such non-probabilistic uncertainty is labeled ambiguity in the decision theory literature, motivating our use of the term ambiguous messages. In our setup, non-probabilistic uncertainty about the communication rule is generated by conditioning messages on privately observed draws from urns with an unknown composition (so-called Ellsberg urns). Echoing this description, real language features many expressions (cryptic sentences, unclear words) whose use cannot be described in probabilistic terms and which are, in this sense, ambiguous in their meaning. The type of ambiguous messages that we study could be seen as a specific instance of vague language. By vagueness one often means that the language is produced according to an unclearly defined rule.[2]

1. See also the following excerpt from a 2001 Congressional hearing speech by A. Greenspan "The members of the Board of Governors and the Reserve Bank presidents foresee an implicit strengthening of activity after the current rebalancing is over, although the central tendency of their individual forecasts for real GDP still shows a substantial slowdown, on balance, for the year as a whole." (Federal Reserve Board's semiannual monetary policy report to the Congress Before the Committee on Banking, Housing, and Urban Affairs, U.S. Senate February 13, 2001)

2. See for example Lipman (2009), Williamson (1996), and Peirce (1902). Lipman (2009), discussing vagueness, gives the example of the word "tall", where the threshold above which

The study of language as an equilibrium phenomenon goes back to the seminal contribution of Crawford and Sobel (1982), which has since spurned a vast literature; theoretical, applied and experimental (see Sobel, 2013; Blume, Lai, and Lim, 2020, for reviews). Key applications include settings with multiple senders or receivers, repeated communication over time, boundedly rational players or image concerns. Equilibrium communication in the baseline Crawford and Sobel (1982) does not feature ambiguity or vagueness but simply imperfect (i.e., coarse) information transmission: intervals of sender types pool on the same messages and thus leave the receiver ex post uncertain.

Kellner and Le Quement (2018) as well as Beauchêne, J. Li, and M. Li (2019) have shown how ambiguity might emerge in addition to pooling in the Crawford and Sobel (1982) model. The communication strategy there combines partitioning with non-probabilistic randomization. A central insight is that rather than hindering communication, language ambiguation—i.e., making language non-probabilistic—can help improve communication by mitigating conflict of interest. The insight is relevant to important applications (see for example Evdokimov and Garfagnini (2019) for an experiment on communication in organizations).

Our experiment aims at testing whether real subjects' response to language ambiguation echoes theory. Does ambiguation affect behavior in the expected direction and in a quantitatively significant way? If so, via which channels? Besides ambiguity averse (or loving) subjects' specific response to ambiguity, behavioral effects could potentially be significant.

We find that ambiguation significantly shifts behavior in the expected direction. We focus our analysis on subjects who demonstrate good Bayesian updating skills when faced with standard partitional messaging rules, and we call these Bayes-Competent. For Bayes-Competent subjects who are ambiguity averse, a significant part of the effect of language ambiguation operates through

someone should be called "tall" is uncertain (words such as "many", or "good" are other examples).

a specific hedging mechanism driven by subjects' desire to reduce ambiguity. Perfect hedging would correspond to the case where agents pick an action, after observing an ambiguous message, which ensures that their expected utility is the same whatever the true randomization probability used by the sender. This effect complements an anchoring effect of similar magnitude. Among Bayes-Competent subjects who are ambiguity neutral, the hedging effect appears to be absent.

In the main treatment task, each subject (also called DM) must choose a number after observing a message issued by an automated process. The message provides information on an unobservable state drawn from $[0, 100]$. DM's payoff decreases linearly in the distance between number and state. We run variations of this task within and between subjects. Our main focus is on the ambiguous variant, which we now describe.

The state $\omega$ is drawn from an (unambiguous) uniform distribution on $[0, 100]$. The latter interval is partitioned into three subintervals $[0, 50)$, $[50, c)$ and $[c, 100]$, for some known $c$. There are three messages, $\star$, $X$, and $\#$. If $\omega \in [0, 50)$, the message sent is $\star$. If instead $\omega$ lies in $[50, c)$ or $[c, 100]$, the message conditions also on an unobservable draw from a so-called Ellsberg urn featuring blue and red balls in unknown proportions. If the draw is red, the message is X if the state lies in $[50, c)$ and $\#$ if it lies in $[c, 100]$. If the draw is blue, the use of X and $\#$ is reversed.

After observing $\star$, DM has a unique posterior, so we expect her to choose 25, the conditional expectation of $\omega$. In contrast, X and $\#$ leave DM with multiple posteriors and her choice should thus depend on her ambiguity attitude and belief updating.

If a DM is ambiguity averse and uses prior by prior updating, a wide range of common models of decision-making under ambiguity (max-min, $\alpha$-max-min, or the smooth model) predict that her optimal action is greater than 75 if $c$ is greater than 75. Similar behavior could arise under a known urn composition if the DM does not reduce compound lotteries. Instead, an ambiguity neutral

DM considering both colors equally likely (at least on average) should choose 75 after X and #.

To gain some intuition, consider the case of a max-min DM choosing an action (a number) after the signal X or #. Such a DM evaluates every action according to its worst-case scenario, the lowest expected utility among all possible urn compositions. Practically, the decision maker needs to consider only the two extreme scenarios, which correspond to a degenerate urn composition (all balls red or all blue). Given the messaging rule, under one of these scenarios the state $\omega$ is uniformly distributed over the interval $[50, c)$, under the other scenario over the interval $[c, 100]$. For low actions, the worst-case scenario corresponds to the high interval, and expected payoffs are increasing in the action. As the action increases, the worst-case scenario at a certain point changes to the low interval and expected payoffs now become decreasing in the action. Overall, the worst-case expected utility is inverse v-shaped and uniquely maximized at the action where the worst-case scenario shifts. At this action, all possible urn compositions yield the same expected utility and thus the decision maker is fully hedged against uncertainty.

To understand that this max-min action is greater then 75, note that for the action 75 the worst-case scenario uniquely corresponds to the high interval, as we explain now. Under the high interval, if 75 is chosen, the loss of $|75 - \omega|$ is uniformly distributed over the interval $[c - 75, 25]$ (with $c - 75 > 0$). Under the low interval, the loss is distributed instead over $[0, 25]$ and any loss in $(0, c - 75])$ is twice as likely as any loss in $(c - 75, 25]$. Thus, the high interval leads to a more adverse distribution of losses (in the sense of first order stochastic dominance) and thus lower expected utility for the action 75. Next, note that under the high interval, since the state is known to be above $c > 75$, expected utility is still increasing in the action at 75. Thus, worst-case expected utility is increasing at 75 as well.[3]

3. Under alternative models such as the smooth model and the $\alpha$-max-min model, the DM is still influenced by worst-case expected utility, though to a lesser extent. Accordingly, the mechanism described above still applies but affects the DM's action to a lesser degree. The

We run a two by two treatment design. The first variable is subjects' knowledge of the composition of the urn (so-called risky vs ambiguous treatments). The second variable is whether subjects are given help in updating their beliefs.

After the main treatment task, subjects perform a set of control tasks checking their (1) ability to update beliefs, (2) anchoring tendency, (3) risk and ambiguity aversion and (4) cognitive ability.

*Literature review.* Starting from Ellsberg (1961), a rich theoretical literature has developed on the subject of decision-making under ambiguity.[4] Decision-making under ambiguity has also been studied experimentally.[5]

A new experimental literature studies responses to ambiguous signals. Epstein and Halevy (2019) study signals of ambiguous precision and distinguish between attitudes towards "prior-ambiguity" and "signal-ambiguity". They find non-indifference to signal-ambiguity and association between attitudes towards

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same applies also if a max-min decision maker would not consider all possible probability distributions, but, say, only those which she considers particularly plausible.

4. Ellsberg (1961) presents a thought-experiment displaying behavior incompatible with subjective expected utility maximization. He rationalizes behavior by introducing ambiguity aversion. The max-min Expected Utility model (Gilboa and Schmeidler, 1989) posits that an ambiguity averse DM facing multiple priors evaluates each action according to its worst-case expected utility across priors and maximizes the thereby constructed lower envelope. The smooth model of ambiguity aversion (Klibanoff, Marinacci, and Mukerji, 2005) incorporates second order beliefs (a prior over priors) and quantifies the degree of ambiguity aversion through a concavity parameter which is a counterpart of the standard risk parameter. The max-min model and the smooth model yield similar predictions in our setup. Given ambiguity averse preferences (defined over an unrestricted domain) and an updating rule, behavior must violate either dynamic consistency or consequentialism (see e.g., Siniscalchi, 2011; Hanany and Klibanoff, 2007; Hanany and Klibanoff, 2009).

5. Fox and Tversky (1995) finds that the effect of ambiguity is greater if only a subset of options features ambiguity. Halevy (2007) shows that ambiguity aversion strongly associates with the failure to reduce compound lotteries. Cubitt, van de Kuilen, and Mukerji (2019) find evidence that choices are more in line with the smooth ambiguity model than with max-min. Dominiak, Duersch, and Lefort (2012) and Bleichrodt, Eichberger, Grant, Kelsey, and C. Li (2018) find that subjects' updating procedure is harder to reconcile with dynamic consistency than with consequentialism.

prior- and signal ambiguity. Shishkin and Ortoleva (2019) and Kops and Pasichnichenko (2020) study the value of ambiguous signals in the case where "all news is bad news".[6] In this case, an ambiguity averse decision maker using prior-by-prior updating assigns a lower valuation to a given bet after *every* signal realization. The key is that for such signals—call them dilation signals—the set of posteriors after any signal realization contains the original set of priors.[7] Shishkin and Ortoleva (2019) compare the willingness to pay for a 50:50 bet with and without being exposed to a dilation signal. The authors find that empirically, decision makers do not assign negative value to dilation signals, in contrast to theoretical predictions. Kops and Pasichnichenko (2020) instead offer a choice between two comparable options, both of which involve being exposed to a dilation signal. The signal provides payoff-relevant information only for the second of two options, and this second option yields slightly higher payoffs in all states. They find that decision makers prefer the first option, where the dilation signal is not payoff-relevant, and, when given a further choice, prefer not to be exposed to the dilation signal.

