Investigating inclusion and disability in teaching staff and pupil discourses in mainstream primary mathematics classrooms in the UK: the case of visual impairment

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Abstract

This study investigates inclusion and disability in the discourses of teaching staff and pupils in British mainstream primary mathematics classrooms with visually impaired (VI) pupils, first in an exploratory phase and then in an experimental phase. The study's theoretical underpinnings are sociocultural: Vygotskian sociocultural theory of learning, with particular emphasis on the notion of mediation; and, the social model of disability. Pertinent role in the study's theoretical framework also plays the theory of embodied cognition. The Vygotskian sociocultural theory of learning affords looking at how the semiotic, the material and the sensory tools that are used in the classroom mediate the mathematical learning of visually impaired pupils and consequently affect the inclusion and enabling of these pupils. The theory of embodied cognition affords a closer look at the construction and expression of mathematical meaning. The social model of disability underpins the study's take on disability as socially constructed. Data collection was conducted in four primary mathematics classrooms in four mainstream schools in Norfolk through: classroom observations; individual interviews with class teachers, teaching assistants and pupils; focussed-group interviews with pupils; written transcripts of the class teachers' contributions in the design of the three experimental lessons; photographs of the pupils' work in the three experimental lessons; and, pupils' evaluation forms of the experimental lesson in two classes. Data analysis is presented under three themes: the role of speech and gesturing in the mathematical learning of visually impaired pupils; the intertwinement of digital and physical resources in the mathematical learning of visually impaired pupils; and, the mathematical contributions of visually impaired pupils in an inclusive primary classroom and the responses of teaching staff and sighted pupils to these contributions. The study concludes with a discussion of theoretical and methodological implications for research in this area as well as implications for policy and practice.

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Chapter 1: Introduction

This doctoral study focuses on inclusion and disability. The two terms are investigated in teaching staff and pupil discourses in mainstream primary mathematics classrooms in the UK. Visual impairment is selected as the characteristic around which inclusion and disability are mainly investigated.

I start this chapter by decomposing the title of the thesis (1.1). I then discuss the discrepancy on disability (1.2) and on inclusion (1.3) in the British educational context. I end the chapter with the aims and contributions of the study (1.4).

1.1 Decomposing the title of the thesis: My inspiration and background

In the next four sections, I explain: the study's focus on inclusion and disability – and, specifically, on visual impairment as a form of disability; the investigation of inclusion and disability in teaching staff and pupil discourses; the contextualisation of these two terms in mainstream primary mathematics classrooms; and, their investigation in the UK.

Why "inclusion" and "disability"?

Since I was a pupil, I have been interested in people coming from a minority background – be it linguistic, cultural, social, or a combination of any of these features. The minority people are often segregated due to their divergence from what is dominant in the particular context. I have always found this situation unfair – why are people who differ, in one or more characteristics, from the majority of people in a particular context segregated and sometimes treated in a diminished way to the majority of the people in that context?

My interest in minority people started in my first year of secondary school. Many of my classmates came from a different linguistic and cultural background – in my home country, Greece, and specifically in my hometown, Xanthi, there are Greek citizens who belong to the Muslim minority in Western Thrace and many of them speak their parents' mother tongue at home.

Most of my minority language classmates – particularly those who did not speak Greek very well – were isolated from the majority of my classmates. As a result, they ended up sitting together in the lessons and being friends amongst themselves, in other words they were ghettoed at school. Apart from social isolation, there were several cases when these pupils also faced racist comments by the 'dominant' groups of pupils: comments on their parents' ethnic origin.

Apart from pupils, there were also cases when some of my minority language classmates were treated unfairly by teaching staff. Some teaching staff tended to neglect these pupils during the lesson – they were not frequently addressing these pupils, either directly (by talking to them) or indirectly (by having eye contact with them or by referring to cases showing that they had taken these pupils into consideration)[1].

Therefore, my minority language classmates were frequently excluded, both academically – in the substantive parts of the lesson, particularly in the pedagogical content aspects of the lesson – and socially – in their interaction with the 'dominant' groups of peers about the lesson or about other topics. Such exclusion occurred despite the fact that my minority language classmates were in the same class with the other pupils attending the same lesson. The academic and social exclusion of my classmates made me develop a sensitivity towards these pupils, for whom I desired inclusion.

My interest in inclusion has thus come since my teenage years. It is also these years from which my master's dissertation, entitled "Minority language students' difficulties with Mathematics: Primary school teachers' beliefs and practices", was influenced and was created. As in my thesis, in my master's dissertation I focused

[1] For example, by referring not only to the mainstream Greek culture but also to these pupils' cultures.

too on minority pupils in inclusive mathematics classrooms. I will now proceed with the origins of my interest in disability.

My interest in disability is far more recent than that in inclusion. It started a decade later, during my undergraduate years. In the third year of my Primary Teacher Education course in the Aristotle University of Thessaloniki, Greece, I attended a part-time course on reading and writing Braille, the language of visually impaired people. This course was not part of my School's curriculum and was taught in an institution that was specialised in people with visual impairment. It was a popular course amongst primary school teacher students in Greece, because the certificate offered upon the course's successful completion gave the students credits in their applications for a teaching position. I therefore decided to take the opportunity and attend this course, with the initial aim to get such a privilege. During the course, though, my motivation increased: I enjoyed the course from the very first lesson, as I was learning a very unique language – a language that has nothing to do with the other languages I was learning (English and French). It is a haptic language, which is read and written by a social group of people across the world – the visually impaired people. I learnt Braille in Greek and in English. Its construction, its history and its impact upon the visually impaired people filled me with enthusiasm to have been learning this language. This is where my interest in disability, and more specifically in visual impairment as a form of disability, started.

My interest in inclusion and disability combined started from a **C**hallenging **A**bleist **P**erspectives on the **Tea**ching of **M**athematics (CAPTeaM) workshop, which I attended in 2015 at the University of East Anglia. CAPTeaM is an international programme involving a partnership between British and Brazilian institutions. It aims to challenge ableist[2] perspectives on the teaching of mathematics (Nardi, Healy, Biza, & Fernandes, 2018). CAPTeaM focuses mostly on engaging pre- and in-service teachers with reflection on classroom vignettes that highlight seminal issues in the learning experiences of disabled[3] learners in mainstream classrooms. In the 2015 workshop, we – the audience – were presented with two types of tasks: Type 1 tasks, which aimed to provoke reflections about disabled learners' mathematical strategies that were valid but whose properties and relations were expressed in unconventional or surprising forms; and, Type 2 tasks, which aimed to provoke reflections about how access to mediational means differently shapes mathematical activity (Nardi et al., 2018). I got excited with this project's inclusive character orientated to disabled learners. That workshop served as a trigger for me to do a PhD in the area of inclusion and disability. I was directly orientated on visual impairment, as this is the form of disability on which I have had experience through learning Braille.

Why "teaching staff and pupil discourses"?

Teaching staff and pupils were chosen as the participants on whom inclusion and disability would be explored. Due to the fact that my study takes place in classrooms, it is the teaching staff and the pupils who constitute the members of a classroom and whom inclusion and disability directly concern.

I investigate inclusion and disability in the discourses of my participants. With "discourses", I denote utterances – expressed through speech but also through gestures, facial expressions and bodily expressions in general – which relate to inclusion and/or disability and which are expressed by the participants either during the lesson or outside the lesson[4]. The discourses may signify the participants' attitude on – and/or experiences of – inclusion and disability.

[2] Ableism is defined as "a network of beliefs, processes and practices that produce a particular kind of self and body (the corporeal standard) that is projected as the perfect, species-typical and therefore essential and fully human. Disability, then, is cast as a diminished state of being human" (Campbell, 2001, p. 44).
[3] I use the term "disabled", as this is the official term in the UK when referring to individuals with physical and/or mental impairments.
[4] Outside the lesson: in my conversations with the participants.

Why "mainstream primary mathematics classrooms"?

I chose "mainstream" classrooms rather than special ones, as inclusive education concerns this type of classrooms: the classrooms in the general education system where disabled and non-disabled pupils are educated together (United Nations, 2006).

The characterisation "primary" classrooms was chosen due to my expertise in primary education. As stated earlier, my undergraduate course was in primary teacher education.

"Mathematics" was selected as the subject of teaching and learning because it is the subject of my expertise in primary education. With Mathematics being amongst my favourite subjects since I was a pupil in primary school and with me coming from a mathematics-orientated school background[5], I selected to attend all the mathematics education modules in my undergraduate course, I did my master's in Mathematics Education, I taught in British primary schools being mostly responsible for Mathematics and I then do a PhD in Mathematics Education.

Why "UK"?

I chose to conduct my doctoral study in the UK, because I found that there is a discrepancy between policy rhetoric and practice in the British educational context, with regard to inclusion and disability. I thus believed that my study in the UK would be meaningful – in aiming to mitigate this discrepancy.

The next two sections concern the discrepancy on inclusion and disability in the British educational context. The first section (1.2) discusses how disability is considered in the "United Nations Convention on the Rights of Persons with Disabilities" (UN CRPD) and how this consideration of disability is translated in the "Special educational needs and disability code of practice: 0 to 25 years" (SEND code of practice), which is the current educational code of practice on special educational needs and disabled people in the UK. I note that the UK ratified the CRPD in 2009 (Office for Disability Issues, 2011) and therefore its relevant policies, one of which is the SEND code of practice, need to resonate with the CRPD. The first section examines the CRPD's and the SEND code of practice's discourses of disability not only in relation to each other but also in relation to the two main models of disability discourse: the medical model and the social model (LoBianco & Sheppard-Jones, 2007). The second section (1.3) discusses inclusive education, particularly the paradox regarding this form of education in policy rhetoric[6] and in practice[7], with a specific focus on the British context.

1.2 Disability: divergence between the CRPD and the SEND code of practice

Over the last 50 years, there have been two main models of disability discourse: the medical model and the social model (LoBianco & Sheppard-Jones, 2007).

The medical model regards disability as a medical condition that is attributed to the individuals with impairments (Bingham, Clarke, Michielsens, & Meer, 2013; Palmer & Harley, 2012). This model considers disability as a personal problem which needs to be remedied in order that the individual can fit within society (Mitra, 2006).

[5] In the Greek educational system, from the second year of high school, the students have to choose a pathway of expertise amongst the following three options: humanities; science; and, engineering and economics. A series of subjects are included in each area of expertise and they are examined in the National Examinations, which allow the students to enter the tertiary education. I chose the science pathway, which includes Mathematics specialised courses.
[6] "Policy rhetoric on inclusive education" concerns how this form of education is endorsed in the CRPD and in other documents of the United Nations, such as those of UNESCO (2008) and UNICEF (2011).
[7] "Practice" in this context refers to how inclusive education is implemented in school classrooms.

The medical model of disability is associated with the integrated approach to the mainstream[8] education of disabled learners. According to this approach, the disabled learners are those who need to accommodate to the demands of the mainstream educational system (UNICEF, 2011).

Contrary to the medical model, the social model of disability endorses the idea that it is the society that disables people with impairments (Bingham et al., 2013; Palmer & Harley, 2012). In other words, the social model considers disability as a problem imposed by the society, which excludes people's full participation in it due to their impairments (Oliver, 2009). Therefore, differentiating impairment from disability, the social model considers disability as a socially constructed phenomenon (Mitra, 2006; Palmer & Harley, 2012). This point is elaborated further by LoBianco and Sheppard-Jones (2007) who argue that disability would not exist if society had not created it or if it removed the barriers that make the individual disabled.

The social model of disability is associated with the inclusive approach to the education of disabled learners. According to this approach, it is the school that needs to transform its culture, policies and practices in order to accommodate every individual's needs (UNICEF, 2011).

The social model of disability underpins the CRPD. This Convention considers that "disability is an evolving concept and […] results from the interaction between persons with impairments and attitudinal and environmental barriers that hinders their full and effective participation in society on an equal basis with others" (United Nations, 2006, p. 1). This excerpt reveals the Convention's consideration of disability as a social construct, since it "acknowledges the importance of the context and environment in enabling or disabling individuals from participating effectively within society" (UNICEF, 2011, p. 5).

While the CRPD does not define disability, the SEND code of practice does. The definition of disability as "a physical or mental impairment which has a long-term and substantial adverse effect on […] [people's] ability to carry out normal day-to-day activities" (Department for Education & Department of Health, 2015, p. 16) reveals a range of explicit and tacit views of disability and disabled people. The explicit point in this definition is the equation of disability with impairment while the implicit ones refer to the consideration of the disabled people as abnormal as well as to the creation of the binary set of "able-disabled" people. This set, in MacLure's words (2003), is a pair of binary oppositions revealing the binary structure of discursive realities, investing people with a particular identity and constituting a way in which meaning and knowledge are produced. Moreover, in the SEND code's definition of disability, there is no reference at all, neither explicitly nor implicitly, to "attitudinal and environmental barriers", which are reported in the CRPD and constitute the causes of disability. Therefore, there is a fundamental divergence between the CRPD and the SEND code of practice, in relation to disability. The SEND code's definition of disability seems to resonate more with the medical model of disability rather than the social model, which the CRPD endorses.

1.3 Inclusive education: benefits in policy documentation and challenges in practice

Inclusive education is the form of education "aimed at offering quality education for all while respecting diversity and the different needs and abilities, characteristics and learning expectations of the students and communities, eliminating all forms of discrimination" (UNESCO, 2008, p. 3). It is the form of education suggested in the Article 24 of the CRPD. Inclusive education refers to the provision of high-quality education to all learners and facilitates the development of more inclusive societies (UNICEF, 2011).

UNICEF (2011) states that inclusive education needs to be implemented, not only because it promotes the disabled people's rights to education, as it is clear in the CRPD, but also because it offers social and educational benefits to all learners. In terms of social benefits, inclusive education makes disabled learners less stigmatised and more socially included while it enriches non-disabled learners with tolerance, acceptance of difference and respect for diversity (UNICEF, 2011). In terms of educational benefits, inclusive education gives disabled learners access to a wider curriculum and it also leads them to higher achievement than in

[8] The word "mainstream" is important here as the medical model is also associated with segregation, which does not take place in mainstream schools but in special ones, designed to respond to the specific impairments of the disabled learners (UNICEF, 2011).

segregated settings (UNICEF, 2011). Simultaneously, UNICEF (2011) states that inclusive education provides educational benefits to the rest of the learners, too, through the changes that it brings in educational planning, implementation and evaluation.

However, despite all aforementioned positive characteristics of inclusive education in policy documentation, the implementation of this form of education is problematic in practice (Slee, 2011). Institutional limitations, such as educational structures, policies and cultures, and pedagogical barriers, such as discriminatory attitudes in class, constitute issues that prevent the implementation of inclusive education in school classrooms (Slee, 2011). In England, specifically, in spite of the attempt to abolish specialist pedagogy in classrooms, the special education traces remain and result in segregation in mainstream settings rather than inclusion (Miles & Ainscow, 2011). Miles and Ainscow (2011) recognise that inclusive schools cannot be achieved "by transplanting special education thinking and practice into mainstream contexts" (p. 3). The problematic implementation of inclusive education in England, at least in some classrooms, is also shown by even more recent evidence, offered by the Children's Commissioner for England (2017). Some evidence is: disabled children's and young people's "difficulties in getting their voices heard and taken seriously" (p. 1); "negative attitudes and stereotypes about disabled people"[9] (p. 1); and, "difficulties with accessibility in mainstream, state funded schools" (p. 1). Therefore, all these elements reveal a discrepancy regarding inclusive education in policy documentation (UNESCO, 2008; UNICEF, 2011; United Nations, 2006) and in practice (school classrooms).

Initial Teacher Education (ITE) around inclusion in England faces a similar divergence between education policies and practice.

In education policies, there are statements such as: "trainee teachers must achieve professional standards before they can be awarded qualified teacher status. The standards ensure that teachers are able to help all pupils, including disabled pupils, to achieve their full potential" (Office for Disability Issues, 2011, p. 73). More specifically: "[t]eachers must learn to vary their teaching to meet the needs of all pupils, including those with SEN" (Office for Disability Issues, 2011, p. 73); and, "[t]eachers must understand how pupils' learning can be affected by their physical, intellectual, linguistic, social, cultural and emotional development" (Office for Disability Issues, 2011, p. 73).

These statements indicate the responsibility for trainee teachers to learn how to teach disabled pupils so that the latter can achieve their full potential. Implicit in these statements is the role of the Universities, which constitute the institutions that provide the trainee teachers with the aforementioned knowledge. The implicit reference to Universities alongside the non-reference to specific ways on trainee teachers' learning about teaching disabled pupils seem to indicate an aspirational – rather than a pragmatic – character of ITE around inclusion in England. The aspirational character, which is not translated to specific measures in practice, is also indicated by the controversy that follows in the next paragraph – and which concerns the amount of significance that the UK pays on inclusion as a part of the ITE.

While in the above statements it is compulsory for trainee teachers to learn how to teach disabled pupils, in the following statement – which also comes from the UK's Initial Report on the UN CRPD (Office for Disability Issues, 2011) – it appears that ITE around inclusion is not compulsory: "All trainees have the opportunity to undertake a specialist area of study, such as the teaching of children with SEN" (Office for Disability Issues, 2011, p. 73). Therefore, ITE around inclusion constitutes an option for trainee teachers to undertake in the University.

Four years later, the Carter Review of Initial Teacher Training (ITT) (Department for Education, 2015) showed that "ITT inadequately prepares new teachers to address special educational needs and disabilities" (Department for Education, 2015, p. 11). This review acknowledges that addressing SEND within ITT programmes can be challenging, especially for the one-year programmes. In order to address the inadequacy

[9] Underestimation of these people's abilities, limited expectations and aspirations for them in educational settings because of their impairments (Children's Commissioner for England, 2017).

of ITE around inclusion, this review makes the following recommendations: "Special educational needs and disabilities should be included in a framework for ITT content" (Department for Education, 2015, p. 11); and, "Wherever possible, all ITT partnerships should build in structured and assessed placements for trainees in special schools or mainstream schools with specialist resourced provision" (Department for Education, 2015, p. 11). Regarding the first recommendation, this review suggested "the development of an advisory framework of initial teacher training core content to include content on SEND" (European Agency for Special Needs and Inclusive Education, 2018, para. 3).

The review also reports that SEND is one of the areas that need to be covered in the ITT courses. More specifically, "ITT should prepare all new teachers to support SEND in their classrooms, providing a solid grounding in the most pertinent issues and setting an expectation for on-going high quality professional development" (Department for Education, 2015, p. 24). As with statements in 2011, this statement is very vague and does not provide specificity in how ITE can be materialised with regard to inclusion. A more specific measure is suggested with regard to tracking progress of pupils with SEND: "Using assessment data and information on pupils' starting points, trainees need to be taught how to establish objectives and design programmes for pupils to enable them to make good progress and access the mainstream curriculum" (Department for Education, 2015, p. 34). However, there is nothing specific about the design of programmes.

Another contradiction between policy and practice arises with regard to the nature of teacher training. While the SEND Code of Practice (Department for Education & Department of Health, 2015) highlights that all teachers are potentially teachers of SEND, in practice the review shows that teachers have not been trained adequately to teach pupils with SEND. The review states the necessity for the translation of this statement of the Department for Education and Department of Health (2015) in the ITE, by emphasising the significance that "all new teachers are given training in how to support children with SEND – this should not be treated as an optional extra but as a priority" (Department for Education, 2015, p. 34). The review reports their belief that "understanding how to teach children with SEND is critical to improving progress and achievement for all children" (Department for Education, 2015, p. 34) and that "ITT [should] recognise […] that good teaching for SEND is good teaching for all children" (Department for Education, 2015, p. 34).

The review emphasises the significance of realism in the topics that can be covered in an ITT programme, especially in one-year ITT programmes. To this realism, it stresses the priority that ITT programmes should put in "the knowledge and understanding new teachers will need to support all children in their classrooms from day one as a qualified teacher" (Department for Education, 2015, p. 35). However, the review then limits this knowledge "to the most common issues they [teachers] will encounter" (Department for Education, 2015, p. 35). These issues are autism, severe learning difficulties and dyslexia. Other issues that need to be addressed in ITT are speech and language development and medical conditions.

While the review stresses the significance for the inclusion of the above-mentioned issues in the ITT, it then states the potential of a challenge in effectively addressing these issues, especially in shorter ITT programmes. To deal with this challenge, the review recommends the emphasis that ITT programmes should put "on instilling a clear expectation of on-going development" (Department for Education, 2015, p. 35). An example for this is that "new teachers should feel confident in knowing where to look and who to speak to for further advice and guidance, such as Special Educational Needs Coordinators (SENCos)" (Department for Education, 2015, p. 35). Therefore, the review implies that teacher knowledge around inclusion should come mostly from teachers' professional development and less from their ITT. This implication raises questions about the extent to which the review itself considers as necessary the topic of inclusion of SEND pupils in the ITT programmes.

The review highlights that a part of ITT programmes should be devoted to trainees' observations of SEND specialists in the classroom and then to their own placements and assessments. More specifically, "[s]chool experiences should be carefully structured to offer opportunities to observe outstanding practice and develop practical strategies that arise from this. Placements where trainees have opportunity to practice and be assessed in a special school or a mainstream school with specialist resourced provision are particularly beneficial" (Department for Education, 2015, p. 35-36).

Three years later, the situation on ITE around inclusion in England has remained similar. The aspirational character has remained, with vague aspirations on ITE around inclusion in England continuing to prevail. For example, "[t]he [UK] government has sharpened the focus on meeting the needs of pupils with special educational needs (SEN) and disabilities within the standards for qualified teacher status" (European Agency for Special Needs and Inclusive Education, 2018, para. 1).

1.4 Aims and contributions of the study

The above two sections (1.2 and 1.3) have revealed two discrepancies between policy rhetoric and practice in the British educational context: one concerns disability and the other regards inclusive education. As for disability, there is evidence that the UK endorses the medical model of disability in its SEND code of practice despite the fact that it has ratified the CRPD, which endorses the social model. As for inclusive education, there is evidence that despite the claim for social and educational benefits which inclusive education offers to all learners, there are barriers to the implementation of this form of education in British classrooms.

The study aims to contribute towards minimising the above discrepancies on disability and inclusive education, by investigating how the two notions are played out in practice and by attempting to eventually bring the practice closer to endorsed principles in legislation[10]. With regard to disability, in this study I endorse that disability is socially constructed rather than a characteristic that an impaired individual necessarily has. With regard to inclusive education, in this study I aim to substantiate the benefits to all learners that are aspired in international legislation as outcomes from the implementation of inclusive education. The study sees the above take on disability and the above aim on inclusive education as contributors to the minimisation of the discrepancies on disability and inclusive education between policy rhetoric and practice.

This study is conducted in two phases: an exploratory phase and an experimental phase. In the exploratory phase, the study evidences the extent to which the educational practice in the UK converges to, or diverges from, the policies which the UK endorses, with regard to inclusion and disability. This substantiation is important because it shows schools', teaching staff's and pupils' attitude to inclusive education and to disability, 11 years after the UK ratified the CRPD. The study's focus on teaching staff's and pupils' attitudes to – and experiences of – inclusion and disability serves as the basis for the creation of inclusive mathematics lessons: the study endorses that, only if the lessons are experienced as inclusive by pupils, does inclusive education take place. It is also this principle that underpins the experimental lessons. Last but not least, the experimental phase evidences any potential discursive shifts of teaching staff and pupils regarding inclusion and disability, which may arise as a result of the co-designed lessons.

In its experimental phase, the study has two focal points. Firstly, it evidences the extent to which the class teachers are ready to participate in – and possibly change their practices towards – designing and implementing inclusive mathematics lessons, in accordance with what was experienced, or not, as inclusive by the pupils in the previous phase of the study. Secondly, it evidences the extent to which the pupils experience these co-designed lessons as inclusive ones and therefore sets the scene towards the creation of even more inclusive mathematics lessons.

The study aims to address issues that are important to the VI community. The VI community is quite a marginalised community that has not yet received the necessary attention in mathematics education research. The study's focus on the VI pupils and its aim for the creation of more inclusive mathematics classrooms bring the VI community at the front, with their needs investigated and met, as much as the needs of the 'dominant' communities are met.

The study also aims to address issues that are important to teaching staff and stakeholders, both of whom are responsible for the inclusion of visually impaired pupils – and also for inclusion as a form of education in

[10] The word "legislation" refers to the CRPD and concerns both disability and inclusive education.

general. The study’s identification of inclusive practices in the classrooms where it takes place along with its presentation of the impact of inclusion upon the class members may make teaching staff and stakeholders reflect on inclusion and, possibly, act accordingly towards it.

Moreover, the study aims to address issues that are important to research projects, such as CAPTeaM, through the identification of classroom episodes that take place in real classrooms. Episodes that come from real classroom situations increase the possibility for teacher engagement compared to episodes from fictional classroom situations. As Watson and Mason (2007) note “…the fundamental issue in working with teachers is to resonate with their experience so that they can imagine themselves ‘doing something’ in their own situation” (p. 208). In the former type of episodes, teachers may find themselves exposed to and may thus bring their experiences forward, resulting in a meaningful reflection on inclusion. The latter type of episodes may be less realistic and may thus bring limited reflections from teachers.

Apart from the educational sector, the study’s contribution to inclusive mathematics classrooms is hoped to bring significant effects to the society, in the long term, as classrooms constitute a micro-level of society. In other words, the creation of inclusive mathematics classrooms can also contribute to the creation of inclusive societies, namely societies which will welcome diversity, will recognise and value people’s different abilities and will eliminate their tendency for homogeneity as well as discrimination against people due to their differences from the so-called normal.

The above aims and contributions of the study are elicited from the study’s exploration of the following two research questions (RQs): (1) How are inclusion and disability constructed in the discourses of teaching staff and pupils in the mathematics classroom? (2) How do collaboratively designed mathematics lessons impact upon teaching staff’s and pupils’ discourses on inclusion and disability?

Chapter 1 is followed by seven chapters: Theoretical Framework (Chapter 2); Literature Review (Chapter 3); Methodology (Chapter 4); Data and analysis (Chapter 5, Chapter 6 and Chapter 7); and, Conclusions (Chapter 8).

Chapter 2: Theoretical framework

2.0 Introduction

The theoretical underpinnings of this study are sociocultural: Vygotskian sociocultural theory of learning, with particular emphasis on the notion of mediation (Vygotsky, 1978; Vygotsky, 1993), and the social model of disability (Oliver, 2009). Pertinent role in the study's theoretical framework plays also the theory of embodied cognition (Gallese & Lakoff, 2005).

This combination of Vygotskian sociocultural theory of learning with the theory of embodied cognition has been used by several researchers, such as Arzarello, Paola, Robutti, & Sabena (2009), Radford, Bardini, Sabena, Diallo, & Simbagoye (2005), and Healy and Fernandes in their research project "Rumo à Educação Matemática Inclusiva" (for example, Healy & Fernandes, 2011). While the first two groups of researchers have used the combination of these two theories in mathematics education but without specific focus on impaired learners, Healy and her colleagues constitute a group of researchers that have used the combination of these two theories in the field of inclusion of VI pupils in the mathematics classroom.

I first start with the study's influences from each of the three theories which comprise the study's theoretical framework: the Vygotskian sociocultural theory of learning (2.1); the theory of embodied cognition (2.2); and, the social model of disability (2.3). I then continue with presenting the study's influences from the CAPTeaM project, which resonates with the study's theoretical underpinnings (2.4). I close this chapter with presenting the study's conceptualisation of "inclusion" and "disability", as well as terms that derive from these two notions (2.5).

2.1 Influences from the Vygotskian sociocultural theory of learning

The study's definition of "mathematical learning" is embedded in the Vygotskian sociocultural theory of learning (Vygotsky, 1978). In particular, I see mathematical learning as a social process which is characterised by the use of semiotic, material and sensory tools that all comprise the culturally developed subject of Mathematics.

While Vygotsky (1978) explicitly considers the first two kinds of tools as forming Mathematics, in his earlier formulations of the notion of mediation, which occurred while he was working with disabled learners, Vygotsky (1993) implicitly considered the sensory organs too as tools which – as the other two kinds of tools – impact upon the individual's cognitive activity. This implicit consideration emanates from his discussion of the idea that organs of the body can be thought of as "instruments" used to sense the world: "the eye, like the ear, is an instrument that can be substituted by another" (Vygotsky, 1997, p. 83). Vygotsky's (1993) attribution of the role of a psychological tool to elements of the body constitutes a strong allusion to the embodied nature of the human intellect. However, while the embodied nature of cognition is clear in Vygotsky's (1993) works, Vygotsky was primarily interested in the sociocultural nature of tools.

As reported in the previous paragraph, Vygotsky's ideas about knowledge mediation have their roots in his experimental work with disabled learners (Vygotsky, 1993). Vygotsky acknowledged that the language of a culture tends to be designed for the able-bodied, which means that it may not be accessible to people who lack of, or have limited access to, a sensory organ. He suggested that, instead of focusing on quantitative differences in achievements between disabled and non-disabled learners, a qualitative perspective should be adopted. For Vygotsky, the inclusion of disabled learners in social (cultural) activities can be fulfilled in the identification of ways to substitute the traditional mediational means with others, which are more suitable to the specific ways in which disabled learners interact with the rest of the world. For example, in the case of VI learners, Vygotsky posited that their inclusion could be achieved through substituting their eyes with another instrument. Just as the inclusion of any other tool in activity, this substitution can be expected to cause a restructuring of the cognitive activity and of the personality of the VI individual (Vygotsky, 1993). "The positive particularity of a child with a disability is created not by the failure of one or other function observed in a normal child but by the new structures which result from this absence […] The blind or deaf child can achieve the same level of development as the normal child, but through a different mode, a distinct path, by other means. And for the pedagogue, it is particularly important to know the uniqueness of the path along with the child should be led" (Vygotsky, 1997, p. 17). Therefore, Vygotsky considered disabled learners as different but not deficient. He focused on what these learners can do rather than what they cannot do.

This understanding of inclusion and disability resonates with the understanding of inclusion as evident in today's international legislation (for example, United Nations, 2006; UNESCO, 2008; UNICEF, 2011). In particular, Vygotsky (1993) acknowledges the importance of designing education systems suitable not only for the able-bodied learners but also appropriate for the inclusion of disabled learners.

I use the constructs of "tools" and "mediation" from the Vygotskian sociocultural theory of learning (Vygotsky, 1978; Vygotsky, 1993). My choice of these constructs – and their particular use – have been influenced by Healy and Fernandes (2011; 2014) who as well use these constructs in their work on inclusive mathematics education with VI pupils.

As indicated earlier in my definition of mathematical learning, I consider as tools the semiotic artefacts, the material artefacts and the sensory organs which are all used in the construction of mathematical meaning. I define all these three groups as tools, because – in line with Vygotsky (1978; 1993) – they all cause a transformation of the individual's cognitive activity.

I define "mediation" as the impact that the particular semiotic, material and sensory tools bring upon the individual's activity. In line with Vygotsky (1978; 1993), these three kinds of tools mediate the mathematical activity of the individual.

In my study, I examine how specifically these three kinds of tools, which are used in the mathematics classroom by teaching staff and pupils, mediate the mathematical learning of VI pupils and consequently affect the inclusion and enabling of these pupils in the mathematics classroom. I also examine whether particular uses of these kinds of mediating tools are, or are not, beneficial not only to the VI pupils but also to the rest of the class too.

2.2 Influences from Gallese and Lakoff's theory of embodied cognition

While the theory of embodied cognition is not used by Gallese and Lakoff specifically for disabled learners, its tenets imply that disability is not a direct implication of an individual's physical impairment. This is extrapolated from Gallese and Lakoff's (2005) understanding of cognition: cognition is embodied and understanding is multimodal.

The combination of these two facts from the theory of embodied cognition (Gallese & Lakoff, 2005) implies that a bodily impairment does not equate to disability: a sensory organ is one out of the multiple modalities through which knowledge is constructed. Therefore, a limited function – or a non-function – of this organ does not by itself stop the individual from such construction, as there are other perceptual modalities too to be utilised. One would say that, in the theory of embodied cognition (Gallese & Lakoff, 2005), what may make an impairment a disability would be a lack of provision of multimodal activities to an impaired individual: indeed, as understanding is multimodal, if the learning experience is provided only through the limited functioning – or the non-functioning – sensory organ, then the individual will be disabled. Therefore, in the theory of embodied cognition (Gallese & Lakoff, 2005), disability can be seen as socially constructed. Implicit as well is the assumption that, as cognition is embodied, a necessary element of inclusion is the provision of opportunities that allow the impaired individual to construct knowledge. Therefore, the theory of embodied cognition seems to resonate too with the understanding of inclusion as evident in today's international legislation (for example, United Nations, 2006; UNESCO, 2008; UNICEF, 2011).

I use the constructs of "embodied nature of concepts" and "embodied imagination" from the theory of embodied cognition (Gallese & Lakoff, 2005). I also use Healy and Fernandes' (2011) construct of "embodied abstraction", which they describe as emanating from the theory of embodied cognition (Gallese & Lakoff, 2005).

I use "embodied nature of concepts" to characterise the study's view that all concepts are embodied (Stylianidou & Nardi, 2019b). I endorse Barsalou's (2009, p. 1282) characterisation of concepts as simulators, which are the distributed neural systems of the multimodal content associated with a particular category.

"Embodied imagination" – by Gallese and Lakoff (2005) – and "embodied abstraction" – by Healy and Fernandes (2011) – are two constructs which emphasise the association of the body with non-physical features. In particular, "embodied imagination" denotes imagination that is "structured by our constant encounter and interaction with the world via our bodies and brains" (Gallese & Lakoff, 2005, p. 456). "Embodied abstraction" denotes an abstraction that is evident in "gestures alongside spoken language" (Healy & Fernandes, 2011, p. 169).

I use "embodied imagination" to characterise a verbal and/or a bodily manifestation which does not come from the practical action that is described in the verbal utterance and/or is shown in the bodily manifestation. In the next paragraph, I present an example of embodied imagination, which is evident in a VI pupil of my study.

Luke's description of Shape X "feels like it's just gonna bob up and down" (Stylianidou & Nardi, 2019a, p. 347) – in the class's assignment by the teacher to explore this shape through touch[11] – "does not come from an actual practical implementation of the rolling of the shape but from imagining the shape doing so" (Stylianidou & Nardi, 2019a, p. 348).

I use "embodied abstraction" to characterise a verbal and/or a bodily manifestation which comes from the practical action that is described in the verbal utterance and/or is shown in the bodily manifestation. In the next three paragraphs, I present three examples of embodied abstraction: the first example is evident in a VI pupil of my study and the other two examples are evident in blind students who participated in Healy and Fernandes' (2011) work.

Luke's gesture for a multiplication jump (Figure 30, p. 124) – in the class's assignment by the teacher to work out 692÷8 on a 'human number line'[12] – comes from the practical action of visually representing division work on a written number line, which Luke experienced in the past. With the firm holding of his whiteboard up, Luke embodily abstracts the multiplication jump that is made in the visual representation of the division work on a written number line. In this case, his gesture constitutes a "simulation" (Barsalou, 2008, p. 618) of a previously experienced activity for working out division calculations on a written number line.

Edson's gestures for folding – in his assignment by Healy and Fernandes (2011) to identify the axes of symmetry of a hexagon on a geoboard (Figure 7 in Healy & Fernandes, 2011, p. 170) – accompanied by verbal utterances come from the practical action of folding two-dimensional, cartonic figures, which Edson experienced in the past. In this case, his gestures constitute a "simulation" (Barsalou, 2008, p. 618) of a previously experienced activity. "Edson was able to locate correctly the two axes of symmetry for the hexagon he was working with, seemingly by mentally simulating the process of folding" (Healy & Fernandes, 2011, p. 170).

Leandro's gesture for area – in his assignment by Healy and Fernandes (2011) to identify the area of an "[in]"complete" concrete representation of a rectangle" (Healy & Fernandes, 2011, p. 164) – accompanied

[11] Details are found on p. 110-111.
[12] Details are found on p. 123.

by a verbal utterance come from the practical action of finding the area of a complete physical rectangle, which Leandro experienced in the past. In this case, his gesture constitutes a "simulation" (Barsalou, 2008, p. 618) of a previously experienced activity for finding the area. "Leandro's gesture serves as external evidence of an imagined re-enaction, a simulation, of a past experience" (Healy & Fernandes, 2011, p. 165).

Luke's, Edson's and Leandro's cases are embodied abstractions: they are manifestations of previously experienced activities, which are now relived through simulation. In this respect, they are characterised by some form of repetition, now in non-physical contexts.

On the other hand, embodied imaginations are not manifestations of previously experienced activities, which are now relived through simulation. In the first example with Luke, Luke could not have experienced the bobbing of up and down of a shape which cannot be practically placed in a position to roll – it is made of Wikki Stix and also Luke had never encountered this shape before. Therefore, embodied imaginations are not characterised by some form of repetition.

However, they constitute simulations, but not of experiences which were previously relived with the same object. Instead, they constitute simulations of previously relived experiences with another object: that to which the user attributes an embodied metaphor. In the first example with Luke, Luke visualises an object bobbing up and down in terms of the mathematical abstraction of Shape X (he considers that an object bobbing up and down is Shape X).

While metaphors for mathematical learning are often used to make sense of new (mathematical) ideas or objects associating them with already existing (often non-mathematical) ones (Presmeg, 1998), "blind people are naturally engaged in the opposite, namely using metaphors to make sense of ideas or objects in their context in terms of already existing mathematical ideas" (Figueiras & Arcavi, 2014, p. 126). Luke verifies this statement of Figueiras and Arcavi (2014), making the case that their statement is applicable too to people with some vision – and not just to people with blindness.

2.3 Influences from Oliver's social model of disability

Mike Oliver (2009) is a key advocate of the social model of disability, according to which disability is socially constructed. His understanding of disability resonates with the understanding of this notion as evident in today's international legislation (for example, United Nations, 2006; UNESCO, 2008; UNICEF, 2011): in fact, these legislations seem to be based on Oliver's social model of disability in their understanding of disability. The social model of disability was elaborated in Chapter 1 (section 1.2).

I see disability as socially constructed in line with Oliver's social model of disability (Oliver, 2009). Drawing upon this model's conceptualisation of disability, I use the constructs of "enabling" and "disabling", which are presented in 2.5.

2.4 Influences from the CAPTeaM project

Apart from the three aforementioned theories, this study is also influenced from the CAPTeaM project (Nardi et al., 2018), which has sociocultural and embodied theory underpinnings too. I particularly use the constructs of "valuing" and "attuning", which originate in the data analysis of this project (in Nardi et al., 2018) and, in particular, in the analysis of teachers' mathematical discourses as the teachers engage in the Type 1 and Type 2 tasks. I also use the construct of "incorporating", which is similar to the construct of "classroom management" that is used by Nardi et al. (2018). I use the constructs of "valuing", "attuning" and "incorporating" in a different setting – and with more groups of participants – compared to CAPTeaM: I use them in the mainstream mathematics classroom and to analyse class teachers', teaching assistants' (TAs') and sighted pupils' discourses which act as a response to VI pupils' mathematical contributions. Before I present my use of these three constructs, I will present what "valuing", "attuning" and "classroom management" mean according to Nardi et al. (2018).

Nardi et al. (2018) use “valuing and attuning” to denote “to what extent the respondent attunes to and values the disabled learner’s contribution(s), and how, if at all, s/he attends to the particularities of their mathematical agency (Type I) or adapts to the restriction imposed on the communication (Type II)” (p. 154). These researchers use “classroom management” to denote “how the respondent reflects on classroom management in relation to engaging all students in class after the contribution has been made (Type I) or after undertaking Type II tasks” (Nardi et al., 2018, p. 154).

I use “valuing” when I discern that a class teacher, a teaching assistant or a sighted pupil values a VI pupil’s mathematical contribution. With “valuing”, I denote appreciating and approving the VI pupil’s mathematical contribution as mathematically rigorous and of potential benefit to the rest of the class too. I use “attuning” when I discern that a class teacher, a teaching assistant or a sighted pupil attunes to a VI pupil’s mathematical contribution. With “attuning”, I denote engaging in understanding the VI pupil’s mathematical contribution, by embodying it, be the contribution verbal or bodily. I use “incorporating” when I discern that a class teacher incorporates a VI pupil’s mathematical contribution into the lesson. With “incorporating”, I denote bringing the mathematical contribution of the VI pupil to the rest of the class too as an integral part of the lesson.

Nardi et al. (2018) have identified two camps of teachers, each of which is characterised by a specific take on the mathematical contributions of disabled learners. The first camp of teachers focuses on “what for them is a more conventional description, involving using well-defined and familiar terms” (Nardi et al., 2018, p. 157). The second camp of teachers focuses on “developing […] [the disabled learner’s mathematical contribution] in its own right” (Nardi et al., 2018, p. 158). While in Nardi et al.’s (2018) work these two camps concern teachers, in my work I use these camps to also include teaching assistants and sighted pupils too. I use the constructs of “valuing”, “attuning” and “incorporating” to characterise sighted members’ discourses in any of the two camps of teachers that Nardi et al. (2018) have identified.

2.5 The study’s conceptualisation of “inclusion” and “disability” and of deriving terms

I see “inclusion” as taking place in the mainstream mathematics classroom when the VI pupil is invited to participate in the lesson on an equal basis with everyone else in class and is also valued equally to the rest of the class (Stylianidou & Nardi, 2019b).

One assumption underpinning this study is that equitable participation does not necessarily equate with the use of the same sensory, material and semiotic tools between the VI pupil and the sighted pupils. In this respect, I endorse that equitable participation can be achieved with the use of various sensory, material and semiotic tools (Stylianidou & Nardi, 2019b). With equitable value, I denote “the value attributed to VI pupils, even in those contributions which differ from the dominant, institutional ones, potentially as a result of the different tools through which VI pupils may construct and convey mathematical meaning” (Stylianidou & Nardi, 2019b, p. 4688). Reference to participation and value as contributors to inclusive education is present in international documents (for example, United Nations, 2006) as well as in research studies (for example, Nardi et al., 2018). I consider equitable participation and equitable value as interrelated elements of inclusive education: my study’s rationale is that one element exclusive of the other does not suffice for implementation of inclusive education (Stylianidou & Nardi, 2019b). In line with Nardi et al. (2018), I endorse that the placement of the VI pupil in the mainstream mathematics classroom does not necessarily equate with the inclusion of this pupil: “access alone does not guarantee quality education” (Nardi et al., 2018, p. 147).

I see “disability” as a socially constructed term, which is created by society and which occurs when environmental and attitudinal arrangements do not meet the needs of people with impairments (Stylianidou & Nardi, 2018). In this respect, I differentiate “impairment” from “disability”, considering impairment as “deterioration in the functioning of a body part, organ, or system that can be temporary or permanent and can result from injury or disease”[13]. In line with LoBianco and Sheppard-Jones (2007), I endorse that physical impairment can be of far less consequence if society removes the barriers that make an individual disabled.

[13] Extracted from: https://dictionary.cambridge.org/dictionary/english/impairment.

In my data analysis, I use the following terms which derive from "inclusion" and "disability": "including", "excluding", "enabling" and "disabling". I use the term "including" when a sighted member[14] invites the VI pupil to participate in the mathematics lesson alongside sighted pupils and when a sighted member values the VI pupil's mathematical contribution equally to sighted pupils. I use the term "excluding" when a sighted member does not invite the VI pupil to participate in the mathematics lesson alongside sighted pupils, when the conditions (e.g. the task, the activity, the context etc) do not allow the VI pupil to participate and when a sighted member does not value the VI pupil's mathematical contribution equally to sighted pupils. I use the term "enabling" when a sighted member acts in a way that meets the VI pupil's perceptual needs in the mathematics lesson. I define perceptual needs as the needs that relate to the pupil's accessibility to the mathematics lesson. I use the term "disabling" when a sighted member acts in a way that does not meet the VI pupil's perceptual needs in the mathematics lesson.

Having presented the theoretical underpinnings of the study, I will now proceed into the literature review (Chapter 3).

[14] A sighted member may be a teacher, a teaching assistant or a sighted pupil.

Chapter 3: Literature review

3.0 Introduction

In this chapter, I review literature on the inclusion of VI pupils in the mathematics classroom. I first review implementation issues towards inclusion of VI pupils in the mathematics classroom (3.1). In the following three sections (3.2-3.4), I review literature that corresponds to the themes which I explore in Chapters 5, 6 and 7. These themes are: speech and gestures (3.2); physical and digital resources used in mathematics lessons (3.3); and, VI pupils' mathematical contributions and teaching staff's and sighted pupils' responses to these contributions (3.4). Finally, in 3.5, I outline how the research questions that this study explores emerged from the reading of the literature presented in 3.1-3.4.

3.1 Challenges of implementing inclusion of VI pupils in the mathematics classroom

As discussed in Chapter 1, however beneficial inclusive education appears to be in international policy – for example, in United Nations (2006), in UNESCO (2008) and in UNICEF (2011) – the implementation of this form of education is often problematic in practice. The problematic implementation of inclusive education is also evident in the mathematics education literature with regard to VI pupils.

In the Maltesian context, Buhagiar and Tanti (2011) report "widespread implementation difficulties when it comes to translating inclusion policies into practices" (p. 72). The negative response of a leading Maltese disability lecturer and researcher in the question whether inclusive education works in Malta characteristically indicates this discrepancy between inclusion policy and practice. His negative response is justified through the following factors: the usual placement of disabled pupils at the back or side of the classroom, sitting on their own in the company of the facilitator; the rare interaction of disabled pupils with the teacher or their peers during the lesson; and, the frequent assignment for disabled pupils to do a different – easier – work compared to their peers. This researcher emphasises on the feeling of loneliness that the above circumstances must create to disabled pupils. He concludes with the fact that disabled pupils "were physically present in the class, but they did not seem to be part of it" (Buhagiar & Tanti, 2011, p. 72).

Curricular and institutional arrangements contribute to the discrepancy between inclusion policy and practice. Bartolo (2003) reports that, despite their increasing education in mainstream schools, disabled pupils "often remain excluded from the unchanged, one-size-fits all curricula, organization and activities of schools dedicated to their normative function" (p. 170).

Moreover, there are different meanings attributed to inclusion and to its implementation in the mathematics classroom. On the one hand, there are studies which advocate the necessity of both the classroom teacher and a support teacher – or, in other words, a facilitator teacher – for the inclusion of VI pupils. These studies, such as (Argyropoulos & Stamouli, 2006) and (Buhagiar & Tanti, 2011), conceptualise inclusion as consisting of a co-teaching scheme. In these studies, inclusion is based on a collaboration between the class teacher and the support teacher. This collaboration is manifested in the creation of a classroom environment which cultivates interaction and knowledge exchange in a way that the dyad 'support teacher-VI pupil' does not operate at a different level compared to the set 'class teacher-sighted pupils'. For Buhagiar and Tanti (2011), the successful collaboration between the class teacher and the facilitator comprises of the following impacts to the VI pupils: the VI pupils' following the mathematics lesson at the same pace as their classmates; the VI pupils' following the teacher's explanations and/or instructions; and, the VI pupils' listening to and participating in all the discussions in class. On the other hand, there are studies, such as (Bayram, Corlu, Aydın, Ortaçtepe, & Alapala, 2015), (Pinho, Castro, Alves, & Lima, 2016) and (Sticken & Kapperman, 1998), which consider the class teacher as the only one responsible for the inclusion of VI pupils and which do not mention the support teacher at all. Sticken and Kapperman (1998) report complexities when inclusion is implemented by the class teacher and the support teacher. They particularly report that the support teacher often interferes in the class teacher's role and, as a result, the mathematics education of the VI pupil becomes very complicated.

A main issue reported by studies which endorse that inclusive education needs to be implemented solely by the class teacher is the limited teacher training towards including VI pupils in the mathematics classroom. The limited teacher training may indicate that the inclusion of VI pupils in the mathematics classroom is still at its early stage and is not practised in the way it is conceptualised in international legislation – for example, in United Nations (2006), in UNESCO (2008) and in UNICEF (2011). The limited teacher training in combination with the training of support staff – and the work of the latter with the VI pupils in the mathematics classroom – may indicate that the inclusion of VI pupils is a transplantation of special education in mainstream mathematics settings (Slee, 2011).

Inherent to the limited teacher training and augmented support staff training is the existence of ableism in schools and the notion of the "normal" student. Ableism is "the network of beliefs, processes and practices that produce a particular kind of self and body (the corporeal standard) that is projected as the perfect, species-typical and therefore essential and fully human" (Campbell, 2001, p. 44) and holds a perspective on disability "as a diminished state of being human" (ibid.). Schools cannot become inclusive when they are underpinned by the notion of the "normal" student: this notion legitimises exclusion, since it separates students who differ from the sociopolitical connotations of this type of student as problematic and as in need of remediation (Healy & Powell, 2013).

Until recently, the mathematics learning potential of disabled students has been underestimated (Gervasoni & Lindenskov, 2011). As Borgioli (2008) points out, disabled learners' mathematical performance has been defined by their internal disability and not by external factors, "such as a mismatch between the learner and the task, conceptually fragile curriculum and/or instruction, inadequate social and emotional support structures, etc" (Borgioli, 2008, p. 137). "Internal disability" is a term that the school attributes to disabled learners because they do not learn in the way that the school expects.

With regard to VI pupils, their low mathematical performance often goes beyond their visual impairment and is particularly attributed to additional disabilities, such as learning disabilities, that these pupils may have (Klingenberg, Fosse, & Augestad, 2012). It is indeed considered common that VI pupils also have additional impairments (View, 2020) which impact upon their mathematical performance. However, rather than taking the additional disabilities as internal to the VI pupils, Klingenberg et al. (2012) advocate that we need to focus on the environmental and attitudinal factors which may cause the VI pupils to experience mathematical difficulties.

As Buhagiar and Tanti (2011) advocate, inclusion is implemented successfully when the visual impairment does not hinder the VI pupils' mathematical learning. Klingenberg et al.'s (2012) perspective in combination with Buhagiar and Tanti's (2011) one leads to the advocation that the successful inclusion of VI pupils takes place when the classroom environment and attitudes are such that do not allow the impairments to hinder the VI pupils' mathematical learning.

In line with the above researchers, Healy and Fernandes (2014) advocate that "[s]tudents' participation (or non-participation) depends on the nature of the learning situations in which they encounter mathematics and upon the development of pedagogies which recognise potential rather than deficiency" (p. 150).

The problematic implementation of inclusive education in the mathematics classroom has also been evident in the experiences of VI pupils, which are reported in Bayram et al.'s (2015) study conducted with three VI participants aged between 18 and 24 years old in Turkey. More specifically, VI pupils report experiences of ableism in the classroom, with their teachers believing that the VI pupils cannot learn mathematics. Ignorance and lack of interest about VI pupils' mathematical success were amongst the main manifestations of ableism in the VI pupils' mathematical learning experiences. Ableism had detrimental impact on the VI pupils' belief in their ability to learn mathematics.

Limitations in teacher training also impact teaching and therefore the experiences of the VI pupils in Bayram et al.'s (2015) study. These limitations are particularly evident from the lack of information that teachers have about learning materials for VI pupils and also about teaching methods in the mathematics classroom.

With regard to teaching methods, VI pupils also report some exceptions of teachers, who attempted to implement an effective teaching method in the mathematics classroom: explaining the lesson twice (once to the rest of the class and once, individually, to the VI pupil). While this method was beneficial for the VI pupils, it was time-consuming for the teachers and its pace was inadequate for the mathematical understanding of the sighted pupils. As a result, the teachers gave this method up without trying to adjust it to the benefit of every pupil (Bayram et al., 2015).

In the mathematical learning experiences of the VI pupils in Bayram et al.'s (2015) study, the lack of teacher training is linked to "teachers' negative attitudes towards inclusive education" (Bayram et al., 2015, p. 217). This negativity was manifested through unwillingness, fear and lack of interest. In particular, teachers did not want to take on the responsibility of teaching VI pupils, were afraid to update their knowledge and were not interested in learning about materials for VI pupils.

Bayram et al. (2015) attribute teacher negativity to a lack of interest about what VI pupils experience in the mathematics classroom and to a lack of knowledge about effective teaching methods and materials.

Nardi et al. (2018) see the teachers' attitudes towards inclusive education as consistent with – or embedded into – the institutions in which they teach. If the institutions do not enculturate the necessity of inclusion and (instead) cultivate the notion of the "normal" student, then the teachers may become uneasy about including VI pupils in the mathematics classroom.

In Nardi et al.'s (2018) view, teachers' unease may not be overcome if the educational structures to which the teachers belong (physical structures, such as buildings, and curricular structures, such as textbooks and materials, policies and assessment methods) sustain the notion of the "normal" student. Therefore, in Nardi et al.'s (2018) view, the issue of teachers' attitudes towards inclusive education is explicitly extended beyond the individual (teacher) and includes the sociopolitical impact of the educational institution.

Bayram et al.'s (2015) emphasis on the individual and Nardi et al.'s (2018) emphasis on the institution are also reflected upon these two groups of researchers' suggestions towards the creation of more inclusive school mathematics. Their suggestions are discussed later on in this section.

In order that the discrepancy between inclusion policies and practices can be overcome, teachers' role is argued to be significant – in (Buhagiar & Tanti, 2011) and in (Bayram et al., 2015).

Buhagiar and Tanti (2011) emphasise the fact that the existence of policy documents which prompt schools to become inclusive institutions does not guarantee the inclusion of VI pupils in the mathematics classroom. The teachers' "insistence and ability to translate inclusion policies and school backing into a classroom ambience in which all students, disabled or not, could feel part of and participate fully" (Buhagiar & Tanti, 2011, p. 72) are what Buhagiar and Tanti (2011) consider as the necessary factors for including VI pupils in the mathematics classroom.

Bayram et al. (2015) state that the classroom environment needs to be conducive to the VI pupils' learning and that the teachers need to be effective – motivated and able to address the learning styles of their pupils. To this aim, Bayram et al. (2015) suggest the implementation of need-specific professional development activities and the creation of opportunities for interaction between experienced special education teachers and mathematics teachers.

Nardi, Healy, Biza, & Fernandes (2016) extend the debate towards the minimisation of the above discrepancy by emphasising the institution's role. They particularly emphasise the requirement for deconstruction of the notion of the “normal classroom”. To this aim, they suggest the creation of a type of task which involves “imagining what a truly inclusive mathematics classroom might look like” (p. 354).

Having reviewed the general aspects of inclusion of VI pupils in the mathematics classroom, I will now review literature that corresponds to the specific themes that are identified within the mathematics lesson – and that I explore in Chapters 5, 6 and 7. These themes are: speech and gestures (3.2); physical and digital resources used in mathematics lessons (3.3); and, VI pupils’ mathematical contributions and teaching staff’s and sighted pupils’ responses to these contributions (3.4).

3.2 Speech and gestures

In this section, I discuss the following: speech and gestures of VI pupils (3.2a); and, speech and gestures of teaching staff (3.2b).

3.2a Speech and gestures of VI pupils

Back in history, there have been two main views on speech-gesture relations: the formalist view and the sensualist view (Kendon, 2008). In the formalist view, gestures are considered as spontaneous and idiosyncratic and as not constructed according to any standard forms. These characteristics that the formalist view attributes to gestures show that, in this view, gestures are not part of language.

It was only in the mid-1980s that gestures started to be considered as an integral part of language. McNeill (1985; 1992) shows how gestures are used while speaking. For Kendon (2008) though gestures were still considered as an appendage to speech: they were not considered as language itself.

However, further back in history, in the Age of Enlightenment, it was argued that speech came from gestures. LeBaron and Streeck (2000) report that, in the view of the sensualist Condillac, first we perceive the world through our bodies and then we transform our perceptions into sharable signs – through speech – in order to communicate our perceptions to others. Therefore, this view suggests that it is the gestures that gave rise to speech.

Impacts from the formalist view on speech-gesture relations are evident today too. As Rotman (2009) states, there is a general tendency to devalue communication systems which make use of the visual modality as compared to orally based ones and to assume that language should be identified with speaking, while communications using body movements are judged more primitive and non-intellectual.

Impacts from the sensualist view on speech-gesture relations are evident today too and resonate with the finding that all human cognition is embodied (by Gallese & Lakoff, 2005).

The above-described speech-gesture relations have also been reflected in the varying conceptualisations of gestures. Here I only report McNeill’s (1992) conceptualisation of gestures. In McNeill’s (1992) conceptualisation of gestures, gestures are part of what a person says – gestures are part of the same semiotic system as speech. Particularly, McNeill defines gestures as “movements of the arms and hands…closely synchronised with the flow of speech” (McNeill, 1992, p. 11).

McNeill (1992) indicates four types of gestures: deictic, iconic, metaphoric and beat gestures. “*Deictic* gestures are pointing movements, which are prototypically performed with the pointing finger, although any extensible object or body part can be used, including the head, nose, or chin, as well as manipulated artifacts” (McNeill, 1992, p. 80). “An iconic gesture […] is one that, in its execution and manner of performance, refers to a concrete event, object, or action that is also referred to in speech at the same time” (McNeill, 1992, p. 76-77). In other words, “[a] gesture is *iconic* if it bears a close formal relationship to the semantic content of speech” (McNeill, 1992, p. 78). “*Metaphoric* gestures are similar to iconic in that they present imagery, but present an image of an abstract concept, such as knowledge, language itself, the genre of the narrative, etc” (McNeill, 1992, p. 80). Finally, “[*b*]*eat* are defined as movements that do not present a discernible meaning, and they can be recognized positively in terms of their prototypical movement characteristics" (McNeill, 1992, p. 80).

Research on gesture use of VI pupils has been done primarily in non-mathematical settings – for example, by Goldin-Meadow (2003) and by Iverson and Goldin-Meadow (1998). It is only in the last decade that research on gesture use of VI pupils has started to take place in mathematical settings – for example, by Healy and Fernandes (2011) and by Sedaghatjou (2018). Out of the studies that examine gesture use of VI pupils in mathematical settings, it is only Healy and Fernandes' work that focuses on inclusive settings.

Goldin-Meadow (2003) and Iverson and Goldin-Meadow (1998) have found that VI pupils use deictic, iconic and beat gestures in non-mathematical settings. In mathematical settings, Healy and Fernandes (2011) have found that VI pupils use metaphoric gestures too. Therefore, the findings from the overall literature on gesture use of VI pupils suggest the use of all the four types of gestures by VI pupils.

