Bridging atomistic simulations and thermodynamic hydration models of aqueous electrolyte solutions

Cite as: J. Chem. Phys. **156**, 024502 (2022); https://doi.org/10.1063/5.0074970 Submitted: 12 October 2021 • Accepted: 21 December 2021 • Accepted Manuscript Online: 22 December 2021 • Published Online: 13 January 2022

Xiangwen Wang, Simon L. Clegg and 🔟 Devis Di Tommaso



ARTICLES YOU MAY BE INTERESTED IN

Interfacial water asymmetry at ideal electrochemical interfaces The Journal of Chemical Physics **156**, 014705 (2022); https://doi.org/10.1063/5.0076038

Accurate parameterization of the kinetic energy functional The Journal of Chemical Physics **156**, 024110 (2022); https://doi.org/10.1063/5.0063629

Accurate parameterization of the kinetic energy functional for calculations using exactexchange

The Journal of Chemical Physics 156, 024107 (2022); https://doi.org/10.1063/5.0065217





J. Chem. Phys. **156**, 024502 (2022); https://doi.org/10.1063/5.0074970 © 2022 Author(s).

Bridging atomistic simulations and thermodynamic hydration models of aqueous electrolyte solutions



AFFILIATIONS

¹ Department of Chemistry, Queen Mary University of London, Mile End Road, London El 4NS, United Kingdom ²School of Environmental Sciences, University of East Anglia, Norwich NR4 7TJ, United Kingdom

^{a)}Authors to whom correspondence should be addressed: <code>s.clegg@uea.ac.uk</code> and <code>d.ditomaso@qmul.ac.uk</code>

ABSTRACT

Chemical thermodynamic models of solvent and solute activities predict the equilibrium behavior of aqueous solutions. However, these models are semi-empirical. They represent micro-scale ion and solvent behaviors controlling the macroscopic properties using small numbers of parameters whose values are obtained by fitting to activities and other partial derivatives of the Gibbs energy measured for the bulk solutions. We have conducted atomistic simulations of aqueous electrolyte solutions (MgCl₂ and CaCl₂) to determine the parameters of thermodynamic hydration models. We have implemented a cooperative hydration model to categorize the water molecules in electrolyte solutions into different subpopulations. The value of the electrolyte-specific parameter, k, was determined from the ion-affected subpopulation with the lowest absolute value of the free energy of removing the water molecule. The other equilibrium constant parameter, K_1 , associated with the first degree of hydration, was computed from the free energy of hydration of hydrated clusters. The hydration number, h, was determined from a reorientation dynamic analysis of the water subpopulations compared to bulk-like behavior. The reparameterized models [R. H. Stokes and R. H. Robinson, J. Solution Chem. **2**, 173 (1973) and Balomenos *et al.*, Fluid Phase Equilib. **243**, 29 (2006)] using the computed values of the parameters lead to the osmotic coefficients of MgCl₂ solutions that are consistent with measurements. Such an approach removes the dependence on the availability of experimental data and could lead to aqueous thermodynamic models capable of estimating the values of solute and solvent activities as well as thermal and volumetric properties for a wide range of compositions and concentrations.

Published under an exclusive license by AIP Publishing. https://doi.org/10.1063/5.0074970

I. INTRODUCTION

Aqueous thermodynamic models of solvent and solute activities are used to predict the equilibrium behavior and chemical speciation of natural aqueous solutions (oceans, brines, groundwater, and atmospheric aerosols) and industrial systems (ionic liquids, underground contaminants, and fluids used for oil and gas processing).^{1–6} Some of the most widely used thermodynamic models to estimate the activity coefficients of electrolyte solutions are semi-empirical.⁵ They attempt to represent key micro-scale ion and solvent behaviors that control the macroscopic properties in a simplified way using small numbers of parameters whose values are obtained by fitting to activities and other partial derivatives of the Gibbs energy that have been measured for the bulk solutions. Examples include the Pitzer² and Pitzer–Simonson–Clegg models,^{3,4} which are applied in aqueous geochemistry and atmospheric science, and the non-random two-liquid (NRTL) models of Chen,^{5,6} used to describe mixed solvent systems of industrial solutions. However, key molecular-level behaviors, such as hydration and ion pairing, are generally not treated explicitly in these semi-empirical models, and when extrapolated beyond the range of concentration to which they were fitted, they tend to give inaccurate thermodynamic properties.⁷ In contrast to these approaches, some thermodynamic hydration models attempt to represent the microphysical hydration processes more directly within their theoretical framework (*vide infra*) through the incorporation of hydration (formation of solvent shells around ions).⁸ Although their fundamental assumption (the ion hydration is the principal determinant of activity) is