In contrast to Shishkin and Ortoleva (2019) and Kops and Pasichnichenko (2020), in our experiment signals have positive ex ante value—a relevant case for many applications—as they always reveal whether $\omega$ lies above or below 50. Yet, between our ambiguous treatment and an alternative in which X or # would be merged, a DM would prefer the latter. Without commitment, the ambiguity contained in X or # has a negative value ex ante. To reconcile Shishkin and Ortoleva (2019) with our findings, one could posit that DMs ignore only signals that are not valuable from an ex ante perspective.

A rich body of work studies behavioral biases in belief updating (see for example Kahneman and Tversky, 1974; Jörg Oechssler, Roider, and Schmitz, 2009). Anchoring occurs when irrelevant information becomes a reference point distorting peoples' belief updating and action choice. For example, exposure to

6. For theoretical discussions of the value of information under ambiguity aversion see Siniscalchi (2011) and J. Li (2020).

7. See Seidenfeld and Wasserman (1993) for the first definition of dilations.

a random integer might affect guesses on the percentage of African countries in the UN. Cognitive sophistication has been shown to negatively correlate with such bias (see Jörg Oechssler, Roider, and Schmitz, 2009; Bergman, Ellingsen, Johannesson, and Svensson, 2010).

There exists a lively literature on strategic information transmission and various forms of vagueness, both theoretical and experimental. A key message of this literature is that vagueness can help incentivize more informative communication by senders and ultimately improve welfare. There are two strands of literature, (a) papers that take a non-probabilistic uncertainty approach and (b) papers that take an expected utility approach.

We build on the non-probabilistic uncertainty approach proposed in Kellner and Le Quement (2018) and Beauchêne, J. Li, and M. Li (2019).[8] The other set of contributions studies language vagueness in an expected utility framework. A common aspect is the presence of some known noise process that generates randomness in the message received by the receiver or in the interpretation thereof. Garbling, by generating pooling of sender (S) types, can induce the receiver (R) to react less adversarially to messages, in turn incentivizing S to reveal more. In the partitional equilibria of Blume, Board, and Kawamura (2007), R's expectation is a weighted average of the conditional expectation without transmission error and the ex ante mean, implying a beneficial upwards distortion of R's action. In Blume and Board (2014), the sender abstains from choosing messages that minimize the effect of exogenous channel noise, thereby similarly achieving a conflict mitigating effect. Giovannoni and Xiong (2019), building on a framework by Blume and Board (2013), studies cheap talk under

8. Both build on Bose and Renou (2014), where a principal can use an Ellsbergian device to make the agents face ambiguity. Kellner and Le Quement (2018) considers the case of cheap talk with an ambiguous distribution of the state, and find that in sender optimal equilibria, the sender randomizes between partitional strategies and thereby hedges against exogenous ambiguity. See also Colo (2021) for a complementary analysis of cheap talk under exogenous ambiguity. Ambiguity in strategic settings has been studied in general in Azrieli and Teper (2011), Bade (2011) and Riedel and Sass (2014). See also Lo (1996) and Klibanoff (2001) on equilibrium in ambiguous beliefs.

language barriers which restrict messages that can be respectively sent and understood. Under the standard communication protocol, language barriers weakly improve equilibrium welfare by relaxing incentive conditions.

Comparing the expected utility approach to the approach of Kellner and Le Quement (2018) and Beauchêne, J. Li, and M. Li (2019), a number of clear differences appear. In the latter, noise is entirely endogenous, it does not come in the form of a well specified stochastic process, and it affects R's actions via a different channel, namely hedging against ambiguity. [9]

Language vagueness has also been studied experimentally. Closest to the above contributions, Blume, Lai, and Lim (2021) find that—as predicted by theory—garbling of information via a mediator can improve reporting incentives, by advantageously distorting the receiver's response to information. Vagueness has also been shown to have value through other types of channels. Agranov and Schotter (2012) study a privately informed principal sharing information with agents about potentially asymmetric payoffs in a coordination game. Coarse communication helps by hiding payoff asymmetries which hurt coordination by weakening focal points. Serra-Garcia, Van Damme, and Potters (2011) study sequential public good games in which a leader privately informed about the good's value can communicate with others. Leaders prefer using vague messages rather than explicitly lying, and achieve the same efficient outcome as if lying due to imperfect updating by followers. In Blume, Lai, and Lim (2019), exogenous garbling of sender messages improves incentives to reveal unfavorable information by mitigating risk through the option to plausibly deny specific interpretations. Finally, Evdokimov and Garfagnini (2019) investigate experimentally communication within organizations à la Alonso, Dessein, and Matouschek (2008) and conjecture that receivers' biased

9. Lipman (2009), studying cheap talk under aligned interests, argues that explaining the value of vagueness requires assuming bounded rationality. Jäger, Metzger, and Riedel (2011) consider a sender with an infinite type space endowed with a finite message set. Equilibrium language is by definition imprecise, and the paper identifies efficient languages.

behavior might originate in ambiguous communication as in Kellner and Le Quement (2018).

## 2. Experimental design

### *2.1. Main treatment task*

In the main treatment task, the state of the world $\omega$ is given by a number between 0 and 100, which is drawn from a uniform distribution on $[0;100]$.[10] An automated process generates an informative signal (also called message) about the state. Upon observing the signal, a subject has to choose a point estimate of the true state and is rewarded in money according to the distance between the state and her estimate. Denoting the chosen action by $a$, the payoff function is simply given by $-|\omega - a|$. Accordingly, given a unique probability distribution of $\omega$, the subject's expected payoff maximizing action is the expected value of the state.[11]

We now describe the signal generating process in more detail. The state space $[0,100]$ is partitioned into three adjacent intervals $[0,50)$, $[50,c)$ and $[c,100]$, which we call intervals 1, 2 and 3, respectively. Moreover, there is an urn containing 100 balls which can be either red or blue. Before a message is sent, a ball is drawn randomly from the urn, the color of which is not observed by the subject. Let $\theta$ be a random variable that takes either value $r$ if the drawn ball is red or $b$ if it is blue.

The message sent depends on $\omega$ and $\theta$ as follows. If $\omega \in [0,50)$, the signal sent is $\star$ no matter the value of $\theta$. If $\omega$ lies in intervals 2 or 3, the emitted message depends on $\omega$ and on the value of $\theta$. If $\theta = r$, then the sent message

10. Within the experiment all random draws are simulated using the random number generator of zTree (Fischbacher, 2007).

11. Computerizing the sender makes the problem under consideration a decision problem as opposed to a game between a strategic sender and a receiver. This leaves no scope for other-regarding or moral preferences (Gneezy, 2005; Wang, Spezio, and Camerer, 2010).

is X if the state is in interval $[50, c)$ while the message is # if the state is in interval $[c, 100]$. If, on the other hand, $\theta = b$, then the messaging rule on these two intervals is reversed; i.e., the message is # if the state is in interval $[50, c)$ and the message is X if the state is in interval $[c, 100]$.

We will refer to this task as MAIN-TREATMENT. Participants make nine decisions in MAIN-TREATMENT. The value of $c$ changes with each repetition and is drawn from the set $\{54, 64, 86, 96\}$. The values of $c$ are assigned in random order, in a way that guarantees that each subject was assigned each value at least twice across the nine iterations. We introduce two independent dimensions of between subjects variation. These are described below.

The first dimension of between subjects variation has to do with what subjects know concerning the distribution of colors in the urn. In the so-called RISKY environment, the subject knows that there are 50 red and 50 blue balls in the urn. In the so-called AMBIGUOUS environment, the subject has no information regarding the proportion of red and blue balls in the urn.

The second dimension of between subjects variation is whether or not we provide subjects with help in forming beliefs. Across the RISKY and AMBIGUOUS environments, in case help is provided, we assist subjects in evaluating the conditional probabilities of the different messages across intervals 1-3. In the RISKY environment, we point out that messages $X$ and # each have a conditional probability $\frac{1}{2}$ of being sent both on intervals 2 and 3 (and 0 on interval 1). In the AMBIGUOUS environment, subjects are asked to propose a potential composition of the urn. Given this composition, we provide the conditional probabilities of the different messages in the different intervals. Subjects are asked to repeat this procedure for several possible urn compositions (at least two, at most four).

### *2.2. Control tasks*

Our control tasks cover four themes: Agents' belief updating abilities, their anchoring propensity, their attitudes to risk and ambiguity, and finally their general understanding and ability.

*2.2.1. Belief updating ability.*

*Choice after message $\star$ in the main treatment task.* In the main treatment task, as message $\star$ does not depend on draws from the Ellsberg Urn, it allows us to evaluate to which extent the participants follow Bayesian updating in the absence of ambiguity. The control variable *starchoice* records the difference between the choice after this message and the Bayesian choice 25.

*Only red balls task.* The decision is identical to the MAIN-TREATMENT task, with the difference that subjects are told that the urn contains only red balls. The task is repeated three times and each repetition features a new independent draw of the state.

*Belief elicitation task.* This set of controls is referred to as BELIEFS. We explicitly elicit subjects' probabilistic beliefs over the actual interval within which the state is contained. Subjects face the same signal generating process as in the MAIN-TREATMENT task. However, the value for $c$ is fixed at 80, *and* they now are informed about the distribution of colors in the urn, independent of the previously encountered MAIN-TREATMENT task, in which they were faced with either a risky or an ambiguous urn.