Alongside gesture use, speech use is important too in the inclusion of VI pupils. As with gesture use, it is only Healy and Fernandes' work that focuses on speech use of VI pupils in inclusive mathematical settings.

Since my focus is on inclusive mathematical settings, I limit my review of speech and gesture use of VI pupils to Healy and Fernandes' work.

Healy and Fernandes (2014) differentiate themselves from McNeill's (1992) conceptualisation of gestures: they oppose to solely view gestures as part of what a person says, in other words to solely view gestures together with speech. They instead suggest viewing gestures also as separate entities, which are not necessarily subordinated to speech. They suggest that gestures themselves are indicators of cognition and often occur without speech. Therefore, while McNeill (1992) considers gestures as having a secondary role in communication and cognition, Healy and Fernandes explore the possibility for them having a primary role in communication and cognition.

Healy and Fernandes (2014) endorse Alibali's (2005) definition of gestures, according to which gestures are "movements of the hands and arms that are produced when engaged in effortful cognitive activity" (Alibali, 2005, p. 309, cited in Healy & Fernandes, 2014, p. 126-127). Based on this definition, Healy and Fernandes (2014) consider gestures as "the ways physical objects are explored by the hands" (p. 127).

Healy and Fernandes (2014) also endorse Rotman's (2009) division of gestures along three modes of embodiment: the semiotic mode; the instrumental mode; and, the immersive mode. The semiotic mode concerns the signifying/affective body: the body that "expresses, communicates, speaks, writes, gives signs, uses language and the apparatus of codes to construct, convey or mediate significance, meaning, affect" (Rotman, 2009, para. 3). The instrumental mode concerns the functioning body: the body that "becomes a mechanical apparatus" (Rotman, 2009, para. 3). Finally, the immersive mode concerns the experiencing/participating body: the body that "presents, performs, enjoys and experiences itself through enacted and participatory activities" (Rotman, 2009, para. 3).

Healy and Fernandes (2014) agree with Rotman's (2009) and Merleau-Ponty's (1945/1962, p. 215) view of speech-gesture relations – according to which speech is species of gesture, perceived by auditory rather than visual means (Healy & Fernandes, 2014). They also do not attribute any intellectual superiority to any of these two forms of expression.

Having shed light on the speech-gesture relations across the history, as well as on Healy and Fernandes' conceptualisation of gestures, I will now proceed with the particular contributions by Healy and Fernandes into the speech and gesture use of VI pupils in mathematics.

Healy and Fernandes (2014) found that VI learners use speech and gestures in two cases:

- to create mathematical meaning, thus gestures and speech are directed for the learners themselves, as part of their thinking-in-action

- to communicate their mathematical meaning to others.

Based on the gesture use of VI pupils, Healy, Ramos, Fernandes, & Peixoto (2016) suggest that, in order to include VI pupils in the mathematics classroom, we need to see what these pupils have to say with their hands as much as we see what they say with their mouths and not to consider speaking with their hands as less intellectual to speaking with their mouths.

In Healy and Fernandes' work with VI pupils, speech and gestures are used as simultaneous mediational resources throughout the learners' activities, and, as co-temporal simultaneous productions (Goldin-Meadow, 2003), their roles in thinking and communicating are difficult to separate (Healy et al., 2016).

In Healy and Fernandes' work, the gestures of some VI pupils were a clearer indicator of these pupils' mathematical understanding than their speech while in other VI pupils' mathematical discourses, gestures emerged even without any speech. Both cases contradict to a common, previously-described, position that gesture is an add-on to speech and they offer a reflection upon the origin and formation of language (Healy et al., 2016).

3.2b Speech and gestures of teaching staff

The focus of speech and gesture on sighted teaching staff takes place in three cases in the literature: in teaching staff's communication with VI pupils about the latter's mathematical meaning making; in teaching staff's engagement with mathematical activities whilst being deprived of one of their sensory organs; and, more generally, in teaching staff's teaching VI pupils in the mainstream mathematics classroom. I note that this third case includes some general pieces of advice for speech and gesture use of teaching staff in the mainstream mathematics classroom with VI pupils – and does not stem from observations of teachers during their mathematics teaching in the classroom.

The use of speech and gesture in teaching staff's communication with VI pupils about the latter's mathematical meaning making is illustrated differently amongst Healy and Fernandes (2014), Nardi et al. (2018) and Quek and Oliveira (2013). While Healy and Fernandes (2014) and Nardi et al. (2018) focus on the attunement of teaching staff in their attempt to feel what the VI pupils feel when they create and communicate their mathematical meaning making, Quek and Oliveira (2013) focus on the class teachers' speech and gesture use during their teaching of mathematics in a classroom that has a VI pupil. I first start with Healy and Fernandes' (2014) and Nardi et al.'s (2018) work and then continue with Quek and Oliveira's (2013) work.

In their work with VI pupils, Healy and Fernandes found similarities in the gesturing activity of the VI pupils and the sighted researchers[15] (Healy & Fernandes, 2014). When a researcher wanted to communicate a feeling to a VI pupil, she picked up the VI pupil's hand in hers and dynamically traced along the objects that she was referring to: she did not simply point to these objects. An even more striking finding of Healy and Fernandes is the similarity in the gesture use of the sighted researchers and the VI pupils when the former did not use gestures purposefully in their communication with the latter. Particularly, "[t]he researchers consistently ran their hands along the edges they were talking about (from vertex to vertex), just as the blind students did, even when they were not purposely attempting to show the edges to others" (Healy & Fernandes, 2014, p. 147). Healy and Fernandes attribute this similarity in gesture use to the researchers' attempts to 'feel' the feelings of the VI pupils, who experience the world in different, tactile ways: in line with Merleau-Ponty (1945/1962), the researchers attempted to feel almost as if they temporarily inhabited their VI pupils' bodies. In this case, the researchers also acted as learners, as they attempted to appropriate the VI pupils' ways of experiencing the world. Healy and Fernandes concluded that the researchers' similar gesture use to their VI pupils allowed them to interpret – through sensing – what their VI pupils were thinking. This conclusion suggests that, to create more inclusive mathematics classrooms, we need to use the VI pupils' speech and gestures as a way to rethink our own mathematical perspectives (Healy & Fernandes, 2011).

[15] The sighted researchers acted as teachers.

Nardi et al. (2018) phrase the above attitude of teachers' appropriation of VI pupils' mathematical meaning making through embodiment as "attuning". While Healy and Fernandes' (2014) attuning is manifested in the researchers[16] and during their direct communication with VI pupils, Nardi and her colleagues' attuning is manifested in teachers and does not take place in direct communication with VI pupils: it instead occurs during the teachers' engagement with VI pupils' mathematical contributions that were inserted into mathematics tasks and are presented as classroom episodes (in Type 1 tasks); and, during the teachers' engagement with mathematical activities whilst temporarily being deprived of one sensory organ (in Type 2 tasks). Before I present how attuning is manifested in each of the two types of tasks, I will explain what these types of tasks are.

Type I and II tasks are research-informed and situation specific tasks. These tasks invite teachers to reflect on mathematics teaching situations that are grounded on seminal learning and teaching issues and are likely to occur in actual practice (Nardi et al., 2018). They are tailored to the challenges of teaching mathematics to disabled learners. Type I tasks involve teachers reflecting on mathematical contributions of disabled learners as if they were the teachers of the particular learners. Type II tasks involve teachers reflecting on how access to different mediational means differently shapes mathematical activity.

In the former type of tasks, attuning occurs as a result of teachers' attempt to understand the VI pupils' mathematical meaning making (thus it occurs similarly to Healy and Fernandes' attuning). In the latter type of tasks, attuning occurs necessarily within the teachers – and without any presence of VI pupils – in their attempt to solve mathematical tasks whilst temporarily being deprived of one sensory organ. Therefore, this second case of attuning differs from Healy and Fernandes' one, as there is no presence of VI pupils. We can say that attuning comes more naturally in the second case: the teachers have to find an alternative route – to the one they are used to – in order to solve the mathematical tasks, and attuning comes naturally to them.

I would like to further elaborate this second case of attuning and to talk about a pattern which appears when teachers are asked to solve mathematical tasks whilst temporarily being deprived of one sensory organ. The lack of access to the visual field seems to make the teacher, who acts as a learner, shift to mental work. In other words, the lack of access to the visual field seems to make the learner substitute their eyes with another tool, which is the mental simulation of the working out of the mathematical task. For example, Carol, who is one of the participating teachers in the CAPTeaM project (Nardi et al., 2018) and who is asked to work out the multiplication 237x36 whilst temporarily being deprived of the visual field, resorts to mental working out of this multiplication. The alternative route of mental working out – to the commonly used one when having access to the visual field, which is the visual method of working out the calculation on paper – is highlighted by another teacher, Melanie, who particularly comments: "I think in all the groups, the person with their eyes shut tried to reproduce the algorithm in their head. So I am wondering, how does a person who is blind imagine this calculation?" (Nardi et al., 2018, p. 164).

Therefore, the lack of function of the visual tool, which necessitates the identification of an alternative tool and, hence, and, according to Vygotsky (1993), an alternative route to engage with the mathematical task, seems to make the learner resort to mental working out of the task. As Melanie highlights, this practice may be of particular significance to VI pupils and, therefore, and in line with Healy and Fernandes' (2011) association between the VI pupils' mathematical practices and the creation of more inclusive mathematics classrooms[17], to the inclusion of these pupils in the mathematics classroom.

The association between the lack of access to the visual field and the mental working out of mathematics is evident in the literature for the VI pupils too. In particular, Buhagiar and Tanti (2011) report that "Debbie [who is the VI pupil] had to do a big chunk of her work mentally because […] she was still learning Braille" (p. 70). In this example, the mental working out seems to have been developed for the VI pupil due to the circumstances in which the pupil was taught mathematics and which seemed inappropriate to her – the

[16] The researchers acted as teachers.
[17] Healy and Fernandes (2011) particularly argue that the development of inclusive classroom practices "can only be achieved if we better understand the mathematical learning processes of blind students" (p. 158).

circumstances were the non-mastery in Braille. Again, as in the previous example of teachers, the mental working out seems to have been the alternative route for the VI pupil to engage with the mathematical task and occurred due to external factors (the non-teaching of Braille at a mastery level for the VI pupil).

Both Healy and Fernandes and Nardi and her colleagues see attuning in the gestures of VI pupils and they both consider it as a necessary element for the creation of more inclusive mathematics classrooms.

Quek and Oliveira (2013) focus on the class teachers' speech and gesture use during their teaching of mathematics to a class that has a VI pupil. Their work is embedded in three main ideas: time synchrony for both production and uptake of deixis in relation to speech (1); teacher's simultaneous production of speech and gestures (2); and, student's reception of both speech and gesture during instruction (3). These researchers have developed a Haptic Deictic System which allows the VI pupil to tactilely access the teacher's point of reference through a Haptic Glove. The Haptic Glove contains vibration motors that allow the VI pupil to locate the deictic focus of the teacher.

This system was implemented in a small group consisting of a teacher, a VI pupil and a sighted pupil and had positive results towards the accessibility of the teacher's instruction to the VI pupil. It particularly allowed the VI pupil to participate in the classroom as much as his sighted peer and it also allowed the teacher to be flexible in the type of speech and gesture that he uses. The Haptic Deictic System allowed the same instructional discourse to be addressed to both the VI and the sighted pupil, without any disruptions. If the Haptic Deictic System – or any other system that would allow the VI pupil to access the teacher's gestures – was not there, the teacher would have to be very careful in his speech and gesture use in order to make the VI pupil included in the lesson.

Thanks to the aforementioned beneficial results from the trialling of the Haptic Deictic System in a small group, Quek and Oliveira (2013) envisage to implement it in an inclusive mathematics classroom. However, it is a costly system and may not be financially possible to be widely implemented in mainstream classrooms with VI pupils. Also, the fact of a VI pupil being the only pupil who uses a special system may bring negative impact upon this pupil in the classroom. For these two reasons, an alternative – less costly and less individualistic – way may instead be more beneficial towards the creation of mathematical discourse of teaching staff that allows its accessibility, engagement and participation of VI pupils.

The last case of speech and gesture use by teaching staff includes some general pieces of advice for teaching staff to follow when they teach mathematics in mainstream classrooms that have one or more VI pupils. Sticken and Kapperman (1998) report the necessity for avoidance of deictic words, such as "this", "that" and "there". They also state that verbal clarity of what is written on the board is essential, whether the object is a word, a diagram, et cetera.

3.3 Resources

I use "resources" as the umbrella term for tasks and tools. In particular, "resources" can refer to tasks, to tools and to both tasks and tools.

I endorse Healy, Fernandes, & Frant's (2013) use of – and differentiation between – tasks and tools. "Tasks are proposed to motivate learners to engage with practices associated with the set of artefacts that have, historically and culturally, come to represent the body of knowledge we call mathematics" (Healy et al., 2013, p. 63). Tools mediate the resolution of the tasks (Healy et al., 2013).

Inclusive mathematics resources are designed to rely on one of the following sensorial forms: on tactile perception; on auditory perception; and, on multisensory modalities (for example, touch, hearing and vision).

3.3a Tactile resources

In this section, I will first briefly talk about tactile perception and I will then proceed to tactile resources.

Perceiving spatial relations by touch absorbs concentration and time, and thus may interfere with the processing of the mathematical concepts (Leuders, 2016).

As touch requires more working memory than vision and increases the processing load, the pupils need to be well accustomed to their resource in order to handle it automatically and reduce the processing load. Otherwise, their processing capacity is too small to focus on the concrete resource and the formal mathematical concept at the same time (Leuders, 2016).

Unlike vision, touch does not provide at once a preliminary overview of the mathematical object. As Ochaita and Rosa (1995) put it, vision is wholistic and touch is gradual, allowing the exploration of an object from its individual parts to its whole.

In the view of VI people, touch seems to be a very effective sense, often more effective than vision. A VI mathematician, Morin, argues that haptic perception "allows one to comprehend information at once" (Jackson, 2002, p. 1248). Similarly, another VI person states that his hands can tell him more about the moon than a sighted person's eyes or telescopes – "my hands would tell me more of what goes on in the moon than your eyes or your telescopes" (Diderot, 1916/1749, p. 77).

These two VI people's views on the affordances of tactile perception are verified in an experiment with 16 geometrical solids that was carried out by Lakatos and Marks (1999). Lakatos and Marks (1999) found that "solids with a similar global shape but with different local features were judged to be *less* similar by touch than by sight, while touch may attribute greater importance to local features and is more attuned to them than sight" (Figueiras & Arcavi, 2014, p. 125). Figueiras and Arcavi (2014) particularly advocate that this finding is related to the fact that "hands are optimized to seek particular types of spatial information" (p. 125). In particular, haptic exploration promotes "connection and synthesis of mathematical properties, supporting the sense-making of others' descriptions of mathematical ideas" (Figueiras & Arcavi, 2014, p. 126).

The above benefits from tactile perception may not be limited to VI pupils. Figueiras and Arcavi (2014) particularly suggest that "haptic experiences could be very productive also for students with normal eyesight" (p. 132). This suggestion contradicts the long-held practices of tactile perception being a characteristic of solely the VI pupils as well as the long-held beliefs that tactile perception is a more primitive and less intellectual form of perception. It instead resonates with Healy et al.'s (2013) suggestion for multimodal resources and for the opportunities that these resources provide to both VI and sighted pupils in their access to mathematics through various modalities, one of which is touch. Benefits from tactile perception in sighted pupils are also elicited within Argyropoulos and Stamouli's (2006) work. Apart from the enhancement of the sighted peers' interactions with the VI pupil, there seems to have been another reason for the sighted peers' positive experiences in the mathematics classroom: the reason is the possible benefits which the sighted peers have speculated that they arise in case these pupils too were provided with the opportunity to engage with these resources. A sighted peer characteristically reports: "I think a lot of […] [these resources] could help me as well" (Argyropoulos & Stamouli, 2006, p. 132). This is related to Healy et al.'s (2013) argument on the mathematical benefits that multimodal resources – through their accessibility via different modalities – could bring to sighted pupils, allowing them to develop a range of ways to think mathematically (Healy et al., 2013). Based on sighted pupils' speculations and Healy et al.'s (2013) argument, a reasonable next step could be the design of tactile resources in a way that they are addressed to all pupils.

Resources that encourage tactile perception are the most frequently designed and are provided solely to VI pupils. They often act as adaptations of visual resources that are used by sighted pupils.

Tactile resources can be used as adaptations of visual resources for spatial perception, as both touch and vision enable such perception (Leuders, 2016). Tactile resources though need to be well adapted to touch: they should be clearly structured and emphasise the features representing the mathematical content (Leuders, 2016).

Leuders (2016) concludes that, while tactile resources are very important in mathematics education for VI pupils, “their design and use in the classroom need to be well deliberated” (p. 45).

Tactile resources have been successfully used in Argyropoulos and Stamouli’s (2006) work. More specifically, tactile 2D shapes were designed and provided to the VI pupil in the mathematics classroom. These were adaptations of the visual resources that the sighted peers used in the classroom.

Argyropoulos and Stamouli (2006) report positive experiences of the VI pupil, sighted peers and the class teacher, with the trial of the tactile resources in the mathematics classroom. In all these class members’ accounts, the benefits of the tactile resources are on the creation of a classroom atmosphere which allowed the interaction between the VI pupil and the rest of the class members – Argyropoulos and Stamouli (2006) note that this interaction did not occur before, as the VI pupil was working with the support teacher at a different level to that developed between the sighted peers and the class teacher. The following phrases are illustrative of this common experience of the class members: “I feel the interaction which was developed between Nefeli and the rest of the students facilitated the procedure of our lessons and released many of our tensions” (p. 132) (in the class teacher’s account); “I feel more active and I think that my relationship with the classroom teacher has improved” (p. 132) (in the VI pupil’s account); “It was amazing to see all the stuff that Nefeli was using with the researchers. […] I liked this way of interacting in class” (p. 132) (in sighted peers’ accounts). The benefit of tactile resources upon the interaction between the VI pupil and the rest of the class members was also evident in the headmistress’s account: “All students have showed an improvement in interacting with each other” (p. 132).

One would say, therefore, that in Argyropoulos and Stamouli’s (2006) study it is more the restructuring of the classroom norms – than the actual materials – that have brought beneficial experiences in the inclusion of the VI pupil. The creation of a classroom in a way that allowed interaction between the VI pupil and the rest of the class members was evaluated as a very significant benefit from the inclusion of the VI pupil.

The resources facilitated this type of classroom in that they allowed independent exploration by the VI pupil, thus they brought the VI pupil closer to her sighted peers, who too access mathematics independently. Therefore, it is concluded that, to create an inclusive mathematics classroom – that is, a classroom which allows the interaction between the VI pupil and the rest of the class and moves away from, as Argyropoulos and Stamouli (2006) put it, the two-level system of VI pupil-support teacher and sighted pupils-class teacher – the resources need to be designed in a way that allows independent access by the VI pupil.

Apart from the enhancement of the VI pupil’s relationship with her class teacher, there was another reason behind the VI pupil’s positive experience with the tactile resources. The reason was the tactility, which was also the element behind the design of these resources. Here I will talk about tactility in the experiences of the VI pupil.

The tactility which the VI pupil experienced from her engagement with the particular resources was a positive factor on the VI pupil’s mathematical learning experience. The VI pupil’s excitement with the resources – reported by both the VI pupil herself and the researchers – is related to the tactility that these resources allow. It is noted that the VI pupil was not often provided with tactile resources, however she was experienced in tactile perception – “[s]he was very skilful in using her fingers to investigate and recognize shapes” (Argyropoulos & Stamouli, 2006, p. 130). While her tactile perception seemed not to be encouraged in the mathematics classroom for the VI pupil prior to the researchers’ intervention, the VI pupil’s familiarity with this form of perception was evident and was probably developed in other settings. There is an implication here about the ‘acceptability’ of tactile perception in the educational institutions. This is probably related to my earlier discussion on the ‘acceptability’ of gestures as a form of mathematical construction and expression.

Three of the previously discussed points – tactility as a feature of tactile resources, tactility as a design element of resources and rare encouragement of tactile perception in the mathematics classroom – are also verified in Healy and Fernandes’ work. In particular, Healy and Fernandes designed tactile resources, such as representations of shapes in foldable cardboard, and trialled them with VI pupils. Their design of resources aimed to favour tactile exploration of mathematical objects (Healy & Fernandes, 2014), as the participating VI pupils reported that “it was rare for them to interact with representations of geometrical shapes” (Healy & Fernandes, 2014, p. 134). “The students had explained to us that it was rare for them to interact with representations of geometrical shapes, and an important aspect of designing the tasks was to produce tactile resources that would make this possible” (Healy & Fernandes, 2014, p. 134).

As in Argyropoulos and Stamouli’s (2006) study, in Healy and Fernandes’ work (2011; 2014), the resources generated positive impact upon the VI pupils’ mathematical learning experiences. Again, central to the positivity in their experiences was tactility which, as stated earlier, was rarely encouraged in the classroom.

Tactile exploration generated interesting mathematical contributions of VI pupils, which differ from those of sighted peers. An example of such contributions is Andre’s meaning making of a square-based pyramid: “*I would tell them that the base is square and that as they move up the sides they become smaller until they form a point, here, on top* (moves his finger simultaneously up the edges of a triangle face in the same manner as before)” (Healy & Fernandes, 2011, p. 168). Healy and Fernandes attribute such contributions to the bodily tools through which VI pupils access mathematical objects. These researchers take that “it is not only the material and semiotic tools […] that impact upon the practices […] Equally important are the bodily resources through which tool and task are experienced, with different sensory-motor systems potentially affording different modes of acting mathematically and, hence, different paths by which mathematical meanings might be appropriated” (Healy et al., 2013, p. 68).

3.3b Auditory resources

In this section, I will first briefly talk about auditory perception and I will then proceed to auditory resources.

Auditory perception is the most useful modality for the processing of temporal information (Leuders, 2016). It also has the advantage of memory compared to tactile perception: while memory for haptic-spatial structures has a low capacity in VI and sighted pupils, the memory of VI pupils for auditory, sequential information is above average (Leuders, 2016).

Leuders (2016) suggests the use of auditory resources in the teaching of numbers. Auditory resources could be beneficial for the VI pupils, as VI pupils can determine the number of beats in a rhythm very effectively and they also use this ability in counting and calculating spontaneously, without any formal instruction (Leuders, 2016).

It has been particularly found that VI pupils can memorise temporal auditory information better than sighted pupils and that many VI pupils use auditory representations for mental calculations: they were spontaneously counting rhythmically in ones or in groups, although auditory representations for number were not used in school. This may be caused by the fact that the VI pupils do not use finger counting (Leuders, 2016).

However, there are some concerns too about auditory resources in the teaching of number. Pupils with dyscalculia often do not develop beyond counting strategies for simple calculations. Counting strategies can be an impediment to the understanding of calculation because they keep pupils from thinking about the quantities and operations and have them focus on the sequence of number words instead. Auditory representations could have the unwanted effect of prompting counting strategies because of their temporal structure: countable sounds must be played one after the other. This problem is deteriorated by the fact that tactile resources can also prompt counting strategies because the counted objects are often touched one by one. A resource that enables VI pupils to perceive number simultaneously would be very valuable (Leuders, 2016).

Leuders (2016) though suggests that auditory perception does not necessarily prompt counting strategies in calculation. This is because the cognitive result from auditory perception is relatively similar to simultaneous perception offered by vision: quasi-simultaneous wholes (for example, rhythms) are created through the cooperation of auditory cortex and short-term memory. Therefore, auditory representations of number are a useful addition for teaching number to VI pupils. They may serve the same purpose as dot patterns for sighted pupils: creating mental representations of number that include important structures such as part-whole relations. Leuders (2016) suggests that tactile resources, such as raised-line drawings and manipulatives, should be supplemented with acoustic resources for VI pupils.

Resources that encourage auditory perception have only recently started to become suggested for creation and trialling. Unlike the tactile resources, the auditory resources are used by both the VI and the sighted pupils in mathematics.

Auditory resources include pre-produced recordings (for example, music or natural sounds, but not speech – sonification is included here too) and sounds produced by pupils (for example, by clapping or singing) (Leuders, 2016).

Leuders (2016) develops an evaluation procedure in her exploration of the best way to adapt a task of linear patterns in a first-grade German textbook for a VI pupil in an inclusive mathematics classroom. The evaluation procedure consists of 4 steps: task analysis (Step 1); adaptation outline (Step 2); usability for inclusive settings (Step 3); and, criteria from mathematics education (Step 4).

With regard to the textbook example, in Step 2, Leuders (2016) suggests the acoustic representation of the linear patterns – by using different sounds. Her suggestion is based on the temporal nature of the linear patterns, which optimises auditory perception compared to tactile perception. Indeed, in case of a tactile adaptation, such as counters with different textures instead of different colours for VI pupils, tactile perception might lead VI pupils to processing overload: VI pupils would need to count the textured dots one by one, then to remember the number of each group (‘red’ and ‘blue’), while planning and enacting tactile strategies at the same time.

On the other hand, linear patterns are easier to be perceived by VI pupils when presented auditorily (Leuders, 2016). This is because patterns with a repetitive structure create a rhythm when presented acoustically. Thus, they are easier to notice and to reproduce than patterns with growing numbers, which prompt a stronger focus on counting activities.

Auditory resources are more beneficial than tactile resources in Step 3 too. When using the dots and counters with different textures to represent the colours, the VI pupils would need to learn the texture-colour combinations (for example, blue=smooth) in order to communicate with other pupils, and they would have no access at all to the sighted pupils’ printed visual patterns. Also, understanding and continuing a pattern by touch takes much longer and requires more concentration than by sight, so the VI pupils would likely work slower than their sighted peers. On the other hand, in body music, all pupils can work together effortlessly, and the VI pupils may even be more successful than the sighted peers in recognising the patterns because they are more familiar with the analysis of auditory input (Leuders, 2016).

In Step 4, the question is “Does the [acoustic] material adhere to criteria for good teaching material in general maths education? Or would the sighted children still be better off with their visual material?” (Leuders, 2016, p. 49).

Leuders (2016) reports three criteria for good teaching resources:

- One aspect for good teaching resources is the usability in varied classroom situations and the continuability throughout at least one school year. If clapping is used to represent number, it also should be able to represent the important structure of fives and tens in the decimal system. For fives, this can be achieved by rhythmic grouping. Groups of five are best represented using six beats per bar, with a rest on the last beat of each bar. First-graders can use this pattern to clap numbers from 5 to 20. For longer units like tens or hundreds, they can use a rattling sound, which can be understood as representative of many knocks or claps.

- Good teaching resources should support the documentation of the learner's results and store them for later review. Acoustic patterns can be written down, represented by dots of different sizes (for volume), with gaps of different sizes in between (for duration or rests). The VI pupils can use Braille dots, they can do raised-line drawings or use felt stickers.
- An important aim in the use of teaching resources is helping pupils to focus on the abstract mathematical structure instead of the concrete perception. The translation between different sensory channels can be very effective for this purpose. Modal transfer can foster pupils' understanding of the abstract meaning of number beyond dot patterns.

Based on the above evaluation procedure, Leuders (2016) concludes that, in the example of the linear patterns, acoustic teaching resources are useful. In other situations, for example in geometry, tactile resources may be favoured (Leuders, 2016).

Vines et al. (2019) use auditory resources in practice. They particularly use sonification, which is non-verbal, audio medium to convey information. In their study, sonifications represent plots and are implemented to VI and sighted students via virtual learning environments. Sonifications enable users to listen to data and to auditorily process the information within the graph. They correspond to the different dimensions of the data displayed in a graph with different characteristics of sound, which include the pitch, the loudness and the timbre of the note (Vines et al., 2019). However, Vines et al. (2019) note that correspondence of a dimension of the data to more than one aspect of sound is also possible.

Vines et al. (2019) produced sonifications of graphs in advance and gave them to sighted and VI students in MP3 files. Customisation of sonifications, such as change of the duration, was not made possible to students. Considering the above characteristics of sonifications, Vines et al. (2019) explore the impact of sonifications of graphs upon VI and sighted students. Potential benefits to sighted students are also explored.

Vines et al. (2019) endorse Hermann and Ritter's (1999, p. 5) argument that sonifications allow data "to be experienced in a new way [– auditorily –], which bears the advantage of a deeper and possibly richer understanding of data structures". This endorsement is similar to Healy et al.'s (2013) perspective about benefits of multimodal resources to both VI and sighted students – coming from the opportunities that these resources provide for mathematical access via various sensory tools.

As suggested in Leuders (2016), in Vines et al. (2019) acoustic resources are the same between sighted and VI students. The 'sameness' of the resources overcomes the limitations – time and funding (Leuders, 2016) as well as adequacy and availability (Whitburn, 2014) – that may arise from the adaptation of resources solely to VI pupils, in other words from resources which are different between sighted and VI pupils.

Vines et al. (2019) supplemented auditory resources with tactile and visual resources. Participants listened to the sonification and were then asked to 'sketch' an interpretation of the graph which the sonification represented. The sketching was done tactilely by VI students – using Wikki-Stix or a DRAFTSMAN Tactile Drawing Board – and visually by sighted students – using paper and pen/pencil (Figure 1 in Vines et al., 2019, p. 26). The supplementation of auditory resources with tactile and visual resources is in line with the Step 4 of Leuders' (2016) evaluation procedure: particularly, with the necessity for the documentation of the auditory input.

At the end, Vines et al. (2019) invited the participants to engage with the plot using the visual version (for the sighted students) and the tactile version (for the VI students).

The first phase of their study concerned the exploration of the effectiveness of sonifications in similar activities to those that might be found in a distance learning setting and involved a small group of participants. The second phase of their study concerned the use of sonifications in an actual distance learning setting – an Open University module – and involved all students enrolled on this module.

Findings from their Phase 1 suggest the appropriateness of sonifications for augmenting visual teaching resources. The sonifications give the general sense of the shapes which form the plot in a timely fashion. They particularly enable the participants to get an overview of the relationships depicted in the graph and also to know where to look when the participants interrogate the plot in the tactile diagram.

The tactile diagram enables the participants to interrogate the plot and also to flexibly explore the plot in the way that they wish. On the other hand, the sonification makes the participants engage with the information presented in a fixed order. These findings on the different benefits between the sonification and the tactile diagram are consistent with Leuders' (2016) findings on auditory and tactile resources, respectively.

Findings from their Phase 2 indicate the possibility – and worthiness – to offer sonifications to all students in an actual distance-learning setting.

While "only a minority of students appeared to get a benefit from the audio graphs, […] the audio graphs exposed the students to important concepts in accessibility as they saw graphs of data points represented in an alternative format: audio" (Vines et al., 2019, p. 35).

Moreover, further technical development of the production of the audio graphs is found to be desirable. This consists of: removal of unnecessary crackle; and, consideration of a user-friendly software, which would allow the students to customise the sonifications to match their preferences.

An interesting suggestion is the provision of guidance about how to interpret sonifications – in particular, guidance on what is and what is not reasonable to pick up from the specific sonification.

Findings from both phases show that sonifications enabled most of the participants to get the gist of the plot.

A striking finding is a relation between music interest and sonification interest: particularly, "participants who enjoyed listening to music seemed to more easily work with the sonifications" (Vines et al., 2019, p. 35).

Another striking finding is a relation between experience with sonification and success in mathematical interpretation: particularly, "[g]reater experience with sonifications should only increase participants' ability to interpret plots and graphs given in this format" (Vines et al., 2019, p. 35).

3.3c Multimodal resources

Multimodal resources encourage multisensorial forms of access and have only recently started to become suggested for creation and trialling. As the auditory resources, the multimodal resources are suggested to be used by both the VI and the sighted pupils in the mathematics classroom. The difference between the multimodal and the auditory resources is that the former resources do not limit the class's access – to those – on the auditory perception but provide choices for the class to access them with the sensory tool that is preferable to them. Multimodal resources "are designed to facilitate multiple ways of interacting with […] [mathematical] objects and relations and to respect the diverse experiences of the students" (Healy et al., 2013, p. 62). This benefit is achieved thanks to the various sensory tools which multimodal resources allow and which, in line with Vygotsky (1993), bring a restructuring of the individual's cognitive activity.

In Healy and Fernandes' work, multimodal resources represent mathematical ideas "through colour, sound, music, movement and texture, and hence appeal to different sensory canals, and particularly to the skin, the ears and the eyes" (Healy et al., 2013, p. 62). Multimodal resources aim to be accessible to all pupils: to VI pupils, through touch and hearing; to deaf pupils, through touch and vision; and, to pupils who can both see and hear, through vision, touch and hearing (Healy et al., 2013). Healy et al. (2013) argue that multimodal resources are beneficial for pupils without impairments in that they allow them "to have a variety of ways to think mathematically" (p. 62).

Multimodal resources can be either physical or digital. Healy et al. (2013) present an example of a physical multimodal resource and an example of a digital multimodal resource.

The physical multimodal resource – MATRIZMAT – enabled visual stimuli to deaf pupils and tactile stimuli to VI pupils. For the deaf pupils, the numbers were written on foam-rubber rectangles and placed in the cells of the matrices, which were represented through plastic boxes of 5cm by 5cm by 3cm joined to each other by magnets fixed to each of the boxes' four sides (Figure 1 in Healy et al., 2013, p. 65). For the VI pupils, the numbers were written in Braille on the lids of the boxes and were stuck onto the top of the relevant boxes (Figure 2 in Healy et al., 2013, p. 65).

The above physical resource enabled VI pupils "to develop efficient ways of expressing matrix structure" (Healy et al., 2013, p. 65). It was particularly beneficial for the VI pupils in that it emphasised the spatial layout of the elements in question. On the other hand, Healy et al. (2013) report that the representation of matrices in Braille, which the VI pupils have been used to, caused them difficulties in their locating the elements in different matrices that should be added to each other.

The digital multimodal resource – "a digitally controlled board made up of a rectangular matrix of pins, each of which represented a point on the plane" (Healy et al., 2013, p. 66) – was beneficial for the VI pupils in that it enabled the pupils to feel the graph as it was created (Healy et al., 2013). The digitality of the tool seems to have been particularly beneficial: the digitality enabled the VI pupils to experience a dynamic view of the function and to understand the dependence relationship between the independent and dependent variables. If the graph was presented in a physical format, then the physicality of the tool would allow the VI pupils to experience only "static representations of the locations of particular points on the Cartesian plane or static representations of the graphs of specific functions" (Healy et al., 2013, p. 66).

The tactility of the tool seems to have been another particularly beneficial feature: the tactility enabled the VI pupils to access the graph with their hands, which constitute an essential sensory tool for these pupils. Therefore, digitality and tactility constituted the two important features of the tool that allowed a positive experience of the VI pupils.

Following the beneficial impact of this digital tool upon the VI students' mathematical learning experiences, Healy et al. (2013) suggest that the tool could also be used by sighted students. However, these researchers confess that this digital tool constitutes an expensive option.

From the above two examples of tools, it seems that physical tools are more 'financially realistic' to be used in the mainstream mathematics classroom. Therefore, a next step could be the design of physical tools and their trialling by both the sighted and the VI pupils in the classroom. This step has started to be thought of by mathematics teachers too – for example, Taiana, who is a teacher participating in the CAPTeaM project, suggested the use of the VI pupil's resources by the entire class (Nardi et al., 2018). Further details on CAPTeaM are going to follow in the third – and last – theme of my literature review chapter: VI's mathematical contributions and teachers' reactions to those contributions. Before I move on to this last theme though, I will briefly discuss another issue on resources: assistive technology in inclusive mathematics classrooms.

3.3d Assistive technology in inclusive mathematics classrooms

Assistive technology constitutes a significant element of digital tools in the mathematics classroom with VI pupils. It particularly provides accessible instruction to VI pupils in a way that will prompt their independence (Freeland, Emerson, Curtis, & Fogarty, 2010; Kapperman, Sticken, & Heinze, 2002; Kelly, 2009; Kelly & Smith, 2011; Zhou, Parker, Smith, & Griffin-Shirley, 2011) and their motivation (Campana & Ouimet, 2015; Shah, 2011).

However, despite the aforementioned benefits of assistive technology to VI pupils, in the literature concerning the inclusion of VI pupils in the mathematics classroom, the use of assistive technology has its limitations. Some of the limitations are attributed to the assistive technology per se and others are attributed to the use made by teachers.

With regard to the former limitations, VI pupils report that the Job Access With Speech (JAWS) screen reading software which they used to read online documents interpreted some mathematical notations as images and leaves those parts as a space while reading (Bayram et al., 2015). VI pupils also report that JAWS is not designed for mathematics, as it only reads text format and does not recognise mathematical notation. For example, it reads 3·5 as three dot five and not as multiplication (Bayram et al., 2015).

With regard to the latter limitations, literature shows that teachers "do not know how to access or use assistive technology in their lessons" (Bayram et al., 2015, p. 213) and that teachers' lack of skills in assistive technology impacts too upon the inclusion of VI pupils in the mathematics classroom. In particular, Freeland et al. (2010) report that teachers who are skilful in using assistive technology are effective too. Similarly, Freire, Linhalis, Bianchini, Fortes, & Pimentel (2010), as well as Zhou et al. (2011), also indicate that such teachers manage to facilitate a motivating classroom environment.

3.4 VI pupils' mathematical contributions and sighted people's responses to these contributions

Literature on VI pupils' mathematical contributions and sighted people's responses to these contributions focuses on teachers, in their engagement outside the classroom with mathematical contributions of VI pupils. My review of this theme focuses on: characteristics of VI pupils' mathematical contributions as evident in studies on inclusive mathematics education (3.4a); and, teachers' responses to mathematical contributions of VI pupils (3.4b).

3.4a VI pupils' mathematical contributions

Healy and Fernandes' work (2011; 2014), which was earlier discussed in the themes "speech and gestures" and "resources", is central too in the theme of VI pupils' mathematical contributions. As stated earlier, Healy and Fernandes work with VI pupils and design mathematics resources aiming to favour tactile exploration of mathematical objects.

Healy and Fernandes have found that gestures play a central role in VI pupils' mathematical contributions. There are some central characteristics on the mathematical contributions of their VI pupils which need to be taken into account in the inclusion of this group of pupils:

- VI pupils' mathematical contributions are often different to commonly used mathematical contributions in the specific mathematical concepts. The central factor that causes frequent differences between sighted and VI pupils' mathematical contributions is the sensory tool through which sighted and VI pupils access the mathematical object. According to Vygotsky (1993), a sensory tool – just as any material and semiotic tool – is expected to cause a restructuring of the individual's cognitive activity. So one would need to be open to mathematical contributions that differ from those expected, to evaluate their substance and include them in the classroom, contributing to mathematical diversity and offering the class the opportunity to experience mathematics from various points of view, which may lead the class to better understand the mathematical concept.
- Gestures often prevail compared to speech (with speech being insufficient for the expression of mathematical meaning). So one would need to consider the gestural part to make sense of the VI pupils' mathematical contributions. This would require teachers to shift their perspectives towards gestures being an equally significant element to speech in one's mathematical discourse. In other words, this would require teachers to consider embodiment as central to mathematical cognition.
- Gestures are often repeated, being used at different times as part of VI pupils' expression of their meaning making of the same mathematical concept. So one would need to consider the abstraction element that gestures may play in VI pupils' mathematical contributions. Healy and Fernandes (2011) define "embodied abstractions" as occurring when the VI pupil shows some conscious appreciation of the generalised relationships within the mathematical concept in question. In this case, gestures are

“outward signs of *imagined re-enactments* of previous doings with the things and the other people involved in the activity in question” (Healy, 2015, p. 305).
- Mathematical terms are often not used by VI pupils. However, the VI pupils’ understanding of these terms is evident – and is expressed through gestures. So one would need to focus on the substance of the VI pupils’ mathematical contributions.

The above are characteristics of VI pupils’ mathematical contributions – with the mathematical contributions presented by Healy and Fernandes – which need to be taken into account in the creation of more inclusive mathematics classrooms. How are VI pupils’ mathematical contributions though evaluated by teachers? The CAPTeaM project provides some answers to this question.

3.4b Teachers’ responses to VI pupils’ mathematical contributions

In their work with mathematics teachers on the latter’s reflections upon specific mathematical contributions of VI pupils – the mathematical contributions are extracted from Healy and Fernandes’ project “Rumo à Educação Matemática Inclusiva” – Nardi et al. (2018) have identified two camps to which teachers belong: one camp focuses on a more conventional description, which involves the use of well-defined and familiar mathematical terms; and, the other camp focuses on developing the VI pupil’s mathematical contribution in its own right. In both camps, teachers may value the VI pupil’s mathematical contribution – and this shows appreciation of the contribution. The difference between the two camps is on the “attuning” element of the CAPTeaM’s analysis: the difference is on the extent to which teachers are willing – and ready – to incorporate the VI pupil’s mathematical contribution into their lesson.

Central characteristics of the first camp are the following:
- The use of the “proper mathematical definition” (Nardi et al., 2018, p. 155), which ensures the “certain rigour” (p. 155) in the mathematical contribution. The mathematical terms which the teacher expects to hear constitute important components that generate the “proper mathematical definition”.

Therefore, in this view of teachers, a mathematical contribution that does not include the mathematical terms which the teacher – and probably the school and the National Curriculum – expects to hear is not considered as a “proper mathematical contribution” and it lacks a “certain rigour”. A particular teacher from this camp, Marco, suggests using the VI pupil’s mathematical contribution “as an opportunity to define some of the terms” (Nardi et al., 2018, p. 155).
- The use of the definition included in the school textbook.

In (Nardi et al., 2018), the definition of a 3D solid in the school textbook consists of the number of faces, edges and vertices.
A view of a particular teacher from this camp, Cesar, who endorses this characteristic – the use of the definition included in the school textbook –, is slightly different from the above teacher’s view. Cesar does not seem to consider the VI pupil’s mathematical contribution as improper or as lacking rigour. Therefore, while this teacher seems to consider the VI pupil’s mathematical contribution as equally mathematically significant to the one that is taught in the classroom, he suggests that he would bring the VI pupil closer to the mathematics that he teaches. This teacher’s view seems to be based more on the educational institution – in which this teacher teaches and which promotes certain ways to teach mathematics – than on the teacher’s personal judgement. On the other hand, the first teacher’s view seems to contain personal elements (probably as additional to institutional elements).
The second teacher’s view indicates the great impact that educational institutions play upon the teacher – and his/her inclusion practices. To create inclusive mathematics classrooms, we need to reform the educational institutions which dictate specific models of inclusion.

A central characteristic of the second camp is the following:
- The attempt to feel the mathematical object in the way that the VI pupil had. Unlike the first camp of teachers, who confined themselves to judging the VI pupil’s mathematical contribution, the second camp did another step too: that of trying to enter the VI pupil’s position and experience the mathematical object in the VI pupil’s way. The fact of engaging with the VI pupil’s mathematical contribution not from a distance – as a contribution presented to them – but from within themselves

– by appropriating the contribution – seems to have made this camp of teachers realise the significance of the mathematical contribution and to suggest taking it in its own right in their mathematics classroom.

Having reviewed literature that corresponds to the themes which I explore in Chapters 5, 6 and 7, I will now present how my study is embedded in each of these three themes.

3.5 The research questions explored in this study in the light of the literature presented in 3.1-3.4

Healy and Fernandes (2011) argue that the development of inclusive classroom practices "can only be achieved if we better understand the mathematical learning processes of blind students" (p. 158). From this statement as well as throughout their work, it is clear that a prerequisite for the development of inclusive classroom practices is the acknowledgement of blind students' particular needs.

While Healy and Fernandes investigate the needs of VI students outside the mainstream classroom – and in a group consisting of only VI students – I investigate the needs of this group of students in the mainstream classroom – and in classes consisting of VI students, sighted students, class teachers and teaching assistants. In my exploration of the needs of VI pupils – and therefore in the development of inclusive classroom practices –, I focus particularly on the role of this classroom context, namely when the VI pupils are educated in the classroom together with sighted pupils.

This focus also emanates from Argyropoulos and Stamouli's (2006) work, which indicates the role of classroom context upon the inclusion of VI pupils. While their work is limited to one action research intervention in one mathematics classroom, I explore the role of classroom context in more than one classroom.

In this respect, my exploration of speech and gesture use, physical and digital resources and mathematical contributions is not limited to VI pupils but is extended to everyone in class.

In the rest of this chapter, I talk about how my study is embedded in each of the three previously described themes: speech and gestures (3.5a); resources (3.5b); and, VI pupils' mathematical contributions and sighted people's responses to these contributions (3.5c).

3.5a How my study is embedded in speech and gestures

I argue that it is vital to shed light on the speech and gesture use of the sighted members of the mathematics classroom, as this impacts and shapes the mathematical meaning making of VI pupils. As Healy and Fernandes too argue, the processes of concretion and abstraction are social, related to the interactions in the classroom (Healy & Fernandes, 2011). Therefore, the mathematical meaning making of VI pupils is shaped too by the interactions in the classroom. I argue that focusing on the speech and gesture use of the sighted members will give a more complete picture with regard to the mathematical meaning making of VI pupils and, alongside the already existing work on speech and gesture use by VI pupils done by Healy and Fernandes, will provide a more complete picture on the development of inclusive classroom practices, which constitutes my – and Healy and Fernandes' – study's aims.

More specifically, examining the current use of speech and gesture of the sighted members in the mathematics classroom can provide insights into successful strategies of speech and gesture use as well as strategies that do not lead to the inclusion of VI pupils – thus strategies to be avoided in the mathematics classroom. It is also important to explore whether there are any strategies that sometimes lead to the inclusion of VI pupils and sometimes do not lead to it and the factors behind this varying impact.

I explore the role that speech and gestures separately play in the discourses of the sighted members of the mathematics classroom and I then suggest effective ways of speech and gesture use by the sighted members towards the VI pupil's inclusion.

Apart from providing a more complete picture on the development of inclusive classroom practices, the exploration of speech and gesture use on sighted members' mathematical discourses aims to also contribute to the exploration of the relationships between sensory experience and mathematical knowledge, which is the third aim of Healy and Fernandes' (2011) work and which – in Healy and Fernandes' work – is tailored to VI pupils.

In order to contribute to this aim, I consider speech and gesture as two separate, distinct elements of expression. In particular, I observe the role of speech in relation to gesture and the other way around: that is, if both speech and gesture carry the same meaning or if one of them is subordinant to the other. In this respect, I take Healy and Fernandes' view that gestures are not always used together with speech, thus serving a subordinated role to speech or representing what is told. In this way, as Healy and Fernandes do, this study is distanced from McNeill's (1992) view that gestures should be treated as supplements to speech.

The separate investigation between speech and gesture can further strengthen the embodied non-dualistic approach of body and brain: that gestures may be the only meaningful contributors to conveying meaning – and that speech may be non-existent or, if existent, insufficient in conveying meaning.

What if teaching staff, who are considered in the classroom as the More Knowledgeable Other, use solely visual forms of communication or use visual and verbal forms of communication with the verbal ones being insufficient for the learners' understanding?

Does this suggest that the More Knowledgeable Other uses primitive and non-intellectual forms of communication?

In the second phase of my study, I design mathematics lessons with the class teachers. The design is based on findings from the first phase of the study as to how inclusive and enabling the speech and gesture use by the sighted members is for the VI pupils and also on findings from the literature as to successful use of speech and gesture in the inclusive mathematics classroom.

I will close my study's location in the speech and gesture theme with my conceptualisation of gestures. Unlike Healy and Fernandes, who focus their consideration of gestures on hand and arm movements, I expand this consideration of gestures in my study. Particularly, I include as gestures any bodily expressions that carry some meaning and entail some form of communication. In my study, gestures include: hand and arm movements (as in Healy and Fernandes' work); facial expressions; head movements; and, shoulder movements. I argue that the expansion of definition of gestures is important for the creation of more inclusive mathematics classrooms, as all the above cases of bodily expression have an impact upon the inclusion and enabling of VI pupils.

My conceptualisation of gestures resonates with Rotman's (2009) definition of gesture: "any body-movement that can be identified, repeated, and assigned significance or affect as a sign, a function, or an experience" (para. 3). Rotman divides gestures along three modes of embodiment: the semiotic mode; the instrumental mode; and, the immersive mode.

Finally, as Healy and Fernandes do, I use the classification of the four gesture types proposed by McNeill (1992): iconic gestures; metaphoric gestures; deictic gestures; and, beat gestures.

I chose to use McNeill's (1992) and Rotman's (2009) systems of classification of gestures. I found these two systems complementary in my work on gesture analysis. More specifically, McNeill's (1992) system offers very specific characterisations to gestures while Rotman's (2009) system offers broader characterisations of gestures. Depending on the purpose that each gesture played in my study, I chose one of the two systems to characterise this gesture. Both McNeill's (1992) and Rotman's (2009) systems have been successfully used in Healy and Fernandes' (2014) work to analyse the gestures of VI pupils in the mathematics classroom. It seems that Healy and Fernandes (2014) first embedded themselves in Rotman's (2009) system, identifying the broader category of gestures that they investigate, and then within this category they used McNeill's (1992) system to discern the various gestures. In other words, Healy and Fernandes (2014) seem to have used Rotman's (2009) system to ground the gestures under a specific general prism and, within this general prism, they seem to have used McNeill's (1992) system to zoom in and discern the specific gestures. More specifically, Healy and Fernandes (2014) used a single characterisation of gestures from Rotman's (2009) study across all the gestures of VI pupils – they classified all gestures as of "semiotic" mode in Rotman's (2009, para. 3) words – and then made variations in their characterisations of gestures using McNeill's (1992) classification of gestures – they sometimes discerned deictic gestures, other times iconic gestures, other times metaphoric gestures and other times beat gestures.

3.5b How my study is embedded in resources

Resources also impact upon the inclusion and enabling of VI pupils in the mathematics classroom. More specifically, Argyropoulos and Stamouli (2006) found that "the difficulties which Nefeli [the VI pupil] was facing were based more on the lack of material rather than on her blindness" (p. 131).

In my investigation of inclusion and disability in the discourses of teaching staff and pupils in the mathematics classroom, I aim to also contribute to the resources that are used in the classroom.

Unlike Leuders (2016), who investigates the issue of inclusive mathematics resources in a theoretical way, I opt for the alternative way to investigate this issue: that is, the "empirical research on the effect the specific materials have on learning" (Leuders, 2016, p. 43).

In particular, I investigate the kinds of resources that are used in the mathematics classroom – be they tactile, auditory, multimodal and/or mediated through assistive technology – and the impact that these resources have upon the inclusion and enabling of VI pupils.

In the second phase of my study, I design resources with the class teachers for the experimental mathematics lessons. The design is based on findings from the first phase of the study – as to how inclusive and enabling the resources are for the VI pupils – and also on findings from the literature – as to the characteristics in the design of particular kinds of resources.

The previously-presented literature on resources in the section 3.3 focuses on the use of either physical or digital tools in inclusive mathematics settings. However, none of these studies focuses on the intertwinement of physical and digital tools, which is highly likely to occur in today's mathematics classroom. The intertwined contributions of physical and digital tools for the teaching and learning of mathematics is a 2019's Special Issue that the journal "Digital Experiences in Mathematics Education" deals with, in the attempt to explore the contributions of digital and physical tools as they appear – and are used – together in the mathematics classroom. It is to this issue that my study primarily aims to contribute, with regard to resources.

3.5c How my study is embedded in VI pupils' mathematical contributions and sighted people's responses to these contributions

Sighted members' responses to VI pupils' mathematical contributions also impact upon the inclusion of VI pupils in the mathematics classroom.

I aim to contribute to this theme, by investigating the sighted members' responses to VI pupils' mathematical contributions in the mathematics classroom. I examine the responses of not only the teachers (unlike CAPTeaM) but also of the rest of the class members too (namely, the sighted peers and the teaching assistant) to VI pupils' mathematical contributions. I also explore whether there are any mathematical benefits to the entire class from the mathematical contributions of VI pupils.

Moreover, I aim to contribute to characteristics of VI pupils' mathematical contributions which, in line with Healy and Fernandes (2014), is a necessary element for the creation of more inclusive mathematics classrooms.

In the second phase of my study, I design mathematics lessons with the class teachers. These lessons aim to show whether there are any shifts – or, in Nardi et al.'s (2018) words, any resignifications – in the sighted members' responses to the mathematical contributions of VI pupils, in light of the particular resources and of other design principles that we follow on the lesson design. They also aim to show whether there are any mathematical benefits to the entire class from the mathematical contributions of VI pupils as well as from the design principles of the lesson.

So, in the light of a-c, this study explores the following research questions:

- RQ1: How are inclusion and disability constructed in the discourses of teaching staff and pupils in the mathematics classroom?
- RQ2: How do collaboratively designed mathematics lessons impact upon teaching staff's and pupils' discourses on inclusion and disability?

Chapter 4 presents the study's methodology.

Chapter 4: Methodology

4.0 Introduction

I start this chapter by describing the research design of the study, with particular emphasis on the study's two phases (4.1). I continue with the context and the participants of the study (4.2). An account of the data collection then follows (4.3). This consists of two main sections: an account of the participating schools' discourses on inclusion and disability as they are evident in the schools' policy documents (4.3a); and, presentation of the research methods of the study, in each of its phases (4.3b). I then present the ethical considerations of the study (4.4) and close this chapter with an account of the data analysis methods (4.5).

4.1 The research design of the study

This doctoral study investigates inclusion and disability in the discourses[18] of teaching staff and pupils in British mainstream primary mathematics classrooms with VI pupils, first in an exploratory phase (Phase 1) and then in an experimental phase (Phase 2).

In Phase 1, I investigate how class teachers, teaching assistants and sighted pupils consider inclusion and disability in the context of the mathematics classroom and with regard to the VI pupils. I examine whether there are any variations amongst different participants of the same classroom in their consideration of inclusion and disability. I also identify how consistent the participants' discourses are with the discourses on inclusion and disability in: the participating schools' policies; the SEND code of practice (Department for Education & Department of Health, 2015), which is the UK's educational code of practice for children and young people with Special Educational Needs and/or Disabilities; and, the international policies on inclusion and disability (United Nations, 2006; UNESCO, 2008; and, UNICEF, 2011). In Phase 1, I used classroom observations, focussed-group interviews and individual interviews.

In Phase 2, I examine evidence from Phase 1 on inclusion and disability and, based on this evidence – and with the aim of bringing the practice closer to endorsed principles in international legislation on inclusion and disability –, I design mathematics lessons with the participating teachers and then they trial them in the classroom. These lessons aim to be experienced as enabling and inclusive by the VI pupils and as beneficial by every pupil, with the study exploring, and aiming to provide specific evidence on, the social and educational benefits to all pupils – emanating from the implementation of inclusive education in the classroom. They also serve as the trigger for me to examine potential discursive shifts of participants regarding inclusion and disability. In Phase 2, I used written transcripts of the class teachers' contributions in the design of the three experimental lessons, classroom observations, focussed-group interviews, individual interviews, photographs of the pupils' work in the three experimental lessons and pupils' evaluation forms of the experimental lesson in two classes.

The study addresses the following two research questions: How are inclusion and disability constructed in the discourses of teaching staff and pupils in the mathematics classroom? How do collaboratively designed mathematics lessons impact upon teaching staff's and pupils' discourses on inclusion and disability? The first research question is explored in both phases of the study while the second research question is explored in Phase 2.

Inclusion and disability are investigated in the following three themes, which have been found to be central on VI pupils' inclusion and enabling: speech and gestures; digital and physical resources; and, mathematical contributions. Each of these themes is discussed in a separate chapter (Chapters 5, 6 and 7, respectively) and

[18] As explained in Chapter 1, by "discourses", I denote utterances – expressed through speech but also through gestures, facial expressions and bodily expressions in general – which relate to inclusion and/or disability and which are expressed by the participants either during the lesson or outside the lesson. The discourses may signify the participants' attitude on – and/or experiences of – inclusion and disability.

references to both phases of the study are included under the same theme. This is done to ensure that the second research question can be clearly answered with regard to each of the three themes.

4.2 Context and participants of the study

Data collection was conducted in four primary mathematics classrooms (Y1, Y3 and two Y5 classes; pupils' ages varied from six to ten) in four mainstream schools in Norfolk. The VI pupils' presence and the willingness of teaching staff and pupils to participate in the study constituted the main criteria for the selection of participants. A significant factor for the selection of the classrooms was the non-existence of more impairments in each VI pupil – it is common for VI pupils to have other impairments apart from their visual impairment (View, 2020), but I sought participants with visual impairment only. There were two reasons behind this selection: a personal/circumstantial reason and an academic reason. The personal/circumstantial reason is my lack of experience with impairments other than visual impairment. As reported in Chapter 1, I only have experience with visual impairment – through my learning of Braille. Therefore, it would be difficult for me to conduct research on pupils with impairments that are unfamiliar to me. The academic reason is a conjecture that I made in my doctoral application: whether visual impairment opens up mathematical constructions that may differ from those of the sighted pupils and consequently whether a generation of mathematical benefits to everybody stems from the inclusion of VI pupils in the mathematics classroom. This conjecture could probably not have been made if I had focussed on pupils with additional impairments, as it would have been difficult for me to detect the specific impairment(s) that would open up for different mathematical constructions. Another significant factor for the selection of the classrooms was the existence of pupils with as severe a visual impairment as possible. This is because I had initially thought that these pupils' inclusion and enabling may be more difficult due to significant adaptations required for them. Norfolk was selected because I knew that the Team Leader for the education of VI pupils in Norfolk is interested in the study and would be happy to help me with the list of schools that suited my needs.

There were 11 mainstream primary schools in Norfolk that had severely VI pupils for whom visual impairment was the only type of impairment. Out of these schools, there were four schools that expressed willingness to participate in the study. It is these four schools that my data collection took place. I collected data after securing ethical approval by the University of East Anglia's Research Ethics Committee and I have ensured participant anonymity, confidentiality and right to withdraw from the study.

There is one VI pupil in three of the classes and two in the fourth. Most of the participating VI pupils have severe visual impairment and none of them is blind in both their eyes. Two pupils have congenital visual impairment while three have adventitious visual impairment.[19]

Every class has at least one teaching assistant but the teaching assistant's role differs from class to class. While two of the classes have a teaching assistant supporting the VI pupils almost exclusively, in the other two the teaching assistants support pupils who need help at particular instances and their role does not focus on supporting the VI pupils specifically.