different from that of the ion interaction and local composition models, they are also highly simplified and contain empirical fitted parameters.⁹ Properties such as water activity and freezing point depression of simple electrolyte solutions have been shown to correlate very simply with concentration to calculate a range of hydration numbers, which gives some encouragement to the use of these approaches.¹⁰

Explicit water molecular dynamics (MD) simulations and static density functional theory (DFT) calculations, with a dielectric continuum description of the solution environment, can provide an atomistic view of the processes of ion hydration and ion association in solution. These atomistic simulation methods have been extensively applied to quantify the energetics of ion-water and ion-ion interactions and to compute single-particle, pair, and collective dynamical properties of aqueous electrolyte solutions.¹¹⁻¹⁴ Therefore, advances in the field may be achieved if the parameters contained in thermodynamic hydration models could be obtained from atomistic simulations of aqueous electrolyte solutions rather than experimental data. Such an approach, if successful, would remove the dependence on the availability of experimental data and pave the way to develop aqueous thermodynamic models capable of estimating the values of solute and solvent activities as well as thermal and volumetric properties for a wide range of compositions and concentrations. Alternative approaches to link the microscopic structure to the thermodynamic properties are based on the liquid state theory.¹⁵ These methods determine the osmotic coefficients from MD generated ensembles by integrating the molecular structure of electrolyte solutions, described in terms of the interionic radial distribution functions.^{9,16,17} These methods have, in principle, fewer approximations to obtain thermodynamic properties but most likely rely on the quality of the ab initio and force field used to describe the ion-water and ion-ion interactions.

This study aims to link the thermodynamic models used to estimate activity coefficients of electrolyte solutions with the molecularlevel processes of ion hydration and ion association. We have achieved this objective by computing the parameters of aqueous thermodynamic hydration models from MD simulations and DFT calculations. We have conducted MD simulations and DFT calculations of hydrated alkaline earth metal ions (Mg^{2+} and Ca^{2+}) and aqueous electrolyte solutions (MgCl₂ and CaCl₂) with different concentrations, with the results of the simulations being used to determine the parameters of thermodynamic hydration models: the hydration number *h*, the hydration stages *n*, and the chemical equilibrium constant parameters K and k. The values of the parameters computed from this first principles approach were then used directly in the hydration models of Stokes and Robinson¹⁸ and Balomenos et al.,¹⁹ which were then applied to compute osmotic coefficients of aqueous MgCl₂ solutions.

A. Overview of the thermodynamic stepwise hydration model

The stepwise hydration-equilibrium models introduced by Stokes and Robinson (S & R),¹⁸ and later improved by Schönert,^{20,21} assume that a cation can possess various discrete degrees of hydration related by stepwise hydration equilibria. Cations (*C*) become hydrated through a stepwise process where a single water

molecule binds to the ion-water cluster during each degree of hydration,

$$\mathbf{C} \cdot (\mathbf{H}_2 \mathbf{O})_{i-1} + \mathbf{H}_2 \mathbf{O} \rightleftharpoons \mathbf{C} \cdot (\mathbf{H}_2 \mathbf{O})_i. \tag{1}$$

In Eq. (1), each hydration step *i* is controlled by an equilibrium constant relating the activities of the hydrated species and water,

$$K_i = \frac{a_i}{a_{i-1}a_W}.$$

The competition between neighboring solvent molecules in the coordinate shells will reduce the strength of binding from one hydration step to the next. For each successive step, the assumption is that the standard free energy change for attachment of a water molecule decreases by a constant amount $RT\ln(k)$, where k is an electrolyte-specific parameter. This simplified assumption works in the R & S's electrolyte solution model but may not be realistic. It may also be that the model can represent activity data using equilibrium constants K that are related to each other in different ways. In the R & S approach, the equilibrium constants for each step are related by

$$K_{1} = K,$$

$$K_{2} = kK,$$

$$\dots$$

$$K_{i} = k^{i-1}K,$$

$$\dots$$

$$K_{n} = k^{n-1}K,$$
(3)

where *i* is the number of sites of hydration and goes from 1 to *n*. If c_i is the stoichiometric molar concentration of the *i*th hydrated species, then the average hydration number *h* is then defined as

$$h = \frac{\sum ic_i}{\sum c_i}.$$
 (4)