The nature of the decision after receiving a signal differs from that encountered in the MAIN-TREATMENT task. A specific task is repeated twice with minimal modification. In the first variant of the task, subjects can choose between a fixed option A and a list of versions of Option B, each being indexed by a value of $x \in (0, 1)$. Option A yields 100 ECU if the state is in interval 2 and 0 otherwise. Option $(B, x)$ yields 100 ECU with probability $x$ and otherwise nothing. The values of $x$ considered are $\{.1, .2, .3, .35, .4, .45, .5, .55, .6, .65, .7, .75, .8, .9.\}$. We chose this grid to be sufficiently fine in the region of interest. The second variant of the task is identical, except that option A yields 100 ECU if the state is in interval 3 and 0 otherwise.

In the two above variants of the task, the value of $x$ at which the subject switches from option A to option B indicates the probability that she attributes to the respective interval (2 in the first task, 3 in the second). An expected utility DM should attribute probability 0 to intervals 2 and 3 after message $\star$. The probability assigned to interval 2 should be .6 after $X$ and #. The probability assigned to interval 3 should be .4 after $X$ and #. After $X$ and #, if the elicited probabilities for intervals 2 and 3 do not add up to one, this could be a sign that, perhaps due to difficulties in updating, participants consider also the risky treatment in fact as ambiguous.

*2.2.2. Anchoring propensity.* The Anchoring 1 and Anchoring 2 control tasks test subjects' anchoring propensity. A concern in our setup is that the partitioning of the $[0, 100]$ interval potentially makes threshold $c$ an anchor. Subjects who anchor might display a tendency to choose an action close to $c$. Anchoring 1 and Anchoring 2 provide simplified environments in comparison to the Main-Treatment task. The expectation is that subjects who anchor in the treatment also anchor in these simpler tasks. Anchoring 1 and Anchoring 2 share the following basic features. The task is iterated 3 times with an independent draw of the state in each repetition. The value of $c$ changes across periods. Each subject observes three out of the four values in the set $\{54, 64, 86, 96\}$. Observed values and their order are randomly determined.

In the Anchoring 1 control task, we reduce the signal space to $\{\star, X\}$ and subjects are informed that they will receive signal $\star$ if the state is in interval 1 and X if it is either in interval 2 or in interval 3. Threshold $c$ has thus lost its significance, i.e. it should not affect the action taken by DM in response to messages X and #. Both of these messages contain only the information that $\omega \geq 50$, whatever the value of $c$.

In the Anchoring 2 control task, the messaging rule conditions on the color of the ball drawn from the urn. For subjects participating in a risky treatment, the urn is known to contain 50% red balls. For subjects participating in ambiguous treatments, the composition of the urn is unknown. As usual the message is $\star$ if the state is in interval 1. If the drawn ball is red and the state is either in interval 2 or 3, then the message is X. If on the other hand the ball is blue and the state is either in interval 2 or 3, then the message is #. Again, note that the threshold $c$ should not affect the action taken by DM in response to messages X and #. Both of these messages contain only the information that $\omega \geq 50$, whatever the value of $c$.

*2.2.3. Risk and ambiguity attitude.* In the Uncertainty-Attitudes tasks, we elicit risk and ambiguity aversion within the same framework in order to construct a risk corrected measure of ambiguity aversion, our control variable of interest.

In the risk aversion elicitation task, subjects face multiple similar choices. The first Option, A, is indexed by a value of $x \in (0,1)$. Option (A,x) yields $x$ ECU for sure. We consider a grid of equally spaced values for $x$ given by $0, 5, \ldots, 100$. The payoff of the other Option, B, depends on a draw from an urn containing 50% white balls and 50% black balls. It yields 100 ECU if the drawn ball is white and otherwise 0. One expects that decision makers chooses B for low values of $x$, A otherwise. The exact switching point indicates their risk attitude.

The ambiguity aversion elicitation task is similar in structure to the one used for risk aversion and comes in two similar variants. The first variant is identical to the risk aversion task, with the difference that the composition of the urn determining the payoff of Option B is now unknown. The second variant is identical to the first variant, with the only difference that Option B now yields 100 ECU if the drawn ball is black and 0 otherwise. In all Uncertainty-Attitudes tasks, participants indeed switched only once. This allows us to partition ambiguity attitudes set into three categories as follows. We define a subject as *Ambiguity averse*, *Ambiguity neutral*, and *Ambiguity loving* if her switching point in both ambiguity aversion tasks is located after, at, or before the switching point in the risk aversion task, respectively.[12]

*2.2.4. General understanding and ability.*

12. No subjects were not classifiable according to this rule. For the vast majority of subjects, switching points differed by no more than one unit across the two versions of the ambiguous task.

*Pre-treatment control questions.* Before the main task, subjects have to answer questions concerning the signaling rule and pay-off calculations which verify that they understood the instructions. To make guessing more tedious, subjects are told that after mistakes, they will be asked to answer the concerned questions again, without being told where they made a mistake. This procedure aims at incentivizing subjects to think more carefully about the correct answers.

*Cognitive ability tests.* Subjects perform two standard non-verbal tasks, which both provide general measures of cognitive ability: The Raven's matrices test (Raven) and the cognitive reflection test (CRT, Frederick, 2005). The latter measures in particular subjects' proneness to give answers governed by impulses rather than deliberation. The task is numerical, which matches the nature of our experiment.

*2.2.5. Overview of controls.* The control tasks Red Balls Only, Anchoring 1 and Anchoring 2 are closely related to the Main-Treatment task, in the sense that each introduces one specific modification to the messaging rule used in the Main-Treatment. Table 1 summarizes these four tasks.

Table 1. Signal generating process in the Main-Treatment task and 3 selected control tasks

| | Color | [0,50) | [50,c) | [c,100] | Urn composition |
|---|---|---|---|---|---|
| Main-Treatment | red ball | $\star$ | X | # | risky or ambiguous |
| | blue ball | $\star$ | # | X | |
| Red Balls Only | red ball | $\star$ | X | # | all balls are red |
| | blue ball | $\star$ | # | X | |
| Anchoring 1 | red ball | $\star$ | X | X | as in main task |
| | blue ball | $\star$ | X | X | |
| Anchoring 2 | red ball | $\star$ | X | X | as in main task |
| | blue ball | $\star$ | # | # | |

Given that each of the tasks Red Balls Only, Anchoring 1 and Anchoring 2 is repeated three times and that the state belongs to interval 1 with probability one half, the probability that a subject receives three times the message $\star$ is $(.5)^3$. For each of the Red Balls Only, Anchoring 1 and Anchoring 2 control tasks, we thus expect to receive an informative answer

TABLE 2. Summary of experiment

| | |
|---|---|
| MAIN-TREATMENT | Treatments: $c \in \{54, 64, 86, 96\}$<br>Variations: (RISKY / AMBIGUOUS × HELP /NO HELP)<br>9 repetitions |
| MAIN-CONTROL | *Three repetitions for each control task* |
| Further controls | |
| BELIEFS | Belief elicitation |
| UNCERTAINTY-ATTITUDES | Ambiguity and risk aversion test |
| CRT | Cognitive reflection test |
| RAVEN | Raven's matrices 9 items assessment |

for each of the subjects in $1 - (.5)^3 = 87.5\%$ of the cases. We will therefore have a full set of these three controls for approximately 76% of the subjects. As this subset is randomly determined and not correlated with any decisions made by the subjects, we can separately analyze this subset without worrying about selection due to the availability of controls.

Table 2 gives an overview of the complete sequence of control tasks performed by subjects together with elicited variables.

### *2.3. Implementation and procedures*

Payment was done according to the Random Incentive Scheme: At the end of the experiment, subjects received payout for one round of one task taken from the complete set of tasks minus the Raven and CRT tasks. The task and the round were selected fully randomly. At the end of the experiment, subjects were informed of the selection and the obtained payoff, where 100 ECU corresponded to 8€. Subjects also received a fixed payment of 2.50€ for the CRT and RAVEN tasks, and they learned their results for these tasks. In experiments with ambiguity sensitive subjects, there is a theoretical possibility that the random incentive scheme is not incentive compatible (Bade, 2015; Baillon, Halevy, and C. Li, 2022a). In practice, the evidence is mixed. Jörg Oechssler, Rau, and Roomets (2019) find that there is low evidence of hedging across multiple decisions. Baillon, Halevy, and C. Li (2022b)—who explicitly address the incentive compatibility of the RIS—provide evidence that this can be a serious concern if hedging opportunities are fairly straightforward.

See also the discussion in footnote 14 in Cubitt, Kuilen, and Mukerji (2018). As argued below, our main task does not create such straightforward hedging opportunities across repetitions.

The experiment was conducted at the experimental laboratories at Mannheim and Düsseldorf in May 2016 and April 2017, respectively, with a standard student subject pool recruited with ORSEE (Greiner, 2004). In total 119 subjects participated in the experiment. We ran 12 sessions where each session lasted around 45 minutes. The experiment was programmed in z-tree (Fischbacher, 2007). The average payoff was 9.56€.[13]

## 3. Theoretical predictions for the main task

In the risky environment, the messages # and X provide no more information than the fact that $\omega \geq 50$. Indeed, the probability of any of these being sent given $\omega \in [50, c)$ is $\frac{1}{2}$ and the same holds true conditional on $\omega \in [c, 100]$. It follows that DM's best response to these messages is $E[\omega|\omega \in [50, 100]] = 75$. This carries over to the ambiguous environment if DM applies expected utility and, at least on average, considers both colors equally represented.