I have coded the names of classrooms and of teaching staff and have used pseudonyms for the names of pupils. The coding of the classrooms consists of two letter parts and two numerical parts. The first letter part is "S" – signifying the participating "Schools" – and is followed by a number – which is the number of school at the order in which the Phase 1 data was collected. The second letter part is "Y" – signifying the participating "Year group" in each school – and is followed by a number – which is the number of Year Group for the participating class in that school. For example, "S1Y3" is the participating Year 3 class in School 1. The coding of teaching staff consists of a letter part followed by a numerical part and in some cases by a lower case letter too. The letter part is either "T" – for the participating "Teachers" – or "TA" – for the participating "Teaching Assistants". The numerical part signifies the school in which each of the staff teaches. In cases

[19] "Congenital" and "adventitious" have to do with the age of onset of visual impairment. Congenitally VI are the individuals who have been born with visual impairment while adventitiously VI are the individuals whose visual impairment has appeared later in their life.

where there is more than one teacher or teaching assistant in a class, the number is followed by a lower case letter. For example, “TA1a” is one of the teaching assistants of the participating class of School 1. As for the pupils’ pseudonyms, I call “Fred” and “Ian” the two VI pupils of the Y3 class of School 1, “Luke” the VI pupil of the Y5 class of School 2, “Ned” the VI pupil of the Y1 class of School 3 and “Ivor” the VI pupil of the Y5 class of School 4. I also use pseudonyms for the names of the participating sighted pupils.

Fred and Ian are the two VI pupils of S1Y3. Fred has severe, congenital visual impairment in both his eyes. Ian has adventitious visual impairment in one eye and is sighted in the other eye. TA1a works with the two pupils individually, sits in between the two pupils and supports them both perceptually (namely, facilitating their sensory access to materials and resources that may be impeded due to their visual impairment) and substantively (namely, communicating with them about the mathematical content of the lessons). TA1b is the general teaching assistant of the class. There are two teachers in this class, one teaching four days a week – and coded as “T1a” – and the other one teaching once a week – and coded as “T1b”.

Luke is the VI pupil of S2Y5. He is blind in one eye, has reduced vision in the other eye and his visual impairment is adventitious. The class has a general teaching assistant who supports pupils that need help at particular instances and whose role is not on supporting the VI pupil specifically. This teaching assistant is coded as “TA2”. There is one teacher in this class – and this teacher is coded as “T2”.

Ned is the VI pupil of S3Y1 and has severe, congenital visual impairment in both his eyes. The class has two general teaching assistants who support pupils that need help at particular instances and their role does not focus on supporting the VI pupil specifically. One teaching assistant is coded as “TA3a” and the other one is coded as “TA3b”. There are two teachers in this class, one teaching four days a week – and coded as “T3a” – and the other one teaching once a week – and coded as “T3b”.

Ivor is the VI pupil of S4Y5 and has severe, adventitious visual impairment in both his eyes. The class has a general teaching assistant who supports mostly the VI pupil – and is coded as “TA4”. There are two teachers in this class, one teaching twice a week – and coded as “T4a” – and the other one teaching three times a week – and coded as “T4b”.

The codes of the participating classes, of the teaching staff and of the VI pupils are presented in Table 1:

Classes	**Teaching staff**	**VI pupils**
S1Y3	T1a, T1b TA1a, TA1b	Fred, Ian
S2Y5	T2 TA2	Luke
S3Y1	T3a, T3b TA3a, TA3b	Ned
S4Y5	T4a, T4b TA4	Ivor

Table 1: Coding of the classes, of the teaching staff and of the VI pupils.

I do not present the codes of the sighted pupils in Table 1, as I only exemplify a few sighted pupils in this thesis. The codes of these pupils are presented directly in Chapters 5, 6 and 7, where these pupils are exemplified.

4.3 Data collection

4.3a Inclusion and disability policies of the participating schools

Before I started to collect data in the classrooms, I had downloaded the four schools’ educational policies to gather evidence on inclusion and disability in each school. I did this in order to then contextualise data from the participants’ discourses on inclusion and disability in relation to school policy discourses and examine the extent to which the participants' discourses on inclusion and disability are consistent, or not, with the corresponding schools' policies. I also examined the extent to which schools' and participants' discourses on inclusion and disability are consistent, or not, with the national (SEND code of practice – Department for Education & Department of Health, 2015) and international (for example, United Nations, 2006; UNESCO, 2008; and, UNICEF, 2011) policies on inclusion and disability. In order to maintain the anonymity of the participating schools, I do not include quotations of sentences from the schools' policies but I instead refer to the characteristics that the schools' policies have, or not, in common.

Regarding inclusive education, the four schools have mostly commonalities – but also some differences – in their inclusion policies. In all four schools, inclusive education is related to the provision of equal opportunities for all learners. In all four schools, inclusive education is also related to all pupils' access to the same opportunities for learning.

The above two distinctive elements of inclusive education in the schools' policies suggest a differentiation between provision and access. While the provision of equal opportunities seems to be determined exclusively by the teaching staff, access to the curriculum seems to also be determined by the pupils too, whom the notion of access concerns. Also, as indicated in the schools' inclusion policies, both provision and access seem important for the schools' conceptualisation of inclusive education. Both will be later shown not to always be fulfilled in these schools' practice.

In all four schools, differentiation – also referred to as adaptation or adjustment or accommodation or personalisation – seems to be the way for implementation of inclusive education. The only case where a universally designed practice is reported is in S4, but it does not seem to be acknowledged as a universally designed practice. In particular, S4 reports that, in the case of a hearing impaired pupil, the teacher will face the pupil when addressing the class or else the teacher may use a microphone and a transmitter. This report is offered as an example in the paragraph that disabilities can restrict the pupils' ability to participate in the curriculum and can also hinder the delivery of information. Overall, in S4, as in the other three schools, accommodation practices seem to prevail.

In addition, all four schools endorse inclusive education in both academic and social aspects.

Another commonality among the four schools is that the teachers are stated to be primarily responsible for implementing inclusive education and the teaching assistants are stated to secondarily support pupils with needs.

To achieve teacher implementation of inclusion, S4 clearly states the necessity for training and professional development courses for teachers.

Unlike the other three schools, in S4, inclusive education seems to be more an aspirational – than a realistic – form of education. The following words and phrases are some examples from S4's policies that indicate the aspirational nature of inclusive education in that school: "strive", "aims", "seek", "will ensure", "plan strategically over time".

Regarding disability, the four schools have both commonalities and differences in their disability discourses. S1, S2 and S4 define disability and seem to equate disability with impairment. S1 and S2 endorse the definition of disability provided by the Equality Act (2010) while S4 endorses a similar definition of disability, which is provided by the Disability Discrimination Act (1995).

A difference among S1, S2 and S4 is that S1 and S2 often differentiate impairment from disability while S4 does not. S1 and S2 explicitly state that a pupil who is impaired is not necessarily disabled. However, both schools state that a pupil is disabled if his/her ability to carry out normal day-to-day activities is affected. This second statement indicates that S1 and S2 seem to attribute disability to the individual – and this attribution resonates with the medical model of disability. S1 and S2 report the eyesight as one of the aspects that affect normal day to day activity.

On the other hand, S3 does not define disability and refers to this notion only in relation to the creation of special educational needs. S3 refers to disability by endorsing the SEND code of practice (Department for Education & Department of Health, 2015), which defines disability (the definition of disability by the SEND code of practice is discussed in Chapter 1). The fact though that S3 does not define disability seems to show S3's differentiation from the SEND code of practice.

Therefore, S1, S2 and S4 seem to be closer to the national policy on disability (i.e. SEND code of practice, 2015) – and therefore to the medical model of disability – while S3 seems to be closer to international policy on disability (i.e. United Nations, 2006) – and therefore to the social model of disability.

Unlike the other three schools, S4 seems to associate disability with learning difficulties. S4 particularly states that the condition for a child to have learning difficulties is to have a disability that impedes him/her from using educational facilities.

Also, unlike the other three schools, S4 seems to endorse the charity model of disability too in its conception of disability, considering disability as a pity. This is indicated in S4's statement to take the person's disability into account even in cases when this consideration concerns the more favourable treatment of disabled people compared to others. The charity model of disability (Mobility International USA, 2018) is not evident in any of the other three schools' policies.

4.3b Methods of data collected

I collected data through observations of 29 mathematics lessons (33.5 hours in total); individual interviews with 5 class teachers[20] (6 interviews, 2 hours and 10 minutes in total); individual interviews with 4 teaching assistants (6 interviews, 2 hours and 15 minutes in total); focussed-group interviews with 35 pupils (16 interviews, 2 hours in total); 2 ten-minute individual interviews with one pupil; written transcripts of the teaching staff's contributions in the design of the three lessons that constituted the experimental phase of the study; photographs of the pupils' work in the three experimental lessons; and, pupils' evaluation forms of the experimental lesson in two classes. During observations, written notes were kept in all lessons. 21 lessons were audio-recorded and 14 lessons were audio/video-recorded. All interviews were audio-recorded, except four, following interviewee requests. For these, written notes were kept instead. The collected data is presented in Table 2:

Classrooms	Lesson observations	Individual teacher interviews	Individual TA interviews	Focussed-group pupil interviews	Individual pupil interviews	Photographs of pupils' work	Pupils' evaluation forms	Written transcripts of the teaching staff's contributions
S1Y3	8 in Phase 1	1 in Phase 1	1 in Phase 1	3 in Phase 1	No	No	No	No
S2Y5	9 in Phase 1 1 in Phase 2	1 in Phase 1	1 in Phase 1	3 in Phase 1	No	All participating pupils in Phase 2	All participating pupils in Phase 2	In Phase 2
S3Y1	3 in Phase 1 1 in Phase 2	1 in Phase 1 1 in Phase 2	1 in Phase 1 1 in Phase 2	4 in Phase 1 6 in Phase 2	No	All participating pupils in Phase 2	No	In Phase 2
S4Y5	6 in Phase 1 1 in Phase 2	2 in Phase 1	1 in Phase 1 1 in Phase 2	No	1 in Phase 1 1 in Phase 2	All participating pupils in Phase 2	All but one participating pupils in Phase 2	In Phase 2

Table 2: Data collected in each school in accordance with the specific methods used.

[20] In one of the classes (S4Y5) taught by two teachers – on different days – both teachers were interviewed (Table 2).

The types of data collection varied among the schools. This variation occurred because of – and in line with – the preferences of the participants. For example, in cases in which parents of VI pupils did not want their child to be videoed, I did not video-record the lessons. The VI pupils were mostly sitting at the front of the classroom due to their visual impairment and it was not possible for them to be moved at the back of the classroom. As a result, I would not have been able to video-record the particular lessons, as the camera would have captured the VI pupils. The same occurred when teachers did not wish to be video-recorded: I did not video-record the lessons, as the teachers moved around the classroom and would have been captured by the camera.

In Phase 1, I used the following methods: classroom observations; focussed-group interviews; and, individual interviews.

Classroom observations (Jones & Somekh, 2004) were chosen to collect evidence that shows how inclusion and disability are played out in the naturalistic environment of school classrooms. I aimed not to participate in the lesson but to merely sit close to the VI pupil, in order that: the teaching staff's and pupils' participation would be as natural as possible; I would minimise the possibility of personal bias or preconceptions; and, I would observe evidence on inclusion and disability. My aim was to observe the VI pupil as well as observe evidence that may suggest any other pupil's – and teaching staff's – attitudes towards the VI pupil, which would occur either directly, through addressing the VI pupil, or indirectly, through discourses which could affect the VI pupil.

In practice, however, things did not occur exactly in this way. I sometimes shifted my role from a non-participant observer to a participant observer (Jones & Somekh, 2004). This mostly occurred due to teaching staff's and/or pupils' requests for me to participate in specific instances in the lesson.

I always happily accepted to interact with class members during the lesson when I was asked to, because this would help me establish a closer relationship with them and it also felt natural as well as non-disruptive. Establishing a closer relationship with the participants would consequently make the class members feel that I am no more an outsider researcher who had entered their classroom to observe them but that I am instead part of their classroom. This would therefore help them behave in an even more natural way.

From my interaction with VI pupils during the lesson, I gained substantial insight into these pupils':
- mathematical productions;
- perceptual experiences, related to their visual impairment – what they could not access in the lesson and hence needed some help to access / what they could access in the lesson but chose not to access;
- attitudes to their sighted peers – VI pupils expressed to me impressions on their sighted peers in particular instances during the lesson;
- and, attitudes to their teaching staff – VI pupils asked me perceptual and mathematical questions that they had to ask to their teaching staff, and those questions helped me infer elements of VI pupils' attitudes towards their teaching staff.

From my interaction with class teachers during the lesson, I gained insight into treatment of pupils in specific instances of the lesson.

From my interaction with teaching assistants during the lesson, I gained insight into: treatment of pupils in specific instances of the lesson; and, attitudes of VI pupils.

From my interaction with sighted pupils during the lesson, I gained insight into these pupils' mathematical productions.

Occasionally, I took the initiative to interact with the participants during the lesson. I took such initiative in cases when I heard, or saw, a different – to the commonly used – contribution by a pupil and I asked this pupil to elaborate their contributions to me. This helped me gain insight into pupils' mathematical contributions which differed from the common ones that were frequently shared in the classroom and I thus gained a better sense of the mathematical processes that underpin pupils' mathematical contributions. This initiative was beneficial in the design of the Phase 2 lessons.

Throughout my participation during the lesson, I always had in mind to be as impartial an observer as possible. While my participation occurring mostly as a result of class members' initiatives probably made me miss some evidence on inclusion and disability – because I was otherwise preoccupied –, this loss was minimised thanks to my placing of an audio recorder close to the VI pupil and of a video camera in most mathematics lessons. Particularly, things that I had missed in one location in the classroom as a result of my interaction with participants in another location during the lesson were recorded in the videos and audios and I was later able to retrieve them.

My use of a video camera in the classroom was influenced from Quek and Oliveira (2013). I particularly chose a video camera that could capture different aspects of the lesson which occurred in different places in the classroom. Quek and Oliveira (2013) placed three video cameras in the classroom, with each camera capturing a different aspect of the lesson, which occurred in a different place in the classroom. In their work, one camera captured the teacher alongside the teacher's display. A second camera captured the VI pupil alongside the VI pupil's document, which was placed on the top of her desk, and her Haptic Deictic System-aided navigation. Finally, a third camera captured the view of the whole scene at once and enabled the researchers to examine the interaction amongst the participants. As all three cameras captured vital aspects of the lesson – and therefore in my study vital aspects of inclusion and enabling – I wanted to capture as many parts of the classroom as possible. However, due to the practical difficulty of me carrying three cameras with three tripods in each school, I bought one camera but made sure that it could capture an overview of the classroom – as much as possible – and with particular emphasis to the teacher, his/her display, the VI pupil and his work.

Video-recordings, which were also used by Healy and Fernandes (2011; 2014) and Vines et al. (2019), are important as they capture the visual-gestural-somatic expressions of VI pupils. In particular, they "provide a way of recording such utterances for future reference, in a way that writing has traditionally served for recording spoken words. Not only does this technology make it possible for detailed analyses of […] gesture to be undertaken, it also offers the possibility of bringing new language resources to the teaching of mathematics" (Healy et al., 2016, p. 159). In my study, video-recordings are also significant, as they capture the visual-gestural-somatic expressions of the sighted participants too.

The presence of video recorder did not change the behaviour of the pupils in the class. The pupils were concentrated in the lesson and I did not observe any differences in the pupils' behaviour between the first video-recorded lesson and the last video-recorded lesson of the same class. Two factors were particularly significant in maintaining pupils' concentration in the lesson:

- I put the video recorder at a place that would not distract the pupils' attention from the lesson. As the lessons were highly dependent on the Interactive Whiteboard (IWB), I had in mind not to place the video recorder close to it. I instead preferred to place the video recorder close to a corner of the classroom. In this way, I thought that the pupils would almost forget the existence of a video camera in the classroom.
- The teachers were clear from the very beginning that the video recorder is placed only for myself as a researcher and for educational purposes. They asked the pupils to ignore the existence of the video recorder, as it had nothing to do with them.

Focussed-group interviews (Barbour & Schostak, 2004) were chosen to collect evidence on pupils' experiences of learning mathematics. The interviews were semi-structured, giving the opportunity for the pupils to talk freely on issues in question as well as relevant issues which they considered important. I asked a variety of questions to initiate discussion in the focus groups. Some questions were more open, for example what pupils like and do not like in the mathematics lessons in that class, while other questions were more targeted, for example whether tactile resources were useful for these pupils and why. In the classes where focus groups took place, the VI pupil was invited with sighted pupils in a focus group while there were some focus groups consisting of only sighted pupils – as the VI pupils in each classroom were one or two. 2 to 4 pupils participated in each focus group – with the smaller number of pupils typically used for the younger ones and with the bigger number of pupils for the older ones. Focus groups were characterised as much as possible by a range: of genders; of ethnicities; and, in mathematical performance. Therefore, they had an inclusive character in that the VI pupil was together with sighted pupils and in that a mixture in gender, ethnicity and mathematical performance existed in them.

The focussed-group interviews helped me deepen, and expand on, the data collected through classroom observations so that I could: obtain a more integral account of VI pupils' experiences of inclusion; and, examine whether there is any discrepancy in the VI pupils' experiences of – and practices towards – inclusion and disability in the mathematics classroom. The data collected through focus groups was also considered for the design of lessons in the second phase of the study, as its aim was the creation of mathematics lessons which would be inclusive, enabling and enjoyable from everybody.

Focussed-group interviews were used in all but one classes, where an individual interview (Barbour & Schostak, 2004) with the VI pupil was instead conducted, following the teacher's request due to the class's school schedule. The questions to the VI pupil were the same as those asked in the focus groups in the other classes and the interview was semi-structured as were the focussed-group interviews in the other classes. While in that class I asked interview questions only to the VI pupil, I found that the individual interview probably had some benefits compared to the focussed-group interviews. Potential benefits for interviewing a VI pupil individually comparing to interviewing him as part of a focus group were his feeling of comfort to express himself and also his non-influence by another pupil (Barbour & Schostak, 2004). As happened with the data collected through focussed-group interviews, the data collected in the pupil's individual interview were combined with the observation data and were then used for the design of the Phase 2 lesson in that class.

Individual interviews (Barbour & Schostak, 2004) were also chosen to collect evidence on the perspectives of participating class teachers and teaching assistants on inclusion and disability. Experiences with – and training on – teaching mathematics to classes that have VI pupils, also benefits and challenges that the inclusion of VI pupils may have for the teaching staff and for the class were amongst the main things asked in the teaching staff interviews. The interviews were individual and semi-structured: individual, in order that the staff feel comfortable to express themselves and are not influenced by another participant; and, semi-structured, so that the staff are given the opportunity to elaborate on issues in question – and/or talk about relevant issues which are not questioned.

Apart from providing me with information on the teaching staff's background with – and views on – inclusion and disability in the mathematics classroom with regard to VI pupils, interviews were also chosen as a method for triangulation. The data collected through the teaching staff interviews was triangulated with the data collected through the classroom observations, so that I could: obtain a more integral account of teaching staff's attitudes to VI pupils; and, examine whether there are any discrepancies in the teaching staff's perspectives – on inclusion and disability – and corresponding practices in the mathematics classroom.

The list of questions that formed the basic structure of the teacher interviews is inserted below. As the interviews had a semi-structured character, I also asked more questions to the teachers based on their responses to my questions.

- What are your experiences with developing inclusive mathematics classrooms?
- Have you had any training (pre-service and/or in-service) on inclusion? If so, could you give me some details about it? E.g.:
    - How did you identify this training?
    - Who organised it?
    - What was the content of the training?
- What are your experiences with working in mathematics classrooms that have visually impaired learners?
- Have you had any training (pre-service and/or in-service) on including visually impaired learners in mathematics lessons? If so, could you give me some details about it? E.g.:

        - How did you identify this training?
        - Who organised it?
        - What was the content of the training?
    - How do you plan or design the mathematics lessons that you teach and implement in classrooms that have visually impaired learners?
    - How do you find it when you teach mathematics in classrooms that have visually impaired learners?
    - How do your sighted pupils find it when there are visually impaired learners in the mathematics classroom?
    - How do your visually impaired pupils find it when they are taught mathematics in mainstream classrooms?
    - How do you make sure that the visually impaired learners are included in your mathematics lessons?
    - Do you think that the inclusion of visually impaired learners is always feasible in your mathematics lessons? Or, in other words, do you think that the VI pupils are always included in your mathematics lessons?
    - Do you consider visual impairment as a form of disability?
        - If yes:
          What do you think makes visual impairment a disability?
          Do you think that the visually impaired pupils are disabled in your mathematics lessons?
        - If no:
          What do you think makes your visually impaired pupils enabled?

I was influenced from the literature in my choice of specific thematic aspects that constitute the focus of these questions. More specifically, the design of questions around teachers' experiences on inclusion was influenced by the CAPTeaM project (Nardi et al., 2018). The design of questions around pupils' experiences in the mainstream mathematics classroom was influenced by Bayram et al. (2015) and by Argyropoulos and Stamouli (2006). The design of questions around inclusive mathematics materials was influenced by Leuders (2016) and by Healy and Fernandes (2011; 2014).

In the literature on the inclusion of VI pupils in the mathematics classroom, one method that is common with aforementioned methods and that takes place in an exploratory phase is the individual interviews with VI pupils. In particular, Bayram et al. (2015) conducted individual interviews with three VI pupils in their aim to explore the VI pupils' experiences of inclusive mathematics education. In their case too, the interviews were semi-structured and this feature enabled the researchers to obtain a qualitative account of the three VI pupils' experiences. Following these researchers' success of interview use with VI pupils, I too conducted interviews with VI pupils. However, unlike Bayram et al.'s (2015) individual interviews, I conducted focussed-group interviews in which I interviewed VI pupils alongside sighted ones. As mentioned above, exception constitutes a VI pupil with whom an individual interview was conducted, as the alternative of the focussed-group interview was not possible. I conducted focussed-group interviews rather than individual ones as I considered them a research method that enabled me to maintain the inclusive character of my study.

In the literature on the inclusion of VI pupils in the mathematics classroom, it is Bayram et al. (2015) who had an exploratory phase in their study. In other words, it is these researchers who collected data from the naturalistic environment of the mathematics classroom. Although these researchers were not physically present in the classroom, they obtained an account of the implementation of inclusion in the naturalistic environment of the mathematics classroom through the interviews of VI pupils.

The other studies that I reviewed in preparation for my study went directly for an experimental phase, in which the researchers either alone (for example, Quek & Oliveira, 2013) or in collaboration with teachers (for example, Healy & Fernandes, 2011) designed mathematics materials and trialled them with pupils. However, I consider the role of the exploratory phase really important, as it enables researchers to identify issues pertinent to the implementation of inclusion in the naturalistic environment of the mathematics classroom. This research in the naturalistic environment enables the identification of practices that work well towards the inclusion of VI pupils and also practices that do not work well – and therefore need to be modified towards the successful inclusion of VI pupils. Experimental work that is not preceded by exploratory one may have the danger of designing mathematics materials that are not efficient in the specific mathematics classroom. In other words, experimental work may be better contextualised if it is preceded by research done in the specific setting in which the experiment takes place.

In Phase 2, I organised three lessons in total, one per school – I note that S1 did not participate in Phase 2. The limited number of Phase 2 lessons was due to the following factors: the specific period that I had for my data collection (my data collection needed to finish by the end of September 2018) combined with the availability of teachers and my work on preliminary analysis of Phase 1 data and on the design of Phase 2. I conducted Phase 1 until the middle of March 2018, which means that I had the remaining days of March, April, May, June and the first half of July for the primary school pupils. Within the 4-month period, there were three weeks that the schools were closed (two weeks for the Easter break and 1 week for the summer half-term break). I was therefore left with approximately 3 months to devote to Phase 2. During the 4-month period, I spent substantial amount of time analysing the Phase 1 data. I also spent substantial amount of time in the design of each Phase 2 lesson. Substantial amount of time was also spent between myself and each teacher, in the co-design of the lessons. Due to their busy working schedule, it took the teachers time to reflect on the suggested lesson design and to finalise it. Substantial amount of time was also spent on my preliminary analysing the first Phase 2 lesson before I started to design the second one. This time was important to be spent there because preliminary findings from the first Phase 2 lesson informed the design of the second one. Similar amount of time was also spent on my preliminary analysing the second Phase 2 lesson before I started to design the third one. For all the above reasons, I managed to organise only three Phase 2 lessons, one per school. Despite this small number of Phase 2 lessons, these lessons offered significant findings towards the inclusion and enabling of VI pupils.

Regarding the design of the lessons, the methodology in Phase 2 involved the following:

- The teachers informed me about the mathematical topic(s) that they wished the Phase 2 lesson to be on.
- I started to design tasks on each lesson and ways of delivery. I aimed to design lessons that would be inclusive and enabling for the VI pupils and beneficial for everybody, including the teaching staff. To achieve these aims, I got informed from the preliminary Phase 1 findings as to: inclusive practices, identified from my observations of the four classes; sighted pupils', VI pupils' and teaching staff's preferences in a mathematics lesson (e.g. norms, types of tasks, materials etc), identified from the pupils' and teaching staff's interviews. Apart from preliminary Phase 1 findings, the lesson design was also influenced from the literature on the inclusion of VI pupils, reported in Chapter 3. The main characteristics from the literature were: characteristics of visual, auditory and tactile perception and ways to design auditory and tactile tasks; the role of pair work between VI and sighted pupils in the inclusion of VI pupils; and, the consideration of the teacher as the only member of staff who would be responsible for the VI pupils' inclusion.

  As I stated earlier, the three lessons were not designed during the same period. Instead, each lesson, except for the first one, followed a process of design, implementation and preliminary analysis before the next lesson was designed. Preliminary analysis of each lesson was significant before the design of the next lesson, as preliminary findings from one lesson informed the design of the next lesson – this process occurred with the design of the second and third lessons. This iterative process of design, implementation and preliminary analysis helped me design lessons that are gradually more inclusive.
- The teachers then contributed to these tasks and ways of delivery – and the lesson design was finalised. Their suggestions often stemmed from their preferences with regard to what they thought would work better in the mathematical learning of that particular class.

  In some cases, the teachers' suggestions were more realistic to happen in their class than those that I had suggested – e.g. the use of bead strings instead of base ten blocks on the carpet, as bead strings would be more easily manipulable on the carpet.

  Other times, despite the best intentions, the teachers' suggestions ended up obstructing the pupils' mathematical learning (e.g. the use of the same instrument for Tens and Ones[21]: the teacher thought that using the same instrument would be helpful for the representation of numbers but instead the

[21] The relevant task is discussed on p. 104.

pupils were confused with the two sounds, as they were similar – being produced of the same instrument). Even in those cases though, the teachers and I learnt from these suggestions and, in the next lesson, I amended that design principle that hindered the pupils' mathematical learning, aiming for a more inclusive impact.

In other cases, the teachers suggested additional ways of implementing design principles. These additional ways sometimes ended up bringing about mathematical benefits that were beneficial to everybody (e.g. giving plastic shapes to pupils to explore through touch[22]: this suggestion came as an additional one to mine of the Wikki Stix shapes, in the design principle of asking the entire class to explore mathematics through touch). These mathematical benefits often reinforced the mathematical benefits of similar tasks that I suggested, providing a more solid ground on the significance of tactile and auditory mathematical tasks for everybody in the classroom.

- The teachers implemented the lesson and I observed it.

In Phase 2, I used the following methods: written transcripts of the class teachers' contributions in the design of the three experimental lessons; classroom observations; focussed-group interviews; individual interviews; photographs of the pupils' work in the three experimental lessons; and, pupils' evaluation forms of the experimental lesson in two classes.

Written transcripts were chosen to collect evidence on the class teachers' contributions in the design of the three intervention lessons. The three lessons were designed on mathematical topics and learning objectives which the class teachers wanted the class to engage with. I then suggested various mathematical activities – and ways of delivery. The class teachers then contributed to the activities and their delivery. I recorded the teaching staff's contributions through written transcripts, which emerged from emails, and from physical discussion, between the class teachers and myself.

Classroom observations (Jones & Somekh, 2004) were chosen to record evidence that shows whether the collaboratively designed intervention lessons were inclusive and enabling as well as whether they brought any benefits for pupils and teaching staff. As in Phase 1, in Phase 2 too I aimed to be an observer with minimum participation during the class teachers' implementation of the co-designed lessons. However, this was not always possible, and I shifted my role to a participant observer (Jones & Somekh, 2004) when I was asked by the class teacher, or by a pupil, to participate. In rare occasions, the shift of my role occurred on my own initiative, when I heard pupil contributions in their groups and I wanted the pupils to elaborate on such contributions, so that I could be aware of how they constructed these contributions.

Focussed-group interviews (Barbour & Schostak, 2004) were conducted with pupils from one class immediately after the implementation of the first intervention lesson. The focussed-group interviews in Phase 2 consisted of 2 pupils. As in Phase 1, the VI pupil belonged to a focus group with a sighted pupil. The main purpose of the focussed-group interviews in Phase 2 was to elicit pupils' experiences of the intervention lesson in their classroom. Shifts in pupils' discourses on inclusion and disability as well as benefits which pupils might have gained from that lesson were of particular interest in the focus groups. While focussed-group interviews provided insight into pupils' experiences of the intervention lesson, they were limited due to the limited time which I had for interviewing. The time was limited because the class was engaged with the next school subject. As a result, a limited number of focussed-group interviews was conducted and I did not manage to obtain many pupils' experiences of the intervention lesson.

For the other two intervention lessons, I changed how I collected evidence on pupils' experiences of the lesson. I chose to design evaluation forms that would be given to the class by the teacher immediately after the end of the intervention lesson. This method would enable me to obtain more pupils' experiences of the lesson in the limited time that the pupils had until the teaching of the next school subject would start. As with the focussed-group interviews of Phase 2, the evaluation forms had semi-structured questions that aimed to elicit pupils' experiences of the intervention lesson. The evaluation forms were provided in a visual format for every pupil and at a legible font size for the VI pupil. A potential disadvantage of the evaluation form

[22] The relevant task is presented on p. 109.

method over the focussed-group interview method was that the former method might give me superficial insight of the pupils' experiences, as it does not allow me to ask for elaborations and pose further questions to the pupils in order to obtain deeper insight into their experiences. However, I thought that the benefit of obtaining more pupils' views would compensate for the aforementioned potential limitation of the evaluation form method.

The only pupil who was not implemented the evaluation form method was the same pupil of Phase 1, to whom an individual interview (Barbour & Schostak, 2004) was implemented. As an individual interview was the only choice I was provided with in Phase 1 in order to be informed on this pupil's experiences of mathematics in that classroom and as this method worked very well for this pupil – he provided me with substantial insight into his mathematical learning experiences – I decided to use this method again for this pupil, in Phase 2. As with the focussed-group interviews and the evaluation forms, the individual interview was semi-structured.

Individual interviews (Barbour & Schostak, 2004) were also conducted with the teaching staff who had some time for an interview immediately after the implementation of the intervention lesson. The interviews were semi-structured in order to allow the teaching staff to elaborate on issues in question. The main purpose of the individual interviews was to elicit teaching staff's experiences of implementing a collaboratively designed lesson to their class. Shifts in teaching staff's discourses on inclusion and disability as well as benefits which teaching staff might have gained from that lesson – or which teaching staff saw pupils might have gained from that lesson – were of particular focus in the teaching staff interviews.

Finally, photographs were taken to collect evidence on the pupils' work in all three intervention lessons. I took photographs of the pupils' work with my iPad after the end of each intervention lesson. Photographs were appropriate as they could clearly show the pupils' work, which was mostly written with a pencil.

Most of the methods that I used in the experimental phase of the study coincide with the methods that other researchers used too in the experimental phase of their work.

In particular, observations were also reported in Bayram et al.'s (2015) work. They took place during Bayram et al.'s tutoring of one of the VI pupils who participated in one of the interviews. Bayram et al. (2015) report that the observations helped them triangulate the data obtained from the interviews of the VI pupils. However, evidence from the observation data is not included in Bayram et al.'s (2015) paper, therefore the existing literature does not include cases of observation data being triangulated with interview data. I do such triangulation in my study.

Observations were also reported in Argyropoulos and Stamouli's (2006) work. This research method constituted a part of Argyropoulos and Stamouli's action research intervention in their attempt to create a better inclusion of a VI pupil in the mathematics classroom. The observations were done by one of the two researchers and took place in the mathematics classroom. They focused on the researcher's interaction with the VI pupil, with the class teacher and with the support teacher. In this respect, the researcher acted as a participant observer (Jones & Somekh, 2004). There were also cases where the researcher shifted his role to a non-participant observer. These cases occurred when the researcher was keeping notes that were related to the language that the VI pupil used in the mathematics lesson, her tactile perception, her misconceptions, her prior knowledge and her thinking processes. Therefore, in Argyropoulos and Stamouli's work observations focused on the cognitive/developmental processes of the VI pupil. However, in my study, the observations were done with the use of sociocultural lenses.

Below I report two cases in which observations are not reported to have been done during the conduction of the experimental sessions. In particular, Vines et al. (2019) and Quek and Oliveira (2013) were not physically present in the setting where the experimental sessions took place.

More specifically, Vines et al.'s (2019) case concerns the experimental Phase 1 sessions which involved use of sonifications by VI and sighted students in their work with graphs and plots. In these sessions, a facilitator worked one-to-one with each student, by going through each of the examples of plots that Vines et al. (2019) had designed in their attempt to "investigate first year undergraduates' perceptual precision and first-hand access of sonifications of plots and line graphs from STEM modules" (Vines et al., 2019, p. 23). Vines et al. (2019) had access to these sessions through video-recordings. The sessions were video-recorded, enabling Vines et al. (2019) to record gestures of their participants as well as materials and speech.

Quek and Oliveira's (2013) case concerns the experimental courses taught by a teacher to a group of students that consists of one VI student and three sighted students. The courses involved the examination of the effectiveness of the Haptic Deictic System as it was used by the teacher and the VI pupil in a mathematics setting. As in Vines et al.'s (2019) study, Quek and Oliveira (2013) had access to these courses through video-recordings. The courses were video-recorded, enabling Quek and Oliveira (2013) to record gestures of their participants as well as materials and speech.

In my view, the trialling of experimental lessons in which the researchers are not physically present in the particular settings makes these lessons even more 'experimental'. Also, the non-physical presence of researchers in the trialling of the lessons may also make the researchers miss some data which could be significant for analysis. I am concerned whether analysing a video-recorded lesson that does not occur with physical presence in the classroom would provide the same insight with analysing a video-recorded lesson that occurs with physical presence in that classroom. I feel that some information might be missed, however good the video camera may be. As Jones and Somekh (2004) put it, "[n]either audio nor video-recording replaces the need to make field notes, since technology only keeps a partial record and cannot replace the sensitivity of the researcher's 'self', open to nuances of meaning and interpretation" (p. 140).

Individual interviews with class teachers, teaching assistant and VI pupils were also conducted in the literature as part of the experimental phase of the relevant studies. In particular, Argyropoulos and Stamouli (2006) conducted individual interviews with the class teacher, the teaching assistant and the VI pupil after the end of the experimental lesson. The interviews were semi-structured – "open-ended", in Argyropoulos and Stamouli's (2006, p. 132) words. This feature of the interviews enabled these researchers to obtain a qualitative account of – and hence insight into – the class teacher's, the teaching assistant's and the VI pupil's experiences of the action research intervention in the class.

Individual interviews with VI pupils were also conducted by Healy and Fernandes (2011; 2014). However, these researchers did task-based interviews. The aim of these interviews was to engage VI pupils with mathematical tasks and tools which the researchers, alongside teachers, designed. The engagement of VI pupils with the co-designed mathematical tasks and tools made pupils contributors to the experimental work: in particular, VI pupils' engagement helps the researchers and the teachers to later improve the design of the tasks and tools so that these materials can then be used in the mathematics classroom (Healy et al., 2013). The interviews were video-recorded, enabling Healy and Fernandes (2011; 2014) to record gestures of their participants as well as materials and speech.

Focussed-group interviews were also conducted by Healy and Fernandes (2011; 2014). However, unlike my focussed-group interviews, theirs consisted of only VI pupils. As in my case, these researchers' focussed-group interviews served the purpose that the individual interviews served.

Collection of the pupils' work in the experimental lesson was also reported in the literature. In particular, Healy and Fernandes (2011; 2014) and Vines et al. (2019) used still images of the pupils' work which were created from the video-recorded data that these two groups of researchers collected.

Evaluation forms were also used in the literature. In particular, Vines et al. (2019) used them in Phase 2 of their study, in order to gain VI and sighted students' experiences on their engagement with sonifications, which were placed on the website of an Open University module on scatterplots. In Vines et al.'s (2019) work, the evaluation forms were on the form of questionnaires, which included closed questions and were analysed quantitatively. However, in my study, the evaluation forms include open questions and are analysed qualitatively. I consider this as an effective way that enables me to gain deeper insight into the class's experiences of the experimental lesson.

4.4 Ethical considerations of the study

Data collection started after I secured ethical approval by the University of East Anglia's Research Ethics Committee. There were several ethical considerations of my study.

First of all, the study involves children, who constitute a vulnerable group of participants and for whom gatekeeper permission needed to be obtained. Parental information sheets and consent forms were designed and sent to the pupils' parents. More specifically, the teachers who were willing to participate in the study informed the pupils' parents about the study and handed over the parental information sheets and consent forms to them. In all these cases, the VI pupils' parents accepted their child to participate in the study. Most of the sighted pupils' parents also agreed with their child's participation in the study. Data was collected for those children whose parents agreed with their participation in the study.

Central role in the study is played by children with visual impairment. While children in general constitute a vulnerable group of participants, children with visual impairment constitute an even more vulnerable group, with additional ethical considerations related to the impairment of these children. Throughout my data collection, I was cautious not to bring any discomfort to the VI pupils as well as between the VI and the sighted participants.

Ethical considerations were also involved across the entire group of my participants, who consisted of pupils, class teachers and teaching assistants. Prospective participants were given the choice not to participate, either in some part(s) or in the entire study, and were reassured that in case of non-participation no data would be recorded in relation to them. Prospective participants were also clarified that their choice of non-participation would not affect their relationship with me or anyone else at the UEA or their school. I did not want to make prospective participants feel obliged to participate in the study. I instead reassured them that there would be no negative consequences for them if they did not wish to participate. All participant information sheets and consent forms are provided (see Appendix 1).

Prospective participants were also provided with the opportunity to select the individual parts of the study in which they wished to participate. Within some parts, such as within classroom observations, participants could also select the type of data they wished for themselves. The participants were also reassured that they could change their mind at any time on their participation in a specific research method and, if applicable, on the type of data too. Participants who wished to participate in classroom observations but did not wish to be video-recorded were reassured that they would be located in a position that minimised the possibility for them to be captured by the video-camera during the classroom observations. Participants who did not wish to be audio-recorded in the individual interviews agreed that written notes of their responses would be kept during their interviews. Finally, participants who wished to participate in focussed-group interviews agreed that they would be audio-recorded. In every case, anonymity, confidentiality and right to withdraw from the study were guaranteed to the participants.

Ethical considerations existed also for myself, as the conductor of the data collection. Particularly, the fact of me being a female, non-native speaker of English was considered as a possible influence on the conduct of the study. My female identity might help me integrate more quickly in the often female-dominated environments of primary schools. Finally, my identity as a non-native speaker of English could possibly cause some communication problems with the participants, especially the children, many of whom would not be used to my accent. Analogously, characteristics of the participants were also considered to possibly influence the conduct of the study. However, my 2-year work experience in primary British classrooms with children of a variety of ages and from a wide range of cultural, ethnic and religious backgrounds mitigated the impact of these factors.

4.5 Data analysis

The first step for my analysis of the classroom observation data involved organising and coding of the data that I collected from each lesson. As the data collected through classroom observations is of three types – video-recordings, audio-recordings and written notes – and not all the three types of data were collected in each lesson, I followed different approaches to analyse the data collected through classroom observations. The different approaches depended on the type(s) of the data which I collected in each particular lesson.

More specifically, for the lessons which involved data collected through video-recordings, I first transcribed the lessons from the videos. I did a detailed transcription, by writing the particular interval of each video in which the mathematical teaching and learning took place. I paid particular attention to the speech and gestures of the class teachers, the teaching assistants, the sighted pupils and the VI pupils, as well as to the materials that they used in the lesson.

As I kept written notes during all the video-recorded lessons, I then incorporated my notes into particular parts of the corresponding transcribed lessons in case that my notes provided additional information to the videos. This case concerned mostly interactions between the VI pupil and other pupils or the teaching assistant, which did not occur during the class discussion initiated by the class teacher where the participants used loud voices. These interactions involved low voices of the participants, which could not have been captured through the videos. It is to this case that the audio-recordings contributed too.

For the lessons which involved data collected through written notes and/or audio-recordings, I primarily relied on my written notes. I used the audio-recordings in order to retrieve any words that I had probably missed in my note-taking. Primary reliance on my written notes occurred because of the variety of aspects that the notes – as a form of recorded data – can include. They can particularly include speech, gestures, materials and anything else that happens in the classroom. However, audio-recordings can only capture speech.

The second step for my analysis of the classroom observation data was the identification of my unit of analysis. My unit of analysis of the classroom observation data constitutes a classroom episode. I define a classroom episode as a part of the mathematics lesson that has a starting and an ending point – thus can stand alone in the text with relative clarity – and that also has the capacity to convey a key point related to the focus of the study.

Applying this definition to the classroom observation data, I broke each lesson into episodes. In particular, in each lesson, I went through the classroom observation data in the form which I described in Step 1 and I broke them into episodes.

Before I proceed to the third step for my analysis of the classroom observation data, I will briefly talk about the use of episodes as a unit of data analysis in the literature on the inclusion of VI pupils in the mathematics classroom. The successful use of episodes as analytical units by Healy and Fernandes (2011; 2014) impacted on my choice of episodes as the main analytical units of my study.

Episodes have been successfully used as the main unit of data analysis in Healy and Fernandes' (2011; 2014) work. The difference between their work and mine is that in their work the episodes were extracted from a resource room, which is a place in a mainstream school in which the VI pupils learn when they are not educated in the mainstream classroom and alongside their sighted peers. In Healy and Fernandes' work, as well as in my work, episodes are used to convey key points relatively to the foci of the particular studies. For example, in Healy and Fernandes' work, episodes convey findings with regard to mathematical practices of VI pupils as these pupils construct and convey mathematical meaning. In my work, classroom episodes convey findings mostly with regard to: speech and gesture use of sighted participants; digital and physical resources used in the classroom; and, VI pupils' mathematical contributions and sighted people's reactions to those contributions.

In both Healy and Fernandes' work and my work, episodes are significant analytical units because they evidence findings in context. Utterances, for example, could not be successful in this respect as analytical units because they are out of context: although they come from a specific context, their insertion as an analytical unit is out of context. In particular, they do not indicate what happens before and after the specific utterance, which may be significant in the emergence of the particular finding. Therefore, one would probably miss providing a complete account of the particular finding if utterances were used as an analytical unit.

Their advantage of provision of findings in context, in combination with their attribute to stand alone in the text by making sense as they are, makes episodes significant analytical units for another reason too: the reason is the applicability. These two attributes make episodes easily utilised in a variety of settings. For example, episodes can be used with teachers from across the world in order to convey key messages and elicit teachers' reflections upon those. The CAPTeaM project, which I have discussed in previous chapters, is an example of a project that evidences the applicability of episodes. It does this by using episodes with teachers.

The third step for my analysis of the classroom observation data involved coding of each episode. In the classroom observation data of Phase 1, the coding of the episodes consists of three letter parts and three numerical parts. The first letter part is "S" – signifying the participating "Schools" – and is followed by a number – which is the number of the school at the chronological order in which the Phase 1 data was collected and which is further described in 4.2. The second letter part is "L" – signifying the "Lessons" in each school – and is followed by a number – which is the number of the lesson at the chronological order in which it took place in that particular school. As I collected data from one class per school, I do not include the Year Group in my coding of the episodes, in order to avoid unnecessarily long codes. The third letter part is "E" – signifying the "Episodes" in each school – and is followed by a number – which is the number of the episode at the chronological order in which it took place in that particular lesson. For example, S2L3E2 is the second episode from the third lesson of School 2.

In the classroom observation data of Phase 2, the coding of the episodes involves a similar procedure, except for the middle part of the code. In particular, as there was only one Phase 2 lesson per class, the middle part of the code comprises of only one character, "I" – signifying the "Intervention". For example, S2IE2 is the second episode from the intervention lesson of School 2.

The fourth step for my analysis of the classroom observation data involved my labelling of the episodes. As I reported earlier in this section, I consider as episodes any parts of the lesson which have the following two characteristics: a starting and an ending point (1); and, a focus on the two key concepts of the study, which are inclusion and disability (2). I first labelled each episode. The labels illustrated the ways in which inclusion and/or enabling of VI pupils took place in the mathematics lesson along with the impact that these ways had upon the VI pupils. The impact was elicited from the VI pupils' (re)actions during the mathematics lesson. I then collected all the labels, grouped similar labels together and discerned the themes that grouped labels fit to. Afterwards, within each theme and with the help of my labels, I identified the issues that concern each theme.

The above are the four steps which I followed for the analysis of the classroom observation data. I will now proceed with briefly describing the periods in which I conducted data analysis. I will then present the steps which I followed and which made me end up with the particular themes, and the particular episodes, in Chapters 5, 6 and 7.

Analysis of the classroom observation data from Phase 1 started after the end of each lesson. This work, as well as the work that I did in between the end of Phase 1 and the design of each experimental lesson with the teaching staff, helped me identify the main issues from Phase 1 which I wanted to address – when they were negative for the inclusion and enabling of the VI pupil – or keep – when they were positive for the inclusion and enabling of the VI pupil – in the experimental lesson.

In a similar way, analysis of the classroom observation data from Phase 2 was conducted. In particular, I started to analyse the classroom observation data after the end of each experimental lesson. This work helped me identify the main impacts from the Phase 2 lesson upon the teaching staff and pupils, with the impacts being particularly tailored to inclusion and disability. I related positive and negative impacts to particular elements that the teaching staff and I followed in our lesson design. Elements with a positive impact were to be maintained – while elements with a negative impact were to be modified – in the design of the experimental lesson in the next class. However, due to the limited time that I had for analysis during my data collection period, my systematic analysis of the classroom observation data – described in the four steps above – started after the end of both phases of my data collection.

Chapters 5, 6 and 7 emerged from the analysis conducted in the classroom observation data, with classroom episodes constituting the main unit of my data analysis. Each of these chapters focuses on one of the three main themes that emerged from this analysis. In particular, Chapter 5 focuses on speech and gestures, Chapter 6 focuses on the intertwinement of digital and physical resources and Chapter 7 focuses on mathematical contributions.

These themes were also informed by the literature and the theory. The role of the literature in the emergence of the three main themes is described in Chapter 3. I will now describe the role of the theory in the emergence of these themes. The three themes were informed from the combination of the three theories, and the additional analytical constructs, that I use in the study: the Vygotskian sociocultural theory of learning; the theory of embodied cognition; the social model of disability; and, analytic elements from CAPTeaM. Endorsing sociocultural lenses that are combined with embodied cognition, I see learning in a particular way (described in Chapter 2). As through the sociocultural-embodied lenses learning is a process that is mediated by semiotic tools, material tools and sensory tools (Vygotsky, 1978; 1993), I see learning in the inclusive mathematics classrooms with VI pupils as a process that is mediated by these three kinds of tools. In my data, speech and gestures (theme of Chapter 5) were found to constitute the most frequently used semiotic tools that mediate the mathematical learning of VI pupils. Physical and digital resources (theme of Chapter 6) were found to constitute the material tools that mediate the mathematical learning of VI pupils. The mathematical contributions of VI pupils and the sighted people's responses to these contributions (theme of Chapter 7) are informed by the CAPTeaM project: particularly by the way this project aims to contribute towards the creation of more inclusive mathematics classrooms. In all these three themes, the sensory tools play a key role in the mathematical learning, therefore in the inclusion and enabling, of VI pupils. To sum up, the three themes of the study represent the main aspects of the mathematical learning of VI pupils and are informed by the theoretical lenses that the study endorses. In this way, the three themes of the study can be used in the investigation of inclusion of VI pupils in any mathematics classroom under the use of sociocultural and embodied lenses.

Across the three main themes, most of the episodes come from the speech and gesture theme (630 episodes). This is because speech and gestures of sighted participants were the most frequently identified elements in the inclusion and enabling of VI pupils. The least number of episodes comes from the intertwinement of digital and physical resources theme (71 episodes). This is because the intertwinement of these resources constitutes a narrow, specific case of resource use in the classroom. The medium number of episodes comes from the mathematical contributions theme (182 episodes).

I have also identified 159 episodes that concern other issues on inclusion, such as social inclusion of VI pupils and separation of class between sighted and VI pupils. While these issues do not form separate chapters in my thesis, they are brought forward across Chapters 5, 6 and 7 – in the analysis of episodes and in the summaries of these chapters.

The total number of episodes is 1042.

In the data analysis sections of Phase 1, data from individual interviews and from focussed-group interviews are used to support, deepen, expand or contradict to issues that emerge from the analysis of classroom episodes. Data from lesson design, from classroom observations, from individual interviews, from focussed-group interviews and from evaluation forms inform the data analysis sections of Phase 2.

As I explained in Chapter 2, data is analysed through the following theories: the Vygotskian sociocultural theory of learning (Vygotsky, 1978; Vygotsky, 1993); the theory of embodied cognition (Gallese & Lakoff, 2005); and, the social model of disability (Oliver, 2009). Apart from the constructs that are drawn from the three theories, I also use the constructs of "valuing", "attuning" and "incorporating". "Valuing" and "attuning" are two analytic elements of the CAPTeaM project (by Nardi et al., 2018). I use "incorporating" as an alternative analytic element to the respective "classroom management" that is used by Nardi et al. (2018). I note that not all the analytical constructs are always used to analyse a piece of data. A choice of one or more analytical constructs as analytical tools in each piece of data is made purposefully in order to help me convey a particular finding of my study.

My primary supervisor and I went through the following process in order to establish interrater reliability:

- With regard to the data analysis included in the four papers co-authored with me, my primary supervisor and I read/watched/browsed the data and established a factual account of (for example) what happens in a particular episode (as in the transcript or the video / photo excerpts). Then, I wrote an analytical account of the episode. My primary supervisor read this and compared/contrasted it with her analytical account. The process of comparing/contrasting continued in various rounds until a shared account was produced.
- With regard to my primary supervisor's reading of data analysis thesis drafts, I presented to my primary supervisor my factual and analytical accounts for sections of the collected data. My primary supervisor scrutinised these accounts asking for elaborations, clarifications, additional evidence etc. This also occurred in several rounds, with the last two being once the decision had been made to present the thesis findings in the three themes that underpin Chapters 5, 6 and 7 (gestures/speech, digital/physical, mathematical contributions).

As stated earlier in this section, in the three data analysis chapters I present the main issues that I have identified from the classroom observations. Most of the issues have been identified from the first phase of the study, as it is this phase in which I collected most of the classroom observation data.

I illustrate each issue with at least one classroom episode from Phase 1. This one episode is part of a pattern that I observed across the data. In case that the particular issue is illustrated in more than one way in the Phase 1 data, I insert an additional classroom episode that illustrates the issue in a different way.

Apart from the factor of differentiation in the illustration of a particular issue, another factor that determines the number of Phase 1 episodes to be inserted – for the illustration of a particular issue – is the role of the sighted participants. More specifically, if an issue is illustrated differently amongst groups of the sighted participants, such as between sighted pupils and class teachers, I insert an episode from each of the different groups of the sighted participants. Similarly, if an issue is illustrated in the same way amongst groups of the sighted participants, such as between sighted pupils and class teachers, I insert an episode from one group of the sighted participants, for example class teachers, by clarifying that the particular issue is manifested in the same way in the other group of the sighted participants, for example sighted pupils. Also, in case that an issue is not identified in all the groups of the sighted participants, I clarify this too in the data analysis chapters.

In the above ways, answers to my two research questions will be tailored to each group of the sighted participants and with regard to each particular issue.

Phase 2 is discussed towards the end of each of the three data analysis chapters. I refer to Phase 2 in relation to Phase 1. More specifically, after I summarise the issues that were identified from Phase 1 on the particular theme, I then report how these issues were addressed in Phase 2. Reference to Phase 2 is made in this way due to the very small number of the Phase 2 lessons. Despite their small number though, the existent Phase 2 lessons are substantial and enable me to provide answers to my research questions, modestly. The three Phase 2 lessons also set the foundations towards collaborative design of mathematics lessons between teachers and researchers that is based upon findings from the exploratory phase and that aims towards the creation of more inclusive mathematics classrooms.

The Phase 1 and Phase 2 episodes inserted to illustrate a particular issue are mostly from the same classroom. Exception constitutes the S1Y3 class, in which only Phase 1 data was collected.

A significant criterion for my selection of episodes to illustrate a particular issue is the existence of visual data that evidence the issue in the particular episode. Visual data provide additional strength in the substantiation of an issue, making the issue even more solid. In this respect, I mostly select episodes from the video-recorded lessons. However, in case that there are significant differentiations between videorecorded and non-videorecorded episodes in the illustration of the particular issue, I also select an episode from a non-videorecorded lesson in order to illustrate this issue with further clarity, wholistically, by showing all its aspects.

As video-recorded lessons concerned only half of the classes, I try to maintain a balance in my selection of episodes across the four classes. More specifically, in cases in which visual data is necessary for the clarity of the issue, I select episodes from the two classes (S1Y3 and S2Y5) in which video-recorded lessons took place. In cases in which visual data is not necessary for the clarity of the issue, I mostly select episodes from the other two classes (S3Y1 and S4Y5).

Visual data is taken from stills of the videos. When participants' faces are shown in the images, I use pixilation of the faces. I pixelate the faces in such a way that I maintain: the anonymity of the participants (as the participants are not recognisable in the images); and, the facial elements that are pertinent in the analysis (if applicable). With regard to the facial elements, I use light pixilation when participants' faces play a significant role in the purposes of the images. Similarly, I use heavy pixilation when participants' faces happen to be shown in the images but do not play any role in the purposes of the images.

Having described the Methodology of the study, I will now move on to the data analysis in the next three chapters.

Chapter 5: The role of speech and gesturing in the mathematical learning of visually impaired pupils

5.0 Introduction

This chapter presents the different manifestations of speech and gesture use of class teachers and sighted pupils of the mathematics classroom and the impact that the particular manifestations have upon the inclusion, or enabling, of VI pupils.

Central role in this chapter plays the notion of tool mediation. Speech and gestures constitute semiotic tools and are used by class teachers and sighted pupils in the mathematics classroom as elements of mathematical expression. As tools that are used in the mathematical activity, speech and gestures mediate the mathematical learning of VI pupils. In particular, and in line with Vygotsky (1978), speech and gestures transform the cognitive activity of the pupils, who engage with these semiotic tools in the mathematics classroom. More specifically, characteristics of the particular speech and gesture contribute to the restructuring of the pupils' cognitive activity – and therefore to these pupils' mathematical learning.

Speech and gestures do not act alone in the mathematical learning of VI pupils. Sensory organs through which speech and gestures are to be accessed by the VI pupils act as tools too, therefore they too – and alongside the semiotic tools – mediate the mathematical learning of the VI pupils.

The headings 5.1-5.3 indicate the different combinations of speech and gesture use as these combinations were evident in the mathematical discourse of class teachers and sighted pupils: presence of gestures and absence of speech (5.1); presence of gestures and partial absence of speech (5.2); and, presence of gestures and presence of speech (5.3).

Each of these three headings is followed by sub-headings, which indicate the different issues related to the impact that the particular manifestations have upon the inclusion, or enabling, of VI pupils. The phrasing of each issue consists of two parts.

The first part of each of the issues includes the particular manifestation of speech and gesture use of the sighted members. The phrasing of each manifestation is done in a way that shows the mediating role of not only semiotic tools but also sensory tools. The mediating role of sensory tools is indicated in my insertion of a sensory characterisation for access to gestures. I do not insert a sensory characterisation for access to speech, as speech is always accessed auditorily. In each manifestation of speech and gesture use, there is intertwinement of semiotic and sensory tools. More specifically, there is intertwinement of speech and/or gesture and the sensory organ(s) that the speech and/or the gesture require the pupils to use in order to access them and therefore in order to participate in the mathematical learning.

To exemplify the aforementioned characteristics of the first part of each of the issues, I will talk about the issue 5.1.1. The first part of this issue involves the use of visually perceptible gestures without speech. The mediating role of sensory tools is indicated in my insertion of "visually perceptible" for access to gestures. In this issue, there is intertwinement of gesture and vision: vision is required for the pupils to use in order to access the gesture and therefore in order to participate in the mathematical learning.

The second part of each of the issues includes the impact that the particular manifestation of speech and gesture use has upon the inclusion, or enabling, of VI pupils. In all but 5.2.2 issues[23], the second part is phrased with regard to inclusion: particularly in relation to participation, which constitutes one of the two elements of inclusion. Value, which constitutes the second element of inclusion, is not applicable in this chapter and is discussed in Chapter 7. In 5.2.2 issue, the second part is phrased with regard to disability. I phrase some issues with regard to inclusion and other issues with regard to disability, because not both notions

[23] The issue 5.2.2 is "Use of partial speech with visually perceptible gestures provokes the emergence of disability in the lesson".

have a critical role in each issue. The significance of each issue is determined by the notion that is included in the phrasing of each issue. For example, in the issue 5.1.2[24], the fact that the use of auditorily perceptible gestures[25] without speech was a practice of enabling is not as significant as the fact that it was a practice of including.

Each of the sub-headings is followed by one or two episodes, illustrating various facets of the issue under examination, and their analyses. In cases where a sub-heading is illustrated in more than one way across my data, another episode is presented and analysed in order to show a different way that the sub-heading was evident. All the presented episodes come from Phase 1 of the study and from video-recorded lessons.

The presented episodes come from video-recorded lessons because the nature of this chapter is such that visual evidence is necessary for the clarity of the episodes. As in Phase 1 my video-recorded lessons are in School 1 and School 2, I include episodes from these two schools. As the same phenomena were observed in both schools, I maintained a balance between the two schools in my selection of the episodes. More specifically, I selected five episodes from School 1 and four episodes from School 2.