In the S & R hydration model,¹ the partial molar volume of *i*th hydrated species, \overline{V}_i , is considered to be linearly dependent on the degree of hydration,

$$\overline{V}_i = \overline{V}_0 + i\overline{V}_W,\tag{5}$$

where \overline{V}_0 is the partial molar volume of zero hydration degree, and \overline{V}_W is the partial molar volume of a water molecule. The value of the maximum number of hydration sites, *n*, corresponds to the hydration number, *h*. In the S & R model, the water activity and the mean osmotic coefficient depend on the equilibrium constant, the hydration number, and the partial molar volume of different hydration species,²²

$$\ln(a_A) = \ln(1 - c\overline{V}_h) + c(\overline{V}_h - v\overline{V}_W) + \frac{\kappa^3}{24\pi N_A} \cdot \overline{V}_W S(\kappa a), \quad (6)$$

where *a* is the distance of the closest approach in the Debye–Hückel theory, *c* is the stoichiometric molar concentration, *v* is the stoichiometric number of the solute, N_A is the Avogadro constant, κ is the

J. Chem. Phys. **156**, 024502 (2022); doi: 10.1063/5.0074970 Published under an exclusive license by AIP Publishing

ARTICLE

Debye–Hückel reciprocal length expressed in the SI unit system and the molar concentration scale, and the function S(x) comes from the Debye theory,

$$S(x) = \left(\frac{3}{x^3}\right) \left[1 + x - \frac{1}{1+x} - 2\ln(1+x)\right]$$
(7)

For a single salt solute with molal activity, the osmotic coefficient can be then written as

$$\phi = -\frac{\ln(a_A)}{\nu m M_A},\tag{8}$$

where m is the molality of ion and M_A is the molar mass of water.

Schönert's model^{20,21} was developed based upon the S & R approach by considering the hydration of both cations and anions. The anion hydration is assumed to take place with the same stepwise procedure as that of the cation. The hydration equilibrium constant of each step is given by the following expression:

$$K_{p,i} = (k_p)^i \binom{n_p}{i}.$$
(9)

In Eq. (9), k_p is the binding hydration constant, where *p* refers to *c*, the cation, or *a*, the anion. The hydration equilibrium constant $K_{p,i}$ for a hydrated ion will then depend on the distinct combinations of the identical *i* water molecules on the available n_p identical hydration sites. Following Schönert's assumption that the hydrated species form a semi-ideal mixture with the solvent, the average hydration number for each ion is related to the mole fraction of the bulk solvent water x_W ,

$$h_p = \frac{\sum i C_{p,i}}{\sum C_{p,i}} = \frac{n_p k_p x_W}{1 + k_p x_W}.$$
 (10)

The hydration model of Balomenos combines the Pitzer–Debye–Hückel theory,²³ which used statistical thermodynamics to retain a more realistic ionic distribution, removes the charging process, and incorporates a hardcore repulsion factor, with the stepwise hydration model of Schönert.¹⁹ The long-range ionic interaction is described by Pitzer's hardcore repulsion factor,²³ and hydration association is represented by Schönert's two electrolyte parameters hydration sites and the binding hydration constant.^{20,21} In the Balomenos model, the osmotic coefficient is defined as

$$\phi = -\frac{\ln(x_W)}{1000 V_W c} - \frac{\kappa^3}{6Dc(1+\kappa\overline{a})} + \frac{\pi\overline{a}c}{3} \left(2\overline{a}^2 + \frac{\kappa^4}{\left(Dc(1+\kappa\overline{a})\right)^2}\right). \tag{11}$$

In Eq. (11), *c* is the total molar concentration of all ions, $D = 4\pi 1000N_A$, κ is the Debye–Hückel reciprocal length, and \overline{a} is the average distance of the closest approach, which is defined as

$$\frac{\overline{a}}{2} = r_{CA}^{h} = \left[0.5\left(\left(r_{C}\right)^{3} + \left(r_{A}\right)^{3}\right) + h(r_{W})^{3}\right]^{1/3},$$
(12)

where r_{CA}^h is the radius of the average ionic hydration sphere, r_C is the radius of the cation, r_A is the radius of the anion, r_W is the radius

of water, and h is the maximum number of hydration sites (here the hydration number). The Balomenos model uses only one electrolyte parameter, the maximum number of hydration sites, which includes contributions from ion-specific interactions between a single ion and solvent molecules and electrolyte specific interactions between multiple ions and solvent molecules.