We now discuss the ambiguous environment for an ambiguity averse DM. As she does not know the distribution of colors in the urn, a subject faces ambiguity after observing # and X. In this section, we base our prediction on the max-min model, which is simple and widely used. If the DM uses prior by prior updating, the max-min model specifies that she will choose the action with the highest worst-case expected payoff across (updated) priors. In this section, we assume that the subject acts as if considering all proportions of red balls between 0 and 1 possible. Then, the max-min best-response $a^*$ to # and X can be shown to be given as follows.

13. See the Online Appendix for instructions and screenshots.

$$a^* = \begin{cases} 100 - 5\sqrt{100 - c} & \text{if } c \leq 75 \\ 50 + 5\sqrt{c - 50}, & \text{if } c > 75 \end{cases} \quad (1)$$

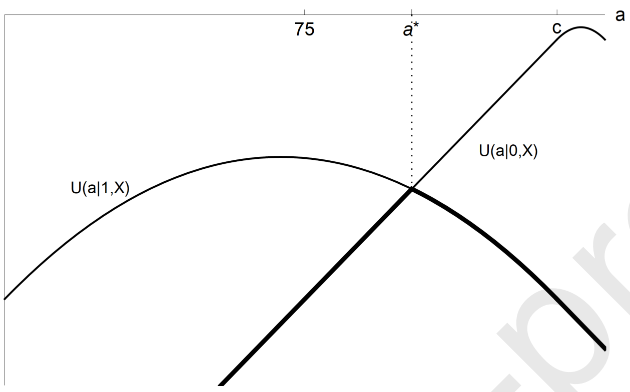



FIGURE 1. Expected utilities given extreme urn compositions

To understand this, simply note that we only need to look at the intersection between two different expected payoff functions. Figure 1 shows these two functions. $U(a|1,X)$ indicates the expected payoff of choosing action $a$ after observing message X under the assumption that all balls in the urn (and hence the drawn ball) are red. $U(a|0,X)$ is the counterpart, assuming that all balls in the urn are blue. Note that $U(a|1,\#) = U(a|0,X)$ and $U(a|0,\#) = U(a|1,X)$. For any action, the highest and lowest expected utility arises under a scenario where all balls have the same color (and is thus captured by one of these two curves). Counterparts for other urn compositions are located between these curves. For a max-min decision maker, guided by the worst-case scenario, the objective function is the lower envelope (in bold) of the two curves, and it is maximized at the point where the two curves intersect. In other words, for this action, the expected utility of the decision maker is the same whether all balls are red or all balls are blue (it is actually the same for any distribution of colors).

By equalizing expected utility across possible urn compositions, the max-min action thus completely hedges the decision maker against ambiguity.[14]

To gain some intuition, observe that one could think about the problem as finding the right compromise between the two actions which are optimal given each of the two possible sub-intervals $[50, c]$ and $[c, 100]$, each of these actions being located on different sides of 75. The smaller sub-interval warrants a larger deviation from 75 than the larger sub-interval. An expected utility decision maker takes into account that the smaller sub-interval is less likely to be the relevant one, so that 75 is her optimal compromise action. Instead, an ambiguity averse decision maker is not concerned with the probability of the smaller interval, but guided by the worst-case scenario. Hence, she behaves as if over-weighting the smaller sub-interval, leading to a deviation from 75. Note that from an ex ante point of view, the decision maker prefers to play 75 after both X and #, but her preferences change after receiving these messages. We refer to Kellner and Le Quement (2018) for further explanations.[15]

For the values of $c$ used in the experiment, Table 3 summarizes the corresponding max-min actions and how much they deviate from 75.

14. For an ambiguity loving "max-max" decision maker, the objective function is the upper envelope of the two curves. As illustrated in the graph, the optimal action for a cutoff above 75 is to state a number above 75. It can be shown that for any value of the threshold $c$, the max-max action is on the same side of 75 as the max-min action, but further away from 75. In that sense, the predicted effect of our form of language ambiguation on the decisions of ambiguity loving subjects is of a similar nature as the predicted effect on ambiguity averse subjects, but simply more extreme.

15. Note that our experiment (which uses the random incentive scheme) does not create any immediate opportunities to hedge across repetitions of the main task. First, we tell participants that the composition of the urn may change at any time. If the receiver in fact believes that the urn could change in every repetition, then she would have no opportunities to hedge at all in the max-min expected utility model. Second, even assuming the subjects believed that the urn remains the same in every period, they face uncertainty about which message they will receive in future iterations of the task. Most importantly, in each future iteration they will receive, with a probability of 50 percent, the $\star$ message, which induces purely risky beliefs about the state in the corresponding round. Thus, the presence of the $\star$ message inhibits hedging across periods.

TABLE 3. Max-min action $a^*$

| $c$ | $c-75$ | $a^*$ | $a^*-75$ |
|---|---|---|---|
| 54 | $-21$ | 66.1 | $-8.9$ |
| 64 | $-11$ | 70 | $-5$ |
| 86 | 11 | 80 | 5 |
| 96 | 21 | 83.90 | 8.9 |

In a theoretical extensions section at the end of the paper, we generalize the result obtained above. If subjects perceive the urn as less than fully ambiguous or rather act according to the smooth model of ambiguity aversion and the $\alpha$-max-min model, the effect of shifting $c$ above 75 remains qualitatively the same though is smaller in magnitude. As to the symmetry of the set of priors entertained by the agent, we believe that we give subjects no reason to adopt an asymmetric set of priors.

Observe that the above predictions may carry over to the risky urn, if decision makers fail to reduce compound lotteries, as demonstrated e.g., in Halevy (2007). A higher aversion towards second-order risk generates similar predictions as the ambiguity aversion model.

We summarize these results by stating the following key prediction:

PREDICTION 1. *In the ambiguous environment:*

(i) *The higher the threshold c, the higher is, on average, the number chosen by participants after messages X and #.*
(ii) *This effect is present for ambiguity averse participants, but not for ambiguity neutral participants.*

These predictions apply to ambiguity averse or neutral decision makers who are quantitatively sophisticated enough to understand the messaging rules and who are free from anchoring biases. We expect anchoring bias to influence choices in the same direction as ambiguity aversion. In contrast, the implications of low sophistication are difficult to predict. In consequence, in our main analysis we restrict ourselves to studying quantitatively sophisticated participants. The regressions that we run on this selected population in order to identify a possible hedging effect (of the type described above) explicitly control for subjects' anchoring tendency as estimated from the anchoring tasks.

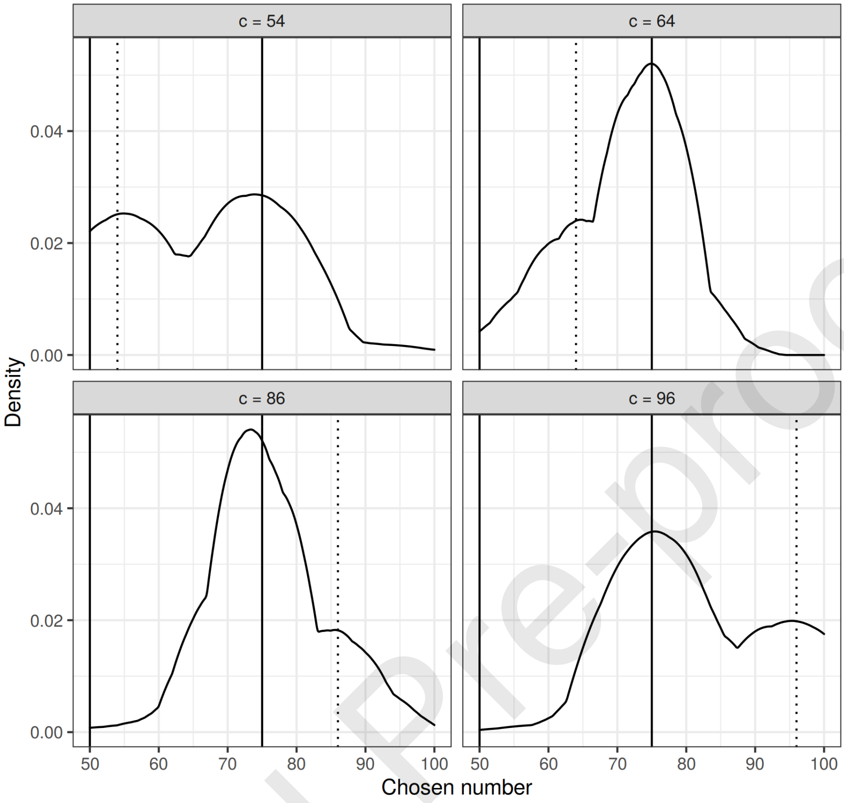
Finally, note that a property of our setup is that whatever the value of threshold $c$ in the ambiguous environment, the DM attaches strictly positive ex ante value to the signal if she evaluates it in terms of her consistent planning ex ante utility, which is her ex ante max-min expected utility anticipating her future (possibly dynamically inconsistent) behavior. We prove this claim in Appendix A.1. The intuition is that although ambiguity leads the DM to take an ex ante suboptimal action whenever the state is above 50, this is more than compensated by the fact that the signal always beneficially reveals whether the state is below or above 50.

## 4. Results

We first present all subjects' behavior pooled across all variants of Main-Treatment (risky and ambiguous, help and no help), after these received either X or #. Figure 2 shows the estimated density of choices using an Epanechnikov kernel pooled over ambiguous and risky urn. Visual inspection reveals that choices are skewed in the direction of $c$. Using Wilcoxon-rank-sum tests making pairwise comparisons of distributions of choices across different levels of $c$, we find significant differences between all pairs of $c$ (p-value$<.01$) except for the comparison pairs ($c \in 86, 96$ and $c \in 54, 64$, p-values 0.820 and 0.827, respectively). Our evidence thus supports Prediction 1 (i). One aspect of the estimated densities is that these visually resemble a mixture of two densities, one centered around $c$ and another centered around 75, though this description is not exhaustive.