Table 3 below illustrates the structure of 5.1-5.3:

			Episodes
			School
Issues	5.1 Presence of gestures and absence of speech	5.1.1 Use of visually perceptible gestures without speech mediates VI pupils' reduced opportunities for participation in the lesson	School 2
		5.1.2 Use of auditorily perceptible gestures without speech mediates VI pupils' participation in the lesson	School 1
		5.1.3 Use of visually perceptible gestures without speech mediates VI pupils' reduced opportunities for participation in the lesson: TA's intervention mediates participation	School 1
	5.2 Presence of gestures and partial absence of speech	5.2.1 Use of partial speech with visually perceptible gestures mediates VI pupils' reduced opportunities for participation in the lesson	School 2
		5.2.2 Use of partial speech with visually perceptible gestures provokes the emergence of disability in the lesson	School 1
		5.2.3 Use of partial speech with visually perceptible gestures mediates VI pupils' reduced opportunities for participation in the lesson: TA's intervention mediates participation	School 1
	5.3 Presence of gestures and presence of speech	5.3.1 Use of visually perceptible gestures that serve an isomorphic role compared to speech mediates VI pupils' participation in the lesson	School 2
			School 2
		5.3.2 Use of visually perceptible gestures that serve an isomorphic role compared to speech mediates VI pupils' participation in the lesson: TA's intervention mediates 'extra' participation	School 1

Table 3: Structure of 5.1-5.3.

[24] The issue 5.1.2 is "Use of auditorily perceptible gestures without speech mediates VI pupils' participation in the lesson".
[25] The auditorily perceptible gestures are the beat gestures in the sense of McNeill (1992).

I continue this chapter with a Synthesis of findings (5.4), where I summarise the findings from Phase 1 with regard to the different manifestations of speech and gesturing. I end this chapter with Phase 2 of the study (5.5), in which I present: how findings identified in Phase 1 – and synthesised in 5.4 – were considered in the design of the Phase 2 lessons; examples from the implementation of the Phase 2 lessons, with the examples tailored to speech and gesture use; and, the impact that the implementation principles on speech and gesture use had upon teaching staff and pupils.

5.1 Presence of gestures and absence of speech

5.1.1 Use of visually perceptible gestures without speech mediates VI pupils' reduced opportunities for participation in the lesson

Episode:

In the Y5 class of School 2, on the Interactive Whiteboard (IWB), in a rectangular yellow screen, three black thick parallel horizontal line segments are displayed. T2 has completed his working out of 38÷4 on the top line segment, which he has used as a number line. He has worked this division out by drawing nine jumps of 4 and one jump of r2 (remainder 2), with the numbers on his number line being as follows: 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 38.
After he has completed this calculation, and has found 9r2 as the quotient of 38÷4, the following takes place in the classroom:
T2: "Ok. Can anyone think of an easier way to do that?"
He then writes 0 on the left of the second line segment and 38 on the right of this segment (Figure 1).
Some pupils do not see T2 writing on the IWB, and Luke does not look at his iPad to access T2's writing on the IWB.
Nobody in the class responds to T2's question.

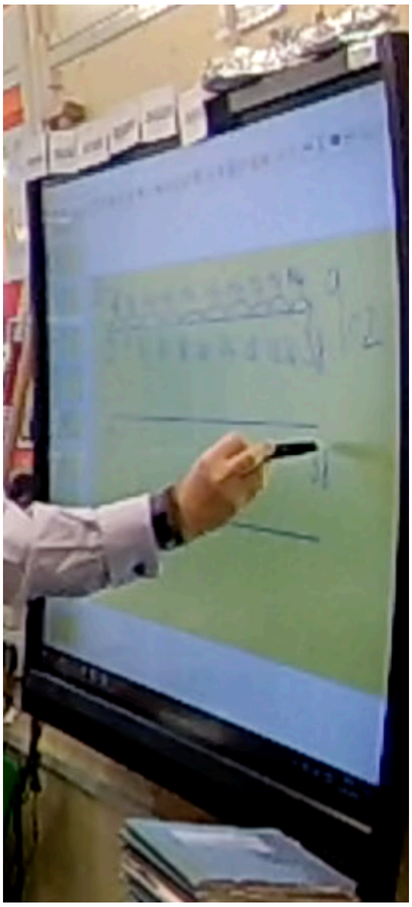

Figure 1: T2 writes 38 on the right side of the second line segment on the IWB without speaking.

Comments:

This episode shows that a sighted person's use of gestures which are not accompanied by speech (as to what the gestures show) mediates the VI pupil's reduced opportunity for participation in the lesson. This impact upon the VI pupil's inclusion occurs because the particular speech and gesture use of the sighted person mediates the VI pupil's reduced access to the lesson.

In this episode, T2 uses three gestures of writing on the IWB: one gesture of writing 0; and, two gestures of writing 38 (one gesture of writing 3 and one gesture of writing 8).

All these three gestures are instrumental. The word "instrumental" comes from Rotman's (2009) system of characterisation of gestures: "[i]n the instrumental mode – pounding a nail, turning a wheel, slicing an apple, swallowing and so on – the body is captured by a machinic circuit, it becomes a mechanical apparatus (more accurately part of a mechanical transduction), the source of what Andre Leroi-Gourhan calls "technics"" (Rotman, 2009, para. 3). In line with this definition, in this episode, writing is instrumental, therefore the gestures used for its manifestation are instrumental too.

T2 makes these three instrumental gestures of writing on the IWB without speech. He does not even tell the class that he writes on the IWB. His use of gestures without speech may impede Luke's access and participation: as T2 does not inform the class about writing on the IWB – and as this writing clarifies T2's earlier question – Luke is not allowed access to this clarification.

In this episode, the particular speech and gesture use of the teacher impacts not only on the VI pupil's inclusion but also on the sighted pupils' inclusion too. In particular, T2's use of gestures that are not accompanied by speech mediates sighted pupils' reduced access to that part of the lesson. T2's semiotic discourse did not allow sighted pupils, who had not incidentally happened to be looking towards T2 at the time that T2 was writing 0 and 38 on the second line segment, to realise that T2 was writing on the IWB. Therefore, the accompaniment of gestures by speech is necessary for the inclusion not only of the VI pupil but also of the sighted pupils too.

T2 did not deliberately use the particular gestures, which were not accompanied by speech, to exclude any member(s) of the class. After all, he gestured on the IWB, which is located at a place to which every pupil can have access – Luke too can have access to T2's display on the IWB, by looking from his iPad, which is connected to T2's computer via a Virtual Network Computing (VNC) connection. Therefore, while T2's gesturing on the IWB was an enabling practice, the non-accompaniment of his gestures by speech made this practice an excluding one, albeit unintentionally.

Gestures seem to be used so spontaneously by T2 that he does not realise using them. Also, he does not seem to realise that gestures carry some meaning – and therefore impact upon Luke's inclusion. This is related to the view of gestures as subordinant to speech and as forms of mathematical expression that are of secondary attention. An example that evidences T2's approach to gestures in relation to inclusion comes from T2's interview. T2 told me that he does not know if Luke can access every part of the mathematics lesson. He also told me that I might be able to tell him if Luke does. I then told him that when his gestures are inaccessible to Luke and he does not accompany his gestures by a verbal expression or he accompanies them by an insufficient verbal expression, such as "this", then Luke cannot access this part of the mathematics lesson. T2 did not seem to have realised these practices of his and he appreciated me mentioning these to him. Below is his response after I mentioned these practices to him: "Oh, ok. That's a good advice […] That's good advice. Yeah, that's good to me […] Yeah, yes. So it's making sure I emphasise what I'm [showing] […] Ok, thank you, thank you for that."

Other indications showing that T2 does not realise using gestures that impact upon Luke's inclusion are: his positive response to my question on whether he thinks that Luke is always included in the mathematics lessons; and, his statement that he is always very mindful and concerned for Luke's needs. As an example of the latter, T2 offered his use of a yellow background and a contrasting colour on the IWB: "all my interactive boards have a yellow background with blue writing, which I think just makes it a bit more gentler – and all of the children actually, on everyone's eyes". This example shows that T2's slide – and writing – on the IWB, which is the case of this episode, took Luke's perceptual needs into account. However, T2 did not realise that he did not direct Luke to follow the demonstration from his iPad.

The above analysis indicates T2's positive attitude towards the inclusion of VI pupils. However, in practice this does not always exist – there are cases in which T2 is not mindful for Luke's needs. I note that T2 received neither pre-service nor in-service training on the inclusion of VI pupils. His lack of training is related to T2 not realising that he is not always mindful. To sum up, T2 is positive towards Luke's inclusion despite limited attention on the inclusion of VI pupils in ITE.

Apart from class teachers, the issue of 5.1.1 is also identified in episodes in which the presence of gestures and absence of speech are evident in sighted peers. These episodes have a similar impact on the opportunities that the VI pupils are offered to access activity (here a key clarification by the teacher) in the classroom.

In the episode that follows in 5.1.2, the presence of an auditorily perceptible gesture appears to facilitate such access.

5.1.2 Use of auditorily perceptible gestures without speech mediates VI pupils' participation in the lesson

Episode:

In the Y3 class of School 1, the class works on their tables on worksheets, in which they are asked to find the sums of given 3-digit numbers and 1-digit numbers. At some point during the class's work on the calculations, T1a takes a maraca from her desk and starts shaking it in order to attract the attention of the class.
As soon as they hear the sound of the shaking maraca, Fred and Ian, as well as some sighted pupils, stop working on their worksheets, raise their hands and move their fingers (Figure 2).
T1a then explains the next task to the class.

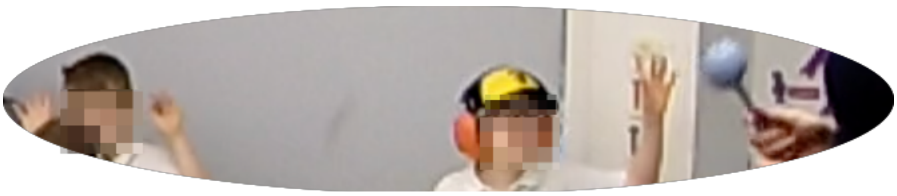

Figure 2: T1a shakes the maraca without speaking. Fred and Ian stop working on their worksheets, raise their hands and move their fingers.

Comments:

This episode shows that a sighted person's use of gestures which are not accompanied by speech (as to what the gestures show) mediates the VI pupil's participation in the lesson. This impact upon the VI pupil's inclusion occurs because the particular speech and gesture use of the sighted person mediates the VI pupil's access to the lesson.

Unlike the gestures of 5.1.1, which are visually perceptible – and only by sighted pupils –, in 5.1.2 the gestures are perceptible auditorily – and by all pupils.

In this episode, T1a uses a gesture of shaking her maraca in order to attract the attention of the class. This gesture is beat. The word "beat" comes from McNeill's (1992) system of characterisation of gestures: beat gestures are perceived auditorily and their significance is based on the auditory feature.

T1a makes this beat gesture of shaking her maraca without speech. She does not even tell the class that she will make this gesture. Her use of gesture without speech affords Fred and Ian the opportunity to participate. Fred and Ian particularly react to T1a's beat gesture: they stop working on their worksheets, raise their hands and move their fingers. Their particular reaction is the one anticipated by T1a. This suggests that the beat gesture has been established in the classroom as a signal for attracting the attention of the class.

In this episode, the particular speech and gesture use of the teacher impacts not only on the VI pupil's inclusion but also on the sighted pupils' inclusion too. In particular, T1a's use of a gesture that is not accompanied by speech mediates the participation of sighted pupils, who had not been looking towards T1a at the time that T1a took her maraca from her desk. As Fred and Ian do, sighted pupils too stop working on their worksheets, raise their hands and move their fingers as soon as they hear T1a's beat gesture.

Therefore, the auditorily perceived gestures are beneficial for the inclusion not only of the VI pupils but also of the sighted pupils too.

Also, unlike the visually perceptible gestures, the auditorily perceptible ones mediate the class's participation in the lesson even if they are not accompanied by speech.

The issue of 5.1.2 is identified only in episodes with class teachers.

In the episode that follows in 5.1.3, the intervention of the TA appears to facilitate the VI pupil's access to the visually perceptible gestures of the teacher.

5.1.3 Use of visually perceptible gestures without speech mediates VI pupils' reduced opportunities for participation in the lesson: TA's intervention mediates participation

Episode:

In the Y3 class of School 1, T1a demonstrates the calculation 238+3 to the class on the IWB. In the beginning, she asks the class how many Hundreds, how many Tens and how many Ones are displayed on the IWB. The IWB displays 238 schematically with pictorial base ten blocks and in columns "100s", "10s" and "1s". Underneath the pictorial representation of 238, there is a pictorial representation of the second addend – 3 – with three units blocks under the 8 units blocks of 238. There is also the symbol "+" as in a column addition. Following pupils' answers, T1a writes the corresponding digit underneath each pictorial representation and then repeats that she does an addition calculation.
T1a then asks Ethan what her number sentence is going to be, by gesturing towards the pictorial representation of 238.
Looking at T1a's gesture, Ethan: "Two hundred. Two…"
T1a: "I've got two hundred – *she points with her index finger towards the pictorial representation of the Hundreds of 238*."
T1a then points with her index finger towards the pictorial representation of the Tens of 238 (Figure 3).
Looking at T1a's gesture, Ethan: "Two hundred and thirty."
T1a then points with her index finger towards the pictorial representation of the Ones of 238.
Looking at T1a's gesture, Ethan: "Eight."
T1a then points with her index finger towards the "+".
Looking at T1a's gesture, Ethan: "Add."
T1a then points with her index finger towards 3, which is the second addend.
Looking at T1a's gesture, Ethan: "Three."
Looking at each of T1a's gestures, TA1a shows 238 add 3 on Fred's iPad, which is connected to T1a's computer through a VNC connection and displays the calculation 238+3 that is on the IWB (Figure 3).
Fred follows the conversation between T1a and Ethan from TA1a and his iPad (Figure 3).

Figure 3: In her interaction with Ethan, T1a points with her index finger towards the pictorial representation of the Tens of 238 without speaking. Looking at T1a's gesture, TA1a shows 238 add 3 on Fred's iPad. Fred follows the conversation between T1a and Ethan from TA1a and his iPad.

Comments:

This episode shows the necessity of the TA's intervention towards the inclusion of the VI pupil. As the teacher's gestures are inaccessible to the VI pupil and as they are not accompanied by speech (as to what the gestures show), the teacher's semiotic discourse would have mediated the VI pupil's lack of participation in the lesson if the TA had not intervened. In that case, the episode would have been inserted in 5.1.1. However, the teacher's gestures were replicated by the sighted TA, who saw the teacher gesturing at an inaccessible place for the VI pupil and made the same gestures at an accessible place for the VI pupil.

In this episode, T1a uses four gestures: one gesture to indicate the pictorial representation of the Tens of 238; one gesture to indicate the pictorial representation of the Ones of 238; one gesture to indicate the addition symbol, "+"; and, one gesture to indicate the second addend, "3".

All these four gestures are deictic. The word "deictic" comes from McNeill's (1992) system of characterisation of gestures: deictic gestures are gestures that are used to indicate a referent.

T1a makes each of these four deictic gestures without speech: she does not even tell the class that she will make these gestures. As in the episode of 5.1.1 with T2, in this episode too T1a does not seem to realise that gestures carry some meaning – and therefore impact upon Fred's inclusion. This is again related to the view of gestures as subordinant to speech and as forms of mathematical expression that are of secondary attention.

Another indication showing T1a's approach to gestures in relation to inclusion is her statement that she is always an inclusive teacher. In particular, T1a told me: "I don't have a problem having them included whatsoever. Everything is blown up, the slides are done, they've got their iPads and the laptops to see what I'm doing." T1a also reported that, when she plans the slides that are projected on the IWB, a vital practice is "thinking about how they are going to know which part I am on. So that's why I've got the 4 different coloured pens. I'll always circle it and say "we're looking at this one", so they know exactly which one to look at and only that within that circle." Therefore, T1a is mindful of the VI pupils' needs when she writes things on the IWB but does not seem to have realised that she should also be mindful about gesturing towards the particular slides. She seems not to realise that she deictically gestures towards the slides and that her deictic gestures impact too upon the VI pupils' inclusion as much as the instrumental gestures – that she refers to – impact.

While T1a does not seem to realise using gestures that impact upon Fred's inclusion, TA1a does. TA1a told me that, unlike the VI pupils, the sighted pupils "can glance at the teacher, see her expressions, watch out her hands."

T1a's use of gestures without speech would impede Fred's participation in the lesson. However, TA1a's intervention of replicating T1a's deictic gestures for Fred on Fred's iPad, which displays T1a's projection on the IWB, mediates Fred's participation in the lesson.

Despite the necessity for the TA's intervention – and the positive impact that her intervention had upon the VI pupil's inclusion –, the TA's intervention in the cases in which the teacher's use of gestures without speech is such that mediates the VI pupil's reduced opportunity for participation in the lesson is limited to this, and to one more, episodes. This suggests that the TA, whose intervention is vital in those cases of the teacher's speech and gesture use, did not intervene in the rest of the cases of 5.1.1 – and, as a result, the VI pupil was not facilitated to participate in the lesson.

However, even when TA1a intervenes, her intervention is not as effective towards the VI pupils' inclusion because there is delay in these pupils' access to T1a's demonstration. TA1a particularly told me that the inclusion of the VI pupils in the mathematics lesson "is difficult because when I'm trying to point to their screen, there's obviously a delay, because I'm not linked up with T1a. By the time T1a has put her hand on a number and then I put my hand on a number, T1a has moved to a next number. There's always gonna be that delay, I suppose." Therefore, TA1a acknowledges that, while her intervention facilitates the VI pupils to access the teacher's demonstration, it is not as effective for the inclusion of the VI pupils due to the fact that it causes a delay in these pupils' access to this demonstration.

The next section (5.2) concerns presence of gestures and partial absence of speech. In the episode that follows in 5.2.1, the presence of a visually perceptible gesture, which is accompanied by partial speech, appears to impede the VI pupil's access to the teacher's activity.

5.2 Presence of gestures and partial absence of speech

5.2.1 Use of partial speech with visually perceptible gestures mediates VI pupils' reduced opportunities for participation in the lesson

Episode:

In the Y5 class of School 2, the class is asked to respond to how they could use resources, which they have on their tables, in order to make a fraction wall. At some point, T2 sees Jessica holding a laminated fraction wall, stands by her and tells the class:
T2: "Use that – *he gestures with his index finger towards the laminated fraction wall (Figure 4)*".
T2's gesture, as well as the laminated fraction wall, are not accessible to Luke, whose sitting place is two tables beyond Jessica's. The laminated fraction wall is not accessible also to sighted pupils whose sitting places are such that do not permit these pupils to face Jessica.

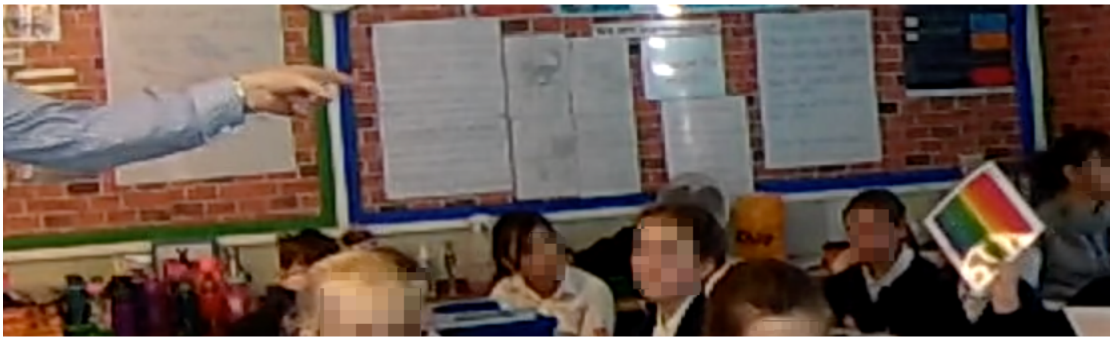

Figure 4: T2 gestures with his index finger towards the laminated fraction wall that Jessica holds while he says "Use that".

Comments:

This episode shows that a sighted person's use of gestures which are accompanied by insufficient speech (as to what the gestures show) mediates the VI pupil's reduced opportunity for participation in the lesson. This impact upon the VI pupil's inclusion occurs because the particular speech and gesture use of the sighted person mediates the VI pupil's reduced access to the lesson.

In this episode, T2 uses a gesture of indicating the laminated fraction wall that Jessica holds. This gesture is deictic, in line with McNeill (1992).

T2 makes this deictic gesture by using partial speech: he tells the class "Use that" but he does not tell them the exact thing to use. The exact thing is shown through his gesture. In this respect, T2's speech is insufficient for the class's participation in the lesson. The only way that T2's semiotic discourse mediates the class's participation is through visual access to his gesture (and to the referent of his gesture). As the gesture is done at an inaccessible place for Luke, T2's discourse mediates Luke's reduced opportunity for participation.

In this episode, the particular speech and gesture use of the teacher impacts not only on the VI pupil's inclusion but also on sighted pupils' inclusion too. In particular, T2's use of a gesture that is accompanied by insufficient speech mediates reduced opportunity for participation of sighted pupils, who sit at places which do not permit these pupils to face Jessica – and to thus access T2's gesture towards the resource that Jessica holds. Therefore, the accompaniment of gestures by sufficient speech is necessary for the inclusion not only of the VI pupil but also of the sighted pupils too.

T2 did not deliberately use the particular gesture, which was accompanied by insufficient speech, to exclude any member(s) of the class. After all, he invited his entire class to share resources which the class could use in order to make a fraction wall. His gesture and speech were part of his attempt to show the class a resource of this kind.

Apart from class teachers, the issue of 5.2.1 is also identified in episodes in which the presence of gestures and partial absence of speech are evident in sighted peers. These episodes have a similar impact on the opportunities that the VI pupils are offered to access activity (here a key indication by the teacher) in the classroom.

In the episode that follows in 5.2.2, the presence of a visually perceptible gesture, which is accompanied by partial speech, appears to provoke the emergence of disability.

5.2.2 Use of partial speech with visually perceptible gestures provokes the emergence of disability in the lesson

Episode:

In the Y3 class of School 1, T1b has demonstrated the working out of the calculation 413-7= using pictorial base ten blocks on the IWB. Fred has followed her demonstration on his iPad, which is connected to T1b's computer through a VNC connection.
After T1b has completed her demonstration, she asks the class to work on one calculation on their whiteboards in the way that she has earlier demonstrated with the calculation 413-7=.
Before she announces the next calculation to the class, she tells the class:
T1b: "Are we listening? I'd like you to do as I'm showing you on the board, not your own personal little method. I'd like you to do like this – *she gestures towards her work of 413-7= on the IWB (Figure 5)*".
After she says "this", Fred looks towards T1b and the IWB.

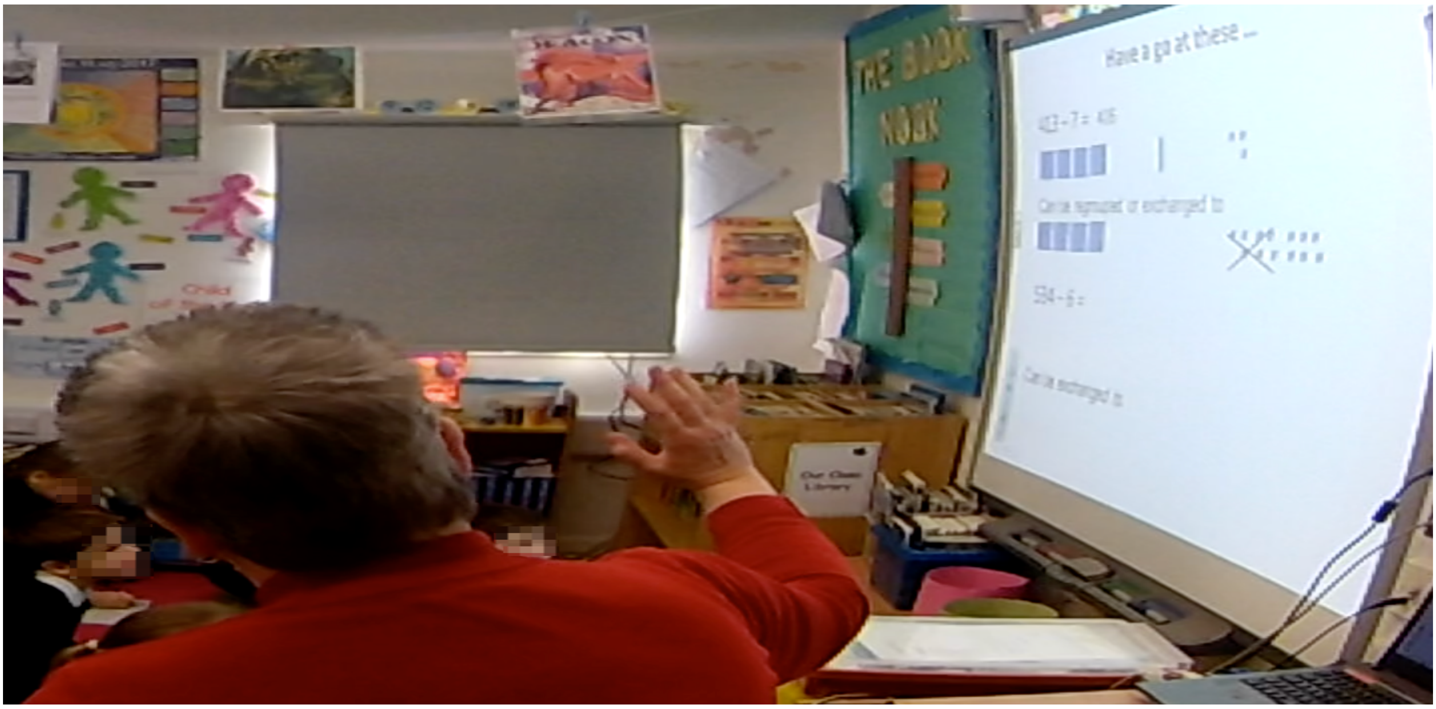

Figure 5: T1b gestures towards her work of 413-7= on the IWB while she says "I'd like you to do like this".

Comments:

This episode shows that a sighted person's use of gestures which are accompanied by insufficient speech (as to what the gestures show) provokes the emergence of disability in the lesson. This impact with regard to disability occurs because the particular speech and gesture use of the sighted person mediates the VI pupil's reaction to this person by using his eyes. The teacher's use of deixis particularly prompts the VI pupil to use his eyes towards the teacher.

In this episode, T1b uses a gesture of indicating her work of 413-7= on the IWB. This gesture is deictic, in line with McNeill (1992).

T1b makes this deictic gesture by using partial speech: she tells the class "I'd like you to do like this" but she does not tell them the exact thing to do. The exact thing is shown through her gesture. In this respect, T1b's speech is insufficient. The only way that T1b's semiotic discourse mediates the class's participation is through visual access to her gesture (and to the referent of her gesture). As the gesture is done at an inaccessible place for Fred, T1b's discourse mediates Fred's reduced opportunity for participation.

The difference in the rationale between this episode and the one of 5.2.1 is the VI pupil's reaction to the teacher's mathematical discourse. While in 5.2.1 the VI pupil did not react to the teacher's mathematical discourse, in 5.2.2 the VI pupil reacted. He particularly reacted by using his limited vision towards the teacher and the IWB. In 5.2.2, the teacher's mathematical discourse provoked the emergence of disability: the VI pupil looked towards the teacher and the IWB but he was made disabled to access the teacher's demonstration. In 5.2.1, the teacher's mathematical discourse did not provoke the emergence of disability: the VI pupil did not use his limited vision towards the teacher, therefore he was not made disabled even though the teacher's demonstration was made inaccessible to the VI pupil.

In 5.2.2, unlike the rest of the issues of Chapter 5, the critical impact from the sighted person's speech and gesture use is with regard to disability (while in the rest of the issues the critical impact from the sighted person's speech and gesture use is with regard to inclusion).

The issue of 5.2.2 is identified only in episodes with class teachers.

In the episode that follows in 5.2.3, the intervention of the TA appears to facilitate the VI pupil's access to the visually perceptible gestures of the teacher.

5.2.3 Use of partial speech with visually perceptible gestures mediates VI pupils' reduced opportunities for participation in the lesson: TA's intervention mediates participation

Episode:

In the Y3 class of School 1, T1a demonstrates the calculation 236+4 to the class by using physical base ten blocks. She gives the blocks to three sighted pupils, who are asked to stand up – facing the rest of the class – and represent the Hundreds, the Tens and the Ones, respectively. Each of the three pupils is given a hat with a place value initial on – "H", "T", "O".
Fred follows T1a's demonstration through physical base ten blocks, which TA1a gives to him in her attempt to look at each of T1a's actions and to follow it for Fred.
After T1a has created 236 in blocks and has then given 4 more blocks of Ones to the Ones pupil, she tells the class:
T1a: "Ok. So, we now have, in here – *she stands behind the Ones pupil and touches him on his shoulders* – 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. *While she counts, she takes each of the 10 blocks of Ones from the Ones pupil.* Is there a problem with that? *She gives the 10 blocks of Ones back to the Ones pupil (Figure 6)*".
T1a's demonstration is not visually accessible to Fred.
Fred has 2 blocks of Hundreds, 3 blocks of Tens and 10 blocks of Ones in front of him.
TA1a touches Fred on his arm to raise his attention and pushes the 10 blocks of Ones to him (Figure 7).

Figure 6: T1a gives the 10 blocks of Ones back to the Ones pupil while she says "Is there a problem with that?".

Figure 7: TA1a pushes the 10 blocks of Ones to Fred.

Comments:

This episode shows the necessity of the TA's intervention towards the inclusion of the VI pupil. As the teacher's gestures are inaccessible to the VI pupil and as they are used with partial speech (as to what the gestures show), the teacher's semiotic mathematical discourse[26] would have mediated the VI pupil's reduced opportunity for participation in the lesson if the TA had not intervened. In that case, the episode would have been inserted in 5.2.1. However, the teacher's gestures were replicated by the sighted TA, who saw the teacher's gesturing being inaccessible for the VI pupil and made the same gestures accessible for the VI pupil.

In this episode, T1a uses three gestures: one gesture of touching the Ones pupil on his shoulders; one block of gestures of taking each of the 10 blocks of Ones from the Ones pupil; and, one gesture of giving the 10 blocks of Ones back to the Ones pupil.

Her first gesture is deictic, in line with McNeill (1992). The rest of her gestures are instrumental, in line with Rotman (2009).

T1a makes each of these gestures by using partial speech. In the first case, she tells the class "in here" but she does not tell them exactly where. In the second case, she counts from 1 to 10 but she does not say what this counting refers to and what she does with this counting. In the third case, she asks the class whether there is a potential problem "with that" but she does not tell them the situation in which a potential problem occurs.

In all these three cases, the missing information is provided through T1a's gestures. In this respect, T1a's speech is insufficient. The only way that T1a's semiotic discourse mediates the class's participation is through visual access to her gestures (and to the referents of her gestures).

As the gestures are done at an inaccessible place for Fred, T1a's discourse mediates Fred's reduced opportunity for participation.

However, TA1a's intervention of pushing the 10 blocks of Ones to Fred mediated Fred's participation.

T1a relies on TA1a for the inclusion of the VI pupils in the mathematics lesson when she performs demonstrations by using physical resources. She particularly told me that she needs TA1a to "mirror me if I am using the manipulatives or the operators". However, T1a expressed her preference for the VI pupils to listen to her rather than TA1a. She particularly told me: "I personally would prefer it if, you know, those children just listen to me […] I'd prefer it if mine was the only voice […] [T]here shouldn't be that they need because of visual impairment someone repeating what I'm saying – there's nothing wrong with their hearing."

[26] Details on the teacher's semiotic mathematical discourse are presented in the next three paragraphs.

While in 5.1.3 the teacher's gestures were not accompanied by any speech, in 5.2.3 the teacher's gestures were accompanied by some speech, however the speech was insufficient for the VI pupil to follow the teacher. Therefore, as in 5.1.3, the TA's intervention is important for the VI pupil to participate in the particular part of the lesson.

The next section (5.3) concerns presence of gestures and presence of speech, with the gestures serving an isomorphic role compared to speech. In the episode that follows in 5.3.1, the presence of this kind of visually perceptible gestures appears to facilitate the VI pupil's access to the teacher's activity.

5.3 Presence of gestures and presence of speech

In this section, I refer to gestures that serve an isomorphic role compared to speech. By "isomorphic role", I denote the same role in which gestures and speech play. The same role takes place when gestures and speech show the same meaning of a word, with the gestures expressing this meaning visually and with the speech expressing this meaning verbally.

5.3.1 Use of visually perceptible gestures that serve an isomorphic role compared to speech mediates VI pupils' participation in the lesson

Episode:

In the Y5 class of School 2, 12 pupils stand up at the front making a horizontal line segment with their bodies and facing the rest of the class. Luke is amongst the pupils standing up.
T2 tells the class that these 12 pupils represent a chocolate. After the class has divided the chocolate in half, T2 says:
T2: "So, we've divided the chocolate in half – *he gestures with his palm for "division in half" as if he cuts the chocolate in two equal pieces (T2 stands at a distance close to the sixth and seventh pupils of the horizontal line segment) (Figure 8)*. How will we write that down? What's that word for the bottom of the fraction? *He gestures with his index finger for the "bottom of the fraction" (Figures 9-11)*[27]. Shout it out."
Pupils: "Denominator."

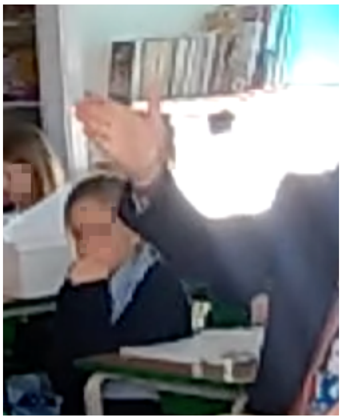

Figure 8: T2 gestures with his palm for "division in half" as if he cuts the chocolate in two equal pieces while he says "So, we've divided the chocolate in half".

[27] Figures 9-11 show the three successive movements that dynamically form T2's gesture for the denominator.

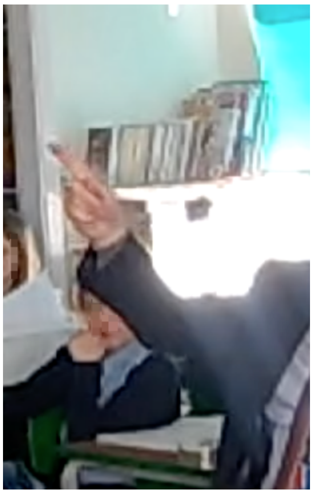

Figure 9: T2 traces the fraction line.

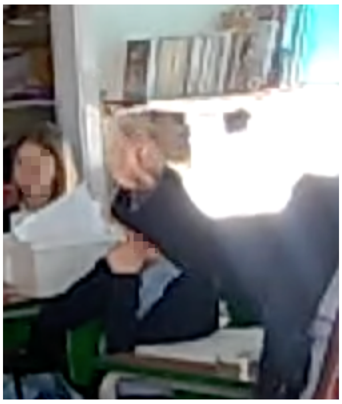

Figure 10: T2 moves his finger lower than the fraction line.

Figure 11: T2 indicates the denominator.

Comments:

This episode shows that a sighted person's use of gestures which are accompanied by sufficient speech (as to what the gestures show) mediates the VI pupil's participation in the lesson. This impact upon the VI pupil's inclusion occurs because the particular speech and gesture use of the sighted person mediates the VI pupil's access to the lesson.

In 5.3.1, the sighted person's speech suffices for the VI pupil's inclusion, with the gestures serving an isomorphic role compared to speech.

In this episode, T2 uses two gestures: one gesture to represent division of the chocolate in half; and, one gesture to represent denominator.

These two gestures are metaphoric. The word "metaphoric" comes from McNeill's (1992) system of characterisation of gestures and its conception resonates with Healy and Fernandes' (2011) interpretation of metaphoric gestures. In particular, Healy and Fernandes (2011) consider a metaphoric gesture as a gesture of a concept for which there is no direct physical manifestation. They offer the example of the limit of a function f(x) as a concept for which a metaphoric gesture can be done. Healy and Fernandes (2011) differentiate a metaphoric gesture from an iconic gesture[28], in that the latter refers to concepts which have a direct physical manifestation.

In this episode, both gestures of T2 are metaphoric in the sense that each of them represents an abstract concept for which there is no direct visual representation. The first gesture represents "division in half" and the second gesture represents "denominator".

The first gesture is also instrumental, in line with McNeill (1992), in the sense that it represents the abstract concept – division in half – with an instrumental action, which is the action of cutting.

T2 makes each of these gestures by using sufficient speech. In the first case, he tells the class "we've divided the chocolate in half" and he does a gesture that represents the division of the chocolate in half. In the second case, he tells the class "bottom of the fraction" and he does a gesture that represents the bottom of the fraction. Therefore, in both of these cases, the gestures serve an isomorphic role compared to speech.

In both of these cases, T2's use of sufficient speech and gesture mediates Luke's participation regardless of the location of the gesture, in other words without the necessity for the gesture to be performed at a visually accessible location for the VI pupil.

As gestures are generally part of sighted people's discourse, the isomorphic way of their use can afford the VI pupil the opportunity to be included in the lesson and can also give flexibility to sighted people, in their gesture use. In other words, the isomorphic gestures constitute the only cases of visually perceptible gestures for which it does not matter whether they are made accessible to the VI pupil. Either accessible or inaccessible, these cases of gestures can afford the VI pupil the opportunity to be included in the lesson.

Apart from class teachers, this aspect of the issue of 5.3.1 is also identified in episodes in which the presence of gestures and presence of speech are evident in sighted peers. These episodes have a similar impact on the opportunities that the VI pupils are offered to access activity (here a key explanation and a key question by the teacher) in the classroom.

In the episode that follows in this same subsection, the presence of visually perceptible gestures that serve an isomorphic role compared to speech is performed on a digital device and appears to facilitate the VI pupil's access to the teacher's activity without the necessity for the VI pupil to use his assistive device.

Episode:

In the Y5 class of School 2, on the IWB, in a rectangular yellow screen, three black thick parallel horizontal line segments are displayed. T2 has nearly completed his working out of 38÷4 on the top line segment, which he has used as a number line. He has worked this division out by drawing nine jumps of 4 and one jump of r2, with the numbers on his number line being as follows: 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 38.
Beth then says that the answer is nine remainder two.
T2: "We've got nine fours – *he writes 9 on the right side of the first number line.* And we've got remainder two – *he writes r2 next to 9 (Figure 12)*".

[28] The word "iconic" also comes from McNeill's (1992) system of characterisation of gestures.

Luke follows T2's writing on his iPad, which is connected to T2's computer through a VNC connection and displays T2's demonstration that is on the IWB.

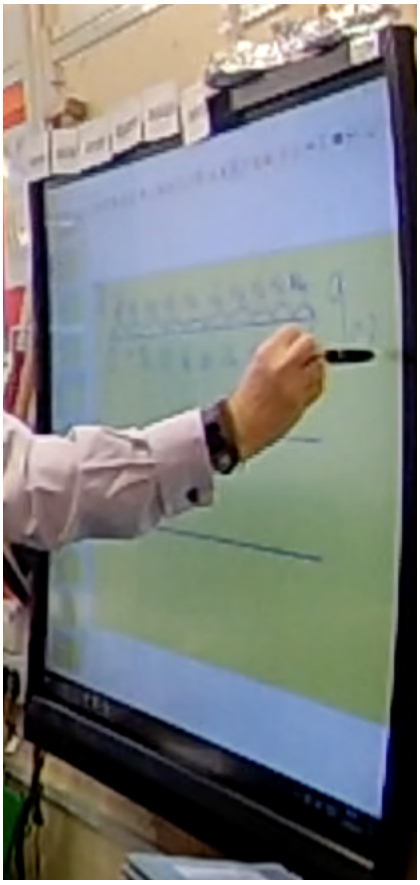

Figure 12: T2 writes 9r2 on the right side of the first number line while he says "We've got nine fours. And we've got remainder two".

Comments:

This episode is representative of 5.3.1 in a slightly different way compared to that of the previous episode.

In this episode, the teacher invites the VI pupil to participate in the lesson through his speech but also through the VI pupil's assistive device, which was not the case of the previous episode.

In this episode, T2 uses three gestures of writing on the IWB: one gesture of writing 9; one gesture of writing r; and, one gesture of writing 2.

All these three gestures are instrumental, in line with Rotman (2009).

T2 makes each of these gestures by using sufficient speech. In the first case, he tells the class "nine" and he writes "9". In the second case, he tells the class "remainder" and he writes "r". In the third case, he tells the class "two" and he writes "2". Therefore, in all these three cases, the gestures serve an isomorphic role compared to speech.

In all these cases, T2's use of sufficient speech and a gesture mediates Luke's participation without the necessity for Luke to use his iPad to access T2's writing on the IWB.

This episode is important to be included in the thesis because it represents cases in which the teacher's use of speech is such that does not necessitate the VI pupil to use his assistive device in order to participate in the lesson. As will be shown in Chapter 6, the use of the assistive device by the VI pupil often brings a range of institutional, environmental and technological issues to the VI pupil. In the cases in which the sighted person's gesture use has an isomorphic role compared to speech, the VI pupil can be asked by the sighted person not to use the assistive device to participate in the lesson.

Apart from class teachers, this aspect of the issue of 5.3.1 is also identified in episodes in which the presence of gestures and presence of speech are evident in sighted peers. These episodes have a similar impact on the opportunities that the VI pupils are offered to access activity (here a key answer by the teacher) in the classroom.

In the episode that follows in 5.3.2, the intervention of the TA appears not to be needed in the VI pupil's access to the teacher's activity.

5.3.2 Use of visually perceptible gestures that serve an isomorphic role compared to speech mediates VI pupils' participation in the lesson: TA's intervention mediates 'extra' participation

Episode:

In the Y3 class of School 1, the class has been working on additions and subtractions of 1-digit numbers to/from 3-digit numbers. After T1a has completed her working out of 214+3 on the IWB, she tells the class:
T1a: "Ok. We're gonna do one more together – *she raises her index finger* – that is gonna be slightly tricky. We've still got 214, this time I'm gonna take away 6. *While she announces the calculation, she writes 214-6= on the IWB (Figure 13).* What am I going to do? Can I take away 6 at the moment from my Ones? *She points towards the pictorial representation of Ones of 214 with her index finger.*"
Fred does not follow T1a's demonstration from his iPad, which is connected to T1a's computer through a VNC connection and displays T1a's demonstration that is on the IWB.
TA1a then shows him T1a's demonstration on his iPad (Figure 14). Fred looks back to his iPad.

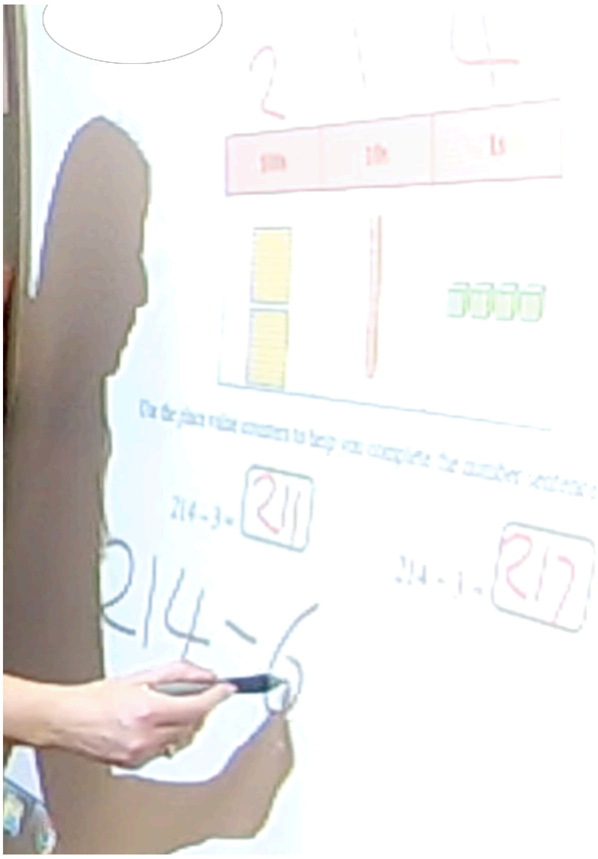

Figure 13: T1a writes 214-6= on the IWB while she says "We've still got 214, this time I'm gonna take away 6".

Figure 14: TA1a shows T1a's demonstration to Fred on Fred's iPad.

Comments:

This episode shows the non-necessity of the TA's intervention for the VI pupil's inclusion. The VI pupil is already included by the class teacher and this is achieved through the teacher's speech.

In this episode, T1a uses three blocks of gestures: one gesture of representing 1; one block of gestures of writing 214-6=; and, one gesture of indicating the pictorial representation of Ones of 214.

The first gesture is metaphoric, in line with McNeill (1992). The second block of gestures are instrumental, in line with Rotman (2009). The third gesture is deictic, in line with McNeill (1992).

T1a makes each of these gestures by using sufficient speech. In the first case, she tells the class "one" and she represents "1". In the second case, she tells the class "We've still got 214, this time I'm gonna take away 6" and she writes "214-6=". In the third case, she tells the class "Can I take away 6 at the moment from my Ones?" and she indicates the pictorial representation of Ones of 214.

In all these cases, T1a's use of sufficient speech and gestures mediates Fred's participation without the necessity for TA1a to intervene and without the necessity for Fred to use his iPad to access the calculation on the IWB.

While the teacher's speech and gestures are such that include the VI pupil in the lesson without him needing to use his assistive device, the TA intervenes by making the VI pupil use this device. This issue shows the TA's emphasis on the VI pupil's use of his assistive device – and therefore of his vision – in the mathematics lesson, even when these elements are not needed for the VI pupil's inclusion.

Having analysed the three different manifestations of speech and gesturing of teaching staff and sighted pupils, I will now present a synthesis of findings across these different manifestations (5.4).

5.4 Synthesis of findings across 5.1-5.3

In Phase 1, I identified three different manifestations of speech and gesturing of sighted pupils and teaching staff in the mainstream mathematics classrooms with VI pupils in which my data collection took place. These manifestations were the following: presence of gestures and absence of speech (5.1); presence of gestures and partial absence of speech – insufficient speech (5.2); and, presence of gestures and presence of speech – sufficient speech (5.3).

The first manifestation of speech and gesturing was operationalised in either of the following two ways: use of visually perceptible gestures without speech; and, use of auditorily perceptible gestures without speech.

The impact of each of the two ways of the first manifestation of speech and gesturing upon the VI pupil's inclusion (and enabling) differed and was determined by two factors: the nature of gesturing; and, the intervention of the TA. More specifically, the use of visually perceptible gestures without speech sometimes did not facilitate the VI pupil's participation in the lesson and other times encouraged this participation. Both of these impacts occurred when the sighted person (teacher and/or peer) gestured at an inaccessible place for the VI pupil, with the VI pupil not being offered the opportunity to access the gesture.

While the use of visually perceptible gestures without speech is an excluding practice, its impact upon the VI pupil's inclusion was determined by the intervention, or not, of the TA. While the VI pupil was discouraged from participating in the lesson when the TA did not intervene, he was facilitated to participate in the lesson when the TA intervened.

In the latter case, the TA enabled the VI pupil to access the gesture that he was not offered the opportunity by the teacher/peer to access. In this case, the TA acted as a mediator between the teacher/peer and the VI pupil. The mediating role of the TA was operationalised in her alerting the VI pupil about the gesture that was performed by the teacher/peer. Her alerting was made either by replicating the gesture at an accessible place for the VI pupil or by asking the VI pupil to use his assistive device to access the teacher's/peer's gesture.

In this way of the first manifestation of speech and gesturing, the TA's role was vital in the inclusion (and enabling) of the VI pupil. This is not the case of the second way of the first manifestation of speech and gesturing.

More specifically, the use of auditorily perceptible gestures without speech facilitated the VI pupil's participation in the lesson. This impact occurred when the sighted person (teacher and/or peer) gestured at an accessible or an inaccessible place for the VI pupil, with the VI pupil not being directly asked to access the gesture.

The auditory gesture allows the VI pupil to participate in the lesson through their sense of hearing. It has the privilege of being performed at a visually inaccessible place for the VI pupil, in other words, it gives flexibility to the teacher/peer regarding their location.

The use of auditorily perceptible gestures without speech is an inclusive practice. Unlike the previous practice, this one does not necessitate the intervention of the TA in order to fulfil the inclusion (and enabling) of the VI pupil. It is instead performed directly from the teacher/peer.

The second manifestation of speech and gesturing was operationalised in the following way: use of partial speech with visually perceptible gestures.

The impact of this way of the second manifestation of speech and gesturing upon the VI pupil's inclusion (and enabling) differed and was determined by the following factor: the intervention of the TA. More specifically, the use of partial speech with visually perceptible gestures sometimes did not facilitate the VI pupil's participation; other times encouraged the emergence of disability in the lesson; and, other times facilitated the VI pupil's participation in the lesson. All these three impacts occurred when the sighted person (teacher and/or peer) gestured at an inaccessible place for the VI pupil, with the sighted person's speech being insufficient for the participation of the VI pupil in the lesson.

While the use of partial speech with visually perceptible gestures is an excluding practice, its impact upon the VI pupil's inclusion was determined by the intervention, or not, of the TA. While the VI pupil was discouraged from participating in the lesson when the TA did not intervene, he was facilitated to participate in the lesson when the TA intervened. In the latter case, the TA enabled the VI pupil to access the gesture that he was offered the opportunity by the teacher/peer – through insufficient speech – to access.

The mediating role of the TA was operationalised in the same way as in the first manifestation of speech and visual gesturing. Also, in this second manifestation too, the TA's role was vital in the inclusion (and enabling) of the VI pupil.

In this second manifestation of speech and gesturing, there was a significant impact with regard to disability: the use of partial speech with visually perceptible gestures often made the VI pupil disabled in the lesson. This impact was evident in a very specific case of exclusion: when the VI pupil looked towards the teacher/peer, as a response to the teacher's/peer's insufficient speech. The teacher's/peer's insufficient speech often encouraged the VI pupil to look towards them, in order to participate in the lesson: due to the fact of their gesturing at an inaccessible place for the VI pupil, the VI pupil was made disabled, as he could not access that gesture.

The third manifestation of speech and gesturing was operationalised in the following way: use of visually perceptible gestures that serve an isomorphic role compared to speech.

The impact of this way of the third manifestation of speech and gesturing upon the VI pupil's inclusion (and enabling) was the following: this practice facilitated the VI pupil's participation in the lesson. This impact occurred when the sighted person (teacher and/or peer) gestured at an accessible or an inaccessible place for the VI pupil, with the sighted person using sufficient speech.

As the auditory gestures, this case of visually perceptible gestures has the privilege of being performed at a visually inaccessible place for the VI pupil, in other words, it gives flexibility to the teacher/peer regarding their location.

The use of visually perceptible gestures that serve an isomorphic role compared to speech is an inclusive practice. As with the auditory gestures, this practice too does not necessitate the intervention of the TA in order to fulfil the inclusion (and enabling) of the VI pupil. It is instead performed directly from the teacher/peer.

However, despite the aforementioned fact about the non-necessity of the TA to intervene, the TA often intervened.

As in the other two cases of the TA's intervention, in this case too, the TA acted as a mediator between the teacher/peer and the VI pupil. The mediating role of the TA was operationalised in the same way as in the other two manifestations of speech and gesturing.

Across the three manifestations of speech and gesturing: the TA's intervention is vital in the first two manifestations of speech and visual gesturing; and, the TA's intervention is not necessary in the third manifestation of speech and visual gesturing.

5.5 Phase 2 of the study

While there were 26 lessons from Phase 1 of the study, there were only 3 lessons from Phase 2. The reasons for the small number of Phase 2 lessons are indicated in Chapter 4. However, despite the small number of Phase 2 lessons, there were significant findings from Phase 2 of the study.

In this section, I present: how findings identified in Phase 1 – and synthesised in 5.4 – were considered in the design of the Phase 2 lessons; examples from the implementation of the Phase 2 lessons, with the examples tailored to speech and gesture use; and, the impact that the implementation principles on speech and gesture use had upon teaching staff and pupils.

In the design of the Phase 2 lessons, the class teachers and I decided not to involve the teaching assistants in the inclusion and enabling of the VI pupils. Instead, we decided to design the lessons in a way that the class teachers were responsible for the inclusion and enabling of these pupils. This principle was already followed in the Phase 1 lessons in S2Y5 and S3Y1 but was not followed in S4Y5, as the teaching assistant was consistently involved in the inclusion and enabling of the VI pupil.

This design principle denoted that the class teachers and I needed to find ways of speech and gesture use that would be inclusive and enabling for the VI pupils – and in a way that the teaching assistants would not be needed as the teaching staff that fulfil the inclusion and enabling of these pupils.

Findings from Phase 1 – as well as literature – on speech and gesture use informed the design of principles for speech and gesture use. More specifically, the class teachers and I decided that the visually perceptible gestures needed to be made visually accessible to the VI pupils (and to the rest of the class too) and/or be accompanied by speech that would be sufficient for the inclusion of the VI pupils (in other words, gestures of isomorphic nature compared to speech).

As we were aware that standing right at the front of the VI pupils to gesture would not always be feasible, we paid particular attention to the use of clear speech that would suffice for the inclusion of the VI pupils irrespectively of the location of the class teachers. In this case, the class teachers would primarily have in mind to use clear, sufficient speech instead of also thinking about their location to gesture.

With regard to auditorily perceptible gestures, we considered this kind of gestures in the design of auditory mathematical tasks: namely, mathematical tasks that could be conveyed, and worked on, through auditory means. We decided that this kind of gestures can be performed irrespectively to speech and to the location of the class teachers in the classroom.

Regarding the implementation of the Phase 2 lessons, the class teachers were the only members of teaching staff who implemented inclusion and enabling of the VI pupils in the mathematics classroom. This means that the issues identified in Phase 1 with regard to the teaching assistants' interventions were not evident in Phase 2.

In practice, the use of visually perceptible gestures without speech – and with the gestures made inaccessible to the VI pupils – occurred but concerned specific sighted pupils rather than the entire class. I insert as an example the case in which T2 made a facial expression of disappointment and had his palms facing upwards – as a reaction to the fact that a sighted pupil repeatedly did not focus on him. The fact that the teacher's gestures were not accessible to the VI pupil did not seem to really impact upon the VI pupil's inclusion, as the teacher's discourse did not seem to concern the entire class but only the particular sighted pupil.

The use of visually perceptible gestures without speech – and with the gestures made inaccessible to the VI pupils – was identified only in the class teachers.

The above impact of the first manifestation of speech and visual gesturing shows the positive impact that the co-designed lesson had upon T2's discourse on inclusion. Also, unlike in Phase 1, the teachers' use of visually perceptible gestures without speech – and with the gestures made inaccessible to the VI pupils – concerned specific sighted pupils and not the entire class. Therefore, it did not really affect the inclusion of the VI pupil.

Unlike in Phase 1, the first manifestation of speech and visual gesturing did not exist in Phase 2 for sighted pupils. The inclusive nature of my study probably made the sighted pupils more aware of the VI pupil's needs and this awareness can explain the non-existence of this manifestation in sighted pupils in Phase 2. Therefore, there seems to have been a positive impact of my presence in the classroom, alongside the co-designed inclusive lesson, upon sighted pupils' discourses on inclusion.

Similarly to the aforementioned manifestation of speech and gesture use, the use of visually perceptible gestures with partial speech – and with the gestures made inaccessible to the VI pupils – occurred but concerned specific sighted pupils. I insert as an example the case in which T2 told Georgia: "You are looking there – *he gestures with his index finger towards Zak, who sits in Luke's table and whom Georgia has been looking at*. You try to get people's attention." The fact that his gesture was not accessible to the VI pupil did not seem to really impact upon the VI pupil's inclusion, as the teacher's discourse did not seem to concern the entire class but only the particular sighted pupil.

The above impact of the second manifestation of speech and visual gesturing shows the positive impact that the co-designed lesson had upon T2's discourse on inclusion. Also, unlike in Phase 1, T2's gestures concerned specific sighted pupils and not the entire class. Therefore, they did not really affect the inclusion of the VI pupil.

Apart from the class teachers, the use of visually perceptible gestures with partial speech – and with the gestures made inaccessible to the VI pupils – was also identified in sighted peers. However, in the case of sighted peers, the aforementioned manifestation of speech and gesture use in Phase 2 often mediated VI pupils' reduced opportunities for participation in the lesson.

The use of visually perceptible gestures with partial speech – and with the gestures made inaccessible to the VI pupils – also provoked the emergence of disability in the mathematics lesson. This occurred in one episode, in S2Y5. While in Phase 2 an important principle of the co-design of the lesson was to enable Luke to access every part of the lesson, in this episode T2's use of partial speech with visually perceptible gestures made Luke disabled. Therefore, it seems that there was no impact of the co-designed lesson on T2's discourses on disability.

The auditorily perceptible gestures of the class teachers and of the sighted pupils mediated participation of the VI pupils in the mathematics lessons. While in Phase 1 the auditory gestures were used as routines in 'non-mathematical' parts of the lesson (for example, as signs for silence), in Phase 2 they were also used as manifestations of 'mathematical' parts of the lesson (for example, as signs in number sequences). As in Phase 1, in Phase 2 too the use of auditorily perceptible gestures was beneficial for the inclusion not only of the VI pupil but also of the sighted pupils too.

As with the auditorily perceptible gestures, the use of isomorphic gestures of the class teachers and of the sighted pupils mediated participation of the VI pupils in the mathematics lessons.

I would now like to pinpoint an aspect regarding the use of isomorphic gestures in Phase 2, which did not exist in Phase 1: the importance of gestures for the VI pupil despite the fact that the speech was sufficient for the inclusion of the VI pupil. There was a Phase 2 episode in which T3a explained the increasing and decreasing number sequences to the class, by accompanying her speech with gestures that show "increasing" and "decreasing", respectively. These gestures were visually accessible to the VI pupil and played a catalytic role in his understanding of increasing and decreasing number sequences (this role is shown in Stylianidou & Nardi, 2019b). Therefore, gestures seem to play a significant role for the VI pupil, even when they serve the same role as speech in a sighted person's mathematical discourse. This is not the case with the sighted pupils, for whom speech is the part of the discourse that this group of pupils pays significant attention to and re-enacts.