II. COMPUTATIONAL DETAILS

A. Classic molecular dynamics simulation

We used classical MD simulations of magnesium chloride solutions, MgCl₂(aq), and calcium chloride solutions, CaCl₂ (aq), with concentrations ranging from 0.1 to 2.8 mol.kg⁻¹. These simulations have been used to characterize the structure and low-frequency dynamics of water and estimate the hydration number, h, of this electrolyte as a function of concentration. Furthermore, free-energy difference calculations from ensembles generated by MD simulation have also been conducted to evaluate the electrolyte-specific parameter, k. All MD simulations were carried out using GRO-MACS (v. 2019.4).²⁴ The models of aqueous electrolyte solution were based on the full atomistic treatment of the solute and solvent molecules. Water molecules were described with the three-site SPC/E water model,²⁵ while "scaled charge" Empirical Continuum Correction (ECC) force fields were used to describe the ion-water and ion-ion interactions.²⁶⁻²⁸ The fast electronic polarization in ECC force fields is considered in a mean-field approach and implemented numerically by scaling the charges of the ions. Here, we have used the Lennard-Jones potentials for MgCl₂(aq) and CaCl₂(aq) parameterized by Duboué-Dijon et al.,26,29 in which the charges for the cations (magnesium and calcium) and chlorine ions are set to +1.7 and -0.85, respectively. We have used these force fields among several others available in the literature for divalent cations because they have shown to provide an accurate structural description of concentrated aqueous MgCl₂ and CaCl₂ solutions. Duboué-Dijon et al. have parameterized and benchmarked this force field to experimental neutron scattering patterns of concentrated (3 m) MgCl₂(aq).³⁰ A similar benchmarking against neutron scattering data on concentrated CaCl₂(aq) solutions together with ion-pairing free energy profiles from ab initio molecular dynamics has also been conducted by Martinek et al.²⁹ Moreover, a detailed assessment of ten interatomic potentials for hydrated Mg²⁺ by Di Tommaso and co-workers also concluded that the Duboué-Dijon ECC model used in this work provided the best overall agreement with quantum mechanical and experimental reference data.3

The MD simulations were conducted using the following computational protocol: A cubic box was filled with ~800 water molecules. The MD simulations were conducted in the isothermal-isobaric ensemble (NPT, P = 1 atm and T = 300 K) for 1 ns to generate an equilibrated aqueous solution. The last configuration was then used to generate MgCl₂(aq) and CaCl₂(aq), with concentrations ranging from 0.1 to 2.8 mol kg⁻¹, by randomly replacing N water molecules with N/3 Mg²⁺ and 2N/3 Cl⁻ ions and by making sure that the initial configuration did not contain contact ion pairs. The volume of the simulation box was further minimized in the NPT ensemble, followed by a production period of 1 ns. Configurations were saved every 1 ps. Long-range electrostatic interactions were accounted for using the Particle–Mesh Ewald (PME) method with a 12 Å cutoff for nonbonded interactions. Simulations in the NPT ensemble used the velocity-rescale thermostat³² and Parrinello–Rahman pressure coupling methods³³ with a 2 ps time constant. Bonds involving hydrogen atoms were constrained by the LINear Constraint Solver (LINCS) algorithm.³⁴

The free energy calculations of removing a specific water molecule in the electrolyte solution were conducted with the GROMACS "bar" module which implements the Bennett Acceptance Ratio (BAR) method combined with the PLUMED plugin (v. 2.4.1).^{35,36} To avoid the selected water molecule moving between different coordination shells during the MD simulation, we imposed two harmonic bias potentials, both with a constant of 1000 kJ mol⁻¹, along the reaction coordinates defined by the distances between the cation and the water and between the anion and the water.