Given the complexity of the main treatment task, we would expect that behavior differs not only between ambiguity averse and ambiguity neutral participants, but also between subjects displaying different levels of quantitative sophistication (basic conceptual understanding of signaling rules, ability to

FIGURE 2. Density of choices in MAIN-TREATMENT




*Note:* The figure shows the smoothed density of choices for the four different values of the threshold $c \in 54, 64, 86, 96$ using an Epanechnikov kernel in MAIN-TREATMENT when receiving an ambiguous message. There are significant differences between all pairs of $c$ (p-value$<$ .01) except for the comparisons ($c \in 86, 96$ and $c \in 54, 64$, p-values 0.820 and 0.827, respectively)

do Bayesian updating). As a measure for this quantitative sophistication, we classify participants as “Bayes-Competent” based on their decisions in the two instances where only one layer of uncertainty is involved in the message. One instance is when they see the $\star$ message in the main treatment. Another instance is when they perform the RED BALLS ONLY control task. The total number of decisions, across these two instances, is at least 6. Across these instances, a decision is marked as correct if and only if the DM chooses the exact expected value of the state conditional on the observed message. To allow for occasional errors, we require a subject to be correct in these tasks 80 percent of the time in order to be classified as “Bayes-Competent”.

This results in 47.90 percent of 119 subjects in our sample being classified as Bayes-Competent.[16] In the main analysis presented in what follows, we focus exclusively on such participants. Within this population, we study separately ambiguity averse and neutral subjects. Ambiguity averse subjects constitute 53.78 percent of the full population of subjects and 56.14 percent of the population of subjects classified as Bayes-Competent. The corresponding frequencies of ambiguity neutral subjects are respectively 27.73 percent and 24.56 percent. The corresponding frequencies of ambiguity loving subjects are respectively 18.49 percent and 19.30 percent.

In the following analysis, we adopt a panel regression framework accounting for the panel structure of our data and differentiating out the decisions in MAIN-TREATMENT. The independent exogenous variable is $c$, the dependent variable is the chosen action $a$. We estimate the following linear panel model:

$$a_{it} = \alpha + \beta_1 \mathbf{c_{it}} + \beta_2 Anchor_{it} + \beta_3 \mathbf{c_{it}} \times Anchor_{it} + \varepsilon_{it} \tag{2}$$

In the above regression, the second and the third variable control for subjects' individual tendency to anchor their decision on the threshold $c$, and thus allow us to differentiate out the behavior in our ANCHORING 1 and ANCHORING 2 anchoring control tasks. Coefficient $\beta_3$ captures the part of the effect of $c$ which is due to anchoring. Instead, coefficient $\beta_1$ captures the part of the effect of $c$ that is not due to anchoring. This lies at the core of our analysis as it should be expected to capture the hedging effect of $c$ when ambiguity averse decision makers face an ambiguous urn.

Panel A of Table 4 reports regression results for ambiguity averse Bayes-Competent subjects, pooling Help and No Help treatments. Column (1) shows coefficients when pooling both urn types, while columns (2) and (3) show the

16. As a robustness check, we have also tried close variations of our classification rule, where a decision is classified as correct if it is located within a small region around the optimal choice. This yields essentially the same classification of subjects into Bayes-Competent and non Bayes-Competent.

TABLE 4. Effect of c by urn type and ambiguity attitude and more than 80% of Bayes control correct

| **Panel A: Ambiguity averse** | Pooled | Ambiguous | Risky |
|---|---|---|---|
| c | 0.116** | 0.126** | 0.096 |
| | (0.049) | (0.061) | (0.078) |
| Anchoring control | -6.145 | -0.940 | -12.095 |
| | (4.674) | (5.906) | (7.385) |
| c x Anchoring control | 0.086 | 0.035 | 0.146 |
| | (0.060) | (0.076) | (0.094) |
| Num.Obs. | 272 | 144 | 128 |
| R2 | 0.151 | 0.137 | 0.188 |
| R2 Adj. | 0.030 | 0.013 | 0.053 |
| **Panel B: Ambiguity neutral** | Pooled | Ambiguous | Risky |
| c | 0.024 | 0.015 | -0.161 |
| | (0.073) | (0.248) | (0.172) |
| Anchoring control | -16.503** | -26.525 | 0.583 |
| | (6.691) | (19.010) | (18.766) |
| c x Anchoring control | 0.193** | 0.289 | 0.257 |
| | (0.092) | (0.281) | (0.253) |
| Num.Obs. | 112 | 30 | 109 |
| R2 | 0.182 | 0.251 | 0.225 |
| R2 Adj. | 0.044 | 0.055 | 0.128 |

*Note:* This table reports the effects of $c$ on the choice of *Bayes-Competent* subjects using fixed effect panel regressions. Column (1) reports the effects when pooling all urn types, column (3) and (4) report the effects for the ambiguous and the risky urn types, respectively. In Panel A we present the results for subjects classified as Ambiguity averse by our control task, while Panel B reports the effects of for the Ambiguity neutral subjects. Significance is reported at the following levels: *** $< .001$, ** $< .01$, * $< .05$.

results when considering separately the *Ambiguous* (32 subjects) and the *Risky* urn treatments (28 subjects). The coefficient on $c$ is positive and significant in the pooled regression and in the ambiguous urn regression. The coefficient on $c$ in the risky urn regression is slightly smaller and not significantly different from zero. This in line with our Prediction 1(ii). We summarize the results of Panel A of Table 4 in what follows:

RESULT 1. *Among Bayes-Competent and ambiguity averse subjects facing an ambiguous urn, the threshold $c$ has a significantly positive influence on the chosen number, after controlling for their susceptibility to anchoring.*

Notice that our point estimate of .116 for $\beta_1$ in Panel A of Table 4 is below the upper bound suggested by the theoretical predictions in equation 1, which imply a slope between .354 (for $c$=86 or 64) and .417 (for $c$=96 or 54). This can be explained by subjects' risk attitude and by the fact that subjects may perceive less ambiguity or be less ambiguity averse than assumed in section 3 (see discussion in section 5). Alternatively, some—but not all—of the ambiguity averse decision makers may resort to a dynamically consistent updating procedure (as in Hanany and Klibanoff, 2007; Hanany and Klibanoff, 2009.)

In the light of Halevy (2007), who finds that ambiguity subjects treat purely risky compound lotteries as if they were ambiguous, it is worth noting that (restricting ourselves to Bayes-Competent subjects) we find that ambiguity averse subjects behave differently in the ambiguous and the risky treatments. Abdellaoui, Klibanoff, and Placido (2015) and Aydogan, Berger, and Bosetti (2019) provide a way to reconcile our results with Halevy (2007). Both of these papers find that the positive correlation between ambiguity aversion and non-reduction of compound lotteries is weaker for quantitatively sophisticated decision makers. To the extent that the subjects that we classify as Bayes-Competent subjects are indeed on average more quantitatively sophisticated, our results are in line with these two latter papers.

Panel B of Table 4 reports regression results for ambiguity neutral Bayes-Competent subjects, pooling Help and No Help treatments. Results reveal that such subjects' decisions are not significantly affected by $c$ when controlling for the anchoring effect of $c$. This is true when considering only the ambiguous urn or only the risky urn, or both together. We summarize the results of Panel B of Table 4 in what follows:

Result 2. *Among Bayes-Competent and ambiguity neutral subjects facing an ambiguous urn, the threshold $c$ does not have a significantly positive influence on the chosen number, after controlling for their susceptibility to anchoring.*

Taken together, Results 1 and 2 thus provide support for Prediction 1(ii) for the population of quantitatively sophisticated subjects. In Appendix E, we provide a counterpart of the above two regression tables for Bayes-Competent ambiguity loving subjects. Point estimates are in line with what we would expect from max-max decision-making.

We present results for subjects who are not Bayes-Competent in Appendix B. Such subjects react to the threshold $c$ even when controlling for anchoring, but the effect appears to be overall orthogonal to their ambiguity attitude. There are not enough observations for ambiguity loving subjects to meaningfully interpret their results. As a robustness check, we also conducted a median split by the performance in the Cognitive Reflection Test instead of using Bayes-competence as a measure of sophistication. We find qualitatively and quantitatively similar results to those obtained in our main analysis. These are presented in Table D.1 in the Appendix. Finally, we find no significant qualitative or quantitative differences between Help and No Help treatments, as reported in Table C.1 in the Appendix.

## 5. Alternative models of ambiguity aversion

This section provides an extended theoretical analysis of the effect of ambiguation on decision making. We extend the analysis in two ways, by parameterizing the level of ambiguity and by considering alternative models of decision making. First, we relax the assumption that the decision maker acts as if she considers all urn composition possible when evaluating available actions in the main experiment. Second, we consider two further models of decision making under ambiguity, the smooth model (Klibanoff, Marinacci, and Mukerji, 2005) and the $\alpha$-max-min model (Ghirardato, Maccheroni, and Marinacci, 2004). We first present the interim utility of the DM after receiving a message $m$, as it arises under each of these models.