In this chapter, I presented the various manifestations of speech and gesture use of the sighted members of the mainstream mathematics classroom with VI pupils. These various manifestations were significant to be explored, as they impact and shape the mathematical meaning making of VI pupils as well as contribute to the development of inclusive classroom practices.

In particular, this chapter provides insights into successful strategies of speech and gesture use as well as strategies that do not lead to the inclusion of VI pupils. In this respect, my findings on the various manifestations of speech and gesture use of the sighted members of the mathematics classroom contribute to the existing literature on speech and gesture use in inclusive mathematics settings.

More specifically, they come to further illustrate, and expand, Sticken and Kapperman's (1998) general pieces of advice to be followed in an inclusive mathematics classroom. They particularly illustrate that in practice – in the mathematics classroom – the insufficient speech is not always avoided. This suggests that insufficient speech is an internal practice of sighted people, probably because they are used to sighted interlocutors. Of course, as Sticken and Kapperman (1998) point out, the avoidance of partial speech and the necessity for sufficient speech are vital practices towards the creation of more inclusive mathematics classrooms.

In my study, I examined the different cases in which partial speech and sufficient speech took place in the classroom and, then, based on the inclusive practices from my data as well as from the literature, I co-designed lessons with the teachers in order to examine potential shifts towards inclusive practices of speech and gesture use.

Apart from contributing to the development of inclusive classroom practices, the various manifestations of speech and gesture use of the sighted members also contribute to the relationships between sensory experience and mathematical knowledge.

To this contribution, I consider speech and gesture as two separate, distinct elements of expression, in line with the sensualist view of speech-gesture relations, undertaken also by Healy and Fernandes (2014). This consideration was elaborated in Chapter 3.

I particularly found that, in cases of presence of gestures and absence of speech (5.1) and of presence of gestures and partial absence of speech (5.2), gestures played a vital role in mathematical communication: such that the non-accessibility to those discouraged the VI pupil from participating in the lesson.

This finding strengthens the significance of gestures in mathematical communication. It opposes to McNeill's (1992) view that gestures should be treated as supplements to speech: as 5.1 shows, gestures can be separate, distinct elements of expression and are not always used together with speech.

This finding further strengthens the embodied, non-dualistic approach of body and brain. In 5.1, with gestures being the only meaningful elements of expression (as speech is absent) and, in 5.2, with gestures being necessary elements of expression (as speech is insufficient to convey mathematical meaning), gestures are vital forms of expression.

The fact that gestures are used in this way by sighted members leads to the following deductions: that gestures constitute vital forms of expression of not only VI people (gestures being vital forms of expression of VI pupils is shown by Healy and Fernandes, 2014) but of sighted people too.

The fact that gestures are vital forms of expression of sighted members constitutes another piece of evidence on the significance of gestures as distinct forms of mathematical expression. In this way, this fact further reinforces the consideration of gestures as elements of mathematical expression. This consideration is particularly necessary for the VI pupils, who – in line with Healy and Fernandes (2014) – use gestures in their mathematical meaning making and expression. As Healy et al. (2016) suggest, in order to include VI pupils in the mathematics classroom, we need to see what these pupils have to say with their hands as much as we see what they say with their mouths and we should not consider speaking with their hands as less intellectual to speaking with their mouths.

As elaborated in Chapter 3, my conceptualisation of gestures is expanded compared to that of Healy and Fernandes (2014). While Healy and Fernandes (2014) focus their consideration of gestures on hand and arm movements, I include as gestures any bodily expressions that carry some meaning and entail some form of communication. My conceptualisation of gestures resonates with Rotman's (2009) definition of gesture: "any body-movement that can be identified, repeated, and assigned significance or affect as a sign, a function, or an experience" (para. 3).

Having concluded the theme of speech and gestures, I will now move into the theme of resources (Chapter 6).

Chapter 6: Intertwinement of digital and physical resources in the mathematical learning of visually impaired pupils

6.0 Introduction

In this chapter, I present the interface – and occasional intertwinement – of physical and digital resources in the ways that mathematical meaning is constructed and conveyed in mainstream mathematics classrooms where VI pupils are present. This theme emerged from my analysis concerning the resources that are used in mainstream mathematics classrooms with VI pupils.

Central role in this chapter plays the notion of tool mediation. Physical and digital resources constitute material tools and are used in the mathematics classroom as sources of mathematical learning. As tools that are used in the mathematical activity, physical and digital resources mediate the mathematical learning of VI pupils.

In this chapter, mediation is used to connect the different resources as they intertwine in the mathematical learning of VI pupils. In line with Vygotsky (1978), one resource mediates learning in a way that is determined by the resource itself (i.e. by characteristics of the particular resource) and by the user (i.e. by the way in which the user accesses the resource, e.g. by the sensory tool that s/he uses to access the resource).

Using the above definition of mediation in the intertwinement of two resources, one resource mediates learning in the two ways described above, plus in line with the second resource. For example, in 6.1, the heading "Digital resources mediate VI pupils' visual access to the teacher's physical demonstration" denotes that a digital resource, which is used as a resource of accessing the teacher's physical demonstration, mediates learning in a way that is determined by the resource itself (i.e. the digital resource), by the user (i.e. by the way in which the VI pupil accesses the digital resource) and in line with the teacher's physical demonstration (i.e. by the way the teacher uses the physical resource in her physical demonstration). These three components make the digital resource act as a mediating tool in the VI pupil's accessing of the teacher's physical demonstration.

The headings 6.1-6.4 indicate the intertwined contributions of physical and digital tools as these contributions were evident in the mathematical learning experiences of VI pupils: digital resources mediate VI pupils' visual access to the teacher's physical demonstration (6.1); digital resources mediate VI pupils' visual access to the teacher's hybrid physical-digital demonstration[29] (6.2); hybrid physical-digital resources[30] mediate VI pupils' visual and tactile access to physical resources (6.3); and, physical resources mediate VI pupils' visual and tactile access to the teacher's digital demonstration (6.4). The four different cases of intertwined contributions of physical and digital tools constitute the four different ways in which physical and digital tools co-existed at the same time in the same part of a mathematical task in the classroom. In this respect, physical and digital tools illustrated a specific part of a mathematical task in particular ways.

The phrasing of each heading is done in a way that shows the mediating role of not only material tools but also sensory tools. The mediating role of sensory tools is indicated in my insertion of a sensory characterisation for access to physical and digital resources.

In each intertwined contribution of physical and digital tools, there is intertwinement of material tools and sensory tools. More specifically, there is intertwinement of physical and/or digital tools and the sensory organ(s) that the physical and/or digital tools require the pupils to use in order to access them and therefore in order to participate in the mathematical learning.

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[29] By "hybrid physical-digital demonstration", I denote a demonstration that involves both physical resources and digital resources.
[30] By "hybrid physical-digital resources", I denote resources that operate with the use of both physical components and digital components.

The four headings, 6.1-6.4, are phrased with regard to participation, which constitutes one element of inclusion (value is another element and is a focus in Chapter 7).

Each of the four headings is followed by one or more episodes, illustrating various facets of the issue under examination, and their analyses. In cases where a heading is illustrated in more than one way across my data, another episode is presented and analysed in order to show a different way that the heading was evident. All the presented episodes come from Phase 1 of the study.

While the headings 6.1-6.4 indicate the intertwined contributions of physical and digital tools, the episodes illustrate intertwinement of varying degrees of success with regard to inclusion. The reasons behind this variability are explored in the comments accompanying each episode.

This chapter includes episodes from Schools 1, 3 and 4. School 2 is not included in this chapter, as the intertwinement of physical and digital resources was not evident in the data collected in that school.

Apart from a heading, which indicates a particular type of intertwinement of physical and digital resources, each episode is also characterised according to the 'location' of the particular type of intertwinement. The different 'locations' of the four types of intertwinement are: in teacher-pupil interactions; in pupil (inter)actions; and, in teacher demonstrations. Therefore, each heading is represented by one or more episodes, which are identified in a particular 'location'. Common to all episodes is a sharp focus on the mathematical learning experiences of the VI pupils.

Table 4 below illustrates the structure of 6.1-6.4:

<table>
<tr><td colspan="3" rowspan="2"></td><th colspan="2">Episodes</th></tr>
<tr><th>School</th><th>Type of data</th></tr>
<tr><td rowspan="10">Issues</td><td rowspan="3">6.1 Digital resources mediate VI pupils' visual access to the teacher's physical demonstration</td><td rowspan="3">Teacher-pupil interactions</td><td>School 1</td><td>Lesson video-recording and lesson note-taking</td></tr>
<tr><td>School 1</td><td>Lesson video-recording and lesson note-taking</td></tr>
<tr><td>School 1</td><td>Lesson video-recording and lesson note-taking</td></tr>
<tr><td>6.2 Digital resources mediate VI pupils' visual access to the teacher's hybrid physical-digital demonstration</td><td>Teacher-pupil interactions</td><td>School 1</td><td>Lesson video-recording and lesson note-taking</td></tr>
<tr><td rowspan="2">6.3 Hybrid physical-digital resources mediate VI pupils' visual and tactile access to physical resources</td><td rowspan="2">Pupil (inter)actions</td><td>School 4</td><td>Lesson audio-recording and lesson note-taking</td></tr>
<tr><td>School 3</td><td>Lesson note-taking</td></tr>
<tr><td rowspan="4">6.4 Physical resources mediate VI pupils' visual and tactile access to the teacher's digital demonstration</td><td rowspan="3">Teacher-pupil interactions</td><td>School 3</td><td>Lesson note-taking</td></tr>
<tr><td>School 1</td><td>Lesson video-recording and lesson note-taking</td></tr>
<tr><td>School 1</td><td>Lesson video-recording and lesson note-taking</td></tr>
<tr><td>Teacher demonstrations</td><td>School 3</td><td>Lesson note-taking</td></tr>
</table>

Table 4: Structure of 6.1-6.4.

I continue this chapter with a Synthesis of findings (6.5). I first summarise the findings from Phase 1 with regard to the intertwined contributions of physical and digital resources in the mathematical learning experiences of VI pupils. I then discuss the obstacles that impeded the successful intertwinement. I close the chapter with Phase 2 of the study (6.6), in which I present: how findings identified in Phase 1 – and synthesised in 6.5 – were considered in the design of the Phase 2 lessons; examples from the implementation of the Phase 2 lessons, with the examples tailored to resources; and, the impact that the implementation of these resources had upon teaching staff and pupils.

6.1 Digital resources mediate VI pupils' visual access to the teacher's physical demonstration

Episodes from teacher-pupil interactions:

Episode:

In the Y3 class of School 1, T1a demonstrates the calculation 240-70 to the class, by using physical base ten blocks. She gives the blocks to three sighted pupils, who are asked to stand up – facing the rest of the class – and represent the Hundreds, the Tens and the Ones, respectively. Each of the three pupils is given a hat with a place value initial on – "H", "T", "O".
After T1a has given 2 blocks of Hundreds to the Hundreds pupil and 4 blocks of Tens to the Tens pupil, she asks the class what she has to do in order to subtract 70.
Fred follows T1a's physical demonstration on his iPad. Using the zooming in function of the iPad's camera, Fred holds his iPad towards T1a's physical demonstration. He sits up, with his elbows in the air (Figure 15).
Following a pupil's response, T1a takes 10 blocks of Tens from her Dienes box and then asks the class why she is exchanging 1 block of Hundreds for 10 blocks of Tens. Pupils attempt to respond to T1a's question and T1a responds back to them to challenge their answers.
At some point during T1a's response to a pupil, Fred changes his posture in his following of T1a's physical demonstration: he now has his back stretched and leaning towards TA1a, with his right elbow on his table (Figure 16). He continues to follow T1a's physical demonstration on his iPad.
T1a then asks the class again why she is exchanging 1 block of Hundreds for 10 blocks of Tens. Attempting to respond to T1a's question, Chris says that there cannot be a Hundred in the Tens' column.
T1a then responds to Chris: "Right. Yeah, you are quite right. I can't move my Hundred into my Tens' column – *she moves the block of Hundreds, which she holds and earlier took from the Hundreds pupil, towards the blocks of Tens that the Tens pupil holds.* It's not allowed."
Before T1a responds to Chris, Fred moves his body as in his previous posture, namely as in Figure 15 (Figure 17). He continues to hold his iPad towards T1a's physical demonstration.

Figure 15: Fred sits up, with his elbows in the air. He holds his iPad towards T1a's physical demonstration.

Figure 16: Fred has his back stretched and leaning towards TA1a, with his right elbow on his table. He continues to follow T1a's physical demonstration on his iPad.

Figure 17: Fred moves his body as in his posture in Figure 15. He continues to hold his iPad towards T1a's physical demonstration.

Comments:

This episode shows that, while the iPad mediates Fred's access to the teacher's physical demonstration – therefore there is successful intertwinement of physical (concrete base ten blocks, teacher's working out of 240-70 with physical blocks) and digital (iPad, display of teacher's working out of 240-70 on the iPad) –, the iPad as a mediating tool is tiring for Fred. It is particularly tiring for his back and arms.

In this episode, it is not a technological reason but it is instead an environmental reason that makes this intertwinement tiring for Fred: it is the posture that the teacher takes to demonstrate the calculation to the class. Her posture limits Fred to sit in a specific way – such that allows him to access the teacher's physical demonstration. As her demonstration is in a higher level than Fred's body, Fred has to hold his iPad towards the teacher's demonstration.

The issue of bodily discomfort associated with the use of the iPad as a mediating tool for Fred's access to the teacher's physical demonstration is also reported by TA1a: "[W]ith Fred holding the iPad as a camera trying to see what the teacher is doing in the front of the class, his arms ache, because he's having to hold the iPad up". TA1a additionally reports that "obviously the image shakes" as a result of Fred's holding the iPad towards the teacher's physical demonstration.

T1a's reliance on technology stems from the emphasis that the school puts on technology as the way to include the VI pupils. T1a particularly states that "[if] we didn't have the technology, I don't know how I would include them. We'd find a way, but it wouldn't be…as easy or as successful as it is now". Therefore, the school shapes T1a's personal philosophy on inclusion and has an impact on the way that she works with VI pupils.

I note that T1a received neither pre-service nor in-service training on the inclusion of VI pupils. She received training on inclusion – as a general topic – during her Postgraduate Certificate in Education (PGCE) course. She particularly said that her training on inclusion was not compulsory but that T1a "chose to do it as an option". She chose to do it because earlier she had worked at a college 1:1 with children who were "removed from their home environment" and "had problems being included in mainstream education". As an in-service teacher, she reported that she has learnt about inclusion from the school's SENDCO, whom she "can go to and speak to about inclusivity and what needs to happen", and "from other teachers [in the classroom], seeing what they do".

T1a emphasised that she learnt more things about inclusion during classroom teaching at school rather than in the university, "because the number of days you spent actually at [the university] […] on the PGCE course has been substantially reduced". "The government decided it wanted people in, teachers in the classrooms more, than doing classroom learning. So I think mine is probably the first year that they did that. So, rapidly reduced down the amount of days at [the university] […] I think that's how you learn most of it, from seeing what they do, learning from other teachers".

While the teacher expects Fred to use his iPad to follow her physical demonstrations, in this episode Fred's use of his iPad is not necessary for his inclusion. This occurs because the teacher's visually perceptible gestures have an isomorphic role compared to her speech, allowing Fred to access the teacher's demonstration by relying on her speech (this resonates with 5.3).

The teacher's use of sufficient speech[31] removes the necessity for Fred to use his iPad to be included in the lesson and it would therefore remove the bodily discomfort associated with using his iPad. Other times though the teacher uses visually perceptible gestures and insufficient speech[32] (this resonates with 5.2) that make the iPad necessary for Fred's inclusion in her physical demonstrations, therefore his bodily discomfort inevitable.

This episode suggests that teachers should do their physical demonstrations in a way that minimises the emergence of bodily discomfort to VI pupils.

- They could do these demonstrations in a way that does not necessitate the VI pupils to hold their iPads in the air or to stretch their bodies (therefore, the bodily discomfort of VI pupils can be minimised)
- They could do these demonstrations right at the front of the VI pupils, with the demonstrations being accessible to the rest of the class too (an example is shown in the last episode of 6.4)
- They could do these demonstrations by using sufficient speech, with the VI pupils relying on the teachers' speech to be included in the lesson

To sum up, while the iPad mediates VI pupils' access to the teacher's physical demonstration – and allows them to independently access the demonstration as their sighted peers do –, its use as a visualising mediating tool is not always efficient. In other words, while the iPad is an inclusive tool for the VI pupils – as it allows them to participate in the lesson as much as their sighted peers –, it is not always as effective for them. In this episode, the limited effectiveness of the iPad is evidenced in the VI pupil's bodily posture – in the second

[31] As in Chapter 5, in this chapter too "sufficient speech" denotes the speech that conveys mathematical meaning in a way that allows the interlocutor to constructively engage with it: the interlocutor does not need to access accompanied gestures in order to be able to constructively engage with the semiotic discourse.

[32] As in Chapter 5, in this chapter too "insufficient speech" denotes the speech that conveys mathematical meaning in a way that does not allow the interlocutor to constructively engage with it: the interlocutor needs to access the accompanied gestures in order to be able to constructively engage with the semiotic discourse.

episode of 6.4, it is evidenced in the VI pupil's eyes. The limited effectiveness of the iPad for the VI pupils' inclusion necessitates the teachers' identification of alternative ways to include the VI pupils in the lesson.

Episode:

After T1a has responded to Chris in the previous episode, she asks the class again why she is exchanging 1 block of Hundreds for 10 blocks of Tens.
Before T1a completes her question, Fred puts his iPad away and makes a facial expression of tiredness and disappointment (Figure 18).
Below is what happens next:
T1a: "Why am I doing that? Why 10 Tens, why not 9 Tens, why not 11 Tens? Why am I swapping 10 Tens?"
Following a pupil's answer, T1a: "You've got to exchange it for something with the same value. So, a Hundred – *she shows the block of Hundreds, which she holds on her left hand* – is exactly the same – *she starts to place the 10 blocks of Tens, which she holds on her right hand, above the block of Hundreds* – as 10 Tens. If I line all these up, on here… – *she continues to place the blocks of Tens above the block of Hundreds.*"
TA1a looks at T1a's physical demonstration. After T1a says "on here", TA1a taps Fred on his right shoulder and lifts the iPad up, giving it to Fred to follow T1a's physical demonstration with the zooming in function of his camera.
Fred takes the iPad by making a facial expression of unhappiness (Figure 19) and follows T1a's physical demonstration.

Figure 18: Before T1a completes her question, Fred puts his iPad away and makes a facial expression of tiredness and disappointment.

Figure 19: TA1a gives the iPad to Fred to follow T1a's physical demonstration. Fred takes the iPad by making a facial expression of unhappiness.

Comments:

In this episode, the intertwinement of physical (concrete base ten blocks, teacher's working out of 10 Tens as 1 Hundred) and digital (iPad, display of teacher's working out of 10 Tens as 1 Hundred on the iPad) works but with Fred's reluctancy to use his iPad, due to the iPad's inappropriateness for his bodily posture.

This episode complements the focus of the previous episode: in this episode, it is even clearer that the iPad is associated with the emergence of bodily discomfort to Fred. The bodily discomfort is so much augmented after Fred's continuous holding of his iPad towards the teacher's physical demonstration that prompts Fred to give the iPad up as a mediating tool for his inclusion in the lesson.

In this episode, the teacher's speech is insufficient for Fred's inclusion and therefore necessitates Fred to use his iPad in order to be included in the teacher's physical demonstration. This shows the teacher's emphasis on vision in the mathematical learning of Fred: her teaching is such that would not allow Fred to access it through a physical tool. Therefore, the TA's intervention is vital for Fred's inclusion.

With the teacher using visually perceptible gestures and insufficient speech and with Fred not using his iPad to follow the teacher's physical demonstration, there should be something to be done in order to include Fred in the lesson. The TA realises that Fred is not included in the lesson and she tries to include him. However, she tries to include him through his iPad, which is the tool through which the teacher tries to include him too and, simultaneously, the tool that resulted in Fred's non-participation in the lesson.

The TA's intervention was necessary for Fred's inclusion, however the way in which she intervened was not as effective for Fred's inclusion. The way in which the TA intervened, which coincides with the way that the teacher includes Fred in her physical demonstrations, shows the TA's – and the teacher's – emphasis on vision for Fred's mathematical learning: the digital tool, iPad, is used as a visualising mediating tool for Fred's inclusion.

In this episode, the TA's intervention could have been more effective for Fred's inclusion if it had not involved Fred's use of his iPad. For example, the TA could have followed the teacher's physical demonstration for Fred, by giving him the physical blocks that the teacher shows each time. In other words, the TA could have been the 'eyes' for Fred – and in this way she could have made the iPad unnecessary for Fred's inclusion. Simultaneously, Fred could have followed the teacher's physical demonstration through physical blocks, which would have enabled him to use not only his vision but also his touch to access the teacher's demonstration (unlike the iPad, which acts as a visualising tool). This more effective way of the TA's intervention – effective for Fred's inclusion – is evident in the episode that follows.

However beneficial the TA's intervention, which involves Fred's use of physical tools to access the teacher's physical demonstration, is, it does have its limitations, which are going to be discussed in the next episode and which involve for example delay in Fred's accessing the teacher's physical demonstration.

Episode:

In the Y3 class of School 1, T1a demonstrates the calculation 236+4 to the class, by using physical base ten blocks. She gives the blocks to three sighted pupils, who are asked to stand up – facing the rest of the class – and represent the Hundreds, the Tens and the Ones, respectively. Each of the three pupils is given a hat with a place value initial on – "H", "T", "O". In the beginning, Fred uses the zooming in function of his iPad's camera to follow T1a's demonstration. Ian's computer does not have such a function and Ian cannot access T1a's demonstration with his assistive device. TA1a, who sits in between Ian and Fred, opens the box of base ten blocks, which is placed on the table in front of her, in order to help Ian follow T1a's demonstration. As soon as she unclicks the box to open, Fred looks at it and puts his iPad away (Figure 20).
TA1a insists that Fred should follow T1a's demonstration from his iPad. Fred refuses. TA1a then gives him physical base ten blocks, as she does with Ian. Fred follows, both visually and tactilely (Figure 21).

There is delay in the VI pupils' following T1a's demonstration, with TA1a trying to look at each of T1a's actions and follow it for each VI pupil.

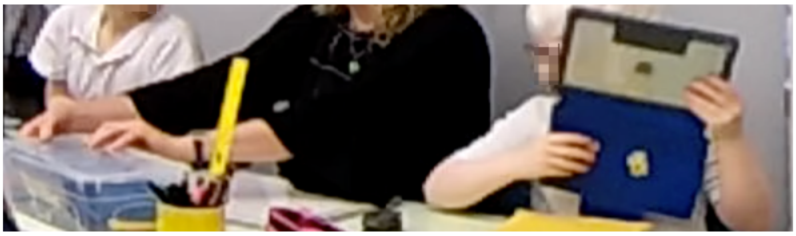

Figure 20: Fred moves the iPad's case to cover its screen when TA1a opens the box with the physical blocks.

Figure 21: Fred works visually and tactilely with the physical blocks. TA1a has given him 220 while the teacher has given 230 to the three sighted pupils – TA1a missed the teacher's earlier giving of 1 block of Ten to the Tens pupil.

Comments:

In this episode, the intertwinement of physical (concrete base ten blocks, teacher's working out of 236+4 with physical blocks) and digital (iPad, display of teacher's working out of 236+4 on the iPad) works until Fred sees the TA opening the box of physical base ten blocks.

While the previous two episodes simply showed the inefficiency of the iPad (which is evidenced in Fred's bodily posture), in this episode an additional issue is also shown on top of that inefficiency: Fred's preference for physical (base ten blocks) over digital (iPad) for his inclusion in the teacher's physical demonstration. This preference may be attributed to the fact that:

- The physical resources do not limit Fred to stand in a specific posture (thus do not cause bodily discomfort to Fred), as Fred is the one who controls the physical tool (thus he can control his body too in his manipulation of the physical tool)
- The physical resources do not make Fred differ from his sighted peers: the stood sighted peers and the teacher use physical resources, thus Fred is not made to feel different from them
- The physical resources allow Fred to use his touch too apart from his vision. However, Fred highly relies on his limited vision to work with the physical blocks, which is related to the tacit sociomathematical norms established in his classroom as to the prevalence of vision in the mathematical learning.

As in the previous episode, in this episode too the TA prioritises Fred's inclusion through the visualising digital tool, which again indicates her consideration of vision as the prevalent sense for mathematical learning. Only when she sees Fred continuously refusing to use his iPad, does she include him through physical tools.

TA1a's priority for Fred's inclusion through the visualising digital tool is also evidenced in TA1a's interview. TA1a said: "If the teacher is [teaching] a lesson at the front, obviously with Fred and Ian having to watch, and it's a little bit further away from where they see, Fred will use his iPad on camera mode, can zoom in". TA1a is also hopeful that Ian will be given an iPad too to follow the teacher's physical demonstration visually: "Ian at the moment doesn't have that iPad, but hopefully he will get the iPad and he'll be able to zoom in and out, just like Fred does."

However beneficial the TA's intervention was for Fred – as he is included through physical base ten blocks which he prefers over the digital visualising iPad –, it had its problems for the two VI pupils' inclusion. The main problem was the delay in the VI pupils' following the teacher's physical demonstration, as the TA tried to look at each of the teacher's actions and to follow it for each VI pupil. This delay, which arises from the TA acting as a mediator between the teacher and the VI pupils, reduced the opportunity for VI pupils' access to the teacher's demonstration. TA1a also told me that she gets stressful when she turns from one VI pupil to the other trying to follow the teacher's physical demonstration.

In order for this problem to be overcome, there should be no 'adult mediator' between the teacher and the VI pupils, as this results to a time lag between the teacher's demonstration and the VI pupils' access to that demonstration. In this respect, the iPad is appropriate, however it causes the issues reported in the previous two episodes as well as in the second episode of 6.4.

As the previous episode, this episode suggests too that the teacher could be the only member of teaching staff responsible for the VI pupils' inclusion and that the VI pupils can be successfully included in the teacher's physical demonstrations through physical tools. A necessary factor for this to happen is the teacher's use of sufficient speech for the VI pupils' inclusion. In this respect, neither the iPad nor the TA are needed for the VI pupils' inclusion – these two compensate for the teacher's use of gestures and insufficient speech.

6.2 Digital resources mediate VI pupils' visual access to the teacher's hybrid physical-digital demonstration

Episode from teacher-pupil interactions:

Episode:

In the Y3 class of School 1, T1a demonstrates the calculation 127+6 with physical base ten blocks on a cardboard under the visualiser and projects her work on the IWB (Figure 22).
Fred follows from his iPad, which is connected to T1a's computer through a VNC connection and displays T1a's demonstration. After T1a projects 1 block of Hundreds, 2 blocks of Tens and 13 blocks of Ones on the IWB, she tells the class that she now has 13 in her Ones and asks them what to do next in order to find the sum. Following a pupil's suggestion, she transfers 3 physical blocks of Ones to the Tens of the sum, saying "Take 3 out of my Ones to my Tens?". Her gesture, as well as the three blocks of Ones in the Tens' column, appear on the IWB and on Fred's iPad. Fred though looks towards the IWB. T1a asks him whether her demonstration appears on his iPad. Fred responds positively and looks at his iPad.

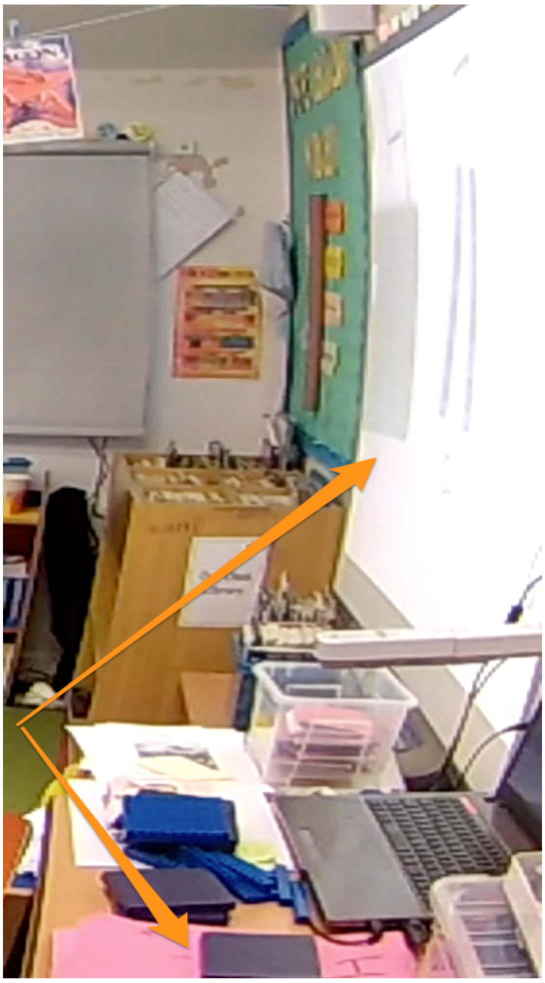

Figure 22: 127 demonstrated with physical blocks on the visualiser and projected on the IWB.

Comments:

In this episode, the intertwinement of hybrid physical-digital [visualiser, physical cardboard and physical blocks (with the cardboard and the blocks placed on the base of the visualiser), digital representation of the cardboard, of the blocks, and of teacher's actions to the blocks, on the IWB] and digital (iPad, display of the cardboard, of the blocks, and of the teacher's actions to the blocks, on the iPad) works until Fred stops following the teacher's demonstration on his iPad and looks instead towards the IWB.

The teacher's practice has been inclusive for Fred. It is particularly significant in that it allows the teacher to use gestures that are not accompanied by sufficient speech, which is something that sighted people often do in their discourse – as they are used to having sighted people as interlocutors. Therefore, teachers' hybrid physical-digital demonstrations make teachers flexible in their mathematical discourse – they do not need to be concerned about their use of gestures and speech, as their gestures are made accessible to the VI pupils on their iPads.

While though this is an inclusive practice, it does have its limitations. The most important limitation is that teachers need to constantly gesture under the visualiser, which is something that is not feasible to be done frequently unless there is another reason too for teachers to use the visualiser. The reason can be a demonstration with physical tools that is projected on the IWB, which is the case of this episode.

Another limitation, which is evident in this episode, is on Fred's attitude to his iPad. The fact that Fred looks towards the IWB, from which he cannot access the teacher's demonstration, and his simultaneous abandonment of his digital tool (iPad), through which the teacher has included him, may be attributed to his wish to resemble his sighted peers, who look directly from the IWB, and to simultaneously show – to his classroom environment – of his negativity in being the only one who is asked to use a special tool for his inclusion.

While the VI pupil's wish to resemble his sighted peers is not explicitly reported by Fred or by his teaching staff, it is explicitly reported by TA4. In particular, TA4 told me that "Ivor is sometimes reluctant to use the assistive devices because he wants to look like his sighted peers". She also told me that, without the assistive devices, "he feels that he is not very different to his sighted peers".

Therefore, in this episode, while the teacher's practice mediates Fred's access to her gestures and to her demonstrations, it is not as efficient for Fred's inclusion.

6.3 Hybrid physical-digital resources mediate VI pupils' visual and tactile access to physical resources

Episodes from pupil (inter)actions:

Episode:

In the Y5 class of School 4, the class are given a worksheet with printed angles and are asked to name each angle according to its type (acute, right, obtuse, reflex). T4a has put Wikki Stix above the printed angles of Ivor's worksheet. TA4 is not in the classroom.
At some point during the class's work on the worksheet, T4a passes from Ivor's table.
She notices that Ivor struggles with remembering the names of the types of the angles and she then asks that they two write a reminder on Ivor's maths book with the name of each type of angles and a definition for it. She tells Ivor: "It doesn't matter what you write, as long as you can remember it."
After they have written a pictorial definition for reflex angles, in which T4a has drawn a reflex angle after "Reflex=", T4a asks Ivor to see this reflex angle on his visualiser.
Ivor places his maths book on the base of his visualiser. He uses the zooming in function of the visualiser's camera, which he points towards his maths book. He navigates his maths book in such a way that the reflex angle appears on the visualiser's screen, zoomed in.
T4a then places a ruler underneath – and in parallel with – the horizontal line segment of this reflex angle on Ivor's maths book in order to compare the reflex angle to a straight line. She says that the reflex angle goes down compared to the straight line.
Ivor sees T4a's demonstration on the visualiser's screen.

Comments:

In this episode, the intertwinement of physical (maths book, reflex angle drawn on the maths book) and hybrid physical-digital (visualiser, digital representation of the maths book and of the reflex angle on the visualiser's screen), which is made possible via the visualiser, works successfully for the inclusion of Ivor in his accessing of a reflex angle.

This intertwinement works for Ivor, as the visualiser – with its zooming in function – allows Ivor to visually access the angle of his maths book, which he could otherwise struggle to access if he did not have the hybrid physical-digital tool.

What Ivor experiences with the visualiser is a zoomed in version of the reflex angle that is drawn on his physical maths book. Ivor told me that with the visualiser "I can zoom in as much as I want". The visualiser also allows Ivor to navigate on the zoomed in reflex angle, by moving his physical book in respective directions.

This intertwinement is also experienced by the teacher, who acts on the reflex angle of Ivor's maths book by placing the ruler horizontally – underneath the horizontal line segment of the angle – to help Ivor understand the reflex angle and distinguish it from the straight line. The teacher's experience of this intertwinement makes the teacher more aware of Ivor's needs and therefore more inclusive as a teacher.

As in the previous episode, in this episode too the visualiser is an inclusive tool for the VI pupil. In this episode, it is useful in Ivor's accessing of a mathematical object drawn on a physical resource: particularly, a mathematical object that is not designed in a way that it is visually accessible to Ivor.

While the visualiser is a very useful mediating tool for Ivor's inclusion, it is often not desirable by Ivor. This occurs because Ivor is the only pupil in the class who uses this tool in the lesson.

The visualiser also does not facilitate collaborative work between Ivor and his sighted peers. In particular, T4a told me that sighted pupils find it challenging if they work with Ivor in pairs or in groups "because if you have Ivor, he may then have to put it under the visualiser, he has to then adjust it so he's got it, but then the others need to move it or they don't need that, so he is a couple of steps behind".

Episode:

In the Y1 class of School 3, T3a asks the class to sit on their tables and work on a time task with their partners. She asks Ned to go with his partner to the table where the visualiser is and instructs him to work on the visualiser. She gives A4 worksheets to the pairs of sighted pupils and an A3 worksheet to Ned and his sighted partner.
After he has placed the A3 worksheet on the base of the visualiser, Ned looks at the task from the paper per se. He does not look at the screen of the visualiser, in which he can have a zoomed in version of the task.

Comments:

In this episode, the intertwinement of physical (A3 paper, time task printed on this paper) and hybrid physical-digital (visualiser, digital representation of the paper and of the time task on the visualiser's screen), which is made possible via the visualiser, is not experienced by Ned in his work on the time task with his sighted partner.

This intertwinement is not experienced because Ned looks only at the physical resource – the time task from the A3 paper – and not at the digital resource – how the time task from the A3 paper appears on the visualiser's screen, zoomed in. This occurs because the time task in the paper is already designed to meet Ned's perceptual needs, thus the visualiser is not needed for him.

Unlike the teacher in the previous episode, who herself realised the need for the VI pupil to use the visualiser, the teacher in this episode does not seem to have realised that the VI pupil does not need the visualiser to access the time task. In this way, she tries to include him with a tool that he does not need.

T3a told me that she struggles to support Ned in the mathematics lesson. This struggle stems from the lack of training on the inclusion of VI pupils. I note that T3a received neither pre-service nor in-service training on the inclusion of VI pupils. She received training on inclusion – as a general topic – during her PGCE course. She particularly said that the university had a general course (lectures) about inclusive learning. This course referred to pupils with disabilities in general. As an in-service teacher, she reported that she has learnt about inclusion from the following stakeholders as the ones who help her to include the VI pupil:

- The reception teacher, who had Ned when he was in Reception
- Ned's mum, who tells her what helps Ned
- TA3a, who advises her about how to support Ned in the classroom. The school has asked TA3a to have a training on visual impairment.

As T1a, T3a also seems to believe that the School-based support on the inclusion of VI pupils is much better than the University-based support. She confessed that, in the University, she has learnt nothing practical –

but only theoretical stuff – on inclusion. However, in School, she has learnt more practical stuff, which she said are more useful for her.

However, despite her lack of training, T3a tries to include Ned in the mathematics lesson. She particularly reports that she uses her logic to include him, with one of her reported practices being asking Ned to use the visualiser.

This indicates T3a's positive attitude towards the inclusion of VI pupils. However, in practice this way of inclusion is not always successful. To sum up, T3a is positive towards Ned's inclusion despite limited attention on the inclusion of VI pupils in ITE.

6.4 Physical resources mediate VI pupils' visual and tactile access to the teacher's digital demonstration

Episodes from teacher-pupil interactions:

Episode:

In the Y1 class of School 3, the class are given a worksheet with clock faces and are asked to write what time it is underneath each clock face. T3a gives Ned a whiteboard in which she writes "o'clock" and "half past" for him, with large font. Earlier, T3a wrote the same phrases on the IWB. Ned looks from his whiteboard while the rest of the pupils from the IWB. All aim to spell the time phrases on their worksheets correctly.

Comments:

In this episode, the intertwinement of digital (IWB, time phrases written on the IWB) and physical (whiteboard, time phrases written on the whiteboard) works successfully for the inclusion of Ned in the lesson. Ned is included in the mathematical task albeit through a different mediating tool (a whiteboard, physical) to the one that his sighted peers use (IWB, digital). The physical tool is vital thanks to the flexibility in its location as well as in size (of the artefact as well as of the mathematical representation on it).

This episode is similar to the one that follows, but in the following episode the emphasis is placed on the TA, not on the teacher, for the VI pupil's inclusion through a physical tool in a teacher's digital demonstration.

Episode:

In the Y3 class of School 1, T1a writes out loud 176+40 as a column addition on the IWB.
Fred's iPad is connected to T1a's computer through a VNC connection and displays T1a's writing that is on the IWB.
Fred has been rubbing his eyes since T1a's earlier demonstration on the IWB, which had involved the class to find the missing number of a bar model whose given numbers were 176 and 40.
TA1a writes the column addition on her whiteboard while T1a is writing it on the IWB (Figure 23).
Fred continues to rub his eyes (Figure 23). He then makes a semi-circular movement with his head, as if his nape hurts (Figure 24).
Below is what happens next:
T1a tells the class: "You must line – *she vertically gestures up and down towards the Ones' column.* […] You must line your Ones up – *she vertically gestures towards the Ones' column.* You must line your Tens up – *she vertically gestures towards the Tens' column.* You must line your Hundreds up – *she vertically gestures towards the Hundreds' column.*"
T1a then draws a horizontal line for equality, as in the column addition.
As soon as T1a has started to speak, Fred turns his head towards TA1a. TA1a then taps him on his right arm and shows him the Ones', the Tens' and the Hundreds' columns on her whiteboard. Fred looks towards TA1a and follows T1a's working out of the calculation from TA1a and her whiteboard (Figure 25).

Figure 23: TA1a writes the column addition on her whiteboard while T1a is writing it on the IWB. Fred continues to rub his eyes.

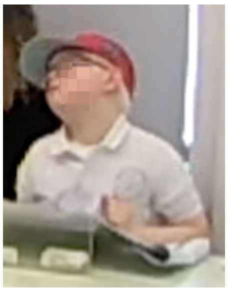

Figure 24: Fred makes a semi-circular movement with his head, as if his nape hurts.

Figure 25: Fred follows T1a's working out of the calculation from TA1a and her whiteboard. TA1a looks at T1a's demonstration on the IWB.

Comments:

In this episode, the intertwinement of digital (IWB, representation of 176+40 on the IWB) and physical (whiteboard, representation of 176+40 on the whiteboard) is fulfilled thanks to the TA and it works successfully for Fred's inclusion in the lesson.

In this episode, the teacher's speech is insufficient for Fred's inclusion and therefore necessitates Fred to use his iPad in order to be included in the teacher's digital demonstration. As in the second episode of 6.1, this episode too shows the teacher's emphasis on vision in the mathematical learning of Fred: her teaching is such that would not allow Fred to access it through a physical tool. Therefore, the TA's intervention is vital for Fred's inclusion.

Therefore, the teacher's reliance on Fred's use of his visualising assistive device to follow her demonstrations takes place both in her physical demonstrations (episodes of 6.1) and in her digital demonstrations (episodes of 6.4). The prevalence of vision for Fred's mathematical learning is therefore clear in any type of the teacher's demonstrations.

With the teacher using visually perceptible gestures and insufficient speech and with Fred not using his iPad to follow the teacher's digital demonstration, there should be something to be done in order to include Fred in the lesson. The TA realises that Fred is not included in the lesson and she tries to include him. Unlike in the second episode of 6.1, in this episode she tries to include him through a physical tool, the whiteboard. His inclusion through this physical tool is successful because the physical tool does not have the drawbacks of the digital tool – making Fred's eyes hurt.

Her use of the whiteboard is one of the practices which TA1a reported that she implements to make sure that the VI pupils are included in the mathematics lesson. She particularly told me: "I have the whiteboard and, if it's a number line, I quickly draw a number line. That's why I've got a really big whiteboard and just quickly draw it on the whiteboard so that they've got it in front of them".

An advantage of the whiteboard compared to the VI pupils' digital devices is that it allows mathematical work to be written with as big a font size as appropriate for the VI pupils' needs. However, despite this advantage, TA1a told me that sighted pupils could face a difficulty working with VI ones in the mathematics lesson as they would write on the whiteboard using a font size that is appropriate only to them: "if children are not experienced in working with Fred, they would just write normally on the whiteboard, on the maths book, and they would just assume that Fred will read it. But I think that this is lack of knowledge, isn't it?"

An additional advantage of the whiteboard compared to the VI pupils' digital devices is that it allows the VI pupils to focus on one mathematical representation. TA1a told me that "on the whiteboard they've just got one sum, one calculation to concentrate on. On T1a's board [and therefore on the VI pupils' digital devices], it probably becomes quite overcrowded, quite messy […] So I think the whiteboard, having the really big white clean board helps because I can just focus on one particular sum, on one particular operation and it doesn't become too busy or overcrowded".

However beneficial the TA's intervention, which involves Fred's use of a physical tool to access the teacher's digital demonstration, is, it does have its limitations, some of which are the VI pupils' dependence upon the TA and their not believing in their mathematical abilities.[33] Fred's dependence upon TA1a was also reported to me by T1a, who added that Fred works better when TA1a is not in the classroom.

Episode:

In the Y3 class of School 1, T1a demonstrates the calculation 214 – 3 to the class on the IWB. In the beginning, she asks the class how many Hundreds, how many Tens and how many Ones are on the 3-column table projected on the IWB. The table presents 214 schematically with pictorial base ten blocks and in columns "100s", "10s" and "1s" (Figure 26).
Following a pupil's answer, she writes the corresponding digit above each column. Then, telling the class that she will take away 3 from 214, she crosses out three blocks of Ones with a red whiteboard pen (the blocks of Ones are in green). She then writes "211" in red, in the box, which is in black, for the answer to 214 – 3.
Fred's iPad is in front of him and displays T1a's work that is on the IWB.[34] After T1a tells the class that she will take 3 away from 214, TA1a gives Fred 214 in physical base ten blocks. When T1a crosses out 3 blocks

[33] These limitations were evident in other episodes.
[34] It is connected to T1a's computer through a VNC connection.

of Ones on the IWB, Fred takes 4 physical blocks of Ones away with his right hand, without looking at his iPad (Figure 27).

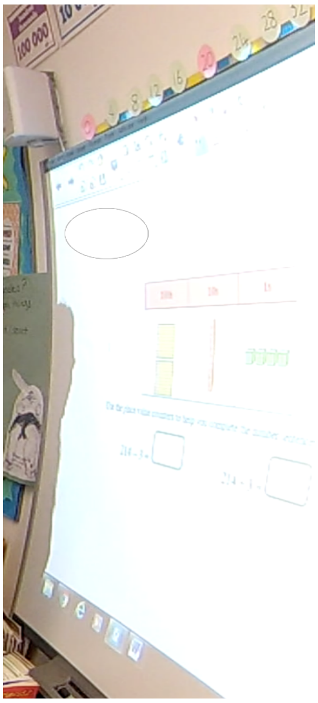

Figure 26: The IWB with the 3-column table in which 214 is represented with pictorial base ten blocks.

Figure 27: Fred pushes 4 blocks of Ones – instead of 3 – with his right hand, without looking at his iPad.

Comments:

In this episode, the intertwinement of digital (IWB, working out of 214-3 on the IWB) and physical (concrete base ten blocks, working out of 214-3 with the blocks) does not work successfully due to the fact that Fred mistakenly follows the teacher's digital demonstration with his physical resources – despite the fact that the teacher is clear that she will take 3 away from 214.

Fred's taking 4, not 3, away may be attributed to one, or a combination, of the following:

- His unfamiliarity with using physical tools to follow teacher demonstrations. He is mostly asked to follow teacher demonstrations through a digital tool.[35]
- His use of vision in his mathematical work with the physical tools.[36] His reliance on his limited vision seems to have prompted him to consider the 4 blocks as 3.
  If Fred had relied on his touch, this mistake could have been avoided.
  The teacher's role is vital in cultivating to Fred that touch can be an equally important sensory tool to vision in Fred's, and in everybody's, mathematical learning.

[35] This evidences the teacher's emphasis on vision in Fred's mathematical learning, which seems to be cultivated in Fred's attitude too.

[36] This also evidences the teacher's emphasis on vision in Fred's mathematical learning, which seems to be cultivated in Fred's attitude too.

The use of physical base ten blocks is one of the practices that TA1a reported as one through which she makes sure that the VI pupils are included in the mathematics lesson. However, she emphasises the visual aspect of this practice: “everything has to be very visual”.

Episode from teacher demonstrations:

Episode:

In the Y1 class of School 3, T3b displays small clock faces on the IWB. The class sit on their carpet places, with Ned sitting on the first line. T3b sits in front of Ned and faces the class. The digital clock faces are in a size which Ned cannot access from his sitting place. Holding a big physical clock, T3b shows on it each time which is on the IWB and he then asks the class to tell him what time he shows. Ned constantly has his hand up to respond.

Comments:

In this episode, the intertwinement of digital (IWB, clock faces represented on the IWB) and physical (concrete clock, clock faces represented by the teacher on the concrete clock) works successfully for Ned’s inclusion in the lesson. This is because the digital clock faces are displayed only for the teacher – for his awareness of the particular times that he has to show to the class – and the physical clock is used for conveying the mathematical task to all in class. Ned would not have been enabled to access the time if the teacher had worked with the digital clock faces at their current size, as they are not accessible to Ned from his sitting place.

The physical tool is vital thanks to its flexibility in its location and in size (of the artefact as well as of the mathematical representation on it).

6.5 Synthesis of findings across 6.1-6.4

In Phase 1, I identified four different cases of intertwined contributions of physical and digital resources in mainstream primary mathematics classrooms with VI pupils. These cases were the following: digital resources mediate VI pupils’ visual access to the teacher’s physical demonstration (6.1); digital resources mediate VI pupils’ visual access to the teacher’s hybrid physical-digital demonstration (6.2); hybrid physical-digital resources mediate VI pupils’ visual and tactile access to physical resources (6.3); and, physical resources mediate VI pupils’ visual and tactile access to the teacher’s digital demonstration (6.4).

6.1 and 6.2 were manifested in teacher-pupil interactions. 6.3 was manifested in pupil (inter)actions. 6.4 was manifested sometimes in teacher-pupil interactions and other times in teacher demonstrations.

In this section, I will first summarise the findings from Phase 1 with regard to the intertwined contributions of physical and digital resources in the mathematical learning experiences of VI pupils and will then discuss the obstacles that impeded the successful intertwinement.

With regard to 6.1, a digital resource allows direct and independent access to the teacher’s physical demonstration. This access is achieved thanks to the external camera on the digital resource, as well as the zooming in function of this camera. In this respect, the digital resource acts as a magnifier of the teacher’s physical demonstration.

A necessary factor for the digital resource to mediate access to the teacher’s physical demonstration is the external camera of the VI pupil’s digital resource to point towards the teacher’s physical demonstration, so that the teacher’s physical demonstration is displayed on the screen of the digital resource.

Despite these advantages, this type of intertwinement has some limitations, whose presence impedes the realisation of intertwinement and limits the two kinds of tools as merely co-existing but not in synergy. The first limitation concerns the VI pupil's bodily posture. There is often a bodily discomfort for the VI pupil when he holds his iPad towards the teacher's physical demonstration. His bodily discomfort is evident in his back and arms. It occurred as a result of the teacher's position in her physical demonstration: her position limits the VI pupil to sit in a particular way – such that allows him to access the teacher's physical demonstration. When her demonstration is in a higher level than the VI pupil's body, the VI pupil has to hold his digital resource lifted upwards. This limitation often made the VI pupil physically tired. As a result, he often gave up the digital resource as a mediating tool for his inclusion in the lesson.

A second limitation in this type of intertwinement is the VI pupil's preference for physical resources over digital ones for his inclusion in the teacher's physical demonstration. This preference is probably related to the first limitation, described above. Indeed, the physical resource does not make the VI pupil stand in a specific posture (thus does not cause bodily discomfort to the VI pupil), as the VI pupil is the one who controls the physical resource (thus he can control his body too in his manipulation of the physical resource). Another potential reason for the VI pupil's preference for physical resources over digital ones is that the physical resource does not make the VI pupil differ from his sighted peers: the sighted peers and the teacher use the physical resource, thus the VI pupil is not made to feel different from them. A third possible reason for the VI pupil's preference for physical resources over digital ones is that the physical resource 'allows' the VI pupil to use his touch too apart from his vision.

A third limitation in this type of intertwinement is the TA's intervention. The TA's intervention is often manifested in the TA's asking the VI pupil to use his digital resource, which the VI pupil had given up for the reasons stated earlier (i.e. because of his bodily discomfort and/or because of his preference for physical resources). Her intervention does not seem to result into changing the VI pupil's mind about not using the digital resource.

The TA's intervention has a further limitation: the delay in the VI pupil's following the teacher's physical demonstration, as the TA tries to look at each of the teacher's actions and follow it for the VI pupil. This delay often deprived the opportunity of the VI pupil to access the teacher's physical demonstration.

With regard to 6.2, a digital resource allows direct and independent access to the teacher's hybrid physical-digital demonstration. This access is achieved thanks to the connection of the VI pupil's digital resource with the teacher's computer.

Pertinent role in hybrid physical-digital demonstrations plays the teacher's use of a visualiser. The visualiser, which allows the teacher to use and manipulate physical resources and to then display her physical work in a digital version (i.e. on the IWB), is particularly significant for the VI pupil's accessing of the teacher's gestures. The VI pupil's accessing of the teacher's gestures is a major concern reported by Quek and Oliveira (2013), who propose ways to overcome this problem through their development of the Haptic Deictic System. The visualiser, alongside the VNC connection, allows the VI pupil direct access – to the teacher's gestures – on his digital resource. It is also particularly significant for the teacher too: it allows the teacher to use gestures that are not accompanied by sufficient speech. In other words, it offers flexibility to the teacher in her speech and gesture use.

Despite these advantages, this type of intertwinement of digital resources and hybrid physical-digital resources is not always successful. This type of intertwinement stops working for the VI pupil's inclusion when the VI pupil abandons his digital resource and instead looks towards the IWB, from which he cannot access the teacher's demonstration. In this respect, the VI pupil is made disabled by the teacher. The disabling is done unintentionally: the teacher encourages the VI pupil to use a digital device to access his teacher's demonstration. The VI pupil though may wish to be not very different from his sighted peers: he may not like being the only one who is asked to use a special resource for his inclusion.

With regard to 6.3, a hybrid physical-digital resource allows direct and independent access to a physical resource. This access is achieved thanks to the VI pupil's visualiser, which allows the VI pupil to visually access a physical resource that is placed on the base of the visualiser. With its zooming in function, it particularly allows the VI pupil to access a zoomed in version of the physical resource. The visualiser also allows the VI pupil to navigate on the zoomed in version, by moving the physical resource in respective directions.

This type of intertwinement is particularly significant when the mathematical objects in the physical resource are not designed in a way that they are visually accessible to the VI pupil.

Despite these advantages, this type of intertwinement of hybrid physical-digital resources and physical resources has some limitations. These limitations may impede intertwinement of hybrid physical-digital resources and physical resources and may limit their productive synergy.

- The first limitation is that the visualiser is often not desirable by the VI pupil. This seems to occur because the VI pupil is the only pupil in the class who uses this resource in the lesson.
- A second limitation is on the teacher's awareness of the necessity of this resource for the VI pupil. The teacher often did not realise that the VI pupil does not need the visualiser to access the physical resource. In this way, she tries to include the VI pupil with a resource that he does not need.

With regard to 6.4, a physical resource allows direct and independent access to the teacher's digital demonstration. This access is achieved thanks to:

- the flexibility in the location of the physical resource (the physical resource can be brought right at the front of the VI pupil);
- the flexibility in the size of the artefact as well as of the mathematical representation on the artefact (the physical resource, as well as the mathematical representation in that resource, can be created at a size that is visually accessible to the VI pupil);
- the non-emergence of eye discomfort to the VI pupil (eye discomfort often emerges in the use of digital resources due to their digitality);
- and, the VI pupil's use of touch to access the mathematical representation (when the mathematical representation can be tactile).

Despite these advantages, this type of intertwinement of physical resources and digital resources has some limitations. These limitations may impede intertwinement of physical resources and digital resources and may limit their productive synergy.

- The first limitation is on the VI pupil's often mistakenly following the teacher's digital demonstration with his physical resource. His mistaken following of the teacher's digital demonstration may be attributed to the VI pupil's non-familiarity with using a physical resource to follow the teacher's demonstration. In line with Leuders (2016), the non-familiarity with a physical resource occupies parts of the VI pupil's attention – as he is trying to familiarise himself with the resource – and may distract the VI pupil from using it productively towards performing a mathematical task. His mistaken following of the teacher's digital demonstration may also be attributed to the VI pupil's use of vision in his mathematical work with the physical resource. The VI pupil's reliance on his limited vision seems to deprive the opportunity of the VI pupil to correctly perform the teacher's digital demonstration.
- The second limitation is on the TA's intervention. The TA's intervention often discourages the VI pupil from working independently and from believing in his mathematical abilities.

The above findings on 6.1-6.4 indicate that physical and digital tools may have been simultaneously present in the lessons but were often not productively intertwined. This seems to be due to circumstantial and systemic obstacles.

- circumstantial obstacles: the bodily position of the teacher, in her physical demonstrations. Her bodily position often resulted in the emergence of bodily discomfort to the VI pupil in his holding of his iPad towards the teacher. This bodily discomfort often made the VI pupil physically tired and resulted in him giving up his iPad.

- systemic obstacles:
    - the VI pupil's reluctance to use his iPad, even when the iPad does not cause bodily discomfort to the VI pupil. This obstacle is systemic because it is rooted in the school's consideration of difference. The school does not always seem to cultivate positive connotations of difference with regard to visual impairment.
      One way that this is evident is through the provision of special digital resources to the VI pupils – iPad and computer. These special digital resources are asked to be used by the VI pupils in almost every part of the mathematics lesson. In this respect, the VI pupils are differentiated from the rest of the class, as they are the only ones who cannot see most of the teacher's demonstrations. The special digital resources are provided to the VI pupils with this underpinning idea: to make these pupils access what their sighted peers access, which is what their limited vision does not allow them to access. In this respect, their difference is not celebrated: instead, their difference is shown more as a deficit, which needs to be overcome – and it is overcome through the special digital resources, as these resources enable the VI pupils to access what the rest of the class accesses.
      Another way that this is evident is through the consideration of vision as the prevalent sense in the mathematical learning. Again, the VI pupils' difference is not celebrated, as the VI pupils are asked to use their limited vision to construct mathematical meaning. They are rarely asked to use other, more developed senses, such as touch, in their mathematical learning.
    - the appropriate adjustment of a physical resource to the needs of the VI pupil: this may result in the non-necessity for the VI pupil to use the visualiser. This obstacle is systemic because it is rooted in the school's consideration of inclusion and, in particular, of the class teacher's role in the inclusion of VI pupils. No pre-service and in-service training on the inclusion of VI pupils is provided to class teachers. Instead, training on the VI pupils' inclusion is provided to teaching assistants. These two facts indicate the systemic view that there should be a 'special' person – not the class teacher – responsible for the VI pupils. This indicates that in the institutional view inclusion is a transplantation of special education in mainstream settings (Miles & Ainscow, 2011).
    - the VI pupil's unfamiliarity with using physical resources to follow the teacher's digital demonstration: this may occur because the VI pupil is mostly asked to follow the teacher's digital demonstration through a digital resource – particularly, by using the digital resource as a visualising tool.
    - the VI pupil's use of his vision in his mathematical work with physical resources: this may occur because the teacher emphasises on vision in the VI pupil's mathematical learning. This emphasis seems to be cultivated in the VI pupil's attitude too.

The third and fourth obstacles are systemic because they are rooted in the school's emphasis on vision in the VI pupil's mathematical learning, with touch being considered as of secondary – and probably less-intellectual – sense in the mathematical meaning making of VI pupils. Implicit assertions towards institutional considerations of this role of touch in the mathematical meaning making of VI pupils are evident in (Healy & Fernandes, 2014) and in (Argyropoulos & Stamouli, 2006). Both groups of researchers report that VI pupils are rarely provided with opportunities to use touch in mathematical meaning making: "The students had explained to us that it was rare for them to interact with representations of geometrical shapes, and an important aspect of designing the tasks was to produce tactile materials that would make this possible" (Healy & Fernandes, 2014, p. 134); "According to the two teachers the lack of materials had a great impact on Nefeli's haptic apprehension and for this the researchers prepared and provided the teachers material following exactly the activities suggested in the school textbook" (Argyropoulos & Stamouli, 2006, p. 129).

To optimise the intertwinement of physical and digital resources, we need to overcome these two groups of obstacles. I note that the second group of obstacles are harder to overcome, as they are located in deeply rooted institutional attitudes to mathematical learning. In Phase 2, the class teachers and I attempted to overcome these two groups of obstacles.