Let $P$ denote the set of probabilities of drawing a red ball from the ambiguous urn that the decision maker considers possible. Assuming full Bayesian updating, for any such probability $p \in P$ and given the message $m$ observed by the DM, she will compute a conditional pdf $f(\omega|p,m)$ over states at the interim stage. If $p$ were known, the conditional expected utility of the

DM would then be given by

$$U(a|p,m) = \int_{\Omega} u(|\omega - a|) df(\omega|p,m),$$

where the function $u$ captures the risk attitude of the decision maker. Under the $\alpha$-max-min model, the DM's conditional expected utility function is given by:

$$U^{\alpha}(a|m) = \alpha \min_{p \in P} U(a|p,m) + (1-\alpha) \max_{p \in P} U(a|p,m).$$

Under the smooth model, the DM additionally assigns a second order belief to every possible probability $p \in P$, capturing how likely she considers various urn compositions. This is modeled with a pdf $\mu(p)$ with support $P$. Assuming full Bayesian updating[17], the DM uses Bayes' Rule to compute an updated second-order belief $\mu(p|m)$ after having observed a message $m$. Under the smooth model, the DM's conditional expected utility function is given by:

$$U^{S}(a|m) = \int_{P} \varphi(U(a|p,m)) d\mu(p|m),$$

where $\varphi$ is a function capturing ambiguity attitude. For a linear $\varphi$, representing ambiguity neutrality, the smooth model coincides with expected utility. A concave $\varphi$ corresponds to ambiguity aversion, convex $\varphi$ to ambiguity loving preferences.

We assume symmetry of first and second order beliefs. We let $P = [1/2 - \varepsilon, 1/2 - \varepsilon]$ for $\varepsilon \in (0, 1/2]$ and further assume that $\mu(p)$ is symmetric around 1/2.

In what follows, in discussing the two above models, we focus on the case where $c > 75$ and the DM observes message $X$. Analogous arguments carry over to the cases where $c \in (50, 75)$ and/or the DM instead observes #. Note also that all models trivially predict that the DM chooses 25 upon observing message

17. See Hanany and Klibanoff (2007) and Hanany and Klibanoff (2009) for a dynamically consistent alternative.

$\star$ as the DM faces no ambiguity in this scenario. We impose risk neutrality ($u(|a-\omega|) = |a-\omega|$) to obtain clear results about the involved parameters of the model, but qualitatively similar results apply for the case of risk aversion.[18]

### 5.1. *Behavior under $\alpha$-max-min utility*

To gain some intuition, we consider two special cases (A and B) initially.

We begin with the case (A) where $\varepsilon$ is small but $\alpha = 1$ (the classic max-min model with a smaller set of priors). Figure 3, which assumes $\varepsilon = \frac{1}{6}$, shows the expected utility of each action under each of the two most extreme priors (i.e., urn compositions) considered possible by the DM, that is $U(a|\frac{1}{2} - \varepsilon, X)$ and $U(a|\frac{1}{2} + \varepsilon, X)$ .

The dotted line corresponds to $U(a|\frac{1}{2}, X)$ and is the limit of both curves as

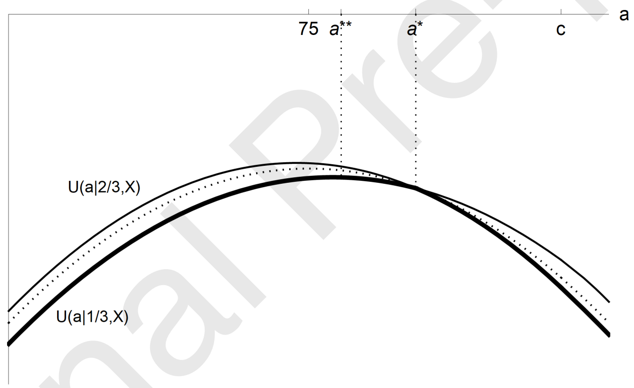



FIGURE 3. max-min preferences with small $\varepsilon$

$\varepsilon$ tends to 0. The decision maker picks the action which maximizes the lower envelope (in bold) of the solid curves. Note that this is now maximised by the value $a^{**}$, the maximizer of $U(a|\frac{1}{2} - \varepsilon, X)$. This action is to the left of $a^*$, the value which perfectly hedges against ambiguity (where the lower envelope still

18. The analysis in section 3 analyses a special case of the above, setting $\varepsilon = 1/2$. Note that simple max-min is a special case of the $\alpha$-max-min model where $\alpha = 1$. Also, assuming constant absolute ambiguity aversion, simple max-min corresponds to the limit of the smooth model as ambiguity aversion approaches infinity.

has a kink). As $a^{**}$ corresponds to a prior below $1/2$, its maximum will however still be above 75. This argument holds true for all possible values of $\varepsilon > 0$.[19]

Consider next the case (B) of $\alpha$ being strictly between 0 and 1, while $\varepsilon = \frac{1}{2}$ (representing maximal ambiguity as assumed in the main model).

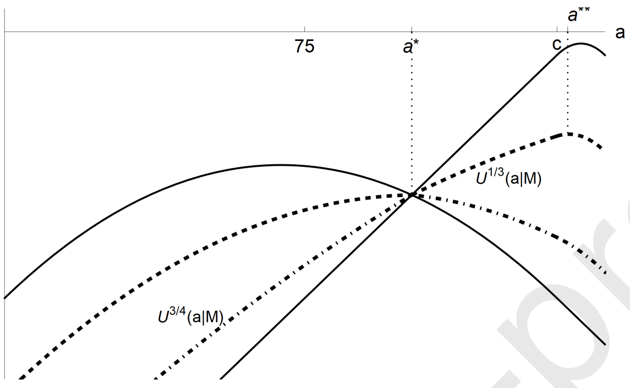



FIGURE 4. Two $\alpha$-max-min objective functions

In Figure 4, we show the objective function $U^{\alpha}(a|X)$ for two values of $\alpha$ (3/4, dot-dashed, and 1/3, dashed). The solid lines correspond to the upper and lower envelope of all possible expected utilities across priors, and recall that $\alpha$ is the weight attached to the lower envelope. When $\alpha = 3/4$ (dot-dashed) the optimal action is $a^*$, resulting in a perfect hedge against ambiguity. For $\alpha = 1/3$, it is however optimal to choose an even higher action, labeled $a^{**}$.

In the circumstances depicted in Figure 4 (i.e. high $\varepsilon$), one can conclude that as $\alpha$ decreases from 1, which is often interpreted as lower ambiguity aversion or less pessimism, the utility maximizing action initially remains at the value that hedges perfectly against ambiguity but eventually starts to increase towards the maximum of the upper envelope. For low values of $\varepsilon$ as considered in Figure 3, less ambiguity aversion may instead lead to lower actions.

19. Additionally, from considering the upper envelope of all curves in the figure, one can conclude that for ambiguity loving max-max preferences ($\alpha = 0$), the optimal action will be the maximiser of $U(a|\frac{1}{2} + \varepsilon, X)$, and hence be below 75.

The proposition below summarizes our insights formally. It allows for intermediate values of both $\alpha$ and $\varepsilon$ and identifies a sufficient condition on these parameters such that c being larger than 75 implies that the optimal action is larger than 75. Note that if $\alpha > \frac{1}{2}$ (capturing preferences which are often considered ambiguity averse or pessimistic), then the condition holds for any $\varepsilon$.

PROPOSITION 1. *Assume $u(|a-\omega|) = |a-\omega|$. Assume that the DM acts according to $\alpha$-max-min model. The DM chooses an action strictly larger than 75 if $\alpha > \frac{1}{2} - \varepsilon\frac{c-75}{100}$*

*Proof.* See AppendixA.2. $\square$

As the case of $\varepsilon = \frac{1}{2}$ (discussed as case B above) illustrates, this condition is not necessary for large values of $\varepsilon$.

### 5.2. Behavior under smooth ambiguity

As in many applications of the smooth model, we assume constant absolute ambiguity aversion by letting $\varphi(u) = -\exp(-\alpha u)$. We further assume that the DM's second order prior is a uniform distribution over $P = [1/2-\varepsilon, 1/2+\varepsilon]$. For this simple specification, we show that ambiguity aversion ensures that the DM's optimal action is above 75 if $c > 75$.

PROPOSITION 2. *Assume that the DM acts according to the smooth model. Assume $u(a,\omega) = -|a-\omega|$ and $\varphi(u) = -\exp(-\alpha u)$ (constant absolute ambiguity aversion). Assume that $\mu$ is uniform over $P = [1/2-\varepsilon, 1/2+\varepsilon]$. If the DM is (strictly) ambiguity averse ($\alpha > 0$) and $c > 75$, then DM chooses an action (strictly) above 75.*

*Proof.* See Appendix A.2 $\square$

The proof proceeds by establishing that the further the DM's action moves from $a^*$ (the action perfectly hedging against ambiguity) towards 75, the more the DM is exposed to ambiguity about the expected utility that she obtains. The optimal action strikes a compromise between two concerns: Increasing the average expected utility (as computed by using the second order belief $\mu$) by moving towards 75 on the one hand, and on the other hand decreasing the entailed ambiguity by moving towards $a^*$.

## 6. Conclusion

In our experiment, language ambiguation affected the decisions of quantitatively sophisticated and ambiguity averse subjects by triggering hedging behavior, in line with the predictions of models of ambiguity aversion. This effect operates in an environment where receivers ex ante benefit from the information contained in messages, though they would benefit if they could selectively ignore the ambiguity created by messages.

As noted in existing contributions, the use of ambiguous communication strategies in sender-receiver games would in principle allow for the emergence of equilibria featuring more informative communication than the standard equilibria predicted by expected-utility theory. Future experimental work should study games where both the sender and the receiver are real subjects, and study equilibrium behavior when the sender is known to privately observe some payoff-irrelevant and ambiguously distributed variable. Would the sender make use of the opportunity to condition his messages on such a variable? Would the receiver anticipate this? Would the overall effect be beneficial to both parties?

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## Appendix A: Proofs

### *A.1. ex ante value of the signal*

We here prove the following result.

Result A.1. *For any $c \in [50, 100]$ and not belonging to $\{50, 75, 100\}$, the DM's consistent planning ex ante utility from observing the message is strictly higher than his ex ante utility from not observing it.*

*Proof.* Assume that $c > 75$. Since the DM chooses the fully hedging action $a^\star$ after # and X and 25 otherwise, her consistent planning ex ante utility, the max-min expected utility given the optimal behavior of future selves, is independent of the urn composition. The max-min utility can be most easily be computed with the urn having equally many red and blue balls, as it leads to a uniform conditional distribution of states over [50,100] after the messages # and X:

$$
\begin{aligned}
&\left(-\frac{1}{100}\right)\begin{pmatrix} \int_0^{25}(25-\omega)d\omega + \int_{25}^{50}(\omega-25)d\omega \\ +\int_{50}^{50+5\sqrt{c-50}}(50+5\sqrt{c-50}-\omega)d\omega \\ +\int_{50+5\sqrt{c-50}}^{100}(\omega-(50+5\sqrt{c-50}))d\omega \end{pmatrix} \\
= \quad & \frac{5}{2}\sqrt{c-50}-\frac{1}{4}c-\frac{25}{4}.
\end{aligned}
$$

Assume that the decision maker does not observe the message. Then she optimally chooses 50 and her ex ante (max-min) expected utility is simply as follows:

$$
\begin{aligned}
&\left(-\frac{1}{100}\right)\left( \int_0^{50}(50-\omega)d\omega + \int_{50}^{100}(\omega-50)d\omega \right) \\
= \quad & -25.
\end{aligned}
$$

It can be easily shown that $\frac{5}{2}\sqrt{c-50}-\frac{1}{4}c-\frac{25}{4} > -25$ for any $c \in (75, 100]$. The same argument can be made for $c \in (50, 75)$ .

□

### A.2. Proof of Proposition 1

By Bayes' rule, for a given prior proportion $p$ of red balls in the urn, let $\pi(p|X)$ denote the conditional probability of a red ball having been drawn from the ambiguous urn given that message $X$ observed. We have

$$\begin{aligned}\pi(p|X) &= \frac{P(\theta=r)P(X\,|\theta=r\,)}{P(\theta=r)P(X\,|\theta=r\,)+P(\theta=b)P(X\,|\theta=b\,)} \\ &= \frac{p\left(\frac{c-50}{100}\right)}{p\left(\frac{c-50}{100}\right)+(1-p)\left(\frac{100-c}{100}\right)}.\end{aligned}$$

Consequently,

$$\begin{aligned}U[a\,|p,X\,] &= \pi(p|X)\frac{1}{c-50}\left(\int_{50}^{c}u(|a-\omega|)d\omega\right) \\ &\quad +(1-\pi(p|X))\frac{1}{100-c}\left(\int_{c}^{100}u(|a-\omega|)d\omega\right).\end{aligned}$$

It is easily shown that for $c>75$, whatever $\varepsilon$ it holds true that $75<a^*<c$, where $a^*$ is the action for which expected utility is the same under all possible priors in $[\frac{1}{2}-\varepsilon,\frac{1}{2}+\varepsilon]$. Note that for $a<a^*$,

$$U^{\alpha}(a)=\alpha U(z|\frac{1}{2}-\varepsilon,X)+(1-\alpha)U(z|\frac{1}{2}+\varepsilon,X).$$

For $a<a^*$, this function is concave as it is a convex combination of concave functions.

Since $a^*<c$, simple calculations establish that

$$\begin{aligned}U^{\alpha}(a) = &-\frac{2\varepsilon\left((c-a)^2+50(a-75)\right)+a^2-150z+6250}{4(c-75)\varepsilon+50} \\ &-\frac{2\alpha(c-100)\varepsilon(25c-(a-100)a-3750)}{4(c-75)^2\varepsilon^2-625}.\end{aligned}$$

The derivative w.r.t. $a$ eveluated at $a=75$ equals:

$$U^{\alpha\prime}(75)=\frac{100\alpha(100-c)\varepsilon}{25^2-4\varepsilon^2(c-75)^2}-\frac{2(50-2(c-75))\varepsilon}{4(c-75)\varepsilon+50}.$$

This is increasing in $\alpha$ as $\varepsilon^2 \leq 1/4$ and $(c-75) < 25$.

Note that $U^{\alpha\prime}(75)$ has its only zero at

$$\alpha = \frac{1}{2} - \varepsilon\frac{c-75}{100}.$$

Thus, if $\alpha$ is larger than this value, we have $U^{\alpha\prime}(75) > 0$. Then $U^{\alpha}(a)$ is increasing at 75. As it is concave at least for $a \in [50, a^*]$ (with $a^* > 75$), the maximizer of this function is above 75.

### *A.3. Proof of Proposition 2*

Recall in what follows that for $c > 75$, whatever $\varepsilon$ it holds true that $75 < a^* < c$, where $a^*$ is the action for which expected utility is the same under all possible priors in $[\frac{1}{2}-\varepsilon, \frac{1}{2}+\varepsilon]$.

By Bayes' rule, using the DM's second order prior $\mu$, the conditional probability assigned to prior $p$ after observing signal $X$ is given by:

$$\begin{aligned}
\mu(p|X) &= \frac{\mu(p)P(X|p)}{\int_{\widetilde{p}}\mu(\widetilde{p})P(X|p)d\widetilde{p}} \\
&= \frac{\left(\frac{1}{2\varepsilon}\right)\left[p\left(\frac{c-50}{100}\right)+(1-p)\left(\frac{100-c}{100}\right)\right]}{\int_{\frac{1}{2}-\varepsilon}^{\frac{1}{2}+\varepsilon}\left(\frac{1}{2\varepsilon}\right)\left[\widetilde{p}\left(\frac{c-50}{100}\right)+(1-\widetilde{p})\left(\frac{100-c}{100}\right)\right]d\widetilde{p}} \\
&= \frac{\left(\frac{1}{2\varepsilon}\right)\left[p\left(\frac{c-50}{100}\right)+(1-p)\left(\frac{100-c}{100}\right)\right]}{(1/4)}.
\end{aligned}$$

The objective function conditional on having received the message $X$ is thus:

$$U^S(a|X) = \int_P \varphi(U(a|p,m))d\mu(p|X),$$

where $U(a|p,m)$ is constructed as explained in the main text.

Let

$$A(a) \equiv \int_P U(a|p,X)\mu(p|X)dp = U(a|.5,X).$$

Noting that $\varphi(u+\tilde{u})=\varphi(u)\cdot\varphi(\tilde{u})$, the objective function rewrites as:

$$U^S(a|X) = \int_P \varphi\left(U(a|p,X) - A(a)\right) \cdot \varphi(A(a))\, \mu(p|X)dp.$$

As $A(a)$ does not depend on $p$, this in turn rewrites as

$$U^S(a|X) = \varphi(A(a) \int_P \varphi\left(U(a|p,X) - A(a)\right)\cdot)\, \mu(p|X)dp.$$

Noting that $\varphi^{-1}(S \cdot \tilde{S}) = \varphi^{-1}(S) + \varphi^{-1}(\tilde{S})$ and recalling that the function $\varphi(x)$ is monotonically increasing in $x$, it follows that:

$$\varphi^{-1}(U^S(a|X)) = A(a) + \varphi^{-1}\left(\int_{1/2-\varepsilon}^{1/2+\varepsilon} \varphi(U(a|p,X) - A(a))\mu(p|X)dp\right).$$

For $a < a^*$, defining the new variable $\hat{u} = U(a|p,X) - A(a)$, we can write the integral inside brackets as:

$$O(a) = \int_{\underline{\hat{u}}(a)}^{\bar{\hat{u}}(a)} \varphi(\hat{u})\hat{f}(\hat{u},a)d\hat{u}.$$

Below, we give closed forms for $\underline{\hat{u}}(a), \bar{\hat{u}}(a)$ and $\hat{f}(\hat{u},a)$ using integration by substitution. For $a < c$, note that

$$\hat{u} = \frac{(100-c)(2p-1)\left(100a - a^2 + 25c - 3750\right)}{50(2(c-75)p - c + 100)}.$$

This is monotonically increasing in $p$ as long as $a < a^*$, so that

$$\underline{\hat{u}}(a) = U(a|1/2-\varepsilon,X) - A(a) = \frac{(c-100)\left(2\left(\frac{1}{2}-\varepsilon\right)-1\right)\left(a^2 - 100a - 25c + 3750\right)}{50\left(2(c-75)\left(\frac{1}{2}-\varepsilon\right) - c + 100\right)}$$

and

$$\bar{\hat{u}}(a) = U(a|1/2+\varepsilon,X) - A(a) = \frac{(c-100)\left(2\left(\varepsilon+\frac{1}{2}\right)-1\right)\left(a^2 - 100a - 25c + 3750\right)}{50\left(2(c-75)\left(\varepsilon+\frac{1}{2}\right) - c + 100\right)}.$$

Using the above presented closed form for $\hat{u}$ and solving for $p$, we get:

$$p(\hat{u},a) = \frac{(c-100)((a-100)a - 25(c + 2(\hat{u}-75)))}{2(c-100)\left(a^2 - 100a - 25c + 3750\right) - 100(c-75)}.$$

From $dp = \frac{\partial p(\hat{u},a)}{\partial \hat{u}} d\hat{u}$ and $\hat{f}(\hat{u},a) = \mu(p(\hat{u},a)|X)\frac{\partial p(\hat{u},a)}{\partial \hat{u}}$ we can compute

$$\hat{f}(\hat{u},a) = \frac{625(c-100)^2\left(a^2-100a-25c+3750\right)^2}{2\varepsilon\left((c-100)\left(a^2-100a-25c+3750\right)-50(c-75)\hat{u}\right)^3}.$$

Recall that we have:

$$\varphi^{-1}(U^S(a|X)) = A(a) + \varphi^{-1}\left(O(a)\right).$$

We want to show that $U^S(a|X)$ is strictly increasing in $a$ for $a \in [50, 75+\delta)$, for some $\delta > 0$. As $\varphi^{-1}$ is monotonic, in order to show that $U^S(a|X)$ is increasing in $a$ on some domain, it suffices to show that the RHS of the above equality is increasing in $a$ on this same domain.