6.6 Phase 2 of the study

In this section, I present: how findings identified in Phase 1 – and synthesised in 6.5 – were considered in the design of the Phase 2 lessons; examples from the implementation of the Phase 2 lessons, with the examples tailored to resources; and, the impact that the implementation of these resources had upon teaching staff and pupils.

In the design of the Phase 2 lessons, the class teachers and I decided not to involve the teaching assistants in the inclusion and enabling of the VI pupils. Instead, we decided to design the lessons in a way that the class teachers were responsible for the inclusion and enabling of these pupils. This principle was already followed in the Phase 1 lessons in S2Y5 and S3Y1 but was not followed in S4Y5, as the teaching assistant was consistently involved in the inclusion and enabling of the VI pupil.

This design principle denoted that the class teachers and I needed to find ways of resource use that would be inclusive and enabling for the VI pupils – and in a way that the teaching assistants would not be needed as the teaching staff that fulfil the inclusion and enabling of these pupils.

Findings from Phase 1 – as well as literature – on resource use informed the design of principles for resource use as well as the design of the resources themselves.

The obstacles that arose in Phase 1 with regard to VI pupils' inclusion through digital resources prompted the class teachers and I to shift towards exploring alternative ways to include these pupils. These ways involved the design of tactile and auditory resources: resources that do not require vision to be accessed but that invite the use of other sensorial modalities by the class.

The obstacles that arose in Phase 1 with regard to VI pupils' mistakenly following the teachers' digital demonstrations with their physical resources prompted the class teachers and I to shift towards designing mathematical tasks that are experienced through touch – and not just by the VI pupils. We increased the classes' familiarity with touch and we showed the significance of touch in mathematical learning.

Below I present auditory and tactile mathematical tasks that the class teachers implemented in the Phase 2 lessons. I first present two auditory tasks, both of which are on number sequences: the first task was co-designed by T3a and myself; and, the second task was co-designed by T4a and myself (and was based on preliminary findings from the implementation of the first auditory task). I then present three tactile tasks. The first task is on number sequences and was co-designed by T3a and myself. The remaining two tasks are on shapes and were co-designed by T2 and myself (and were based on preliminary findings from the implementation of the first tactile task).

First auditory task on number sequences (S3Y1):
In the Y1 class of School 3, T3a plays a single, low sound on a xylophone and tells the class that, when they hear this sound, it is a Ten. She then takes another xylophone, plays a single, high sound and tells the class that, when they hear this sound, it is a Unit. Afterwards, T3a plays various sounds, sometimes by using one xylophone and sometimes by using both xylophones. Each time, she asks the class what number she plays. She then plays number sequences with a pause in between two successive numbers and asks pupils to move to her and play the next numbers. Afterwards, she asks the class if the sequences are increasing or decreasing and by how much.

In this task, T3a expected the class to discern number sequences through the use of hearing – by listening to number sequences represented via musical instruments. She expected to hear mathematical contributions that are due to be given in a Y1 class. She particularly expected the class:

- To discern numbers
- To discern number sequences
- To say the next number in the sequences
- To explore place value

- To say if the sequence increases or decreases – and by how much

The primary aims of this task were: to invite the class to experience number sequences through their sense of hearing – by her incorporating Music into Mathematics; and, to investigate the mathematics that is elicited through the auditory experience.

This task is related to the following statutory requirements of the Y1 National Curriculum:

- "count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number" (Department for Education, 2013, p. 6)
- "count, [']read['] and [']write['] numbers to 100 in numerals; count in multiples of twos" (Department for Education, 2013, p. 6). I enclose "read" and "write" in quotation marks because these terms are contextualised in this task differently – not through the visual sense. More specifically: "read" is contextualised as discerning numbers that are played via musical instruments; and, "write" is contextualised as representing numbers via musical instruments.
- "given a number, identify one more and one less" (Department for Education, 2013, p. 6)
- "identify and represent numbers using objects […] and use the language of: equal to, more than, less than(fewer), most, least" (Department for Education, 2013, p. 6). In this task, the objects are the musical instruments.
- "[']read['] and [']write['] numbers from 1 to 20 in numerals and words" (Department for Education, 2013, p. 6)

This task is also related to the following non-statutory requirement of the Y1 National Curriculum:

- "They recognise and create repeating patterns with objects" (Department for Education, 2013, p. 6)

In this task, T3a suggested playing the number sequences herself using musical instruments and asking the class to play the next number on the musical instruments. Below I explain why these were good suggestions.

Her active involvement in the representation of number sequences facilitated T3a's subsequent positive experiences with this task – and, more generally, with the use of the auditory modality in the construction of mathematical meaning. In particular, T3a emphasised the fact that this task "gave everyone the opportunity to be able to participate whether he could see or not".

The use of the auditory construction of mathematical meaning contributed to T3a's realisation that she should distinguish between focussing on mathematics and looking at the teacher/board. Before this task, T3a – as well as other teachers – confused these two situations: more specifically, they posited that, if the VI pupil does not look at them/board, he is not focussed. In this task, T3a realised that the fact that Ned does not look at T3a does not necessarily make him lose focus: this is because, in this task, mathematical learning is constructed through auditory – not visual – modality. T3a particularly reported: "I did think music still allowed him to access that, so often Ned is not focussed – I mean he is not looking at the board, so he is missing key learning – but, because he wasn't looking necessarily up, I think maybe he can still listen to what was going on, so he could still kind of grasp what was going on."

Her use of musical instruments in the representation of number sequences facilitated T3a's subsequent positive experiences with this task – and, more generally, with the use of music in the mathematics lesson; in other words, the combination of Mathematics and Music. One example that evidences T3a's appreciation of music in the mathematics lesson is the following phrase from T3a's Phase 2 interview: "The music to count on – they [the pupils] love it".

The active involvement – and the use of musical instruments – in the representation of number sequences by T3a, as well as the invitation of the class to play the next number on the musical instruments, were particularly beneficial for both the sighted and the VI pupils. Both mathematical and social benefits arose.

With regard to mathematical benefits, I indicatively report statements from three pupils in their focussed-group interviews:

- Ola[37] said that she particularly liked when T3a played the Tens and Ones – this indicates representation of Tens and Ones with music and easiness in discerning between Tens and Ones through music
- Peter[38] mentioned "ten, one, one" – this was the first number that T3a played with sounds and Peter not only recalled it[39] but also translated each sound into its place value representation, without even being asked to
- Peter mentioned that the pattern was "counting in tens", which is correct – again, the auditory representation of a number sequence was stored in Peter's memory, and the pattern seems to have been clear for him, through listening to the sequence. The fact that Peter recalled twice auditory parts contradicts with the theory that auditory perception is temporary [mentioned in Leuders (2016)]. Auditory parts have been stored in Peter's memory.
- Nicky[40] said that the auditory task made her think of the numbers and count
- Nicky considered it easy to find the pattern, because "I had the numbers in my head, like din, din, din, din, dif and then I was like fiving and sixing and seven, so I knew it" (She had sounds in her head and then counted on them using numerals!)

With regard to social benefits, I indicatively report the following:

- Pupils got familiar with the sense of hearing in the construction of mathematical meaning. For example, Zoe[41] was able to distinguish the One sound from the Ten sound because, as T3a said, the One sound is short ("bit") while the Ten sound is long – I have not thought about it, I would personally distinguish them with the criterion of their volume [high (One) – low (Ten)].
- Pupils realised that Music and Mathematics are not necessarily distinguished subjects from each other, but rather Music can be productively used in Mathematics. For example, Benjamin[42] said that T3a played some songs. He acknowledges though that this activity is, despite the music, about mathematics: "It's like saying the numbers".
- The auditory task made pupils relaxed – they associated music with Mathematics and considered relaxation as an effect of music upon them.
- TA3a acknowledged that this task is very different from the way in which the class has been taught mathematics – and which has emphasised on vision: "[I]t was a very different way of doing things. They've done a lot of mathematics, but lots of it has been visible, I mean manipulatable, so listening to things is a little bit different".

Second auditory task on number sequences (S4Y5):
In the Y5 class of School 4, T4a asks the class to work in pairs. Each pair needs to create a number sequence of five numbers, with the first number being a 2-digit one. T4a tells the class that they will need two sounds for each number: one sound to represent a Ten; and, one sound to represent a Unit. Each sound will need to come from a different musical instrument. One pupil in each pair will represent the Tens and the other pupil will represent the Units. T4a then asks the class to take musical instruments and to start creating, and then practising, their number sequences in their pairs. Afterwards, each pair plays their number sequence to the rest of the class and the rest of the class tries to work out what the numbers are – and what the rule is – in that sequence. T4a tells the class that it is up to them how they record the number sequences.

In this task, T4a expected to hear mathematical contributions that are due to be given in a Y5 class. She particularly expected the class:

- o To discern number sequences
- o To say the rule in the sequences

[37] Ola is considered by the school as a Middle Achieving Pupil (MAP).
[38] Peter is considered by the school as a High Achieving Pupil (HAP).
[39] The auditory representation of numbers was stored in Peter's memory, which was very similar to the effect of tactile numbers in the memory of Ned and Benjamin.
[40] Nicky is considered by the school as a MAP.
[41] Zoe is considered by the school as a HAP.
[42] Benjamin is considered by the school as a very Low Achieving Pupil (LAP-).

- To create number sequences

The primary aims of this task were: to invite the class to experience number sequences through their sense of hearing – by her incorporating Music into Mathematics; to investigate the mathematics that is elicited through the auditory experience; and, to invite the class to construct number sequences in pairs and to then represent these sequences through musical instruments.

This task is related to the following statutory requirement of the Y5 National Curriculum:
- "read, write, order and compare numbers to at least 1000000 and determine the value of each digit" (Department for Education, 2013, p. 31)

This task is also related to the following non-statutory requirements of the Y5 National Curriculum:
- "Pupils identify the place value in large whole numbers" (Department for Education, 2013, p. 31)
- "They should recognise and describe linear number sequences […] and find the term-to-term rule" (Department for Education, 2013, p. 31)
- "They should recognise and describe linear number sequences […] and find the term-to-term rule in words" (Department for Education, 2013, p. 31)

In this task, T4a suggested asking pairs of pupils to construct – and then play – a number sequence using musical instruments. Below I explain why these were good suggestions.

Her invitation of pupils to work together reinforced pair work, which was missing from that class in Phase 1. I indicatively report the following benefits from pair work in VI and sighted pupils:
- The VI pupil was better included in the class, he was no more a separate member from the sighted community of learners. Ivor particularly acknowledged that today's mathematics lesson was "[t]otally different" to the one he normally has. One of the differences that he pointed was that "we were put in partners today". He pinpointed that he likes working with a peer because "[t]hey can help one another". He found "[k]ind of easy and odd" the fact that he did not work with TA4 today in the lesson: "Easy because it was just like stuff and sequences and I just knew what sequences were".
- There was mutual appreciation between the VI pupil and his sighted peer, both mathematically and socially. For example, Ivor helped Frank with the last number of their sequence: While Ivor correctly did not play any Tens for "6", Frank did not play any Ones – he possibly thought that it was Ivor's turn and he did not look at the number. Ivor made a facial expression to Frank showing that it was Frank who had to play that number. Frank played it.
  Therefore, while in Phase 1 Ivor was helped by others – and appeared to be as weak and distracted in Mathematics – in Phase 2 Ivor helped his sighted peer. This finding from Phase 2 illustrates: Ivor's very good understanding of place value; and, very good collaborative skills between Ivor and his partner.
- Pupils found mathematics easier because they "only had to do units (or tens)"
- Pupils liked "working with someone different"
- Pupils liked the collaborative thinking of ideas
- Pupils liked working with a peer because they "could double check [the mathematical work]"
- Pupils found that "teamwork helps to solve things"

I will now present benefits from pair work reported by T4a:
- All children participated in and were actively engaged in the lesson
  T4a particularly said that "all children were genuinely talking to, discussing with each other" / "taking turns" / "trying to solve it together" / "stayed focussed longer because they had to listen to each other"
- T4a said that there seemed to be no pattern in the work of HAPs, MAPs and LAPs in the auditory task. This task showed that ability groups do not firmly exist. More specifically, some LAPs found it easy and some HAPs found it hard. This finding raises discussion on whether the term "ability groups" is accurate.

Benefits from pair work were also reported by TA4:

- TA4 said that "the children were really engaged and were excited, albeit a bit noisy" / "worked in pairs well"
- TA4 appreciated our principle with mixed-ability pairs – and our choice of pairs
- TA4 said that Ivor was "very much included" and that he was "concentrated"

T4a's invitation of pairs of pupils to construct a number sequence reinforced the pupil-centred approach, which was missing from that class in Phase 1.

- This invitation filled pupils with "creativity of making our [their] own pattern"
- "[C]reating our [their] own pattern" helped pupils understand mathematics

T4a's invitation of pairs of pupils to play a number sequence using musical instruments reinforced:

- The pupils' active involvement in the construction of mathematics through music
- The rest of the class's development of the auditory modality in the construction of mathematical meaning. For example, Ivor told me that "listening to the bits of music" and "listen[ing] […] about the sequences" are what made him concentrated today. In Phase 1, Ivor told me that he feels distracted and not very much concentrated in the lesson when he doesn't work with TA4. In today's lesson, he told me that he felt concentrated, even though he did not work with TA4.
  He also told me that, in today's lesson, he did not find it hard to follow the lesson from the class teacher while, in Phase 1, he told me that he finds it hard to follow the maths lesson from his class teacher and that he instead finds it helpful to work with TA4 in Mathematics. He specifically said: "I couldn't keep up with T4a but now I can".

In both cases (both bullet points), this invitation brought similar benefits for the mathematical learning of pupils as in S3Y1, with regard to the auditory task on number sequences.

Both mathematical and social benefits arose.

With regard to mathematical benefits, I indicatively report the following:

- Hearing pupils playing number sequences by using musical instruments helped some pupils understand the pattern
- Instruments helped some pupils discern the Tens from the Units
- Hearing pupils playing number sequences by using musical instruments initiated some pupils' enthusiasm to work out what the numbers were
- The idea of pupils' playing number sequences by using musical instruments prompted pupils' eagerness to construct sequences
- Using music helped pupils to understand sequences. The music made pupils "more confident" with number sequences and patterns
- Hearing pupils playing number sequences by using musical instruments initiated "confusion of who is unit and [who is] ten".
  Pupils enjoyed this – and other kinds of – confusion though
- Instruments helped pupils understand sequences and patterns while until then pupils had not well understood these notions
- "[T]he bits of the numbers" helped pupils understand number sequences and patterns
- Hearing pupils playing number sequences by using musical instruments prompted pupils to "put on brain power to find out answers"
- Pupils found hearing number sequences through musical instruments as a "good way to hear"

With regard to social benefits, I indicatively report the following:

- Pupils really liked the incorporation of music into Maths
- Pupils used devised vocabulary that indicates the incorporation of music into Maths (even LAPs did it). More specifically, they used:
    - "instrument counting"
    - "Music/sequence patterns"
    - "music patterns"

    - “sound numbers”
    - “music numbers”
    - “Musical Numbers”
- Listening to classmates helped pupils understand maths

Hearing pupils play number sequences by using musical instruments initiated some pupils to “pay more attention to what was going on”

Tactile task on number sequences (S3Y1):
In the Y1 class of School 3, T3a introduces the class an A3 worksheet which includes four number sequences in a landscape format. Above the printed numbers, Wikki Stix is stuck so that the number sequences can be felt. T3a moves around the classroom and invites pupils to close their eyes, feel numbers on the worksheet and tell her what the numbers are. She then asks some pupils how many Tens and how many Ones they need for the numbers that they have felt.

In this task, T3a expected to hear mathematical contributions that are due to be given in a Y1 class. She particularly expected the class:

- To ‘read’ numbers. I enclose “read” in quotation marks because this term is contextualised in this task differently – not through the visual sense. More specifically, “read” is contextualised as discerning numbers through the tactile sense.
- To discern number sequences
- To explore place value

The primary aims of this task were: to invite the class to experience number sequences through their sense of touch – by them getting familiar with Wikki Stix, which is often used by VI pupils; and, to investigate the mathematics that is elicited through the tactile experience.

This task is related to the following statutory requirements of the Y1 National Curriculum:

- “count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number" (Department for Education, 2013, p. 6)
- “count, [‘]read[’] […] numbers to 100 in numerals” (Department for Education, 2013, p. 6)
- “identify […] numbers using objects” (Department for Education, 2013, p. 6). In this task, the objects are the Wikki Stix, which are stuck above the printed numbers.
- “read […] numbers from 1 to 20 in numerals and words” (Department for Education, 2013, p. 6)

This task is also related to the following non-statutory requirement of the Y1 National Curriculum:

- “Pupils begin to recognise place value in numbers beyond 20 by reading, […] counting and comparing numbers up to 100, supported by objects” (Department for Education, 2013, p. 6)

First tactile task on shapes (S2Y5):
In the Y5 class of School 2, T2 holds a bag with a range of 2D plastic shapes. He moves around the classroom and invites pupils to put their hands in the bag, pick one shape without taking it out of the bag and without looking at it, feel it and then describe it to the rest of the class. T2 is the only one who has visual access to that shape, which remains in the bag.

In this task, T2 expected the class to describe given shapes through touch. He expected to hear descriptions on shapes that are due to be given in a Y5 class. He particularly expected the class:

- To name the particular shapes (e.g. “rectangle”, “hexagon”)
- To discern whether the particular shapes are 2-D or 3-D
- To say properties of the particular shapes
    - Regarding sides: Number of sides, if any sides are equal to each other, if any sides are parallel to each other, if there are straight and/or curved sides
    - Regarding vertices: Number of vertices
    - Regarding angles: Number of angles, types of angles

The primary aims of this task were: to invite the class to experience shapes through their sense of touch; and, to investigate the mathematics that is elicited through the tactile experience. For these two reasons/aims, T2 was open to the mathematical contributions of the class that stemmed from their tactile experiences and he did not strictly request from the class to respond to all the above mathematical properties.

This task is related to the following statutory requirements of the Y5 National Curriculum:

- o "identify 3-D shapes, including cubes and other cuboids, from 2-D representations" (Department for Education, 2013, p. 37)
- o "know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles" (Department for Education, 2013, p. 37)
- o "use the properties of rectangles to deduce related facts" (Department for Education, 2013, p. 37)
- o "distinguish between regular and irregular polygons based on reasoning about equal sides and angles" (Department for Education, 2013, p. 37)

This task was suggested by T2. Below I explain why this was a good suggestion.

This task was entirely created by T2, under our design principle of asking the entire class to explore mathematics through touch. This was an additional task to the one that I had suggested – the one with Wikki Stix and Shape X – under the same design principle.

The mathematical benefits that came from this task often reinforced the mathematical benefits of the Wikki Stix task that I had suggested. More specifically, pupils reported mathematical benefits that are pertinent to touch and to its characteristics. For example:

- Touch allowed pupils to "count the edges, sides, corners and vertices"
- Touch allowed pupils to realise "the hidden facts on the shapes"
- Touch allowed pupils to "have to feel around to get it"
- Touch allowed pupils to "realis[e] […] the differences"
- Touch allowed pupils to "move the shapes"

Pupils appreciated touch in the mathematical learning not just because it offers them mathematical benefits but also because it offers them social benefits too:

- Pupils liked "touching the shapes and describing them"
- Pupils liked "closing my eyes and feeling the shapes, trying to figure out what they were"
- Pupils liked "picking into the bag and describing it"
- Pupils liked "the different way to learn about shapes"
- Pupils liked "feeling the shape and getting it correct"
- Pupils liked "feeling" the shapes / the "feel" of the shapes
- Pupils liked the "weird" feeling that touch generated to them

Other benefits that arose from this task are the following two:

- This task reinforced gestures as a tool for construction – and expression – of mathematical meaning. Gestures were particularly used by sighted pupils, the VI pupil and T2 in the construction and expression of mathematical meaning.
- This task reinforced the verbal mathematical language that is elicited through tactile experiences (e.g. "it feels").

The above two benefits strengthen: the class's and T2's appreciation of touch in mathematical learning; and, their familiarisation with the mathematics that is created – and expressed – through touch.

Second tactile task on shapes (S2Y5):
In the Y5 class of School 2, T2 asks the class to close their eyes and describe two shapes (Figure 1, Stylianidou & Nardi, 2019a, p. 346), both of which are constructed with Wikki Stix. The shapes are constructed on the same white A4 paper and copies of the paper are given to the class. At some point during the pupils' engagement with the task, T2 also gives circles of various colours and sizes to the class and asks them the

difference between Shape X and these circles. T2 then asks the class to share their mathematical learning experiences from feeling Shape X and the triangle. Afterwards, T2 initiates a class discussion on Braille, which Luke knows.

In this task, T2 expected the class to describe given shapes through touch. He expected to hear descriptions on shapes that are due to be given in a Y5 class. He particularly expected the class:

- To name the particular shapes
  - In the case of Shape X, T2 and I wanted to examine whether the use of touch would help the pupils to discern that the particular shape is not a circle but that it comprises of a curved part and a straight line segment. Being aware of the characteristics of vision and touch – vision is wholistic and touch is gradual, allowing the exploration of an object from its individual parts to its whole (Ochaita & Rosa, 1995) –, we wanted to explore whether vision may generate a misinterpretation of this shape (Shape X at first glance may be perceived as a circle) and whether touch may generate a more accurate interpretation of the shape.
- To discern whether the particular shapes are 2-D or 3-D
- To say properties of the particular shapes
  - Regarding sides: Number of sides, if any sides are equal to each other, if any sides are parallel to each other, if there are any straight and/or curved sides
  - Regarding vertices: Number of vertices
  - Regarding angles: Number of angles, types of angles

The primary aims of this task were: to invite the class to experience shapes through their sense of touch; and, to investigate the mathematics that is elicited through the tactile experience. For these two reasons/aims, T2 was open to the mathematical contributions of the class that stemmed from their tactile experiences and he did not strictly request from the class to respond to all the above mathematical properties.

This task is related to the following statutory requirements of the Y5 National Curriculum:

- "identify 3-D shapes, including cubes and cuboids, from 2-D representations" (Department for Education, 2013, p. 37)
- "know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles" (Department for Education, 2013, p. 37)
- "[identify] angles at a point on a straight line" (Department for Education, 2013, p. 37)
- "distinguish between regular and irregular polygons based on reasoning about equal sides and angles" (Department for Education, 2013, p. 37)

A range of findings arose. First of all, the tactile and the auditory tasks were excitingly experienced by both VI and sighted pupils. No special resources were needed for the VI pupils and everybody accessed the tasks using a sense that is fully developed. Interesting mathematical contributions by both VI and sighted pupils also arose from these types of resources.

Apart from benefits to the pupils, the tactile and the auditory tasks brought benefits to the teaching staff too. The teaching staff were filled with excitement with these resources. They acknowledged the mathematical and social benefits that these tasks brought. They also shifted their considerations on touch and hearing.

While in Phase 1 they considered vision as the prevalent sense in mathematics, with touch and hearing being of secondary importance, in Phase 2 they were more open to – and appreciated – touch and hearing in the mathematical learning. This openness made them diverge from institutional norms, which emphasise vision as the dominantly relevant sense in mathematical learning. They considered other senses as intellectual too and therefore moved away from views that consider certain senses as more intellectual than others. Finally, this openness envisages the design of more tactile and auditory tasks in the mathematics classroom, for the benefit of everybody.

Having concluded the theme of resources, I will now move into the theme of mathematical contributions (Chapter 7).

Chapter 7: The mathematical contributions of visually impaired pupils in an inclusive primary classroom and sighted pupils' and staff's responses to these contributions

7.0 Introduction

This chapter shows how the sighted members of the mathematics classroom – teacher, TA and sighted pupils – responded to mathematical contributions of VI pupils. It also shows characteristics of VI pupils' mathematical contributions.

By "mathematical contribution", I denote a contribution that has a mathematical substance. In particular, a mathematical contribution is made in a specific mathematical task and indicates mathematical meaning.

The headings 7.1-7.7 show the different manifestations of responses to VI pupils' mathematical contributions. "Valuing", "attuning" and "incorporating" constitute the *inclusion elements* that synthesise the different reactions. "Valuing" and "attuning" are two analytic elements of the CAPTeaM project (by Nardi et al., 2018). I use "incorporating" as an alternative analytic element to the respective "managing the classroom" that is used by Nardi et al. (2018). In this chapter, "incorporating" is more appropriate than "managing the classroom", because it is a word that directly characterises – and connects – a sighted person's response to a VI pupil's mathematical contribution.

With "valuing", I denote appreciating and approving the VI pupil's mathematical contribution as mathematically rigorous and of potential benefit to the rest of the class too. With "attuning", I denote engaging in understanding the VI pupil's mathematical contribution, by embodying it, be the contribution verbal or bodily. With "incorporating", I denote bringing the mathematical contribution of the VI pupil to the rest of the class too as an integral part of the lesson.

In cases where there is implicit evidence that an inclusion element takes place, I use the word "implicit" in front of that inclusion element. For example, "implicit valuing" is used when there is indirect evidence that valuing takes place: in other words, when the sighted person does not express his/her appreciation and approval explicitly.

In cases where there is evidence that the opposite manifestation of an inclusion element takes place, I use the prefix "non-" in front of that inclusion element. For example, "non-valuing" is used when there is evidence that lack of valuing takes place.

While "valuing", "attuning" and "incorporating" synthesise the different responses, not all these three elements are necessarily applicable in episodes of VI pupils' mathematical contributions.

Valuing is applicable in every episode. This is because appreciating and approving the VI pupil's mathematical contribution are pertinent to the inclusion of this contribution (the significance of valuing is explained in Chapter 2).

Attuning is applicable when the VI pupil communicates his mathematical meaning making of the particular question. In this case, the sighted person can attune to the VI pupil's mathematical contribution.

Incorporating is applicable when the VI pupil's mathematical contribution takes place as part of the class discussion – and in a mathematical question which is connected with another question. In this case, the sighted person can incorporate the VI pupil's mathematical contribution into the lesson.

The sub-headings under each heading show the different nature of the mathematical questions, in which the specific response emerged. In all sub-headings, there is the word "answer". By "answer", I denote the final product and/or the method that is followed in order for that final product to emerge. The final product is a mathematical word or a mathematical phrase. The method consists of the steps that are followed in order for the final product to emerge.

Each of the sub-headings is followed by one or two episodes, illustrating various facets of the issue under examination, and their analyses. In cases where a sub-heading is illustrated in more than one way across my data, another episode is presented and analysed in order to show a different way that the sub-heading was evident. All the presented episodes come from Phase 1 of the study.

Table 5 below presents the structure of 7.1-7.7:

			Episodes	
			School	**Type of data**
Issues	7.1 Valuing of VI pupils' mathematical contributions	7.1a Valuing occurs when the mathematical question is formed in a way that allows for only one answer, with the VI pupil's mathematical contribution entailing this answer	School 3	Lesson note-taking
		7.1b Valuing occurs when the mathematical question is formed in a way that allows for more than one answers, with the VI pupil's mathematical contribution entailing one of these expected answers	School 2	Lesson video-recording and lesson note-taking
	7.2 Implicit valuing and attuning of VI pupils' mathematical contributions	7.2a Implicit valuing and attuning occur when the mathematical question is formed in a way that allows for only one answer, with the VI pupil's mathematical contribution being different from the answer expected by the teaching staff	School 4	Lesson audio-recording and lesson note-taking
			School 4	Lesson note-taking
	7.3 Valuing and incorporating of VI pupils' mathematical contributions	7.3a Valuing and incorporating occur when the mathematical question is formed in a way that allows for only one answer, with the VI pupil's mathematical contribution entailing this answer	School 1	Lesson video-recording and lesson note-taking
		7.3b Valuing and incorporating occur when the mathematical question is formed in a way that allows for more than one answers, with the VI pupil's mathematical contribution entailing one of these expected answers	School 2	Lesson video-recording and lesson note-taking
	7.4 Valuing, attuning and incorporating of VI pupils' mathematical contributions	7.4a Valuing, attuning and incorporating occur when the mathematical question is formed in a way that allows for only one answer, with the VI pupil's mathematical contribution entailing this answer	School 1	Lesson video-recording and lesson note-taking
		7.4b Valuing, attuning and incorporating occur when the mathematical question is formed in a way that allows for more than one answers, with	School 2	Lesson video-recording and lesson note-taking

		the VI pupil's mathematical contribution entailing one of these answers		
	7.5 Implicit non-valuing but attuning of VI pupils' mathematical contributions	7.5a Implicit non-valuing but attuning occur when the mathematical question is formed in a way that allows for more than one answers, with the VI pupil's mathematical contribution being different from the answers expected by the teaching staff	School 4	Lesson audio-recording and lesson note-taking
	7.6 Non-valuing and non-attuning of VI pupils' mathematical contributions	7.6a Non-valuing and non-attuning occur when the mathematical question is formed in a way that allows for only one answer, with the VI pupil's mathematical contribution being different from the answer expected by the teaching staff	School 1	Lesson video-recording and lesson note-taking
	7.7 Implicit non-valuing, non-attuning and non-incorporating of VI pupils' mathematical contributions	7.7a Implicit non-valuing, non-attuning and non-incorporating occur when the mathematical question is formed in a way that allows for more than one answers but only a specific 'strand' of these answers is considered as correct, with the VI pupil's mathematical contribution not entailed in the particular 'strand' of these answers	School 1	Lesson video-recording and lesson note-taking

Table 5: Structure of 7.1-7.7.

Afterwards, I summarise the findings on sighted pupils' and staff's responses to VI pupils' mathematical contributions (7.8). I continue this chapter with Phase 2 of the study (7.9), in which I present: examples from the implementation of the Phase 2 tasks presented in 6.6, with the examples tailored to VI pupils' mathematical contributions and to class teachers' and sighted pupils' responses to these contributions; the impact that the implementation of these tasks had upon teaching staff and pupils; and, the location of the sighted people's responses to VI pupils' mathematical contributions in the two camps of the CAPTeaM project. I close this chapter with presenting characteristics of VI pupils' mathematical contributions (7.10).

7.1 Valuing of VI pupils' mathematical contributions

This section concerns episodes of valuing. In this section, attuning and incorporating are not applicable.

7.1a Valuing occurs when the mathematical question is formed in a way that allows for only one answer, with the VI pupil's mathematical contribution entailing this answer

Episode:

In the Y1 class of School 3, the class sit on their carpet places, with Ned sitting on the first line. T3a sits in front of Ned and faces the class. Holding a big physical clock, T3a gives instructions to pupils to carry out on that clock and show to the rest of the class.
At some point, T3a instructs "Do a whole turn clockwise".
She then asks Ned to stand up and carry out this instruction on the physical clock.
Ned moves the minute hand, which is on 12, a whole turn clockwise and he then leaves that hand on 12.

T3a: “Good. Well done.”

Comments:

In this episode, T3a’s instruction is such that allows for only one answer, which involves the pupil moving the minute hand in the particular way dictated by the teacher. She also pre-determines the method for the working out of her instruction. The method involves the movement of the minute hand on the physical clock as per T3a’s instruction. In addition, she expects a single final product, which involves the location of the minute hand in 12.

Ned’s mathematical contribution – his moving of the minute hand a whole turn clockwise – coincides with the one expected by the teacher. Valuing – “Good. Well done.” – occurs after Ned acts in the way expected by the teacher.

There are two key features in Ned’s mathematical contribution: Ned’s awareness of what a “whole turn” means; and, Ned’s awareness of what “clockwise” means.

Ned’s mathematical contribution emerged in a setting that considers this pupil’s perceptual needs. More specifically, the physical block is big and is kept right at the front of Ned. These two characteristics of the setting benefitted Ned but also the rest of the class. The tactility, which the hands of the clock stimulate, allowed the class to act on the clock and to feel the turn which the minute hand does and which the teacher instructs.

Engaging with physical objects through touch was reported by pupils of this class in their focus groups as beneficial for these pupils’ mathematical learning. In particular, Ben said that using objects that he can touch in the mathematics lesson helps him do stuff that his teacher asks him to do. Similarly, Elliot and Bob said that using objects that they can touch helps them in their mathematics lesson.

Ned emphasised the element of fun that he feels when he engages with tactile objects in the mathematics lesson. He particularly said that he likes using things that he can touch in the mathematics lesson because “it’s fun”.

The class’s experience with tactility in this task may also be useful for the class’s future meaning making of the notion of a circle. In this episode, Ned may relive the experience of moving the minute hand a whole turn clockwise through “simulation” (Barsalou, 2008, p. 618).

Moving Ned at the front when working on the carpet is a practice that T3a told me that she implements in the mathematics classroom for Ned. She said that this practice comes from her use of her logic to include Ned in the mathematics lesson. T3a told me that, while some mathematical topics are “very visual” and “need to be displayed on the board”, she is in favour of physical resources that promote tactility and inclusion. She offered the example of “3D shapes” and said that she gives Ned “resources he can look and feel”.

7.1b Valuing occurs when the mathematical question is formed in a way that allows for more than one answers, with the VI pupil’s mathematical contribution entailing one of these expected answers

Episode:

In the Y5 class of School 2, the class is asked to work out 50÷2 using chunking on a number line. T2 has started to demonstrate the working out of this calculation on the IWB.
He tells the class that they are going to have to draw their number line.
He also tells the class to write their maths facts. He tells them that he would first of all do 2x10=20, which he writes on the right side of the IWB while he utters it (Figure 28). He then tells the class that he would probably do 5x2=10, which he writes on the IWB too while he utters it (Figure 28).
He then tells the class that he would jump in multiples of 10, because they end in 0.

The class starts working this calculation out on their maths books.
At some point, Luke raises his hand.
T2 sees Luke's hand and goes to Luke.
Luke asks T2 if he could do a direct jump of 25x2 on his number line.
T2: "You could. It's up to you. Remember, though, when you get to…It may not work with bigger numbers."
T2 then tells the class again: "You must write down your multiplication facts. I want to see them, I want to see your thinking."

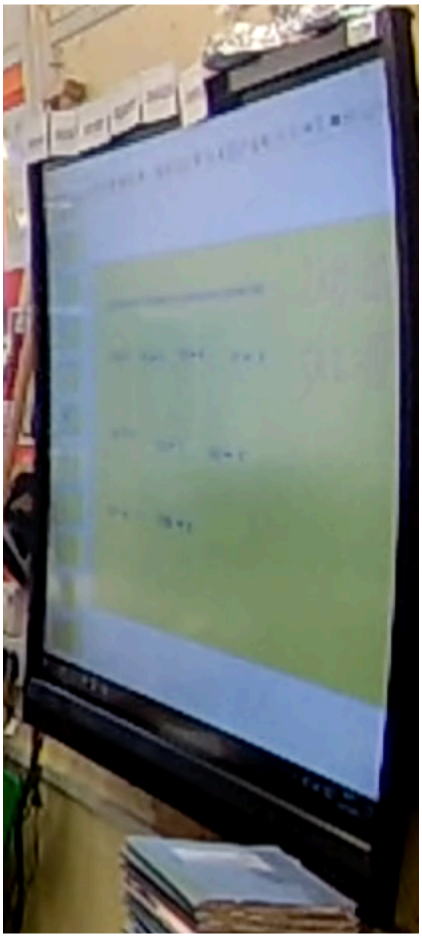

Figure 28: T2 has written 2x10=20 and 5x2=10 in red on the right side of the IWB, as he has started to demonstrate the working out of 50÷2.

Comments:

In this episode, T2's instructions are such that allow for more than one answers, which involve the pupil working out of 50÷2 using whichever multiplication facts they wish, provided that these facts can be used for the particular division calculation. However, T2 dictates the method for the working out of this calculation. The method involves chunking on a number line, with the number line drawn and with the multiplication facts written down.

T2's emphasis on the visual presentation of mathematical work – therefore his consideration of vision as the prevalent sense in the representation of the working out of division calculations – is clear: T2 explicitly tells the class "You must write down your multiplication facts. I want to see them, I want to see your thinking."

His use of different colours for different parts of the task on the IWB is another practice that evidences T2's consideration of vision as the prevalent sense in mathematical learning. In particular, T2 told me: "Well, obviously I do a smartboard presentation, which goes on to Luke's iPad, so he can see that. So I try as much as possible to do as much as I can on the interactive board, so Luke gets sight of that. Trying to make it clear, so all my interactive boards have a yellow background with blue writing, which I think just makes it a bit more gentler – and all of the children actually, on everyone's eyes." T2 emphatically told me that his use of different colour pens for different parts of the task on the IWB is done not only for Luke but also for the rest of the class too as a practice that is helpful for everybody: "they can see the different aspects of what I'm doing".

Luke's mathematical contribution – his working out of 50÷2 by doing a direct, single jump of 25x2 on his number line – resonates with contributions expected by the teacher: the jump resonates with the multiplication facts that can be used to work out 50÷2, and the method is followed. Valuing occurs after Luke acts in a way expected by the teacher.

There is one key feature in Luke's mathematical contribution: Luke's meaning making of division as the inverse operation of multiplication. This element of abstraction, which is evident in Luke's mental mathematical skills, may have been developed as a result of visual impairment: in order to avoid visual work, Luke has developed his mental mathematical skills (this is evident other times too in Luke's work of division calculations).

Luke's mathematical contribution is more sophisticated than the teacher's earlier demonstration of the working out of 50÷2 and of other division calculations. When the teacher had demonstrated the working out of division calculations on a number line, he had used smaller – and more – jumps. Similarly, when the teacher had started to demonstrate the working out of 50÷2 on a number line, he had written smaller multiplication sentences, which would constitute the jumps.

Other times too, Luke works out division calculations mentally. For Luke, the visual representation of his work on the number line came after he had worked out the division calculations mentally. It only served to visually show the teacher his mental work in the method which the teacher has taught (visual number line method). Thus, Luke initially used an alternative method (mental number line method) and he then adjusted it to the method taught by the teacher (visual number line). Therefore, it seems that Luke used the visual number line method only to conform with the sociomathematical norms established in the classroom.

7.2 Implicit valuing and attuning of VI pupils' mathematical contributions

This section concerns episodes of implicit valuing and attuning. In this section, incorporating is not applicable.

7.2a Implicit valuing and attuning occur when the mathematical question is formed in a way that allows for only one answer, with the VI pupil's mathematical contribution being different from the answer expected by the teaching staff

Episode:

In the Y5 class of School 4, the class are given a worksheet with printed angles and are asked to name each angle according to its type (acute, right, obtuse, reflex). T4a has put bright orange Wikki Stix above the printed angles of Ivor's worksheet in order to enable Ivor to visually access the angles. TA4 is not in the classroom.
At some point during the class's work on the worksheet, T4a passes from Ivor's table.
She notices that Ivor is stuck with naming a given angle.
T4a turns Ivor's worksheet upside down and then asks Ivor if it is now more obvious what angle it is.
Then, Ivor: "It looks like a right angle but a bit..."
T4a stops him and says: "Is it? Is it? Is it a sharp corner?"
Ivor: "But it's more wider."
T4a: "So, it's wider than a corner, therefore it's called?"
Ivor does not respond.

T4a: “It’s got a big caboose?”[43]
Ivor: “Obtuse.”
T4a: “It’s obtuse! Yay!”

Comments:

In this episode, T4a’s instruction is such that allows for only one answer, which involves the pupil naming the given angles, by using the relevant mathematical terms on the types of the angles.

Ivor’s mathematical contribution – “It looks like a right angle but a bit…But it’s more wider” – resonates with the contribution expected by T4a: while the verbal mathematical term of the type of the particular angle misses, the pupil’s acknowledgment of the type of that angle is clear – and is expressed through speech (a verbal description of an obtuse angle, through its relation with a right angle).

T4a implicitly values and attunes to Ivor’s mathematical contribution. Her implicit valuing is evident in her paraphrasing of the pupil’s mathematical contribution. T4a particularly says “So, it’s wider than a corner”. While her paraphrasing indicates implicit appreciation and approval of Ivor’s mathematical contribution, T4a uses this paraphrasing to direct Ivor towards uttering the expected mathematical term, “obtuse”. T4a particularly says “So, it’s wider than a corner, therefore it’s called?”.

As Ivor does not respond in his invitation to utter that mathematical term, T4a tries in an alternative way to help Ivor recall the mathematical term “obtuse”. She particularly sings to Ivor the part of the angle song that rhythms with “obtuse”. She specifically says “It’s got a big caboose?”. In this circumstance, Ivor recalls the term. He says “Obtuse”. It is only after Ivor has recalled this term that T4a explicitly values his answer. She says “It’s obtuse! Yay!”.

T4a attunes to Ivor’s mathematical contribution, as she tries to understand his mathematical meaning making. Her attuning is evident in her embodiment of the pupil’s mathematical contribution. She particularly embodies this contribution through speech – “So, it’s wider than a corner”.

What the class misses from not hearing Ivor’s contribution is the relation between the right angle and the obtuse angle.

Ivor’s description of the mathematical term “obtuse” has a stronger mathematical meaning than the term – because in his description of the obtuse angle there is a relation between this type of angle and another type of angle, while in the mathematical term itself the mathematical meaning is not signified.

It is significant to point out T4a’s initial action of turning Ivor’s worksheet upside down. T4a seems to have attributed Ivor’s struggle with naming the angle to the differentiation in the representation of this obtuse angle from the representation of the obtuse angles that Ivor has encountered. Her turning of his worksheet upside down makes the angle represented in the way that previous obtuse angles have been represented.[44] However, Ivor keeps not remembering the mathematical term for the type of the angle. This situation shows that, no matter how the angle is represented, Ivor does not remember the mathematical term for the type of this angle.

The fact that Ivor sometimes does not remember mathematical terms was emphasised to me by T4a. In particular, T4a told me that Ivor “has maths difficulties”, one of which is that “sometimes he can’t recall simple things”. She said that he has concentration issues, apart from his visual impairment, and that he forgets things very quickly. She also said that, if it was only Ivor’s visual impairment, Ivor would achieve better and that his concentration issues affect his learning a lot, on top of his visual impairment.

[43] This is a part of the rhythm for obtuse angles that the class had earlier listened to.
[44] Previous obtuse angles have been represented with the obtuse angles facing up. The obtuse angle of this episode faces down.

T4a's concern about the fact that Ivor does not sometimes recall mathematical terms is also cultivated to Ivor. Ivor is concerned that he forgets mathematical terms. Towards the end of the lesson, Ivor told T4a: "I only forgot one [type of angle] today." Then, T4a told Ivor: "Yes, you only forgot "obtuse". You've done very well, I'm really impressed."

It is also significant to point out the use of Wikki Stix by T4a. This is used for visual purposes rather than tactile purposes: it is used to enable Ivor to see the angle through the bright colour of the Wikki Stix. It is not used to enable him to feel the angle. The particular use of Wikki Stix by T4a suggests T4a's consideration of vision as the prevalent sense for Ivor's mathematical learning: even a tactile object is used to enable visual construction of mathematical meaning.

As discussed earlier, this episode involves non-recalling of a mathematical term that is accompanied by a verbal description of this term. The next episode involves non-recalling of a mathematical term that is accompanied by a gestural representation of this term.

In both episodes, the non-recalling of a mathematical term is accompanied by a relational meaning making of this term. More specifically, while the VI pupil does not recall the mathematical term, he expresses his mathematical meaning making of this term by relating it to another mathematical term. In this episode, he relates his meaning making of the obtuse angle to the right angle. In the next episode, he relates his meaning making of degrees to percentages.

Episode:

In the Y5 class of School 4, T4b asks the class to measure the given angles, which are printed on their worksheets. The sighted pupils have the angles printed on an A4 worksheet. Ivor has each of the angles drawn in a separate, A3 worksheet, with longer and thicker line segments that comprise each angle. TA4 sits next to Ivor.
Ivor works independently, with his big yellow protractor whose signs are in black. After he has measured an acute angle, which he has found 76[45], the following discussion takes place between Ivor and TA4 with regard to the unit of angle measurement:
Ivor: "Is it 76 percent?"
TA4: "Are we doing percent?"
Ivor: "No. How are these things called? *He makes a circular gesture with his index and thumb, which unite on their tips.*"
Looking at Ivor's gesture, TA4 laughs and teases Ivor that he remembers the symbolic representation of the unit of angle measurement but not the name of that unit.
TA4 then says: "Degrees."
Ivor: "Degrees!"
TA4: "Great."

Comments:

In this episode, T4b's instruction is such that allows for only one answer, which involves the pupil working out the given angles using the protractor.

While Ivor finds the correct number of degrees of the given acute angle, he is confused on the name of the unit of angle measurement. However, he remembers the symbolic representation of this unit.

Ivor's mathematical contribution – his verbal part "these things" accompanied by a gestural representation of degrees – (partially)[46] resonates with the contribution expected by the TA: the verbal utterance of the unit

[45] 76°.
[46] I place "partially" in brackets as, in my view, Ivor's contribution resonates with the one expected by the TA, despite the fact that its expression differs from the expression expected by the TA. However, in the TA's view, Ivor's contribution partially resonates with the one that she expects, as the 'proper' mathematical term misses.

of angle measurement misses, however the pupil's acknowledgement of that unit is clear – and is expressed through a gesture.

TA4 implicitly values and attunes to Ivor's mathematical contribution. Her implicit valuing is evident in her considering Ivor's gesture and in her recognising that Ivor's gesture signifies "degrees". While this recognition indicates implicit appreciation and approval of Ivor's mathematical contribution, TA4 considers Ivor's gesture in order to direct Ivor towards uttering the expected mathematical term, "degrees". TA4 particularly tells Ivor that he needs to remember this term and she then says this term to him. It is only after Ivor repeats the term "degrees" that TA4 explicitly values his answer. She says "Great".

TA4 attunes to Ivor's mathematical contribution, as she tries to understand his mathematical meaning making. Her attuning is evident in her embodiment of the pupil's mathematical contribution. She particularly embodies this contribution through speech, by telling Ivor that he remembers the symbolic representation of the unit of angle measurement.

Unlike the previous episode, in this episode the VI pupil's indication of his acknowledgement of the expected mathematical term is made through a gesture (in the previous episode, it is made through speech). In both cases, TA4 attunes to the VI pupil's mathematical contribution through speech.

The implicit nature of her valuing of Ivor's mathematical contribution seems to occur because Ivor does not verbally utter the unit of angle measurement: only after Ivor repeats "Degrees" does TA4 explicitly value his response. Her reactions suggest that the verbal utterance of mathematical terms is considered as of more mathematical rigour than the gestural expression of these terms. This consideration shows the emphasis on speech as the most approved means of mathematical communication and the secondary role that gesture plays in mathematical communication.

The above reaction of the TA may be related to institutional and curricular factors, which require pupils to use the mathematical terms that are taught in the particular period and are included in the National Curriculum.

The fact that Ivor sometimes does not remember mathematical terms seems to be attributed by TA4 to Ivor being "a lower ability pupil" and to him getting "easily distracted in the maths lesson". TA4 particularly told me that "Ivor is anyway a lower ability pupil despite his visual impairment" and that "even if he wasn't visually impaired, he would be in a lower ability group". TA4 also told me that Ivor gets easily distracted in the mathematics lesson and that working one to one with TA4 keeps him more focused. The noise and the too quick pace in the classroom are reported by TA4 as factors that make Ivor distracted in the lesson.

It is significant to say that in Ivor's contribution:

- o The gesture adds more than the speech, with the speech being insufficient for the interlocutor's understanding of the pupil's mathematical contribution (same as in the episode in Stylianidou & Nardi, 2019b)
- o The pupil primarily emphasises on the symbolic representation and secondarily focuses on the verbal mathematical term

While Ivor's mathematical contribution is not explicitly valued by TA4, it would have been explicitly valued if it was written: Ivor would have correctly written down the symbolic representation of degrees. His non-recalling of the mathematical term "degrees" would even not have been evident if he wrote down his answer to the particular angle. This suggests:

- ▪ Whether gesturing in the oral communication should be approved, as it coincides to the written representation, which is approved
- ▪ Whether mathematical terms are really needed – as for example in the written contributions it is mathematical symbols rather than verbal terms which are used

It is also interesting to point out Ivor's initial consideration of the unit of angle measurement as "percent". His consideration of this unit as "percent" does not seem to have been incidental: the symbolic representations of the units of angle measurement and of percentages use the symbol °. Ivor's reference to "percent" as the unit of angle measurement is another indication that evidences his acknowledgement of the unit of angle measurement.

What the class misses from not hearing Ivor's mathematical contribution is the similarity in the units of angles and percentages.

Ivor's initial consideration of the unit of angle measurement as "percent" is an example of a case which indicates that two seemingly irrelevant mathematical terms may be related in a mathematical contribution. It suggests that careful judgement be needed before a conclusion that two mathematical terms are irrelevant to each other is extracted. This suggestion applies to T4a, who told me that in a task on angles Ivor "was just firing random answers out. And he spoke to me about some sort of number line with imagined pins or something – this is how the visual memory can go – 'cause we thought we could do that, he could do 180-80 I think it was […] I don't know how much of it is his visual impairment and how much of that is his ability."

The non-recalling of mathematical terms by VI pupils could be justified below.

The VI pupils seem to focus so much on the visual representation of a term that they do not have 'memory space' to focus on the term too. As a result, when the teaching staff ask a question in which they expect to hear a specific term as the answer, the VI pupils often re-enact the visual representation of the term – and they do this with gestures – but they do not recall the term. Also, when the teaching staff use a term to ask a question relative to its meaning, the VI pupils often respond incorrectly – as they have not associated the term to its meaning. Both cases are possibly attributed to the visual teaching of maths.

On the other hand, sighted pupils, whose vision is good, seem to split their focus between the visual representation of a term and the term itself. As a result, they do not struggle to associate a term to its visual representation.

The non-recalling of mathematical terms and the use of gestures to represent these terms constitute two characteristics of VI pupils' mathematical contributions (another episode with these two characteristics is in Stylianidou & Nardi, 2019b). The combination of these two characteristics is not explicitly valued by the teaching staff.

VI pupils tend to find it easier to express their mathematical understanding through gestures, rather than speech – this may also be related to the way they construct mathematical meaning (i.e. through their body, not formal terminology).

The differentiation in the mathematical construction and expression between VI and sighted pupils is possibly related to the limitation of the VI pupils' vision and their use of their body as a substitute (Vygotsky, 1978) – replacement – to their limited vision.

Factors mentioned by teaching staff like lack of memory and lack of concentration as justifications for some VI pupils' non-use of verbal terminology but instead use of gestures to represent verbal terms may be inaccurate. VI pupils may not pay much attention to verbal references but may instead focus on the bodily expressions in order to construct mathematical meaning. But this mathematical construction and expression deviate from the prevalent institutional mathematical discourse.

The prevalent institutional discourse on inclusion shapes the way that T4b works with Ivor. More specifically, T4b reports that she thinks about adaptations for Ivor: "it's just kind of thinking how we're going to adapt this to his needs. Is he going to need something significantly different or is he going to need something that is just slightly different?".

Her thoughts are associated with the way that inclusion was conceptualised in her training during her PGCE course. T4b particularly said that her training on inclusion was "more of an overview generally on inclusion, differentiation and that kind of thing […] I did about different learning needs, so just a general overview and how that might lead to obviously differentiating lessons to make them inclusive. And different learning styles. Obviously how you can make that into the lesson to make it inclusive." T4b, as well as her pre-service training, limit their conceptualisation of inclusion to adaptations and do not include the aspect of universal design.

7.3 Valuing and incorporating of VI pupils' mathematical contributions

This section concerns episodes of valuing and incorporating. In this section, attuning is not applicable.

7.3a Valuing and incorporating occur when the mathematical question is formed in a way that allows for only one answer, with the VI pupil's mathematical contribution entailing this answer

Episode:

In the Y3 class of School 1, T1a displays 444+10= on the IWB. Fred follows this calculation on his iPad, which is connected to T1a's computer through a VNC connection.
T1a asks Fred to mentally work out[47] what 444 add 10 is.
Fred: "444 add 10 is…400…and 54 – *Fred uses an emphatic tone of voice while he utters 50.*"
T1a: "Perfect. Does everyone agree with Fred? *She writes 454 as the sum of the calculation (Figure 29).*"
Sighted pupils: "Yeah!"
After sighted pupils say "Yeah!", Fred excitedly gestures as if he is the winner (Figure 29).

Figure 29: Following Fred's contribution, T1a has written 454 in red on the IWB as the sum of 444+10=. After sighted pupils say "Yeah!", Fred excitedly gestures as if he is the winner.

Comments:

In this episode, T1a's instruction is such that allows for only one answer, which involves the pupil working out 444+10 mentally. T1a dictates the method for the working out of this calculation. The method involves mental work. T1a expects a single final product, which is 454.

Fred's mathematical contribution resonates with the one expected by the teacher. Valuing and incorporating occur after Fred responds in a way expected by the teacher.

While the final product is single and Fred's response includes this product, there is an interesting element in the pupil's response, which did not seem to be paid particular attention by the teacher and which indicated that the VI pupil's mathematical contribution differed from the sighted peers' mathematical contributions in similar mathematical tasks: his emphasis on 50 while he utters 454. Fred acknowledges the part of the number that changes and he has good meaning making of place value.

[47] T1a asked the class to mentally work out the previous two calculations too.

In his focus group, Fred expressed his enthusiasm with adding Tens. In particular, Fred said that he likes Mathematics because he likes adding Tens together. He then said that he finds it easy when they add Tens together. He also stated: "What makes me happy in maths is when we do adding Tens with Hundreds and Tens. And it's really, really fun because I like adding Tens."

A significant reason why Fred likes addition seems to be the opportunity for mental working out of mathematics. As in this episode, in other episodes too on addition, T1a asked the class to mentally work out calculations. In his focus group, Fred emphasised the 'mental tool' in his working out of mathematics: "My brain is my main tool [to work out maths]". This differs from the sighted peers of his focus group, who emphasised concrete, visual tools, such as "fingers", in their working out of mathematics.

There is one key feature in Fred's mathematical contribution: Fred's meaning making of the sum as the number that results from the modification of one addend in the way indicated by the other addend. This element of abstraction, which is evident in Fred's mental mathematical skills, may have been developed as a result of visual impairment: in order to avoid visual work, Fred has developed his mental mathematical skills (this is evident other times in Fred's work of addition and subtraction calculations).

7.3b Valuing and incorporating occur when the mathematical question is formed in a way that allows for more than one answers, with the VI pupil's mathematical contribution entailing one of these expected answers

Episode:

In the Y5 class of School 2, the class is asked to work out 692÷8 on a 'human number line'. The human number line consists of pupils, who stand up at the front with their contributions written on their whiteboards and face the rest of the class (Figure 30). The pupils whose contributions are the numbers on the number line hold their whiteboards down, as if in a written number line. The pupils whose contributions are the multiplication jumps hold their whiteboards up, as if in a written number line.
After a pupil's contribution of "600", Luke energetically tries to get T2's attention to be picked to suggest the next jump on the number line.
T2 picks him.
Luke: "10 times 8."
T2: "Come on then, Luke. 10 times 8. Off you go. Quick."
Luke stands right next to the 600 pupil and holds his whiteboard up, with 10x8 written on it (Figure 30). He is very happy.
Looking at Luke's whiteboard, T2: "Right. Ok. 10 times 8. What is that?"
Pupils who do not stand up work out what 10x8 is and what the next number on the number line is.
Some pupils who stand up are confused whether they should hold their whiteboards up or not (one example of a sighted pupil is shown in Figure 30). Luke is not confused at all.

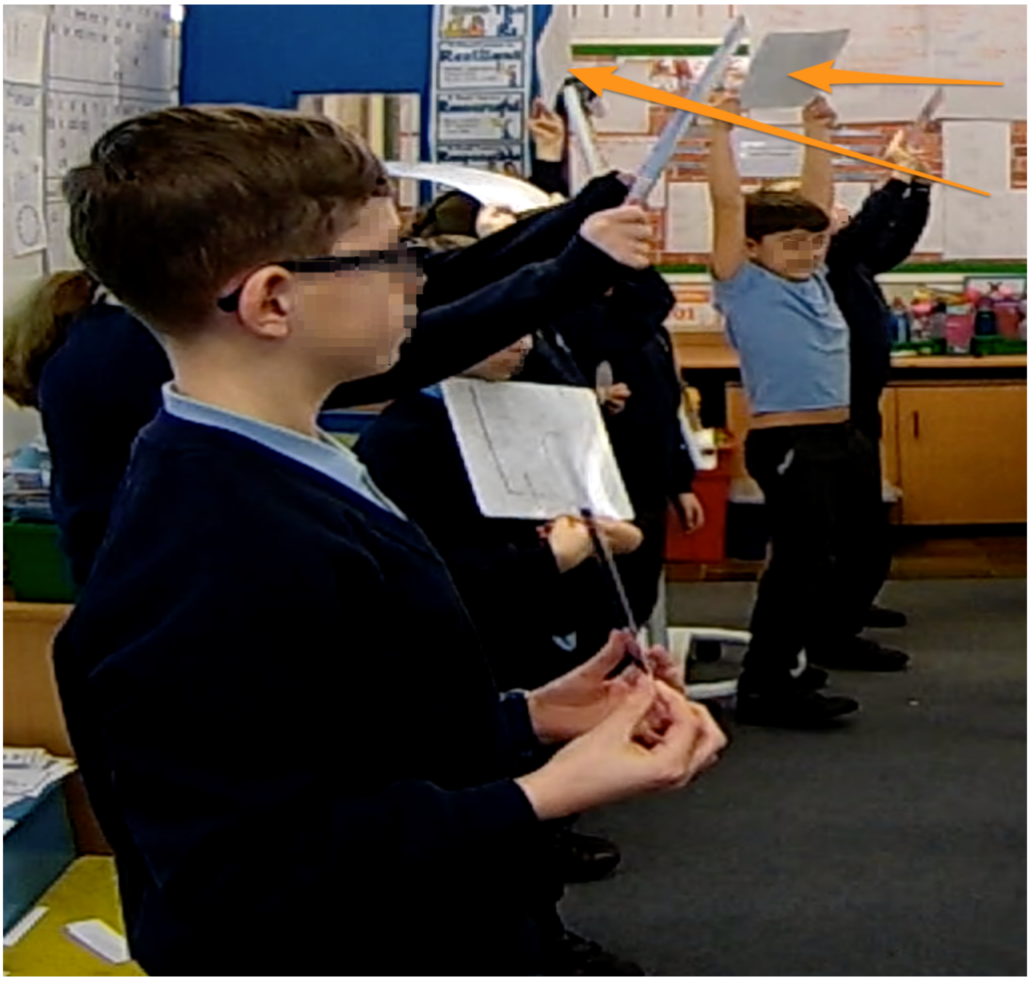

Figure 30: Pupils represent a human number line for the working out of 692÷8. They stand up at the front with their contributions written on their whiteboards and face the rest of the class. Luke, whose contribution is 10x8, holds his whiteboard up (his whiteboard is pointed with the long arrow). A sighted pupil, who stands next to Luke on Luke's left and whose contribution is 680, holds his whiteboard up too (his whiteboard is pointed with the short arrow).