Note first that $A(a)$ is differentiable and concave in $a$ with a maximum at $a = 75$. Consider now $\varphi^{-1}\left(O(a)\right)$. Again, to show that this is increasing in $a$ on some domain, we simply need to show that $O(a)$ is increasing in $a$ on this same domain. Next, we shall show that $O(a)$ is increasing in $a$ for $a \in [50, a^*)$. This, combined with what we established about $A(a)$, implies that $U^S(a|X)$ is strictly increasing in $a$ for $a \in [50, 75+\delta]$, for some $\delta > 0$.

Let us now go back to $O(a)$. We show that it is increasing in $a$ for $a \in [50, a^*]$ by showing that for any $a, \tilde{a} \in [50, a^*]$, if $a > \tilde{a}$ then $\hat{f}(\cdot, a)$ second-order stochastically dominates $\hat{f}(\cdot, \tilde{a})$.

Note first that by construction, the expectation of $\hat{u}$ under $\hat{f}(\cdot, a)$ is 0 for all $a$. Let $\hat{F}(\cdot, a)$ be the corresponding cdf and observe

$$\int_{\underline{\hat{u}}(a)}^{\tilde{u}} \hat{F}(\hat{u},a)d\hat{u} = \frac{\left((c-100)\varepsilon\left(a^2-100a-25c+3750\right)+25\tilde{u}(25-2(c-75)\varepsilon)\right)^2}{2500\varepsilon\left((c-100)\left(a^2-100a-25c+3750\right)-50(c-75)\tilde{u}\right)}.$$

Note that the interval $[\underline{\hat{u}}(a), \bar{\hat{u}}(a)]$ shrinks in $a$ for all $a \in [50, a^*)$ and collapses to a single point at $a^*$. By the standard characterization of stochastic dominance, we need to show that $\forall \tilde{u} \in (\underline{\hat{u}}(a), \bar{\hat{u}}(a))$ the expression $\int_{\underline{\hat{u}}(a)}^{\tilde{u}} \hat{F}(\hat{u},a)d\hat{u}$ is decreasing in $a$.

The derivative of the above expression with respect to $a$ is given by

$$\frac{(a-50)(c-100)\left(\varepsilon T+25\tilde{u}(25-2(c-75)\varepsilon)\right)\left(\varepsilon T-25\tilde{u}(2(c-75)\varepsilon+25)\right)}{1250\varepsilon\left(T-50(c-75)\tilde{u}\right)^2},$$

where $T=\left(a^2-100a-25c+3750\right)(c-100)$.

Note that for any $a\in(50,a^*)$ the denominator is non-zero if $\tilde{u}\in[\underline{\hat{u}}(a),\bar{\hat{u}}(a)]$ Thus, for $a\in(50,a^*)$, the above expression is continuous in $a$,$\tilde{u}$, $\varepsilon$ and $c$. Also, fixing all other variables in an arbitrary way, it has no zeros in $\tilde{u}\in(\underline{\hat{u}}(a),\bar{\hat{u}}(a))$. Finally, simple computation shows that the above expression is strictly negative for some $a\in(50,a^*)$. We can thus conclude that $\int_{\underline{\hat{u}}(a)}^{\tilde{u}}\hat{F}(\hat{u},a)d\hat{u}$ is decreasing in $a$ on the relevant domain, thus establishing second order stochastic dominance as needed.

## Appendix B: Non-Bayes-Competent

Table B.1 reports the result for Non-Bayes-Competent subjects. We show results for ambiguity neutral (Columns 1–3) and averse (columns 4–6) subjects.

TABLE B.1. Effect of $c$ by urn type for Non-Bayes-Competent subjects

| | Ambiguity neutral | | | Ambiguity averse | | |
|---|---|---|---|---|---|---|
| | Pooled | Ambiguous | Risky | Pooled | Ambiguous | Risky |
| c | 0.250*** | 0.231** | 0.252** | 0.170*** | 0.249*** | 0.087 |
| | (0.078) | (0.105) | (0.121) | (0.061) | (0.084) | (0.087) |
| Anchoring control | -18.718** | -17.727* | -19.968* | -19.957*** | -11.474 | -28.714*** |
| | (7.841) | (10.599) | (11.766) | (5.739) | (8.051) | (8.179) |
| c x Anch. ctrl. | 0.263** | 0.232 | 0.296* | 0.273*** | 0.145 | 0.407*** |
| | (0.104) | (0.140) | (0.157) | (0.076) | (0.106) | (0.110) |
| Num.Obs. | 148 | 77 | 71 | 347 | 175 | 172 |
| R2 | 0.362 | 0.335 | 0.391 | 0.260 | 0.241 | 0.294 |
| R2 Adj. | 0.255 | 0.197 | 0.289 | 0.150 | 0.120 | 0.185 |

*Note:* This table reports the effects of $c$ on choices of Non-Bayes-Competent subjects. We employ a fixed effects panel model. Columns (1) to (3) report the effects for ambiguity neutral subjects, column (4) to (6) for ambiguity averse subjects. Significance is reported at the following levels: *** $<.001$, ** $<.01$, * $<.05$.

## Appendix C: Help and No Help treatments

Table C.1 reports the results by help and no-help treatments for subjects who are Bayes-Competent.

Table C.1. Effect of c by help treatments for Bayes-Competent subjects pooled over urn types

| | Ambiguity averse | | Ambiguity neutral | |
|---|---|---|---|---|
| | Help | No help | Help | No help |
| c | 0.178* | 0.327*** | -0.230 | -0.053 |
| | (0.093) | (0.122) | (0.233) | (0.174) |
| Anchoring control | 19.546* | 30.229** | -11.428 | 5.868 |
| | (10.146) | (13.488) | (26.907) | (18.304) |
| c × Anch. ctrl. | 0.081 | -0.113 | 0.492 | 0.210 |
| | (0.133) | (0.180) | (0.364) | (0.246) |
| Num.Obs. | 326 | 170 | 53 | 103 |
| R2 | 0.314 | 0.286 | 0.286 | 0.260 |
| R2 Adj. | 0.307 | 0.273 | 0.242 | 0.238 |

*Note:* This table reports the effects of $c$ on choice who are Bayes Competent. We employ a fixed effects panel model. Columns (1) to (3) report the effects for ambiguity neutral subjects, column (4) to (6) for ambiguity averse subjects. Significance is reported at the following levels: *** $<$ .001, ** $<$ .01, * $<$ .05.

## Appendix D: CRT Median split

As an alternative to measure subjects ability in logical and mathematical tasks, we split the sample by performance in the CRT test. Table D.1 reports the results and finds similar effects for subjects who performed better or equal to the median in the CRT compared to subjects who we classified as Bayes-Competent.

TABLE D.1. Effect of c by performance in the CRT task pooled over urn types

| | CRT $\geq$ median | | | CRT $<$ median | | |
|---|---|---|---|---|---|---|
| | Pooled | Non neutral | Neutral | Pooled | Non neutral | Neutral |
| c | 0.160*** | 0.196*** | 0.090 | 0.169*** | 0.170** | 0.267*** |
| | (0.034) | (0.049) | (0.057) | (0.051) | (0.066) | (0.094) |
| Anchoring control | -6.868** | -6.301 | -7.291 | -22.414*** | -15.324** | -32.288*** |
| | (3.244) | (4.678) | (5.314) | (4.900) | (6.337) | (9.330) |
| c × Anch. ctrl. | 0.091** | 0.084 | 0.103 | 0.303*** | 0.212*** | 0.421*** |
| | (0.043) | (0.061) | (0.070) | (0.065) | (0.082) | (0.127) |
| Num.Obs. | 551 | 308 | 156 | 412 | 223 | 104 |
| R2 | 0.181 | 0.212 | 0.167 | 0.283 | 0.234 | 0.503 |
| R2 Adj. | 0.176 | 0.204 | 0.150 | 0.278 | 0.223 | 0.488 |

*Note:* This table reports the effects of $c$ by performance on the cognitive reflection test (CRT Frederick, 2005). We employ a fixed effects panel model. Columns (1) to (3) report the effects for subjects with equal or above median performance on the CRT, column (4) to (6) with less than median performance. Significance is reported at the following levels: *** $<$ .001, ** $<$ .01, * $<$ .05.

## Appendix E: Ambiguity loving subjects

Table E.1 reports the results for *Ambiguity Loving* subjects who are Bayes-Competent.

TABLE E.1. Effect of c for ambiguity loving subjects

| | Bayes-Competent | | |
|---|---|---|---|
| | Pooled | Ambiguous | Risky |
| c | 0.101 | 0.045 | -0.612*** |
| | (0.103) | (0.065) | (0.203) |
| Anchoring control | -15.630* | -1.190 | -60.526*** |
| | (9.274) | (6.626) | (22.202) |
| c x Anchoring control | 0.205 | 0.000 | 1.220*** |
| | (0.127) | (0.085) | (0.296) |
| Num.Obs. | 84 | 45 | 55 |
| R2 | 0.203 | 0.042 | 0.532 |
| R2 Adj. | 0.055 | -0.171 | 0.463 |

*Note:* This table reports the effects of $c$ on ambiguity loving subjects. We employ a fixed effects panel model. Columns (1) to (3) report the effects for subjects with equal or above median performance on the CRT, column (4) to (6) with less than median performance. Significance is reported at the following levels: *** $<$ .001, ** $<$ .01, * $<$ .05.

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