Comments:

In this episode, T2's instruction is such that allows for more than one answers, which involve the pupil working out 692÷8 using whichever multiplication facts they wish, provided that these facts can be used for the particular division calculation. However, T2 dictates the method for the working out of this calculation. The method is chunking on a number line, with the number line consisting of pupils and with the multiplication jumps and the corresponding numbers on the number line written down.

T2's emphasis on the visual presentation of mathematical work – therefore his consideration of vision as the prevalent sense in the representation of the working out of division calculations – is clear.

Luke's mathematical contribution – his suggestion of 10x8 as the next multiplication jump on the human number line – resonates with contributions expected by the teacher: the jump resonates with the multiplication facts that can be used to work out 692÷8, and the method is followed. Valuing and incorporating occur after Luke acts in a way expected by the teacher.

There is one key feature in Luke's mathematical contribution: Luke's meaning making of division as the inverse operation of multiplication[48]. This element of abstraction, which is evident in Luke's mental mathematical skills, may have been developed as a result of visual impairment: in order to avoid visual work,

[48] Here though Luke does not suggest a single multiplication jump to work out 692÷8 (unlike what he did in the episode of 7.1b): he instead picks the working out of this division calculation from his peer, who has suggested 600, and continues the working out of this calculation, by suggesting the next multiplication jump.

Luke has developed his mental mathematical skills (this is evident other times too in Luke's work of division calculations).

It is significant to point out Luke's meaning making of the (human) number line. Luke is not confused on whether he has to hold his whiteboard up or not – in the human number line for the working out of 692÷8 – while some of his sighted peers are. This shows his great meaning making of the work on a number line.

Luke's action constitutes a manifestation of embodied abstraction of the working out of division calculations on a number line. This may be related to Luke's meticulous focus on the visual representation of the division work on a number line that he can then embodily abstract it (abstract it with his body).

On the other hand, sighted pupils, whose vision is good, do not often seem to focus as much on the particularities of visual representations – and this may explain why they often do not embodily abstract them. A reason behind Luke's embodied abstraction and the lack of embodied abstraction by sighted pupils is that Luke is more used to tactile construction of mathematical meaning than his sighted peers: an advantage of touch compared to vision is that touch is gradual, allowing the meaning making of an object from its individual parts to its whole (Ochaita & Rosa, 1995). This feature, which is more developed to Luke as being VI, may explain Luke's processing of visual representations – by focussing on their parts, too, not just processing them as a whole.

It is also significant to point out the manifestation of enabling in this episode. More specifically, after Luke has suggested 10x8, T2 tells him: "Come on then, Luke […] quick". T2 treats Luke exactly the same as the rest of his class. His word "quick" shows that T2 is not worried about Luke's movement in the classroom. And this is because the classroom is organised in a way that meets Luke's needs – enabling Luke's easy movement in it. T2's sentence does not reveal that T2 is addressed to a VI pupil but to a pupil whose needs are met in class.

T2 told me that he and the sighted pupils treat Luke as "just another pupil". He attributed this treatment to Luke's positive attitude to learning and to his non-use of his impairment as an excuse not to learn. More specifically, T2 told me that he "do[es]n't find it any different" having Luke in the mathematics classroom. He told me that Luke "wants to learn, he wants to get engaged and he doesn't use his impairment as an excuse […] not to learn".

T2 also told me that he makes sure that Luke is always part of the mathematical activity. He particularly reported: "[T]oday, for example, I pulled him out at the front, so he's always part of it – I don't think I'll leave Luke there 'cause he can't do that, you just bring him into everything you do. And that's no different to including children with emotional needs or children with learning difficulties […] So, and it's the same principle as far as I am concerned."

7.4 Valuing, attuning and incorporating of VI pupils' mathematical contributions

7.4a Valuing, attuning and incorporating occur when the mathematical question is formed in a way that allows for only one answer, with the VI pupil's mathematical contribution entailing this answer

Episode:

In the Y3 class of School 1, T1a displays 238+3= on the IWB. She represents the two addends with pictorial base ten blocks and sets these addends in a column addition (Figure 31). Her display is shown on Fred's iPad, which is connected to T1a's computer through a VNC connection.
After T1a has displayed the pictorial representation of the two addends in a column addition and has written 2 underneath the pictorial representation of Hundreds of 238, T1a asks Fred whether she would write anything in her Tens and Ones if she did not have any pictorial representation in the Tens and the Ones.
Fred: "Two zeros. *He gestures two zeros on his maths book.*"

T1a: "I would write two zeros. *T1a writes two zeros after the "2" and underneath the Tens and the Ones, respectively.* Why is Fred telling me I'd write two zeros if I didn't have anything in my Tens or anything in my Ones? Why is Fred saying I have to write two zeros? He is entirely right. Why is he telling me that, though? Why is he telling me that?"
Greg: "0 Ones, 0 Tens."
T1a: "Yeah. I can't just write the "2", can I? Without those – *she gestures towards the two zeros.* Cause otherwise it's just 2, it's not 2 Hundred. I need the other two zeros to be place holders. They've got to be place holders, so I can see the 0 Tens and the 0 Ones."

Figure 31: 238+3 represented with pictorial base ten blocks on the IWB and set in a column addition.

Comments:

In this episode, T1a's instruction is such that allows for only one answer, which involves the pupil working out whether T1a would write anything in her Tens' and her Ones' columns if she did not have any pictorial representation in them.

Fred's mathematical contribution – his verbal part "two zeros" accompanied by a gestural representation of two zeros – resonates with the contribution expected by the teacher: the verbal part "two zeros" is entailed in Fred's mathematical contribution.

T1a values, attunes and incorporates this contribution in her mathematical work.

In this episode, attuning refers to the gestural utterance of Fred – his gesturing of two zeros on his book. The teacher replicates Fred's gesturing in her writing of two zeros on the IWB.

There is a key feature in Fred's mathematical contribution: Fred's meaning making of place value, which involves the necessity of 0 as a place holder and the awareness of Hundreds as accompanied by Tens and Ones.

Pertinent in Fred's mathematical expression of the above meaning making is his gestures. His gestures offer more input on his mathematical contribution than his speech. They particularly show location for each of the two zeros, as placed in two distinctive columns, Tens and Ones.

His gestures may have been developed as a result of visual impairment: in order to avoid visual work, Fred has developed his 'gestural mathematical skills'[49] (this is evident other times too in Fred's mathematical work). The teacher though emphasises on vision in the mathematical work: "so I can see the 0 Tens and the 0 Ones".

In this episode, it is also significant to point out the sighted peers' reactions to the VI pupil's mathematical contribution. Sighted pupils focus on Fred's contribution and one of them responds to the teacher's question that is based upon Fred's contribution.

The sighted pupil's mathematical contribution has different elements compared to the VI pupil's one:

- It includes only speech – no gestures
- It involves the use of mathematical terms "Ones", "Tens" which the VI pupil's mathematical contribution does not include

These two constitute characteristics of sighted pupils' mathematical contributions and are particularly valued by the teachers.

Similarly, the use of gestures and the non-verbal use of mathematical terms, which constitute two characteristics of VI pupils' mathematical contributions, are often not explicitly valued by teaching staff. The use of gestures is valued mostly when gestures are accompanied by speech: such speech that resonates with contributions that teaching staff expect to hear. Gestures are valued mostly when they constitute an accompaniment to speech, in other words when they play a secondary role, with speech being sufficient for the interlocutor's meaning making of the VI pupil's mathematical contribution.

The non-verbal use of mathematical terms is not explicitly valued by the teachers. The teachers explicitly approve contributions which include verbal mathematical terms. This may be related to institutional and curricular factors, which require pupils to use the mathematical terms that are taught in the particular period and are included in the National Curriculum.

7.4b Valuing, attuning and incorporating occur when the mathematical question is formed in a way that allows for more than one answers, with the VI pupil's mathematical contribution entailing one of these answers

Episode:

In the Y5 class of School 2, T2 writes 2/6 on the IWB. T2's writing is shown on Luke's iPad, which is connected to T2's computer through a VNC connection.
T2 asks the class if 4/12 is greater, less or equal to 2/6.
Zon says "equal".
T2 then repeats Zon's answer and asks Zon why these two fractions are equal.
Zon responds that they are equal because they are on the same vertical line on the fraction wall – when the ruler is placed on the fraction wall vertically after any of these fractions.
T2 rewards Zon and then asks the class how else they could do it if they did not have a fraction wall.

[49] Mathematical skills expressed (and constructed) through gestures.

T2: “How else could you work that out?”
Luke is the only pupil who puts his hand up.
T2: “If you did not have a fraction wall, Luke, how could you work that out?”
Luke suggests multiplication.
Luke: “You could do 2 times 2 and 2 times 6?”
T2 looks at the IWB: “Ok. So, if you multiplied this fraction by 2 – *he draws a semi-circle to connect the numerator of 2/6 with the numerator of the second fraction and a semi-circle to connect the denominator of 2/6 with the denominator of the second fraction* –, 2 times 2 equals – *he writes 2x2 above the first semi-circle.*”
Luke: “4.”
T2: “4 – *he writes 4 as the numerator of the second fraction.*”
T2: “And 2x6 equals – *he writes 2x6 below the second semi-circle.*”
Luke: “12.”
T2: “Thank you. You know your tables, your 2 times tables, I’m really proud. *He writes 12 as the denominator of the second fraction.*”
T2: “That’s 4/12.”
T2: “How else could I do it? I don’t want to multiply because we’ve been doing lots of division.”

Comments:

In this episode, T2’s instruction is such that allows for more than one answers, which involve the pupil working out whether 4/12 is greater, less or equal to 2/6 using whichever method they wish.

Unlike the episodes of 7.1b and 7.3b, here T2 does not dictate the method for the comparison of these two fractions. He is instead open to various methods.

Luke’s mathematical contribution – his suggestion of multiplication of each of the terms of 2/6 by the same number, 2 – is welcomed by T2. T2 values, attunes and incorporates Luke’s mathematical contribution in his comparison of 2/6 and 4/12.

However, while he is open to various methods, T2 seems to have a preferred method: that which involves the operation that the class has been working on. This method involves division: the comparison of 4/12 and 2/6 by dividing each of the terms of 4/12 by 2. T2’s preference of this method is probably related to institutional and curricular factors.

There are two key features in Luke’s mathematical contribution: Luke’s meaning making of fractions as consisting of a numerator and a denominator; and, Luke’s meaning making of fraction comparison as a comparison of the numerators and of the denominators. This comparison is done through multiplication.

There is an element of abstraction, which is evident in the relation of two pairs of numbers. This element may have been developed as a result of visual impairment: the limited vision of Luke seems to have prompted him to resort to alternative tools for the meaning making of fraction comparison. In line with Vygotsky (1978), the alternative tools have mediated Luke’s mathematical learning and have therefore contributed to him constructing an alternative meaning of fraction comparison.

There is another element of abstraction, which is evident in the order in which Luke compares the two fractions. Luke ‘went’ from the fraction with the smaller terms to the one with the bigger terms. This differed from the order in which the teacher presented the two fractions.

On the other hand, the sighted peer’s mathematical contribution[50] is concrete and highly visual.

[50] Zon’s mathematical contribution: 4/12 is equal to 2/6, because they are on the same vertical line on the fraction wall – when the ruler is placed on the fraction wall vertically after any of these fractions.

The teacher visualises Luke's mathematical contribution. This shows his emphasis on vision as the prevalent sense in mathematics.

It is particularly interesting that Luke was the only pupil who raised his hand in the teacher's question, which sought for answers that go beyond concretion. Him being the only pupil shows that his sighted peers did not seem to go beyond visual, concrete methods. Luke was also the only pupil who shared in his focus group that he finds nothing hard in maths. The rest of the pupils from his class who participated in focus groups reported that they have mathematical difficulties in specific mathematical topics, such as fractions.

7.5 Implicit non-valuing but attuning of VI pupils' mathematical contributions

This section concerns episodes of implicit non-valuing but attuning. In this section, incorporating is not applicable.

7.5a Implicit non-valuing but attuning occur when the mathematical question is formed in a way that allows for more than one answers, with the VI pupil's mathematical contribution being different from the answers expected by the teaching staff

Episode:

In the Y5 class of School 4, T4a asks the class to point to right angles in objects that are located in the classroom. T4a then moves around the class to see the objects that pupils identify. TA4 sits next to Ivor.
Below is the conversation between TA4 and Ivor after T4a asks the class to point to right angles:
TA4: "So, you need to find something in this room with a right angle."
Ivor: "There! That one! *He gestures towards the whiteboard*[51]*, which is hung on the wall behind him.*"
TA4: "Yeah, well done."
As she moves around the class, T4a hears Ivor's contribution.
Below is the conversation between T4a and Ivor:
T4a: "Feel it, Ivor. What is it doing? Feel it. It's curved. It's not 90 degrees. It's curved, isn't it? There's not a corner. Yeah?"
Ivor feels the whiteboard and then tells T4a: "Yeah."

Comments:

In this episode, T4a's instruction is such that allows for more than one answers, which involve the pupil identifying any objects in the classroom that have at least one right angle. However, the method is implicitly single and is implicitly pre-determined by the teacher. It involves visual identification of right-angled objects.

Ivor's mathematical contribution – his identification of the whiteboard, which is in a shape that may look like a rectangle[52] – does not resonate with the contributions expected by the teacher. The whiteboard has no right angles, as it is a curved shape. However, with a first sight, it may look as if it has 4 right angles.

This is a case of mathematical struggle associated with visual access. While Ivor knows what a right angle is, his use of vision makes him misinterpret an object. Vision is wholistic (Ochaita & Rosa, 1995). However, touch is gradual, allowing the exploration of an object from its individual parts to its whole (Ochaita & Rosa, 1995).

The misinterpretation associated with vision is not limited to the VI pupil. It is evident in the TA too, who is sighted.

[51] The whiteboard's shape is curved but may look like a rectangle.
[52] It may look as if it has the two pairs of opposite sides parallel and equal to each other, with the consecutive sides as if they form a right angle.

T4a seems to have been more careful in her observation of the whiteboard. She seems to have focused on the whiteboard by looking at its individual parts that together form the whole. This is a characteristic of touch but seems to have been used by T4a in her vision too.

Acknowledging the visual impairment of Ivor, T4a asks Ivor to use his touch to realise the curvedness, therefore the non-existence of any right angles on the whiteboard.

T4a does not value Ivor's mathematical contribution, with her non-valuing being expressed implicitly. Her implicit non-valuing is attributed to the fact that Ivor's mathematical contribution does not meet the right-angle criterion of her question.

However, despite her implicit non-valuing, T4a attunes to Ivor's contribution: she realises that his mathematical contribution occurred as a result of vision, not as a result of mathematical inefficiency. To help him overcome his misinterpretation, she asks him to use his touch, which enables one to identify an object by exploring it from its individual parts that together form the whole (Ochaita & Rosa, 1995).

What the class misses from not hearing Ivor's mathematical contribution is Ivor's acknowledgement of what right angle means but his misinterpretation of an object as a result of visual access and the correct interpretation of the same object as a result of tactile access.

As the TA did, sighted pupils too would probably make the same mistake with the VI pupil, for the same reason.

It is interesting that touch was used only when vision was found to be inappropriate as a perceptual sense for Ivor. This shows the prevalence of vision in the mathematical learning of Ivor and the secondary role of touch in this learning.

If Ivor had been instructed from the beginning to use his touch to identify an object that has at least one right angle, he would have realised that the whiteboard is a curved shape and he would have rejected it as an object that meets the task's requirements.

The prevalence of vision in the mathematical learning of Ivor seems to be related to the National Curriculum rather than T4a's preference. T4a told me that she is in favour of touch for Ivor's mathematical learning but she prioritises vision in line with the new curriculum: "I think I've always tried to sort of arrange practical and paper and physical experiences. I think the new curriculum doesn't quite do so much of that, so it's a lot more test-driven and I feel that sometimes we are more paper-based which makes the different sort to him, doesn't it?"

The prevalence of vision in the mathematical learning of Ivor and the secondary role of touch in this learning is also cultivated to – and acknowledged by – Ivor. In particular, Ivor told me that he does not like using objects that he can touch in the mathematics lesson because they make him "distracted" in the lesson. He also told me: "I use them only when I am told to use them. For example, they give me objects to feel them when I can't see them well."

In general, T4a feels that she and T4b do not always include Ivor successfully. She particularly states that, with regard to Ivor's inclusion, "we don't always get it right" and that "[s]ometimes we get it right and sometimes I feel that we don't quite get it right".

This feeling stems from the lack of training on the inclusion of VI pupils. I note that T4a received neither pre-service nor in-service training on the inclusion of VI pupils. She received training on inclusion – as a general topic – during her PGCE course. T4a particularly said that her training on inclusion was about "learning difficulties like dyslexia", "autism and just general behaviour". As an in-service teacher, she reported that, in order to include the VI pupil, she has had to "pick things up from TA4", who has been sent by the school to have a training on visual impairment.

However, despite T4a's lack of training, her feeling indicates T4a's (and T4b's) positive attitude towards – and reflection on – the inclusion of VI pupils. To sum up, T4a is positive towards Ivor's inclusion despite limited attention on the inclusion of VI pupils in ITE.

7.6 Non-valuing and non-attuning of VI pupils' mathematical contributions

This section concerns episodes of non-valuing and non-attuning. In this section, incorporating is not applicable.

7.6a Non-valuing and non-attuning occur when the mathematical question is formed in a way that allows for only one answer, with the VI pupil's mathematical contribution being different from the answer expected by the teaching staff

In the Y3 class of School 1, the class are given a worksheet with addition and subtraction calculations. They are asked to work out the sums and the differences, respectively. The sighted pupils are given A4 worksheets while the VI pupils are given A3 worksheets. The A3 worksheets display the contents of the A4 worksheets enlarged. TA1a sits in between Ian and Fred.
One of the addition calculations is "352+4tens=". Above this calculation, there is a pictorial representation of 352 with blue circles in a 3-column table "100s", "10s" and "1s" (Figure 32).
Ian writes "356" as the sum of this calculation.
TA1a sees this answer and tells Ian that he is wrong.
She takes Ian's ruler and covers with the ruler the part "52+4tens", leaving only the Hundreds digit, "3", uncovered and, afterwards, she points to the 3 blue circles that represent the Hundreds of 352.
After she has explained the rest of the digits of 352 in the same way, she asks Ian to draw 4tens on the pictorial representation of 352.
Ian refuses.

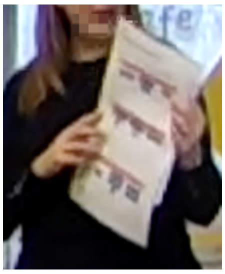

Figure 32: 352 is represented pictorially with blue circles in a 3-column table "100s", "10s" and "1s" in the beginning of the worksheet.

Comments:

In this episode, the nature of the task is such that allows for only one answer, which involves the pupil writing the sums and the differences of the given addition and subtraction calculations, respectively.

Ian's mathematical contribution – "356" – does not resonate with the contribution expected by TA1a, which is 392.

TA1a neither values nor attunes to Ian's mathematical contribution. Her non-valuing is evident in her consideration of Ian's contribution as incorrect.

Her non-attuning is evident in that she does not engage in trying to understand Ian's mathematical meaning making. She instead tries to direct him towards finding 392. Ian's mathematical contribution emerges as a result of visual presentation of the given addition calculation. The class has been used to calculations which are represented only by mathematical symbols – and not by a combination of mathematical symbols and verbal terms. In this respect, Ian's mathematical contribution should be anticipated. Ian considers the given addition calculation as consisting of only mathematical symbols and he ignores the verbal term.

TA1a's non-valuing and non-attuning have negative impact upon Ian's attitude to mathematics. In this episode, Ian refuses to continue to work on the given mathematical task. In his focus group, Ian reported: "Maths is quite hard for me and I try and I get it right – this would make me happy." He also reported: "I like when T1a says "You're trying your best" – I get really happy."

What the class misses from not hearing the Ian's mathematical contribution is the significance that the verbal term "tens" plays in the working out of the given addition calculation.

7.7 Implicit non-valuing, non-attuning and non-incorporating of VI pupils' mathematical contributions

7.7a Implicit non-valuing, non-attuning and non-incorporating occur when the mathematical question is formed in a way that allows for more than one answers but only a specific 'strand' of these answers is considered as correct, with the VI pupil's mathematical contribution not entailed in the particular 'strand' of these answers

Episode:

In the Y3 class of School 1, T1a displays the following four calculations on the IWB: 234+3=, 506+8=, 455+7= and 521+6=. T1a's display is shown on Fred's iPad, which is connected to T1a's computer through a VNC connection.
T1a earlier asked the class to circle – from the above four calculations – the calculations that are the hardest to solve and to then explain why those calculations are hard. The calculations were given to the class on a worksheet.
When she introduced the task, T1a told the class: "There isn't necessarily a right or a wrong answer to this [task]."
In the class discussion, after T1a has written the sums of 234+3= and of 506+8=, she asks the class to put their hands up if they think that 506+8 is slightly harder than 234+3. Some pupils, including Fred, put their hands up.
Then, T1a to the class: "Why was it [506+8] slightly harder than the other one [234+3]?"
Fred has his hand up and T1a picks him.
T1a: "Fred."
Fred: "Because there was a bigger number."
Processing Fred's answer and looking at the IWB, T1a: "It was a bigger number. Does that make it harder? Does it make it harder? Greg, what do you think?"
Sighted pupils respond "No".
Greg: "You have to add up to the Tens a Tens' digit."
T1a, nodding: "It involved some exchanging, didn't it, of your Units for a Ten. The first one didn't, did it? You just had the Units. It involves just you add the Units to Units – there was no exchanging in this one. You had to – *she moves towards the IWB* – 8 add 6 is 14 – *she points to 8, to 6*[53] *and to 14*[54]*, respectively*. You had to then exchange ten of your Units for one Ten, haven't you? *She gestures for 10 Units and 1 Ten.* But there weren't any Tens – were they? – in there, so it was doable."

Comments:

In this episode, T1a's instruction is such that allows for more than one answers, which involve the pupil explaining why 506+8 was slightly harder than 234+3. T1a's openness to more than one answers is clear from the beginning, when she introduced the task to the class.

[53] The "6" from 506.
[54] The "14" from 514.

However, T1a does not seem to be open to Fred's mathematical contribution. She instead approves contributions that belong to a specific 'strand': contributions in which the comparison of the difficulty between two calculations draws upon place value. More specifically, one calculation is more difficult than another one when the former involves exchanging while the latter does not.

Such a contribution is suggested by a sighted peer.[55]

Fred's mathematical contribution – "Because there was a bigger number" – differs from this 'strand' of contributions.

T1a does not value, does not attune and does not incorporate Fred's mathematical contribution.

Her non-valuing, which is implicit, seems to occur because Fred's mathematical contribution does not belong to the particular 'strand' of the answers which T1a expects to hear.

Her non-attuning is evident in that she does not try to engage in understanding Fred's mathematical meaning making. She does not even ask Fred to elaborate his contribution, which is quite vague. She does not even wait for Fred to respond to her question if having a bigger number makes the addition harder. She instead asks a sighted pupil to tell her another explanation.

T1a does not incorporate Fred's mathematical contribution. Her non-incorporating occurs as a result of her implicit non-valuing.

The missed opportunity for the class to engage with Fred's contribution deprives the class of the chance to expand their mathematical horizons and to reflect on an alternative contribution that may enrich their mathematical learning. This seems to concern pupils, such as Rachel, who emphasises the mathematical benefits that she gains when she engages with multiple ways of working out of mathematics. In her focus group, Rachel said: "I like learning new types of maths, like learning new things to do on how to work out things, like many methods to do the same thing. It makes it easier to find out if the answer is correct, so you get different methods and then try doing them and then if most of them are correct then that means that that answer is correct, because you can check it with a different method."

As in the episode of 7.4a, the sighted pupil's mathematical contribution has different elements compared to the VI pupil's one:

- It includes clear, non-vague speech
- It involves the use of mathematical terms – "add", "Tens", "Tens' digit" – which the VI pupil's mathematical contribution does not include

These two characteristics of the sighted pupil's mathematical contribution are particularly valued by the teacher. However, the use of vague verbal expressions and the non-use of mathematical terms, which constitute two characteristics of VI pupils' mathematical contributions, are often not valued by the teacher.

7.8 Synthesis of findings across 7.1-7.7

In this section, I will summarise the different manifestations of the sighted pupils' and staff's responses to VI pupils' mathematical contributions, as these manifestations were identified in Phase 1 of the study.

In this chapter, I presented seven different types of responses to the mathematical contributions of VI pupils. These responses were the following: valuing; implicit valuing and attuning; valuing and incorporating;

[55] Similar reasoning was offered earlier too by another sighted peer (on why 234+3 is easy) and the teacher valued it. Similar reasoning is offered later too by sighted peers (on why 455+7 is slightly harder than 506+8, on why 521+6 is easy, on why 455+7 is the hardest, on why 234+3 is easier than 455+7 and 506+8, and on why 521+6 is the easiest) and the teacher values it.

valuing, attuning and incorporating; implicit non-valuing but attuning; non-valuing and non-attuning; and, implicit non-valuing, non-attuning and non-incorporating.

The non-applicability of inclusion elements did not affect the level of inclusion in which the relevant episodes belong. For example, the “valuing” of VI pupils’ mathematical contributions in cases where attuning and incorporating were not applicable is in the same level of inclusion as the “valuing, attuning and incorporating” of these pupils’ mathematical contributions.

On the other hand, the existence of the opposite manifestation of inclusion elements affected the level of inclusion in which the relevant episodes belong. For example, the “implicit non-valuing but attuning” is located further left from inclusion, on a continuum from exclusion to inclusion.

In many cases, the applicability of attuning is pre-determined by the nature of the mathematical question either explicitly or implicitly and in a way that differs from that expected by the teaching staff. In particular, when the mathematical question is formed in a way that allows for only one answer, then the VI pupil is not invited to communicate his mathematical meaning making but only the final product. Similarly, when the mathematical question is formed in a way that allows for more than one answers, then the VI pupil is often invited to communicate his mathematical meaning making alongside the final product.

While the above two cases constitute the ‘rule’ for the applicability of attuning, there are some exceptions in the applicability of this rule. The exceptions have to do with the VI pupil’s attitude to the particular questions. In the former type of questions, the VI pupil sometimes communicates (part of) his mathematical meaning making alongside the final product.

This occurs when the final product, which the teaching staff expect to hear, is not conveyed by the VI pupil. The VI pupil conveys part of his mathematical meaning making that, alongside the insufficient final product which he utters, form a final product which resonates with the one that the teaching staff expect to hear. So: VI pupil’s final product & VI pupil’s mathematical meaning making resonate with the final product that teaching staff expect to hear.

It also occurs when the final product, which the teaching staff expect to hear, is conveyed by the VI pupil. The VI pupil conveys part of his mathematical meaning making in order to further illustrate the final product. So: final product & VI pupil’s mathematical meaning making ‘exceed’ the final product that teaching staff expect to hear.

In the former case of the VI pupil’s communication of (part of) his mathematical meaning making, attuning is applicable and necessary for the inclusion of the VI pupil’s mathematical contribution.

In the latter case of the VI pupil’s communication of (part of) his mathematical meaning making, attuning is optional but ideal for the inclusion of the VI pupil’s mathematical contribution.

Attuning can be applicable in: verbal mathematical meaning making; gestural mathematical meaning making; and, combination of verbal and gestural mathematical meaning making.

In Phase 1, attuning:

- Either was not applicable, in most of the inclusion cases
- Or was applicable but did not take place, in the exclusion cases

The non-applicability of attuning in most of the inclusion cases is evident both in mathematical questions formed in a way that allows for only one answer and in mathematical questions formed in a way that allows for more than one answers. In both types of questions, the class teachers were primarily interested in the final product and not in the intermediate steps (the method) which the VI pupil followed in order to find the particular product. This primary interest may be attributed to institutional and curricular factors.

The existence of the opposite manifestation to attuning in the exclusion cases is evident too in both types of mathematical questions. In both types of questions, the VI pupil communicated the mathematical method that he followed in order to find the particular product. However, the teaching staff did not embody the VI pupil's method. They instead attuned to specific methods. Their attuning to specific methods may be attributed to institutional and curricular factors.

Most of the Phase 1 episodes took place in mathematical questions which were formed in a way that allows for only one answer.

7.9 Phase 2 of the study

In this section, I present: examples from the implementation of the Phase 2 tasks presented in 6.6, with the examples tailored to VI pupils' mathematical contributions and to class teachers' and sighted pupils' responses to these contributions; the impact that the implementation of these tasks had upon teaching staff and pupils; and, the location of the sighted people's responses to VI pupils' mathematical contributions in the two camps of the CAPTeaM project.

First auditory task on number sequences (S3Y1): Mathematical contribution and response
T3a plays the number sequence 12, 22, 32, 42 with her two xylophones. She utters each number while she plays it, e.g. when she plays each of the three sounds for 12, she says "Ten, Eleven, Twelve", respectively. She then asks the class what the next number is going to be. Ned says that the next number is going to be 52. T3a asks him to play this number on her xylophones. Ned plays 53. T3a values Ned's contribution, by rewarding it.

Second auditory task on number sequences (S4Y5): Mathematical contribution and response
T4a asks Ivor and Frank to play the number sequence, which they have constructed, to the rest of the class, without them revealing to the rest of the class what the numbers are. Ivor and Frank play it, with Ivor using a tambourine and with Frank using a triangle. Each sound of the tambourine represents a Ten and each sound of the triangle represents a Unit. Their number sequence is 27, 20, 13, 6. After they have played it, T4a values their contribution, by rewarding it. She attunes to it, by asking Ivor and Frank if there were actually five numbers (which was the quantity of numbers that T4a had told the class to play in each number sequence) and by telling them that she got confused with what numbers they played. T4a then incorporates Ivor's and Frank's contribution into the lesson, by asking the rest of the class what the number sequence which Ivor and Frank played is.

Tactile task on number sequences (S3Y1): Mathematical contribution and response
T3a invites Ned to close his eyes, feel a number on the worksheet and tell her what the number is. Ned feels it and says "10" (the number on the worksheet is actually 10). T3a values Ned's contribution, by rewarding it.

First tactile task on shapes (S2Y5): Mathematical contribution and response
T2 moves next to Luke and asks him to put his hand in the bag which T2 holds, pick a shape and feel it without taking it out of the bag or seeing it. Luke follows T2's instructions and then says: "Feels like a rectangle". T2 values Luke's contribution, by rewarding it. He attunes to it, by repeating it, and he then asks Luke what the properties of a rectangle are. Luke: "It's got…2…I feel like on the short sides they are equal to each other and then the long sides are equal." T2 values Luke's contribution, by agreeing to it and by enthusiastically saying that he likes it. He then attunes to it, by saying that Luke has actually described short and long. Afterwards, he incorporates Luke's contribution into the lesson, by asking the rest of the class why Luke has described short and long sides for a rectangle and why he didn't say that all the sides are equal.

Second tactile task on shapes (S2Y5): Mathematical contribution and response
When T2 invites the class to share their mathematical learning experiences from the two shapes in Figure 1 (Stylianidou & Nardi, 2019a, p. 346), the conversation turns to Shape X. Luke then proposes the following comparison with the yellow circle (the yellow circle is seen in Figure 2, Stylianidou & Nardi, 2019a, p. 346):

"With the normal circle like this *[he shows and grabs the yellow circle]* feels like, feels like it's gonna roll more *[he positions the yellow circle as if it is ready to roll]*. That one *[he shows Shape X]* feels like it's just gonna bob up and down". T2 values Luke's contribution, by rewarding it. He attunes to it, by saying that Luke went round Shape X and that then he felt a proper circle. He incorporates Luke's contribution into the lesson: he first incorporates Luke's description of a circle, by asking the rest of the class to tell him like what the proper circle is going to roll along; and, he then incorporates Luke's description of Shape X into the lesson, by telling the rest of the class that wheels which have very big kinks in them make passengers of the cars bump up and down. Afterwards, he says that the circle has totally smooth edge and that Shape X almost has vertices.

In Phase 2, the class teachers' and sighted pupils' responses in the exclusion spectrum were minimised. The minimised existence of cases in the exclusion spectrum is evident in the class teachers' openness to a variety of mathematical contributions, which they valued as long as the contributions were mathematically rigorous but not necessarily consistent with contributions that they have been used to. This openness may be attributed to the fact that there were no institutional and curricular factors which underpinned the responses: instead, the only criterion for the approval of mathematical contributions in Phase 2 was their mathematical rigour, irrespectively of any learning objective and of any sensorial modality.

In Phase 2, the class teachers and sighted pupils shifted from responses of the exclusion spectrum to responses of the inclusion spectrum. In other words, they became more inclusive in their responses to the VI pupils' mathematical contributions.

Also, in Phase 2, the attuning element was often applicable. The class teachers shifted towards attuning of mathematical contributions that differed from those which they have been used to in Phase 1 – and which mostly arose from visual construction of mathematical meaning.

Pertinent in the applicability – and emergence – of attuning in Phase 2 was the class teachers' foci on the way in which the pupils constructed the specific mathematical meaning. As reported earlier, in Phase 1, the class teachers mostly focused on the final product of the answer rather than the mathematical meaning making that led to the construction of that product. That focus in Phase 1 may be related to institutional and curricular factors.

In Phase 2, these factors did not interfere in the class teachers' and sighted pupils' responses, probably because the lesson was experimental – and was thus open to principles that differed from those established in the classroom.

In Phase 2, we particularly designed mathematical tasks that allowed for more than one answers. We did this in order to bring various mathematical contributions of pupils into fore.

The nature of such mathematical tasks allowed me to examine any potential differentiation – or, in Nardi et al.'s (2018) wording, "resignification" – in the class teachers' and sighted pupils' responses to the VI pupils' mathematical contributions compared to their responses in Phase 1, on tasks of that nature.

Apart from the nature of mathematical tasks, in Phase 2, pertinent role in the design of mathematical tasks played the various sensorial modalities through which the tasks could be accessed and resolved. While in Phase 1 most of the tasks were designed to be accessed – and resolved – visually by the VI pupils, in Phase 2 tasks were designed to be accessed, and resolved, tactilely and auditorily – and not just by the VI pupils but also by the sighted pupils too.

The above two criteria in the task design contributed to the emergence of a variety of mathematical contributions in the same task in the classroom, in many of which the sensory modalities of touch and hearing shaped the classes' mathematical contributions.

In Phase 2, the class teachers shifted their valuing and incorporating response from contributions that belong to a specific 'strand', which the teaching staff had in mind (and was resonant with institutional and curricular principles), to contributions that are of mathematical rigour even if they differ from those that the teaching staff have been used to.

Pertinent role in this shift seems to have played the design of mathematical tasks which were based on sensorial modalities that the teaching staff were not used to: touch and hearing.

As stated earlier, another factor for this shift seems to have been the experimental nature of the Phase 2 lessons. This nature probably affected the teaching staff towards reacting beyond the institutional and curricular expectations.

In Phase 2, the class teachers shifted towards explicit valuing of mathematical contributions that differed from those which they have been used to in Phase 1 – and which mostly arose from visual construction of mathematical meaning.

They also shifted towards incorporating of mathematical contributions that differed from those which they have been used to in Phase 1 – and which mostly arose from visual construction of mathematical meaning.

Apart from shifts in their responses, there were also shifts in the class teachers' attitudes to sensorial modalities in the mathematical learning. More specifically, in Phase 2, the class teachers shifted towards appreciating touch and hearing as sensorial modalities in mathematics. They recognised – and valued – the mathematical and social benefits that touch and hearing mediate, not just to the VI pupils but also to the rest of the class too.

In Phase 1, they considered touch as the sensorial modality to be used if vision is not appropriate and as playing a secondary role in mathematical learning. Vision was considered as the prevalent sense in mathematical learning. Again, these considerations in Phase 1 may be related to institutional and curricular factors.

The positive, and inclusive, responses of the class teachers and sighted pupils towards tactile and auditory mathematical tasks constitute a small window towards the appreciation of touch and hearing in mathematical learning. It is envisaged that more tasks of such nature will be used in the future too by the particular class teachers but also by the class teachers more broadly.

Having summarised the different responses to VI pupils' mathematical contributions in both phases of the study, I will now move into locating these responses in the two teacher camps of the CAPTeaM project. These camps are presented by Nardi et al. (2018) and are discussed in detail in Chapter 2.

In Phase 1, the teaching staff tended to belong to the first camp of CAPTeaM: the camp which focuses on a more conventional description that involves the use of well-defined and familiar mathematical terms.

In Phase 2, the teaching staff tended to belong to the second camp of CAPTeaM: the camp which focuses on developing the VI pupil's mathematical contribution in its own right.

Central role in the particular location of teaching staff to camps seems to have been played by the institutional and curricular factors. In Phase 1, these factors seem to have determined the teaching staff's responses. In Phase 2, these factors do not seem to have affected the teaching staff, due to the experimental nature of the lessons.

The fact that the teaching staff belonged mostly to the second camp in Phase 2 suggests how differently inclusive the teaching staff are when they are not affected by institutional and curricular factors. In other words, it suggests how these factors often constrain the teaching staff from creating opportunities to engage with differently constructed mathematical contributions and from then developing these contributions in their own right.

With regard to VI pupils' mathematical contributions, as stated in 7.8, in Phase 1 the design of most of the tasks was such that allowed for only one answer, with the mathematical method being pre-determined by the teaching staff. In these cases, the mathematical meaning making of the VI pupils was mostly not communicated, with the final product being the only part of mathematics to be communicated.

In cases in which the VI pupils communicated their mathematical meaning making, there were some characteristics elicited from such mathematical contributions. These characteristics are presented in the section that follows.

7.10 Characteristics of VI pupils' mathematical contributions

I start with findings on the role of speech and gesture in VI pupils' mathematical contributions. I then proceed with findings on other aspects of VI pupils' mathematical contributions.

The role of speech and gesture in VI pupils' mathematical contributions:

- Non-recalling of mathematical terms: use of verbal descriptions, or of gestures, which indicate the expression of mathematical terms
    - VI pupils' mathematical meaning making: primary emphasis on the symbolic representation and secondary focus on the verbal mathematical term

These findings come to complement the relevant findings in the literature: that gestures often prevail compared to speech (with speech being insufficient for the expression of mathematical meaning) and that mathematical terms are often not used by VI pupils but VI pupils' meaning making of these terms is evident – and is expressed through gestures

- Vagueness in verbal mathematical expressions
- Gestures accompany, and often add to, verbal utterances
    - Gestural mathematical skills: often as an alternative to visual construction of mathematical meaning and to visual representation of mathematical work

Mental mathematical skills and embodied abstractions in VI pupils' mathematical contributions:

- Mental mathematical skills: often as an alternative to visual construction of mathematical meaning and, more specifically, to visual representation of mathematical work
    - The limited function of vision results in the substitution of eyes with an alternative tool – the mental meaning making and expression. This substitution results in the emergence of a different mathematical learning trajectory to that caused as a result of vision. This is in line with Vygotsky (1993; 1978).

      The construction of mental mathematical skills particularly results in the formation of abstractions, e.g. division as the inverse operation of multiplication / sum as the number that results from the modification of one addend in the way indicated by the other addend / fraction comparison as a comparison of the numerators and of the denominators

The literature, for example Healy and Fernandes (2011), shows that the VI pupils' mathematical contributions are different to commonly used mathematical contributions because of the different sensory tool through which the VI pupils access mathematics. My work extends/complements this finding: VI pupils' mathematical contributions are often different because the limited function of the visual tool results in the use of a different tool for mathematical access. This different tool is not necessarily sensory but it is often a mental tool. If this mental tool is added along with sensory, semiotic and material tools, it will result in a different path in the construction of mathematical meaning.

My work verifies Melanie[56]'s assumption: the limited access to the visual field results in the VI pupils' shifting to mental work.

- Embodied abstractions of visual mathematical representations

[56] Melanie is one of the participating teachers in the CAPTeaM project (and is reported in Nardi et al., 2018).

Mathematical struggle as a result of visual construction of mathematical meaning and as a result of visual mathematical representation:

- Mathematical struggle as a result of visual construction of mathematical meaning: when the mathematical representation is visually accessed by the VI pupils in a way that their vision mediates a mathematical meaning that does not resonate with the mathematical meaning expected by the teaching staff
- Mathematical struggle as a result of visual mathematical representation: when the visual representation of mathematics is done in a way that it conveys mathematical struggle to the VI pupils. This finding differs from the previous one in that it is not related to these pupils' vision.

Relational meaning making in VI pupils' mathematical contributions:

- Relational meaning making of mathematical terms: e.g. unit of angle measurement in relation to unit of percentage / obtuse angle as an angle in relation to right angle

In line with Healy and Fernandes (2014), the above characteristics of VI pupils' mathematical contributions constitute a necessary element for the creation of more inclusive mathematics classrooms. In other words, these characteristics need to be taken into account for the creation of more inclusive mathematics classrooms.

The study also found that mathematical contributions of VI pupils had the capacity to generate mathematical benefits to everybody in the classroom. However, these benefits were often missed in Phase 1, as the mathematical contributions were either shared only with one class member[57] or were not valued, attuned or incorporated by the teaching staff during the class discussion. The non-valuing, non-attuning and non-incorporating are probably related to institutional and curricular factors, according to which certain ways of mathematical meaning should be approved (and subsequently attuned and incorporated).

In Phase 2, the minimisation of non-valuing, non-attuning and non-incorporating resulted in the emergence of mathematical benefits from the VI pupils' mathematical contributions.

The emergence of mathematical benefits from the VI pupils' mathematical contributions constitutes an additional reason why inclusion of these pupils should be fulfilled: it should be fulfilled not only because it is a legal right of the VI pupils according to the UN CRPD (United Nations, 2006) but also because it is mathematically beneficial to the rest of the class too. Therefore, this study substantiates UNICEF's (2011) aspiration, which is presented in Chapter 1.

Having concluded my last data analysis chapter, I will now move into the Conclusions of the thesis (Chapter 8).

[57] In other words, they did not constitute part of the class discussion.

Chapter 8: Conclusions

In this final chapter, I first summarise the main findings from Chapters 5, 6 and 7 and answer the research questions that this study explored (8.1). I then present implications for theory (8.2) and implications for methodology (8.3) as well as implications for policy and practice (8.4) that emerge from this study. I conclude this chapter with presenting certain limitations of the study and recommending some future research (8.5).

8.1 Summary of findings

This study addressed the following research questions:

> RQ1: How are inclusion and disability constructed in the discourses of teaching staff and pupils in the mathematics classroom?
>
> RQ2: How do collaboratively designed mathematics lessons impact upon teaching staff's and pupils' discourses on inclusion and disability?

The study was conducted in two phases: first in an exploratory phase and then in an experimental phase. The first research question was explored in both phases of the study while the second research question was explored in the second phase of the study. To answer these research questions, data was collected in four primary mathematics classrooms (Y1, Y3 and two Y5 classes; pupils' ages varied from six to ten) in four mainstream schools in Norfolk, UK.

Chapters 5, 6 and 7 have focused on the three main themes with regard to inclusion and disability that emerged from the data analysis: the use of speech and gesture by class teachers and sighted pupils (Chapter 5); the intertwined presence of physical and digital resources in the learning experiences of VI pupils (Chapter 6); and, the mathematical contributions of VI pupils (Chapter 7).

I first summarise the main findings across Chapters 5, 6 and 7 from Phase 1 and I then discuss how these findings informed Phase 2. To this purpose, I associate findings with the sections in Chapters 5, 6 and 7 where relevant episodes are presented. So, for example, 6.4(ii) refers to the second episode presented in section 6.4.

With regard to RQ1, in the observed lessons and in the interviews, **inclusion is often achieved thanks to the intervention of the teaching assistant** [5.1.3, 5.2.3, 6.1(ii), 6.1(iii), 6.4(ii)]. This seems to be rooted in the way that inclusion is endorsed by institutions (the participating schools) and by educational policies at large. In line with Miles and Ainscow (2011, p. 3), inclusion seems to be endorsed as "transplanting special education thinking and practice into mainstream contexts". The responsibility for enacting inclusion often lies largely with the teaching assistants rather than the class teachers. The VI pupils are treated as "special" through the presence of a teaching assistant, whose role is to pay "special" attention to these pupils.

Across observed lessons and interviews, **vision is still considered as the prevalent sense in mathematical learning (e.g. through writing and through reading)**. This seems to be rooted in the view that mathematics is primarily a visual subject. Leuders (2016) as well as Healy and Fernandes (2014) imply the prevalence of vision in mathematical learning in that spatial mathematical data is usually perceived through the eyes. In particular, Leuders (2016, p. 44) reports that "[t]he mathematical content of visual teaching material is usually represented by its spatial structure" while Healy and Fernandes (2014, p. 125) report that "blind learners will use their hands as substitutes for their eyes in order to perceive spatial data".

This study has found that vision is prevalent in every aspect of mathematical learning: in access to mathematical demonstrations [5.2.1, 5.2.2, 5.2.3, 5.3.1(i), 5.3.2, 6.4(ii), 6.4(iv), 7.1a, 7.3b]; in access to worksheets [5.1.2, 6.3(ii), 7.2a(i), 7.2a(ii)]; in use of physical and digital resources [5.1.1, 5.1.3, 5.2.3, 5.3.1(ii), 5.3.2, 6.1(i), 6.1(ii), 6.1(iii), 6.2, 6.3(i), 6.4(i), 6.4(ii), 6.4(iii), 7.3a, 7.4a, 7.4b, 7.5a, 7.7a]; and, in presentation of mathematical work [6.3(i), 7.1b, 7.3b, 7.6a].

The privileging of vision is endorsed by the VI pupils too, who – to be, and feel as, members of the classroom community – attempt to use (their limited) vision even when touch can be used, for example, with physical resources which can mediate tactile mathematical meaning [5.2.3, 6.1(iii), 6.4(iii), 7.2a, 7.5a]. Such attempt to use (limited) vision often proves problematic: for example, it may lead to mathematical errors [6.4(iii), 7.5a].

The privileging of vision seems to work at the expense of making productive use of touch [6.4(iii), 7.5a]. This is because the institutions are such that do not promote touch as a form of mathematical meaning making. As a result, the VI pupils do not practise extensively – and therefore do not always become sufficiently skilled in – tactile construction of mathematical meaning. In line with Leuders (2016, p. 44), "[the pupils] need to be well accustomed to their material in order to handle it automatically and reduce the processing load". Otherwise, too little processing capacity remains for focussing on the physical resources and on the mathematics embedded in these resources (Leuders, 2016).

The study has also found that **speech is considered as the prevalent form of mathematical expression, with gestures being considered as a secondary form of mathematical expression** – namely, as an accompaniment to speech [7.2a(ii), 7.4a]. As a result, gestures are often not (explicitly) valued when they are used by the VI pupils [7.2a(ii), 7.4a].

This seems to be rooted in the formalist view. According to the formalist view, gestures are a secondary, rather neglected part of language: they are seen as spontaneous and idiosyncratic and as not constructed according to any standard forms. As Rotman (2009) states, impacts from the formalist view are evident today too: there is a general tendency to devalue communication systems which make use of the visual modality as compared to orally based ones and to assume that language should be identified with speaking, while communications using body movements are judged more primitive and non-intellectual.

However, despite the fact that gestures are considered as a secondary, less valued form of mathematical expression compared to speech in the mathematical contributions of VI pupils, the study has found that **gestures often play a primary role in the mathematical discourse of the sighted members of the classroom community**. I particularly emphasise the case of gesture use in lieu of speech [5.1.1, 5.1.2, 5.1.3] and the case of gesture use when speech is insufficient to convey mathematical meaning [5.2.1, 5.2.2, 5.2.3, 6.1(ii), 6.4(ii)]. As explained in Chapter 6, by "insufficient speech" I denote the speech that conveys mathematical meaning in a way that does not allow the interlocutor to constructively engage with it: the interlocutor needs to access the accompanied gestures in order to be able to constructively engage with the semiotic discourse.

The primary role of gestures in the mathematical discourse of sighted members of the classroom community indicates that **in the practices of sighted members gestures constitute a significant form of language**. The primary role of gestures in the practices of sighted members seems to resonate with the sensualist view, according to which speech came from gestures, and with the theory of embodied cognition (by Gallese & Lakoff, 2005), which constitutes an impact from the sensualist view.

The combination of the two findings on sighted members' response to gesture use of VI pupils and on the role of gestures in the mathematical discourse of sighted members suggests that, while gestures often constitute a primary form of language in their use by sighted members, they are often not (explicitly) valued when they are used by the VI pupils.

These two findings of the study – namely, the use of gestures by sighted members as a primary form of mathematical expression and the non-valuing of gestures by sighted members when gestures are used by the VI pupils – indicate a conundrum: they indicate how teaching staff embed themselves in the institutional norms but at the same time often deviate from these norms, even if this deviation is done naturally and unwittingly. More specifically, it seems that the teaching staff use gestures so naturally that they do not realise using them. However, when they see gestures used by the VI pupils, they seem to conform to the sociomathematical norms established in the classroom, according to which gestures do not constitute a proper form of mathematical expression.

From the above, I deduce that **attempts at inclusion are often marred by isolation from both a social and an academic (mathematical) point of view**.

From a social point of view, inclusion of VI pupils in mainstream classrooms may imply their isolation in two ways. First, VI pupils are often asked to work with an adult especially assigned for them while the sighted pupils work with the class teacher. As a result, the VI pupils are not provided with opportunities to interact with their sighted peers. The sighted peers are not provided with opportunities to know the VI pupils' needs and vice versa. As Argyropoulos and Stamouli (2006) have put it, it is a class that runs in two separate levels and there are two distinct groups of pupils: the sighted, who work with the class teacher, and the VI, who work with the teaching assistant. Second, VI pupils are often asked to use special resources in order to access the class teacher's demonstrations. While these resources often facilitate the VI pupils' access to the lesson, the way of their use does not create opportunities for social interaction between VI and sighted pupils in the lesson. In this respect, the provision of special resources to VI pupils socially differentiates these pupils from the rest of the class.

From a mathematical point of view, inclusion of VI pupils in mainstream classrooms may imply their isolation in two ways too. First, the use of gestures, which constitute a pertinent form of VI pupils' mathematical expression, is not valued as such by the teaching staff/institution. This may indicate that the VI pupils are not welcomed as VI in class and are welcomed only if they use the prevalent form of mathematical expression – that is, speech. Second, VI pupils are rarely asked to use senses that they have full access to in order to construct mathematical meaning. Instead, they are asked to use their limited vision. This may indicate too that the VI pupils are not welcomed as VI in class and are welcomed only if they use the prevalent sense of mathematical construction – that is, vision.

While the primary notion that was addressed in this thesis is inclusion, the notion of disability was also addressed. The notion of disability was addressed alongside the notion of inclusion. More specifically, manifestations of enabling and of disabling were addressed in the light of manifestations of including and of excluding. Sometimes disabling was identified alongside unintentional exclusion [5.2.2]; other times disabling was identified in attempts at inclusion [6.2].

The latter case is particularly significant and suggests that the particular form that inclusion takes place – i.e. with the provision of special digital resources to the VI pupils – is often undesirable to these pupils, who try to act as their sighted peers even though this action does not allow them to access the lesson [6.2].

Enabling was mostly identified in the efforts of teaching staff towards inclusion [5.1.2, 5.1.3, 5.2.3, 5.3.1(i), 5.3.1(ii), 5.3.2, 6.1(i), 6.1(ii), 6.1(iii), 6.3(i), 6.4(i), 6.4(ii), 6.4(iv), 7.1a, 7.1b, 7.2a(i), 7.2a(ii), 7.3a, 7.3b, 7.4a, 7.4b, 7.7a]. However, there were cases that these efforts were not successful [5.1.1, 6.1(ii), 6.3(ii), 6.4(iii), 7.5a, 7.6a]. Overall, enabling prevailed, with the VI pupils being provided with resources that mediated their access to the lesson. Enabling was primarily achieved through the VI pupils' use of their limited vision rather than other senses.

With regard to RQ1 and RQ2, the Phase 2 lessons aimed to challenge the three main findings from Phase 1: the fulfilment of inclusion thanks to the **intervention of the teaching assistants**; the consideration of **vision as the prevalent sense** of mathematical learning; and, the consideration of **speech as the prevalent form** of mathematical expression, with **gestures** being considered **as a secondary form** of this expression. In Phase 2, inclusion was achieved only by the class teachers – and without the teaching assistants. Vision was not the prevalent sense in the construction of mathematical meaning. Instead, mathematical tasks that encourage touch and hearing were co-designed and distributed to the entire class. Touch and hearing were appreciated by the teaching staff and sighted pupils, both of whom were not used to these senses prior to the experimental phase. The VI pupils felt particularly comfortable in these tasks. Gestures were explicitly valued as a form of mathematical expression and were used by class teachers, sighted pupils and VI pupils as indicators of mathematical meaning. Sighted and VI pupils came closer to each other, with collaboration and learning each other's needs being prevalent. The VI were not the pupils who deviate from the rest of the class but were members of the class, whose mathematical constructions and forms of mathematical expression were brought into fore and valued. They felt proud of being different, with discussions on visual impairment openly taking place in the classroom.

With regard to disability in Phase 2, enabling prevailed while disabling was minimised. The fact that disabling was not eliminated indicates that class teachers are so much used to interacting with sighted interlocutors that they continue to unintentionally disable the VI pupils. Disabling occurred only when the class teachers used partial speech that invited the class to look at them, with their gestures made at an inaccessible place for the VI pupils.

From the above, it is deduced that the collaboratively designed mathematics lessons brought positive impact with regard to inclusion but not as positive with regard to disability.

8.2 Implications for theory

This study has endorsed a theoretical framework that combines the Vygotskian sociocultural theory of learning, the theory of embodied cognition and the social model of disability – and has deployed this framework to analyse evidence collected in a naturalistic context: actual mathematics classrooms. The CAPTeaM study's valuing and attuning categories, which themselves have sociocultural and embodied theory underpinnings, were used as a lens on the data.

8.2.1 Implications emerging from the study's use of Vygotskian sociocultural theory of learning

This study has found that VI pupils' mathematical contributions are often different to mathematical contributions of sighted pupils because the limited function of the visual tool results in the use of a mental tool. This finding complements Healy and Fernandes' (2011) finding that VI pupils' mathematical contributions are different because of the different sensory tools through which these pupils access mathematics. The mental tool could be added to the sensory, material and semiotic tools in the Vygotskian theory.

Moreover, Vygotsky's (1993) acknowledgement that the language of a culture tends to be designed for the able-bodied, which means that it may not be accessible to people who lack of, or have limited access to, a sensory organ, is verified in this study. More specifically, this study has found that the language of the classroom community, which involves speech and gesture use with speech being prevalent, is often not accessible to VI pupils. This occurs when gestures constitute the main form of mathematical expression and are not made accessible to VI pupils.

This study's model of inclusion expands that suggested by Vygotsky (1993). I will first discuss the model of inclusion suggested by Vygotsky (1993) and I will then present how my study expands this model.

Vygotsky suggested that the inclusion of VI pupils can be achieved through a substitution of the traditional mediational means with others, which are suitable to these pupils' needs. This suggestion implies that inclusion is achieved through adaptation: the design of the resources needs to change for the VI pupil in order to meet this pupil's needs. This suggestion also implies a differentiation of the VI pupils from the sighted pupils: the sighted pupils access the resources with their eyes while the VI pupils access the adapted resources with other means, for example with their hands.

Vygotsky's model of inclusion sets out from the notion of a "normal child" and of a prevalent mediational means associated with that normal child: "The positive particularity of a child with a disability is created not by the failure of one or other function observed in a normal child but by the new structures which result from this absence […] The blind or deaf child can achieve the same level of development as the normal child, but through a different mode, a distinct path, by other means. And for the pedagogue, it is particularly important to know the uniqueness of the path along with the child should be led” (Vygotsky, 1997, p. 17). In this respect, Vygotsky implies the prevalence of particular mediational means, which the able-bodied access, and the necessity for the alteration in the form of these mediational means, for the VI pupils to access.

This study’s model of inclusion is not embedded in the above implication. It is instead embedded in the fact that mediational means need to be designed for the entire class on the basis of a sense that is fully developed to everybody.

This study has also indicated the resonance of the Vygotskian sociocultural theory of learning with the inclusion and disability discourse of today’s international legislation on inclusion and disability (e.g. United Nations, 2006).

8.2.2 Implications emerging from the study’s use of the theory of embodied cognition

This study has found that bodily mathematical expressions are conveyed not only by VI pupils but also by sighted members of the classroom too. In both groups, mathematical meaning was often expressed differently in bodily manifestations, which tended to indicate additional aspects of meaning making compared to speech. In other words, bodily mathematical expressions, such as gestures, often complemented verbal mathematical expressions. This finding can further strengthen the non-dualistic approach to how the brain and the body work that embodied cognition theory endorses.

This study has also found that embodied abstraction is evidenced not only in gestures but also in speech (either through specific words/phrases or through tone of voice). Therefore, speech too evidences embodied abstraction that comes from experiences with concrete objects. This finding complements Healy and Fernandes’ (2011) finding that gestures act as embodied abstractions. It also resonates with Healy and Fernandes’ (2011) belief “that all abstractions are embodied and that it is by looking at gestures alongside spoken language that the embodiments associated by different learners with different mathematical ideas become more evident” (p. 169).

This study suggests that the term “embodied abstraction”, which is used by Healy and Fernandes (2011), could become a construct of the theory of embodied cognition (by Gallese & Lakoff, 2005). This is because the term “embodied abstraction” offers a complementary perspective to that offered by the term “embodied imagination”, which Gallese and Lakoff (2005) use.

This study uses both of these terms in the analysis. “Embodied imagination” is used to characterise a verbal and/or a bodily manifestation which does not come from the practical action that is described in the verbal utterance and/or is shown in the bodily manifestation. “Embodied abstraction” is used to characterise a verbal and/or a bodily manifestation which comes from the practical action that is described in the verbal utterance and/or is shown in the bodily manifestation.

Embodied abstractions are manifestations of previously experienced activities, which are now relived through simulation. In this respect, they are characterised by some form of repetition, now in non-physical contexts. On the other hand, embodied imaginations are not manifestations of previously experienced activities, which are now relived through simulation. Therefore, embodied imaginations are not characterised by some form of repetition. However, they constitute simulations, but not of experiences which were previously relived with the same object. Instead, they constitute simulations of previously relived experiences with another object: that to which the user attributes an embodied metaphor.

This study has also indicated the resonance of the theory of embodied cognition with the inclusion and disability discourse of today’s international legislation on inclusion and disability (e.g. United Nations, 2006).

8.2.3 Implications emerging from the study's use of the social model of disability

This study has indicated the resonance of the social model of disability with the disability discourse of today's international legislation on inclusion and disability (e.g. United Nations, 2006).

It has also indicated the appropriateness of the social model of disability as a theoretical lens to examine disability in the context of mainstream mathematics classrooms with VI pupils. The endorsement of the social model of disability as a theoretical lens suggests the deconstruction of disability and the reconstruction of mathematics classrooms in a way that they are enabling for the VI pupils.

8.2.4 Contributions with regard to "valuing", "attuning" and "incorporating"

This study has used the constructs of "valuing" and "attuning", which originate in Nardi et al.'s (2018) work, and the construct of "incorporating", which is similar to that of "managing the classroom" by Nardi et al. (2018). The study has used these three constructs in a different setting – and with more groups of participants – compared to the CAPTeaM project: in the mainstream mathematics classroom and to analyse class teachers', teaching assistants' and sighted pupils' discourses which act as a response to VI pupils' mathematical contributions.

The analyses presented in this study demonstrated that the "valuing" label can be applicable in cases where a mathematical contribution by a VI pupil may be technically incorrect but may at the same time carry potential to reveal alternative ways of tackling a mathematical problem (e.g. in the episode 7.6a).

"Attuning" has been found applicable not only in gestural mathematical discourse but also in verbal mathematical discourse and in a combination of verbal and gestural mathematical discourse. In other words, "attuning" has been found applicable in any type of mathematical discourse, as long as (part of) the mathematical meaning making is indicated.

Mathematical meaning making has been found to be indicated in types of mathematical questions that allow it to be indicated. In particular, mathematical questions that allow for only one answer do not allow mathematical meaning making to be indicated. On the other hand, mathematical questions that allow for more than one answers allow mathematical meaning making to be indicated when the mathematical question is open to more than one method.

The study has found two exceptions in this rule for the applicability of attuning. Both exceptions take place in mathematical questions that do not allow mathematical meaning making to be indicated: however, they involve the indication of mathematical meaning making by the VI pupils. Therefore, attuning is applicable.

The first exception occurs when the final product, which the teaching staff expect to hear, is not conveyed by the VI pupil. The VI pupil conveys part of his mathematical meaning making that, alongside the insufficient final product which he utters, form a final product which resonates with the one that the teaching staff expect to hear. So: VI pupil's final product & VI pupil's mathematical meaning making resonate with the final product that teaching staff expect to hear.

The second exception occurs when the final product, which the teaching staff expect to hear, is conveyed by the VI pupil. The VI pupil conveys part of his mathematical meaning making in order to further illustrate the final product. So: final product & VI pupil's mathematical meaning making 'exceed' the final product that teaching staff expect to hear.

"Incorporating" has been found more appropriate than "managing the classroom", as an analytic element for the inclusion of the VI pupil's mathematical contribution. This is because "incorporating" can directly characterise, and connect, the sighted member's response to the VI pupil's mathematical contribution – as "valuing" and "attuning" can do too.

"Incorporating" has been found applicable when the VI pupil's mathematical contribution takes place as part of the class discussion – and in a mathematical question which is connected with another question.

Moreover, the study has found that "valuing" often takes place implicitly. This happens when the teaching staff and sighted pupils do not explicitly express their appreciation and approval of the VI pupils' mathematical contributions. To this finding, the study suggests the construct "(implicit) valuing" in order to illustrate all cases of valuing: both the explicit and the implicit ones.

The study has also found that the opposite manifestation of an inclusion element often takes place. To this finding, the study suggests the constructs "non-valuing", "non-attuning" and "non-incorporating" in order to illustrate that the opposite manifestation of "valuing", "attuning" and "incorporating", respectively, takes place.

8.3 Implications for methodology

Papers from studies in this research area have used either an exploratory phase (e.g. Bayram et al., 2015) or an experimental phase (e.g. Healy & Fernandes, 2011). This study has used both an exploratory phase and an experimental phase.

It was very important for the study to be conducted in two phases, as the particular area is under-researched. The exploratory phase was significant to investigate how inclusion and enabling are implemented in the classroom: what works and what does not work well with inclusion and enabling in the particular class. The experimental phase was significant to trial some new principles and to investigate the impact that these principles have with regard to inclusion and enabling.

The study argues that it is vital that the exploratory phase is done prior to the experimental phase for the development of changes in this new research area. Changes were tailored to the particular class: not every class is the same with regard to pupils' needs, class members' preferences, institutional possibilities etc. The exploratory phase played the foundation role for the experimental phase to arise.

For the experimental phase to take place, it was very important that data from the exploratory phase is analysed. This is because findings from the exploratory phase inform the experimental phase: particularly, things that worked well for inclusion and enabling need to be maintained in the experimental phase while things that did not work well need to change. Inclusion is very much related to the particular context (class, school, county, country), therefore an experimental phase can be successfully done if there is acknowledgement of norms in the particular classroom (e.g. institutional possibilities and constraints).

In addition to the methodological implications from the study's conduction in two phases, the naturalistic elements in this study, the focus on specific pieces of mathematics and the focus on identifying evidence of benefits to all pupils are significant elements of the methodology of this study. These elements suggest that this study corroborates the power of naturalistic, qualitative methodologies.

Methodological implications also arise from my collaboration with class teachers on the design of mathematics lessons in the experimental phase of the study. The class teachers need to play a central role in the collaboration with the researchers.

More specifically, the lessons need to be co-designed when the class teachers are available and to also be on the mathematical topic(s) and learning objective(s) that the class teachers will have planned to be working on on the day of the lesson implementation. The co-designed lessons need to take into account the availability of the teachers and they also need to be tailored to the teaching and learning expectations that will have been scheduled. In this way, these lessons are aligned with the time preferences of the teachers and do not divert the teachers from their responsibilities.

It is also very important that the lessons are co-designed in a way with which the teachers feel comfortable. New principles can be designed provided that the teachers agree and engage. As the lessons are implemented by the teachers and as they aim to trigger long-lasting changes in the classroom, they need to be substantiated with the teachers' contributions.

8.4 Implications for policy and practice

The study has evidenced the inclusion efforts that participating schools are engaged in but has also identified certain discrepancies in the inclusion discourse between the schools' policies and practice.

While in all four schools' policies inclusive education is related to the provision of equal opportunities for all learners, in practice the enactment of the notion of "provision of equal opportunities" is not always endorsed. The teaching staff often implement practices which are not inclusive and which result in the limited participation of VI pupils in the mathematics lesson. Such practices are identified in all the three focal themes of this thesis and are reported in Chapters 5, 6 and 7. In many of these cases, exclusion occurs unwittingly by the teaching staff: in other words, the teaching staff do not deliberately exclude the VI pupil but their practices are such that result in the VI pupil's exclusion. With regard to class teachers, exclusion often occurs because the class teachers are often not aware of how to include the VI pupil in the mathematics lesson. In order that the notion of "provision of equal opportunities" takes place in the mathematics classroom, training on inclusion of VI pupils could be provided to the class teachers.

While in all four schools' policies inclusive education is related to all pupils' access to the same opportunities for learning, in practice the notion of "access to the same opportunities for learning" is not always fulfilled. VI pupils' access to learning is often mediated by digital resources which are provided to VI pupils and which enable these pupils to access what their sighted peers access. As Chapter 6 has shown, the provision of special digital resources often results in the VI pupils' abandonment of these resources and in their attempting to resemble their sighted peers, who do not use these resources in the mathematics lesson. Therefore, while digital resources enable VI pupils' access to learning, they do not seem to provide the same opportunities for learning as their sighted peers have. In order that the notion of "access to the same opportunities for learning" takes place in the mathematics classroom, opportunities for digital resources to be used by the entire class could be created. For example, we can design research activities which involve collaborative work between VI and sighted pupils and which require every pupil to use a particular digital device (e.g. an iPad) in a mathematics lesson.

While in all four schools' policies the class teachers are stated to be primarily responsible for implementing inclusive education and the teaching assistants are stated to secondarily support pupils with needs, in half of the classes of the study's data collection the situation was reversed.

More specifically, the class teachers were often highly dependent on their teaching assistants for the inclusion of the VI pupils in the mathematics lesson. This is justified by the fact that the class teachers have not had training on the inclusion of VI pupils while the teaching assistants have had training to support VI pupils in the lesson. In order for this principle in the schools' policies to be endorsed in practice, the class teachers could be provided with pre-service and in-service training on the inclusion of VI pupils. If the class teachers are the ones who implement inclusion, the social aspect of inclusion could be more easily achieved too: the VI pupils will belong to the community of learners who are taught by the class teacher.

Apart from the discrepancies between policy and practice, the study has also found a case in which inclusive education is implemented in practice as it is described in the policy but this implementation is often problematic. More specifically, while in all four schools' policies differentiation seems to be the way for implementation of inclusive education, the study has found that in practice a range of problematic issues arise in this way of implementation. On the other hand, when inclusive education is implemented through universally designed practices, the study has found a range of benefits, both for VI and for sighted pupils. These findings suggest that schools' inclusion policies could endorse **universal design** as another way for implementation of inclusive education.

I close this section with an argument relatively to a characteristic of inclusive education in School 4's inclusion policy. In that policy, inclusive education seems to be more an aspirational – than a realisable – form of education. The study argues that, if the above changes occur, inclusive education could become a pragmatic form of education.

Apart from discrepancies with regard to inclusion, the study has also identified discrepancies in the disability discourse between the schools' policies and practice.

While all schools except for School 3 define disability and seem to attribute disability to impairment – therefore to the individual –, in practice teaching staff indicate the social construction of disability. In particular, they mostly act in an enabling way for the VI pupils: that is, they consider the impairment of the VI pupils in a way that they do not disable these pupils. Therefore, in these cases in practice disability does not exist – and also disability is differentiated from impairment. This constitutes an example of teaching staff's differentiation from the institutional discourse. The teaching staff's actions suggest a potential amendment of schools' disability discourse: a shift from the medical model of disability to the social model of disability.

On the other hand, School 3 does not define disability even though it endorses the SEND code of practice (Department for Education & Department of Health, 2015), which defines disability. This constitutes an example of an individual school's differentiation from the national educational policy. This differentiation however occurs implicitly, as the national educational policy is endorsed by the school. Nonetheless, this differentiation suggests that a change in the SEND code's disability discourse may be feasible: a shift from the medical model of disability to the social model of disability. The fact that teaching staff in School 3 endorse the social model of disability in practice may constitute a step closer to the amendment of the SEND code's disability discourse.

School 4 is the only school that seems to endorse the charity model of disability too in its conception of disability. However, this conception is not reflected in practice. Teaching staff in School 4 treat the VI pupil as a pupil without any sense of pity. This action of teaching staff constitutes another example of teaching staff's differentiation from the institutional discourse.

Apart from policy amendments with regard to inclusion and disability, this study suggests the potential for policy amendments with regard to mathematics teaching/learning in the schools and in the National Curriculum. In particular, the study suggests the expansion in the consideration of mathematics with regard to the sense of construction of mathematical meaning and with regard to the form of mathematical expression. Apart from vision, other senses could be included too (e.g. touch and hearing) as senses of construction of mathematical meaning. Apart from speech, other forms of mathematical expression could be included too (e.g. gestures and facial expressions). All these constitute senses of construction of mathematical meaning, and forms of mathematical expression, in resonance with the theory of embodied cognition, therefore they should not be neglected or underestimated.

Implications for practice also emerge from my collaborative work with the teachers. This work triggered changes in the classroom. Even if these changes were probably temporary, small and local, they suggest a frame for future engagement of teachers with small projects, such as action research projects, in which teachers can trial alternative approaches in their classroom, namely changes that promote inclusion and enabling.

In Chapters 5-7, this thesis has illustrated the environment in which teachers work towards the inclusion of VI pupils in British mainstream mathematics classrooms. It has found that the way of the implementation of inclusion is often related to – and often draws upon – the way that inclusion is addressed in the ITE programmes in the UK as well as in the participating schools' policies on inclusion. It has brought forward issues with inclusion and has tried to address them, either by discussing them in Phase 1 or by making alternative suggestions in Phase 2 in order to avoid the re-appearance of these issues.

While the thesis has identified issues with inclusion in practice, it has also identified teachers' interest in inclusion. Despite their limited training on inclusion, the teachers do their best – with the knowledge that they have personally acquired – to include the VI pupils, though this attempt is not always successful.

This thesis has aimed to provide specificity around inclusion that the participating teachers have sought and that constitutes the most helpful way for them to include the VI pupils in their mathematics classroom. One way that this thesis could support teachers to improve the inclusion of VI pupils in their classroom is through reflection on the identified issues on inclusion: how these issues appear, what can be done in order to minimise the re-appearance of these issues. Episodes from this thesis could be used in the ITE of teachers on inclusion as well as in professional development courses for teachers.

It is necessary that ITE should be provided to teachers so that they know how to include the VI pupils. The ITE could include inclusive classroom practices as identified from this study in Chapters 5, 6 and 7. Particular emphasis could be put on designing mathematics lessons under the principle of universal design for learning. More specifically, all pupils' perceptual needs could be considered in the design of the mathematics lessons and we could minimise adaptations for the VI pupils. In this way, the VI pupils will not be considered as diverging from the 'able-bodied' but rather as learners in class whose needs are considered in the lesson design on the same basis to those of their sighted peers.

Apart from the specific practices and principles for lesson design, I could also advise teachers to establish an open dialogue with their VI pupils regarding these pupils' inclusion so that better inclusion is achieved in the classroom. As I mentioned in the early chapters of this thesis, I argue that VI pupils need to have a voice in their inclusion. Inclusive practices need to be tailored to the needs and preferences of the specific VI pupils.

8.5 Limitations of the study and future research

The small number of Phase 2 lessons and the non-videorecorded lessons constitute the two main limitations of the study. In particular, the small number of Phase 2 lessons resulted in modest contributions with regard to the experimental phase of the study. The non-videorecorded lessons denoted that gestures and facial expressions were sometimes missed.

Within the time frame for my data collection and considering the availability of the classes, a bigger number of experimental lessons could not have taken place. The non-videorecorded lessons could not have been avoided either, as they were in accordance with the provision of consent by the participants. The alternative for the latter limitation would have been to conduct data collection only in classes that consented to video-recordings. However, in that case, I would have had to not include classes who were keen to participate – and I did not want to do this. I am grateful for these classes' willingness to participate and honoured to have collected data in them. Also, in that case, my data collection would have been conducted in a smaller number of classes – therefore a smaller spectrum of practices, beliefs and experiences and a greater difficulty in extracting generalisation of findings.

Future research can focus on a collaborative design of more mathematics lessons on the basis of the findings from the three data analysis chapters of this thesis. Given that the exploratory phase of this study was quite long and informative, future research can have a shorter exploratory phase and a lengthier experimental phase. The experimental phase of future research can draw upon the findings of this thesis and upon any new findings from the new exploratory phase. It can start by an iterative process of amending, implementing and analysing the three mathematics lessons that the class teachers and I co-designed for this study.

APPENDICES

APPENDIX 1

Angeliki Stylianidou
PhD student
29.11.2017

Faculty of Social Sciences
School of Education and Lifelong Learning
University of East Anglia
Norwich Research Park
Norwich NR4 7TJ
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Deconstructing disability through/and promoting inclusive education in primary mathematics classrooms: the case of visually impaired learners

PARTICIPANT INFORMATION STATEMENT – class teacher

(1) What is this study about?

You are invited to take part in a research study about the inclusion of visually impaired learners in primary mathematics classrooms. I am interested in how you experience inclusion in your mathematics lessons, what you see as benefits that inclusion may bring to learners and what challenges you may face with teaching mathematics in classrooms that have visually impaired learners. You have been invited to participate in this study because you are a key member of such primary settings and, as a result, your contribution is highly significant for the study. This Participant Information Statement tells you about the study. Knowing what is involved will help you decide if you want to take part in it. Please read this sheet carefully and feel free to ask me questions about anything that you wish to know more about.

Participation in this research study is voluntary. By giving consent to take part in this study, you are telling me that you:

- ✓ Understand what you have read.
- ✓ Agree to take part in the research study as outlined below.
- ✓ Agree to the use of your personal information as described.
- ✓ You have received a copy of this Participant Information Statement to keep.

(2) Who is running the study?

The study is being carried out by the following researcher: Ms Angeliki Stylianidou, PhD student, School of Education and Lifelong Learning, University of East Anglia.
It is being carried out under the supervision of Elena Nardi, Professor in the School of Education and Lifelong Learning, University of East Anglia.

This study is being funded by a University of East Anglia doctoral studentship.

(3) What will the study involve for me?

This is a two-phase study which asks for your participation in each of its two phases. The first phase of the study will involve you teaching your day-to-day mathematics classes as normal. I will be in your mathematics classes for about two consecutive weeks of the first term in 2017-18 and I will be taking notes about how the lesson evolves. You will be able to see any notes I take that are specifically about you. In the first or second week of classroom observations, you will be asked to participate in a one-to-one interview with me where I will ask you about your experiences of teaching in inclusive mathematics classrooms, with a subsequent focus on those that have visually impaired learners. The second phase of the study will involve you participating in mathematics lessons which will aim to be beneficial for your whole class. You will be asked to choose mathematical topics on which you are planning to focus around that time or topics with which pupils in your classroom may struggle. You will then contribute to the design of relevant mathematical activities which I will have started to create on these topics. You will then implement these activities in your classroom and I will be observing the impact that these collaboratively designed activities will have on everyone in there. After the end of each lesson, I will ask you to participate in a one-to-one interview with me to tell me your experiences

and thoughts about the lesson. These will be considered for the design of the next lesson. This second phase will take place in the second term of 2017-18 and is expected to last for about three weeks. Any meeting involved in the design of the activities, and for the interviews, will take place whenever and wherever convenient to you.
In both phases of the study, during the observations, I would like to video record, with your permission, the mathematics sessions so that I can see how all of the class interact with each other and what body language they may use. I will cartoonise you in all the videos, in a way that it is impossible for you to be identified. If you do not want to be videoed as part of the general classroom observations, I will set the video camera in a place to minimise the possibility for you to be visible on screen.
As for the one-to-one interviews, in both phases of the study, I would like the discussion in the interviews to be audio recorded. If you do not consent on this, I can take handwritten notes of your responses during the interviews. The interviews will be conducted in your school, or a convenient location of your choice, at a time that is convenient to you. You will be able to review the transcripts of your interviews, if you wish, to ensure they are an accurate reflection of the discussion.

(4) How much of my time will the study take?

In each phase of the study, each interview is expected to last for 20-30 minutes. As for the classroom observations, they will be part of your normal everyday teaching, so no additional time is required for you. Our meetings for the design of Phase 2 mathematics lessons are not expected to take more than 3 hours in total from your time.

(5) Do I have to be in the study? Can I withdraw from the study once I've started?

Being in this study is completely voluntary. Your decision whether to participate will not affect your current or future relationship with the researcher or anyone else at the University of East Anglia or the school. If you decide to take part in the study and then change your mind, you are free to withdraw at any time, by simply saying this to me. You are also free to stop the interviews at any time. Unless you say that you want me to keep them, any recordings will be erased and the information you have provided will not be included in the study results. You may also refuse to answer any questions that you do not wish to answer during the interviews. If you take part in the observations, you are free to decide not to continue or to withdraw your participation even during the conduction of the observations. In the former case, no data will be recorded about you from that time onwards while in the latter case the collected data about you on that day will not be included in the study.

(6) Are there any risks or costs associated with being in the study?

Apart from giving some of your time to the study, there will be no risks or costs associated with taking part in this study.

(7) Are there any benefits associated with being in the study?

I believe that your participation in both phases of the study will be interesting and beneficial for you. Your experiences of teaching mathematics in classrooms that have visually impaired learners will be vital for the design of mathematics lessons in Phase 2. I aim that your input to these lessons, alongside that of the support staff you work with, your pupils and mine, will result in the creation of mathematics lessons that will be beneficial for the whole class. In addition, I anticipate that your participation in the study will contribute to benefitting other teachers who teach mathematics in classrooms that have visually impaired learners.

(8) What will happen to information about me that is collected during the study?

By providing your consent, you are agreeing to me collecting the data outlined above for the purposes of this research study. Your information will only be used for the purposes outlined in this Participant Information Statement unless you consent otherwise. Data management will follow the 1998 Data Protection Act and the University of East Anglia's Research Data Management Policy (2013). Your information will be stored securely and your identity/information will be kept strictly confidential, except as required by law. Study findings, fully anonymised, may be published.

(9) What if I would like further information about the study?

When you have read this information, I will be available to discuss it with you further and answer any questions you may have. If you would like to know more at any stage during the study, please feel free to contact me at A.Stylianidou@uea.ac.uk or 07769467367.

(10) Will I be told the results of the study?

You have a right to receive feedback about the overall results of this study. You can tell me that you wish to receive feedback by ticking the relevant box on the consent form. This feedback will be in the form of a summary and you will receive it after the study is finished. I will also be happy to discuss with you the progress of the study throughout its duration.

(11) What if I have a complaint or any concerns about the study?

Research involving humans in UK is reviewed by an independent group of people called a Human Research Ethics Committee (HREC). The ethical aspects of this study have been approved under the regulations of the University of East Anglia's School of Education and Lifelong Learning Research Ethics Committee.

If there is a problem, please let me know. You can contact me via the University at the following address:
Angeliki Stylianidou
School of Education and Lifelong Learning
University of East Anglia
Norwich NR4 7TJ
A.Stylianidou@uea.ac.uk

If you would like to speak to someone else, you can contact my supervisor:
Elena Nardi
E.Nardi@uea.ac.uk
+44 (0)1603 59 2631

If you are concerned about the way this study is being conducted or you wish to make a complaint to someone independent from the study, please contact the Head of the School of Education and Lifelong Learning, Professor Richard Andrews, at Richard.Andrews@uea.ac.uk.

(12) OK, I want to take part – what do I do next?

You need to fill in one copy of the consent form until 14/12/2017. Please keep the information sheet and the 2nd copy of the consent form for your information.

This information sheet is for you to keep

PARTICIPANT CONSENT FORM (1st Copy to Researcher)

I, ... [PRINT NAME], agree to take part in this research study.
In giving my consent, I state that:

- ✓ I understand the purpose of the study, what I will be asked to do, and any risks/benefits involved.
- ✓ I have read the Participant Information Statement and have been able to discuss my involvement in the study with the researcher if I wished to do so.
- ✓ The researcher has answered any questions that I had about the study and I am happy with the answers.
- ✓ I understand that being in this study is completely voluntary and I do not have to take part. My decision whether to be in the study will not affect my relationship with the researcher or anyone else at the University of East Anglia or the school, now or in the future.
- ✓ I understand that I can withdraw from the study at any time.
- ✓ I understand that I may stop the interviews at any time if I do not wish to continue, and that unless I indicate otherwise, any recordings will then be erased and the information provided will not be included in the study. I also understand that I may refuse to answer any questions I don't wish to answer.
- ✓ I understand that I may stop being included in the observations if I do not wish to continue, and that unless I indicate otherwise, any field notes and video data about me will then be erased and the information provided will not be included in the study.
- ✓ I understand that personal information about me that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about me will only be told to others with my permission, except as required by law.
- ✓ I understand that the results of this study may be published and that publications will not contain my name or any identifiable information about me.

Regarding the first phase of the study, I consent to:

- **Video recording during the classroom observations** YES ☐ NO ☐
- **Written notes during the classroom observations** YES ☐ NO ☐
- **Audio recording during the interview** YES ☐ NO ☐
- **Written notes during the interview.** This is applicable only if you have not accepted to be audio recorded during the interview. YES ☐ NO ☐
- **Reviewing transcripts after the audio recording of the interview.** This is applicable only if you have accepted to be audio recorded during the interview. YES ☐ NO ☐

Regarding the second phase of the study, I consent to:

- **Participating in the design of mathematics lessons** YES ☐ NO ☐
- **Video recording during the implementation of mathematics lessons** YES ☐ NO ☐
- **Written notes during the implementation of mathematics lessons** YES ☐ NO ☐
- **Audio recording during the interview conducted after the implementation of each lesson** YES ☐ NO ☐
- **Written notes during the interviews.** This is applicable only if you have not accepted to be audio recorded during the interviews. YES ☐ NO ☐
- **Reviewing transcripts after the audio recording of the interviews.** This is applicable only if you have accepted to be audio recorded during the interviews. YES ☐ NO ☐

Regarding both phases of the study:

- **Would you like to receive feedback about the overall results of this study?** YES ☐ NO ☐

If you answered **YES**, please indicate your preferred form of feedback and address:

☐ Postal: ____

____

☐ Email: ____

....
Signature

....
PRINT name

....
Date

PARTICIPANT CONSENT FORM (2nd Copy to Participant)

I, ... [PRINT NAME], agree to take part in this research study.
In giving my consent, I state that:

✓ I understand the purpose of the study, what I will be asked to do, and any risks/benefits involved.
✓ I have read the Participant Information Statement and have been able to discuss my involvement in the study with the researcher if I wished to do so.
✓ The researcher has answered any questions that I had about the study and I am happy with the answers.
✓ I understand that being in this study is completely voluntary and I do not have to take part. My decision whether to be in the study will not affect my relationship with the researcher or anyone else at the University of East Anglia or the school, now or in the future.
✓ I understand that I can withdraw from the study at any time.
✓ I understand that I may stop the interviews at any time if I do not wish to continue, and that unless I indicate otherwise, any recordings will then be erased and the information provided will not be included in the study. I also understand that I may refuse to answer any questions I don't wish to answer.
✓ I understand that I may stop being included in the observations if I do not wish to continue, and that unless I indicate otherwise, any field notes and video data about me will then be erased and the information provided will not be included in the study.
✓ I understand that personal information about me that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about me will only be told to others with my permission, except as required by law.
✓ I understand that the results of this study may be published and that publications will not contain my name or any identifiable information about me.

Regarding the first phase of the study, I consent to:

- **Video recording during the classroom observations** YES ☐ NO ☐
- **Written notes during the classroom observations** YES ☐ NO ☐
- **Audio recording during the interview** YES ☐ NO ☐
- **Written notes during the interview.** This is applicable only if you have not accepted to be audio recorded during the interview. YES ☐ NO ☐
- **Reviewing transcripts after the audio recording of the interview.** This is applicable only if you have accepted to be audio recorded during the interview. YES ☐ NO ☐

Regarding the second phase of the study, I consent to:

- **Participating in the design of mathematics lessons** YES ☐ NO ☐
- **Video recording during the implementation of mathematics lessons** YES ☐ NO ☐
- **Written notes during the implementation of mathematics lessons** YES ☐ NO ☐
- **Audio recording during the interview conducted after the implementation of each lesson** YES ☐ NO ☐
- **Written notes during the interviews.** This is applicable only if you have not accepted to be audio recorded during the interviews. YES ☐ NO ☐
- **Reviewing transcripts after the audio recording of the interviews.** This is applicable only if you have accepted to be audio recorded during the interviews. YES ☐ NO ☐

Regarding both phases of the study:

- **Would you like to receive feedback about the overall results of this study?**
  YES ☐ NO ☐

If you answered **YES**, please indicate your preferred form of feedback and address:

☐ Postal: ____

____

☐ Email: ____

....
Signature

....
PRINT name

....
Date

Angeliki Stylianidou
PhD student
29.11.2017

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Deconstructing disability through/and promoting inclusive education in primary mathematics classrooms: the case of visually impaired learners

PARTICIPANT INFORMATION STATEMENT – support staff

(1) What is this study about?

You are invited to take part in a research study about the inclusion of visually impaired learners in primary mathematics classrooms. I am interested in how you experience inclusion in your mathematics lessons, what you see as benefits that inclusion may bring to learners and what challenges you may face with working in mathematics classrooms that have visually impaired learners. You have been invited to participate in this study because you are a key member of such primary settings and, as a result, your contribution is highly significant for the study. This Participant Information Statement tells you about the study. Knowing what is involved will help you decide if you want to take part in it. Please read this sheet carefully and feel free to ask me questions about anything that you wish to know more about.

Participation in this research study is voluntary. By giving consent to take part in this study, you are telling me that you:

- ✓ Understand what you have read.
- ✓ Agree to take part in the research study as outlined below.
- ✓ Agree to the use of your personal information as described.
- ✓ You have received a copy of this Participant Information Statement to keep.

(2) Who is running the study?

The study is being carried out by the following researcher: Ms Angeliki Stylianidou, PhD student, School of Education and Lifelong Learning, University of East Anglia.
It is being carried out under the supervision of Elena Nardi, Professor in the School of Education and Lifelong Learning, University of East Anglia.

This study is being funded by a University of East Anglia doctoral studentship.

(3) What will the study involve for me?

This is a two-phase study which asks for your participation in each of its two phases. The first phase of the study will involve you working on your day-to-day mathematics classes as normal. I will be in your mathematics classes for about two consecutive weeks of the first term in 2017-18 and I will be taking notes about how the lesson evolves. You will be able to see any notes I take that are specifically about you. In the first or second week of classroom observations, you will be asked to participate in a one-to-one interview with me where I will ask you about your experiences of working in inclusive mathematics classrooms, with a subsequent focus on those that have visually impaired learners. The second phase of the study will involve you participating in mathematics lessons which will aim to be beneficial for your whole class. You will be asked, alongside the class teacher you work with, to choose mathematical topics on which you are planning to focus around that time or topics with which pupils in your classroom may struggle. You will then contribute to the design of relevant mathematical activities which I will have started to create on these topics. These collaboratively designed activities will then be implemented in your classroom by the class teacher and I will be observing the impact that they will have on everyone in there. After the end of each lesson, I will ask you to participate in a one-to-one interview with me to tell me your experiences and thoughts about the lesson. These will be considered for the design of the next lesson. This second phase will take place in the second term of 2017-18 and is expected to last for about three weeks. Any meeting involved in the design of the activities, and for the interviews, will take place whenever and wherever convenient to you. In both phases of the study, during the observations, I would like to video record, with your permission, the mathematics sessions so that I can see how all of the class interact with each other and what body language they may use. I will cartoonise you in all the videos, in a way that it is impossible for you to be identified. If you do not want to be videoed as part of the general classroom observations, I will set the video camera in a place to minimise the possibility for you to be visible on screen.

As for the one-to-one interviews, in both phases of the study, I would like the discussion in the interviews to be audio recorded. If you do not consent on this, I can take handwritten notes of your responses during the interviews. The interviews will be conducted in your school, or a convenient location of your choice, at a time that is convenient to you. You will be able to review the transcripts of your interviews, if you wish, to ensure they are an accurate reflection of the discussion.

(4) How much of my time will the study take?

In each phase of the study, each interview is expected to last for 20-30 minutes. As for the classroom observations, they will be part of your normal everyday work, so no additional time is required for you. Our meetings for the design of Phase 2 mathematics lessons are not expected to take more than 3 hours in total from your time.

(5) Do I have to be in the study? Can I withdraw from the study once I've started?

Being in this study is completely voluntary. Your decision whether to participate will not affect your current or future relationship with the researcher or anyone else at the University of East Anglia or the school. If you decide to take part in the study and then change your mind, you are free to withdraw at any time, by simply saying this to me. You are also free to stop the interviews at any time. Unless you say that you want me to keep them, any recordings will be erased and the information you have provided will not be included in the study results. You may also refuse to answer any questions that you do not wish to answer during the interviews. If you take part in the observations, you are free to decide not to continue or to withdraw your participation even during the conduction of the observations. In the former case, no data will be recorded about you from that time onwards while in the latter case the collected data about you on that day will not be included in the study.

(6) Are there any risks or costs associated with being in the study?

Apart from giving some of your time to the study, there will be no risks or costs associated with taking part in this study.

(7) Are there any benefits associated with being in the study?

I believe that your participation in both phases of the study will be interesting and beneficial for you. Your experiences of working in mathematics classrooms that have visually impaired learners will be vital for the design of mathematics lessons in Phase 2. I aim that your input to these lessons, alongside that of the class teacher you work with, your pupils and mine, will result in the creation of mathematics lessons that will be beneficial for the whole class. In addition, I anticipate that your participation in the study will contribute to benefitting other teaching staff who work in mathematics classrooms that have visually impaired learners.

(8) What will happen to information about me that is collected during the study?

By providing your consent, you are agreeing to me collecting the data outlined above for the purposes of this research study. Your information will only be used for the purposes outlined in this Participant Information Statement unless you consent otherwise. Data management will follow the 1998 Data Protection Act and the University of East Anglia's Research Data Management Policy (2013). Your information will be stored securely and your identity/information will be kept strictly confidential, except as required by law. Study findings, fully anonymised, may be published.

(9) What if I would like further information about the study?

When you have read this information, I will be available to discuss it with you further and answer any questions you may have. If you would like to know more at any stage during the study, please feel free to contact me at A.Stylianidou@uea.ac.uk or 07769467367.

(10) Will I be told the results of the study?

You have a right to receive feedback about the overall results of this study. You can tell me that you wish to receive feedback by ticking the relevant box on the consent form. This feedback will be in the form of a summary and you will receive it after the study is finished. I will also be happy to discuss with you the progress of the study throughout its duration.

(11) What if I have a complaint or any concerns about the study?

Research involving humans in UK is reviewed by an independent group of people called a Human Research Ethics Committee (HREC). The ethical aspects of this study have been approved under the regulations of the University of East Anglia's School of Education and Lifelong Learning Research Ethics Committee.

If there is a problem, please let me know. You can contact me via the University at the following address:
Angeliki Stylianidou
School of Education and Lifelong Learning
University of East Anglia
Norwich NR4 7TJ

A.Stylianidou@uea.ac.uk

If you would like to speak to someone else, you can contact my supervisor:
Elena Nardi
E.Nardi@uea.ac.uk
+44 (0)1603 59 2631

If you are concerned about the way this study is being conducted or you wish to make a complaint to someone independent from the study, please contact the Head of the School of Education and Lifelong Learning, Professor Richard Andrews, at Richard.Andrews@uea.ac.uk.

(12) OK, I want to take part – what do I do next?

You need to fill in one copy of the consent form and give this to the class teacher until 14/12/2017. Please keep the information sheet and the 2nd copy of the consent form for your information.

This information sheet is for you to keep

PARTICIPANT CONSENT FORM (1st Copy to Researcher)

I, ... [PRINT NAME], agree to take part in this research study.
In giving my consent, I state that:

- ✓ I understand the purpose of the study, what I will be asked to do, and any risks/benefits involved.
- ✓ I have read the Participant Information Statement and have been able to discuss my involvement in the study with the researcher if I wished to do so.
- ✓ The researcher has answered any questions that I had about the study and I am happy with the answers.
- ✓ I understand that being in this study is completely voluntary and I do not have to take part. My decision whether to be in the study will not affect my relationship with the researcher or anyone else at the University of East Anglia or the school, now or in the future.
- ✓ I understand that I can withdraw from the study at any time.
- ✓ I understand that I may stop the interviews at any time if I do not wish to continue, and that unless I indicate otherwise, any recordings will then be erased and the information provided will not be included in the study. I also understand that I may refuse to answer any questions I don't wish to answer.
- ✓ I understand that I may stop being included in the observations if I do not wish to continue, and that unless I indicate otherwise, any field notes and video data about me will then be erased and the information provided will not be included in the study.
- ✓ I understand that personal information about me that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about me will only be told to others with my permission, except as required by law.
- ✓ I understand that the results of this study may be published and that publications will not contain my name or any identifiable information about me.

Regarding the first phase of the study, I consent to:

- **Video recording during the classroom observations** YES ☐ NO ☐
- **Written notes during the classroom observations** YES ☐ NO ☐
- **Audio recording during the interview** YES ☐ NO ☐
- **Written notes during the interview.** This is applicable only if you have not accepted to be audio recorded during the interview. YES ☐ NO ☐
- **Reviewing transcripts after the audio recording of the interview.** This is applicable only if you have accepted to be audio recorded during the interview. YES ☐ NO ☐

Regarding the second phase of the study, I consent to:

- **Participating in the design of mathematics lessons** YES ☐ NO ☐
- **Video recording during the implementation of mathematics lessons** YES ☐ NO ☐
- **Written notes during the implementation of mathematics lessons** YES ☐ NO ☐
- **Audio recording during the interview conducted after the implementation of each lesson** YES ☐ NO ☐
- **Written notes during the interviews.** This is applicable only if you have not accepted to be audio recorded during the interviews. YES ☐ NO ☐
- **Reviewing transcripts after the audio recording of the interviews.** This is applicable only if you have accepted to be audio recorded during the interviews. YES ☐ NO ☐

Regarding both phases of the study:

- **Would you like to receive feedback about the overall results of this study?**

YES ☐ NO ☐

If you answered **YES**, please indicate your preferred form of feedback and address:

☐ Postal: ____

____

☐ Email: ____

....
Signature

....
PRINT name

....
Date

PARTICIPANT CONSENT FORM (2nd Copy to Participant)

I, ... [PRINT NAME], agree to take part in this research study.
In giving my consent, I state that:

✓ I understand the purpose of the study, what I will be asked to do, and any risks/benefits involved.
✓ I have read the Participant Information Statement and have been able to discuss my involvement in the study with the researcher if I wished to do so.
✓ The researcher has answered any questions that I had about the study and I am happy with the answers.
✓ I understand that being in this study is completely voluntary and I do not have to take part. My decision whether to be in the study will not affect my relationship with the researcher or anyone else at the University of East Anglia or the school, now or in the future.
✓ I understand that I can withdraw from the study at any time.
✓ I understand that I may stop the interviews at any time if I do not wish to continue, and that unless I indicate otherwise, any recordings will then be erased and the information provided will not be included in the study. I also understand that I may refuse to answer any questions I don't wish to answer.
✓ I understand that I may stop being included in the observations if I do not wish to continue, and that unless I indicate otherwise, any field notes and video data about me will then be erased and the information provided will not be included in the study.
✓ I understand that personal information about me that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about me will only be told to others with my permission, except as required by law.
✓ I understand that the results of this study may be published and that publications will not contain my name or any identifiable information about me.

Regarding the first phase of the study, I consent to:

- **Video recording during the classroom observations** YES ☐ NO ☐
- **Written notes during the classroom observations** YES ☐ NO ☐
- **Audio recording during the interview** YES ☐ NO ☐
- **Written notes during the interview.** This is applicable only if you have not accepted to be audio recorded during the interview. YES ☐ NO ☐
- **Reviewing transcripts after the audio recording of the interview.** This is applicable only if you have accepted to be audio recorded during the interview. YES ☐ NO ☐

Regarding the second phase of the study, I consent to:

- **Participating in the design of mathematics lessons** YES ☐ NO ☐
- **Video recording during the implementation of mathematics lessons** YES ☐ NO ☐
- **Written notes during the implementation of mathematics lessons** YES ☐ NO ☐
- **Audio recording during the interview conducted after the implementation of each lesson** YES ☐ NO ☐
- **Written notes during the interviews.** This is applicable only if you have not accepted to be audio recorded during the interviews. YES ☐ NO ☐
- **Reviewing transcripts after the audio recording of the interviews.** This is applicable only if you have accepted to be audio recorded during the interviews. YES ☐ NO ☐

Regarding both phases of the study:

- **Would you like to receive feedback about the overall results of this study?**
YES ☐ NO ☐

If you answered **YES**, please indicate your preferred form of feedback and address:

☐ Postal: ____
____

☐ Email: ____

....
Signature

....
PRINT name

....
Date

Angeliki Stylianidou
PhD student
29.11.2017

Faculty of Social Sciences
School of Education and Lifelong Learning
University of East Anglia
Norwich Research Park
Norwich NR4 7TJ
United Kingdom

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Web: www.uea.ac.uk

Deconstructing disability through/and promoting inclusive education in primary mathematics classrooms: the case of visually impaired learners

PARENTAL INFORMATION STATEMENT

(1) What is this study about?

Your child is invited to take part in a research study about the inclusion of visually impaired learners in primary mathematics classrooms. I am interested in how your child experiences, thinks and feels about their mathematical learning in their classroom, what benefits they may gain and what challenges/difficulties/problems they may face in their mathematics classroom. Your child has been invited to participate in this study because s/he is a key member of such primary settings and, as a result, their contribution is highly significant for the study. This Parental Information Statement tells you about the study. Knowing what is involved will help you decide if you want to let your child take part in it. Please read this sheet carefully and feel free to ask me questions about anything that you wish to know more about.

Participation in this research study is voluntary. By giving your consent, you are telling me that you:

- ✓ Understand what you have read.
- ✓ Agree for your child to take part in the research study as outlined below.
- ✓ Agree to the use of your child's personal information as described.
- ✓ You have received a copy of this Parental Information Statement to keep.

(2) Who is running the study?

The study is being carried out by the following researcher: Ms Angeliki Stylianidou, PhD student, School of Education and Lifelong Learning, University of East Anglia.
It is being carried out under the supervision of Elena Nardi, Professor in the School of Education and Lifelong Learning, University of East Anglia.

This study is being funded by a University of East Anglia doctoral studentship.

(3) What will the study involve?

This is a two-phase study which asks for your consent on your child's participation in each of its two phases. The first phase of the study will involve your child completing their day-to-day mathematics classes as normal. I will be in your child's mathematics classes for about two consecutive weeks of the first term in 2017-18 and I will be taking notes about how the lesson evolves. You will be able to see any notes I take that are specifically about your child. In the second week of classroom observations, your child will be asked to participate in a focus group with some of their classmates where I will ask them about their views and experiences of their mathematics lessons. The second phase of the study will involve your child participating in mathematics lessons which will aim to be beneficial for your child and the rest of their class. Your child will be observed doing their mathematics classes that will be based on mathematical activities, designed by your child's teaching staff and myself. I will be particularly observing the impact that these collaboratively designed activities will have on everyone in the classroom. After the end of each lesson, your child will be asked to participate in a focus group with some of their classmates where I will ask them what they liked in the particular lesson and what they would like to have been different. Your child's views on each lesson will be considered for the design of the next one. This second phase will take place in the second term of 2017-18 and is expected to last for about three weeks.
In both phases of the study, during the observations, I would like to video record, with your permission, the mathematics sessions so that I can see how all of the class interact with each other and what body language they may use. The body language of your child will also be important for me to comprehend how they learn and understand mathematical concepts. I will cartoonise your child in all the videos, in a way that they will not be identifiable. If you do not want your child to be videoed as part of the general classroom observations, it might be necessary to move them to a position at the back of the classroom to minimise the possibility for them to be visible on screen.

As for the focus group interviews, in both phases of the study, the discussion will be audio recorded and will take place in your child's school at a time that is convenient to the focus group members and the class teacher.

(4) How much of my child's time will the study take?

In each phase of the study, each focus group interview will take 20-30 minutes of your child's time. As for the classroom observations, they will be part of your child's normal everyday learning, so no additional time is required for your child.

(5) Does my child have to be in the study? Can they withdraw from the study once they've started?

Being in this study is completely voluntary. Your decision whether to let your child participate will not affect your/their relationship with the researcher or anyone else at the University of East Anglia or the school, now or in the future. If you decide to let your child take part in the study and then change your mind (or they no longer wish to take part), they are free to withdraw from the study at any time, by simply saying this to me or their class teacher. If your child takes part in focus groups, they are free to stop participating at any stage or to refuse to answer any of the questions. However, it will not be possible to withdraw their individual comments from my records once the group has started, as it is a group discussion. If your child takes part in the observations, they are free to decide not to continue or to withdraw their participation even during the conduction of the observations. In the former case, no data will be recorded about them from that time onwards while in the latter case the collected data about them on that day will not be included in the study.

(6) Are there any risks or costs associated with being in the study?

Apart from your child giving some of their time to the study, there will be no risks or costs associated with taking part in this study.

(7) Are there any benefits associated with being in the study?

I believe that being able to openly and sincerely talk with classmates about their experiences, feelings and thoughts on their mathematics lessons will be interesting for your child. Your child's mathematical experiences, needs and aspirations will be vital for the design of mathematics lessons and their implementation in your child's mathematics classroom. I aim that your child's participation in both phases of the study will make the teaching of mathematics better for all the pupils of your child's classroom. In addition, I anticipate that your child will contribute to benefitting other teachers who teach mathematics in classrooms that have visually impaired learners.

(8) What will happen to information that is collected during the study?

By providing your consent, you are agreeing to me collecting the data outlined above for the purposes of this research study. Your child's information will only be used for the purposes outlined in this Parental Information Statement unless you consent otherwise. Data management will follow the 1998 Data Protection Act and the University of East Anglia's Research Data Management Policy (2013). Your child's information will be stored securely and their identity/information will be kept strictly confidential, except as required by law. Study findings, fully anonymised, may be published.

(9) What if I would like further information about the study?

When you have read this information, I will be available to discuss it with you further and answer any questions you may have. If you would like to know more at any stage during the study, please feel free to contact me at A.Stylianidou@uea.ac.uk or 07769467367.

(10) Will I be told the results of the study?

You and your child have a right to receive feedback about the overall results of this study. You can tell me that you wish to receive feedback by ticking the relevant box on the consent form. This feedback will be in the form of a summary and you will receive it after the study is finished. I will also be happy to discuss with you the progress of the study throughout its duration.

(11) What if I have a complaint or any concerns about the study?

Research involving humans in UK is reviewed by an independent group of people called a Human Research Ethics Committee (HREC). The ethical aspects of this study have been approved under the regulations of the University of East Anglia's School of Education and Lifelong Learning Research Ethics Committee.

If there is a problem, please let me know. You can contact me via the University at the following address:
Angeliki Stylianidou
School of Education and Lifelong Learning
University of East Anglia
Norwich NR4 7TJ
A.Stylianidou@uea.ac.uk

If you would like to speak to someone else, you can contact my supervisor:
Elena Nardi
E.Nardi@uea.ac.uk
+44 (0)1603 59 2631

If you (or your child) are concerned about the way this study is being conducted or you wish to make a complaint to someone independent from the study, please contact the Head of the School of Education and Lifelong Learning, Professor Richard Andrews, at Richard.Andrews@uea.ac.uk.

(12) OK, I am happy for my child to take part – what do I do next?

You need to fill in one copy of the consent form and give this to your child's class teacher until 14/12/2017. Please keep the information sheet and the 2nd copy of the consent form for your information.

This information sheet is for you to keep

PARENT/CARER CONSENT FORM (1st Copy to Researcher)

I, ... [PRINT PARENT'S/CARER'S NAME], consent to my child ……………………………………………………………………………………[PRINT CHILD'S NAME] participating in this research study. In giving my consent, I state that:

✓ I understand the purpose of the study, what my child will be asked to do, and any risks/benefits involved.
✓ I have read the Information Statement and have been able to discuss my child's involvement in the study with the researcher if I wished to do so.
✓ The researcher has answered any questions that I had about the study and I am happy with the answers.
✓ I understand that being in this study is completely voluntary and my child does not have to take part. My decision whether to let them take part in the study will not affect our relationship with the researcher or anyone else at the University of East Anglia or the school, now or in the future.
✓ I understand that my child can withdraw from the study at any time.
✓ I understand that my child may leave the focus groups at any time if they do not wish to continue. I also understand that it will not be possible to withdraw their comments once the group has started, as it is a group discussion.
✓ I understand that my child may stop being included in the observations if they do not wish to continue, and that unless they indicate otherwise, any field notes and video data about my child will then be erased and the information provided will not be included in the study.
✓ I understand that personal information about my child that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about my child will only be told to others with my permission, except as required by law.
✓ I understand that the results of this study may be published and that publications will not contain my child's name or any identifiable information about them.

Regarding the first phase of the study, I consent to:

- **Video recording during the classroom observations** YES ☐ NO ☐
- **Written notes during the classroom observations** YES ☐ NO ☐
- **Audio recording during the focus group interview** YES ☐ NO ☐

Regarding the second phase of the study, I consent to:

- **Video recording during the observations of the collaboratively designed mathematics lessons** YES ☐ NO ☐
- **Written notes during the observations of these mathematics lessons** YES ☐ NO ☐
- **Audio recording during the focus group interviews** YES ☐ NO ☐

Regarding both phases of the study:

- **Would you like to receive feedback about the overall results of this study?** YES ☐ NO ☐

If you answered **YES**, please indicate your preferred form of feedback and address:

☐ Postal: ____

____

☐ Email: ____

....
Signature
....
PRINT name
....
Date

PARENT/CARER CONSENT FORM (2nd Copy to Parent/Carer)

I, ... [PRINT PARENT'S/CARER'S NAME], consent to my child ……………………………………………………………………………………….[PRINT CHILD'S NAME] participating in this research study. In giving my consent, I state that:

✓ I understand the purpose of the study, what my child will be asked to do, and any risks/benefits involved.
✓ I have read the Information Statement and have been able to discuss my child's involvement in the study with the researcher if I wished to do so.
✓ The researcher has answered any questions that I had about the study and I am happy with the answers.
✓ I understand that being in this study is completely voluntary and my child does not have to take part. My decision whether to let them take part in the study will not affect our relationship with the researcher or anyone else at the University of East Anglia or the school, now or in the future.
✓ I understand that my child can withdraw from the study at any time.
✓ I understand that my child may leave the focus groups at any time if they do not wish to continue. I also understand that it will not be possible to withdraw their comments once the group has started, as it is a group discussion.
✓ I understand that my child may stop being included in the observations if they do not wish to continue, and that unless they indicate otherwise, any field notes and video data about my child will then be erased and the information provided will not be included in the study.
✓ I understand that personal information about my child that is collected over the course of this project will be stored securely and will only be used for purposes that I have agreed to. I understand that information about my child will only be told to others with my permission, except as required by law.
✓ I understand that the results of this study may be published and that publications will not contain my child's name or any identifiable information about them.

Regarding the first phase of the study, I consent to:

- **Video recording during the classroom observations** YES ☐ NO ☐
- **Written notes during the classroom observations** YES ☐ NO ☐
- **Audio recording during the focus group interview** YES ☐ NO ☐

Regarding the second phase of the study, I consent to:

- **Video recording during the observations of the collaboratively designed mathematics lessons** YES ☐ NO ☐
- **Written notes during the observations of these mathematics lessons** YES ☐ NO ☐
- **Audio recording during the focus group interviews** YES ☐ NO ☐

Regarding both phases of the study:

- **Would you like to receive feedback about the overall results of this study?** YES ☐ NO ☐

If you answered **YES**, please indicate your preferred form of feedback and address:

☐ Postal: ____

____

☐ Email: ____

....
Signature
....
PRINT name
....
Date

Angeliki Stylianidou
PhD student
29.11.17

Faculty of Social Sciences
School of Education and Lifelong Learning
University of East Anglia
Norwich Research Park
Norwich NR4 7TJ
United Kingdom

Email: A.Stylianidou@uea.ac.uk
Web: www.uea.ac.uk

Study Information Sheet: My maths class

Hello. My name is Angeliki Stylianidou. I am doing a research study to find out what you think and how you feel about your maths classes, what you like from them and what you do not like. I am asking you to be in my study because you are in a maths classroom which I am interested in. Your views about this class are very important for me.

You can decide if you want to take part in the study or not. You don't have to - it's up to you.

This sheet tells you what I will ask you to do if you decide to take part in the study. Please read it carefully so that you can make up your mind about whether you want to take part.

If you decide that you want to be in the study and then you change your mind, that's ok. All you need to do is tell me that you don't want to be in the study anymore.

If you have any questions, you can ask me or your family or someone else who looks after you.

What will happen if I say that I want to be in the study?

If you decide that you want to be in my study, I will ask you to do these things:

- Let me sit in your classroom and make some notes about what happens. This could include things that you say or do. I also want to take some videos of your lessons so that I can look at them later on. It is important for me to video record you during your maths lessons, so that I can see how you learn maths and how you feel about being in your maths classroom.
- Come along to a focus group, in which you will talk with some of your classmates about what you think about your maths lessons. I will ask you to share how you feel in your maths classroom and what you need in order to enjoy your maths lessons in your classroom the most.
- All the information that I will get from the video recording and the focus groups will then help me, your class teacher and support staff to design maths lessons which I hope you will enjoy a lot.
- Let me sit in your classroom during these hopefully exciting maths lessons and make some notes about what happens. This could again include things that you say or do. I also want to video record these lessons so that I can see how you feel about them.
- Come along to a focus group with some of your classmates after the end of each maths lesson and share what you liked and what you did not like about it. Your participation will help us improve the design of this lesson.

If you are happy for me to look at how you do your maths lessons and then you change your mind, that's ok. You can say this to me or your class teacher at any time and I will not observe you anymore.
When I ask you questions, you can choose which ones you want to answer. If you don't want to talk about something, that's ok. You can stop talking to me at any time if you don't want to talk to me anymore.
If you say it's ok, I will make videos of you with a video recorder.
If you say it's ok, I will record what you say with a tape recorder.

Will anyone else know what I say in the study?

I won't tell anyone else what you say to me, except if you talk about someone hurting you or about you hurting yourself or someone else. Then I might need to tell someone to keep you and other people safe.

All of the information that I have about you from the study will be stored in a safe place and I will look after it very carefully. I will write a report about the study and show it to other people but I won't say your name in the report and no one will know that you were in the study.

How long will the study take?
I will be in your maths classes for about two weeks of this first term and three weeks of the second one. If you talk with me and your classmates about what you think your maths class is like, this will take each time about 20 minutes or so.

Are there any good things about being in the study?

I think you'll like talking about your maths classes with your classmates and you will also help me design maths lessons that you and your classmates will hopefully enjoy. In addition, I believe that your participation in the study will make maths teaching better for you and the rest of your classmates and it will also help other teachers who teach maths in a classroom like yours.

Are there any bad things about being in the study?

This study will take up some of your time, but I don't think it will be bad for you or cost you anything.

Will you tell me what you learnt in the study at the end?

Yes, I will if you want me to. There is a question on the next page that asks you if you want me to tell you what I learnt in the study. If you circle 'Yes', when I finish the study I will tell you what I learnt.

What if I am not happy with the study or the people doing the study?

If you are not happy with how I am doing the study or how I treat you, then you or the person who looks after you can:
- **Call** the university on +44 (0) 1603 59 2631
- Write an **email** to E.Nardi@uea.ac.uk
-

OK, I want to take part – what do I do next?

If you're happy to take part in my study, please fill in the two forms below and give number 1 to me. You can keep this letter and the form 2 to remind you about the study.

This sheet is for you to keep.

Study Information Sheet: My maths class
Consent Form 1

If you are happy to be in the study, please
- **write** your **name** in the space below
- **sign** your **name** at the bottom of this page
- put the **date** at the bottom of this page.

You should only say 'yes' to being in the study if you know what it is about and you want to be in it. If you don't want to be in the study, don't sign the form.

I, ...[PRINT NAME], am happy to be in this research study.

In saying 'yes' to being in the study, I am saying that:

✓ I know what the study is about.
✓ I know what I will be asked to do.
✓ Someone has talked to me about the study.

✓ My questions have been answered.
✓ I know that I don't have to be in the study if I don't want to.
✓ I know that I can pull out of the study at any time if I don't want to do it anymore.
✓ I know that I don't have to answer any questions which I don't want to answer.
✓ I know that I won't be observed if I don't want.
✓ I know that the researcher won't tell anyone what I say when we talk to each other unless I talk about being hurt by someone or hurting myself or someone else.

Now I am going to ask you if you are happy to do a few other things in the study. Please circle 'Yes' or 'No' to tell me what you would like.

- Are you happy for me to make **videos** of you? **Yes** **No**
- If you are not happy for me to make videos of you, could I take some **notes** about you? **Yes** **No**
- Are you happy for me to **tape record** your voice? **Yes** **No**
- Do you want me to tell you what I **learnt** in the study? **Yes** **No**

……... **…………………………………………………….**
Signature **Date**

Study Information Sheet: My maths class
Consent Form 2

If you are happy to be in the study, please

- **write** your **name** in the space below
- **sign** your **name** at the bottom of this page
- put the **date** at the bottom of this page.

You should only say 'yes' to being in the study if you know what it is about and you want to be in it. If you don't want to be in the study, don't sign the form.

I, ...[PRINT NAME], am happy to be in this research study.

In saying 'yes' to being in the study, I am saying that:

✓ I know what the study is about.
✓ I know what I will be asked to do.
✓ Someone has talked to me about the study.
✓ My questions have been answered.
✓ I know that I don't have to be in the study if I don't want to.
✓ I know that I can pull out of the study at any time if I don't want to do it anymore.
✓ I know that I don't have to answer any questions which I don't want to answer.
✓ I know that I won't be observed if I don't want.
✓ I know that the researcher won't tell anyone what I say when we talk to each other unless I talk about being hurt by someone or hurting myself or someone else.

Now I am going to ask you if you are happy to do a few other things in the study. Please circle 'Yes' or 'No' to tell me what you would like.

- Are you happy for me to make **videos** of you? **Yes No**
- If you are not happy for me to make videos of you, could I take some **notes** about you? **Yes No**
- Are you happy for me to **tape record** your voice? **Yes No**
- Do you want me to tell you what I **learnt** in the study? **Yes No**

………….. …………………………………………………….
Signature Date

Abbreviations

CAPTeaM: Challenging Ableist Perspectives on the Teaching of Mathematics

CRPD: Convention on the Rights of Persons with Disabilities

HAP: High Achieving Pupil

ITE: Initial Teacher Education

ITT: Initial Teacher Training

IWB: Interactive Whiteboard

JAWS: Job Access with Speech

LAP: Low Achieving Pupil

MAP: Middle Achieving Pupil

PGCE: Postgraduate Certificate in Education

RQ: Research Question

S: School

SEND: Special Educational Needs and Disability

SENDCO: Special Educational Needs and Disabilities Coordinator

T: Teacher

TA: Teaching Assistant

VI: Visually Impaired

VNC: Virtual Network Computing

Y: Year

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