B. DFT calculations

Electronic structure calculations were carried out with the Gaussian09 code.^{37,38} Geometry optimization and frequency calculations were both conducted with the long-range corrected hybrid density functional with damped atom–atom dispersion corrections (B97X-D)³⁹ together with the Pople basis set 6-311++G(d,p). The solvation models used to conduct the solution-phase calculation were the conductor-like polarizable continuum model (CPCM) and the polarizable continuum model using the integral equation formalism variant (PCM).⁴⁰ The free energies of the stepwise process [Eq. (1)] where a single water molecule binds to the ion–water cluster were computed according to the following equation:

$$\Delta G^* = G^*_{C^+(H_2O)_i} - G^*_{C^+(H_2O)_{i-1}} - G^*_{H_2O}, \qquad (13)$$

where G_X^* is the total Gibbs free energy of species $(X = C^+(H_2O)_i, C^+(H_2O)_{i-1}, \text{ or } H_2O)$ in the liquid phase at 300 K. We have used the procedure recommended by Ho *et al.*⁴¹ to apply dielectric continuum solvent models to study chemical reactions in the condensed phase, with the free energies of molecules in the solution obtained from separate gas- and solution-phase calculations. The values of G_X^* were calculated using the following expression:⁴²

$$G_X^* = E_{e,gas} + \delta G_{VRT,gas} + \Delta G_{solv}^* + RT \ln[\tilde{R}T], \qquad (14)$$

where $E_{e,gas}$ is the gas-phase total electronic energy of the gasphase optimized geometry of the species X, $\delta G_{VRT,gas}$ is the vibrational-rotational-translational contribution to the gas-phase Gibbs free energy at 300 K under a standard-state partial pressure of 1 atm, ΔG_{solv}^* is the solvation free energy of the solute corresponding to the transfer from an ideal gas at a concentration of 1 mol dm⁻³ to an ideal solution at a liquid-phase concentration of 1 M, and the $RTln[\tilde{R}T]$ term is the free energy change of 1 mol of an ideal gas from 1 atm to 1 mol dm⁻³ ($RTln[\tilde{R}T]$ = 1.89 kcal mol⁻¹ at 300 K, where $\tilde{R} = 0.082\,052\,\mathrm{K}^{-1}$).^{43,44} We have computed the vibrational-rotational-translational contribution to the optimized gas-phase structure of the hydrated cluster because the solute is not an ideal gas. Consequently, the ideal gas partition functions, particularly the translational and rotational contributions, are unlikely to be valid in solution.⁴¹ When modeling microhydrated molecules and ions, an aspect of the calculations that should be considered is the low-lying energy minima on their potential energy surface. The sampling of multiple configurations of hydrated cluster ions is likely to be important for labile cations, such as K^+ or Na⁺, which display a fast water exchange.⁴⁸ However, Ca²⁺ and, in particular, Mg²⁺ strongly interact with the surrounding water molecules. These hydrated cations are likely to be characterized by a single configuration if the water molecules are part of the first shell.

III. RESULTS AND DISCUSSION

A. Subpopulation water classification method

In an aqueous electrolyte solution, a water molecule can be in the first- or second-coordination shells of the cation/anion, shared by both ions, or part of the bulk. We have implemented a cooperative hydration model to categorize the water molecules in electrolyte solutions into different subpopulations (Fig. 1). Each water molecule in the solution is labeled as W_{ab} , where a, b = 1, 2, B, depending on the position of oxygen and hydrogen atoms from the cation (Mg²⁺ or Ca^{2+}) and the anion (Cl⁻). The subscript *a* in W_{ab} is set to 1 when the oxygen (O) atom is in the first coordination shell of the nearest cation, to 2 when O is in the second coordination shell of the nearest cation, and to B when it is beyond the second shell. Similarly, the subscript b is set to 1 when one of the hydrogen (H) atoms of the water molecule is in the first coordination shell of the nearest anion, to 2 when H is in the second coordination shell of the nearest anion, and to B when it is beyond the second shell. Assignments were made by comparing the distance between oxygen and the nearest cation within the first and second Mg²⁺ coordinate shells and the distance between hydrogen and the nearest chloride within the first and second Cl⁻ coordinate shells (any one of the two hydrogen atoms in the water molecule). We have assumed that water molecules in the



FIG. 1. Definition of water subpopulations for a solvent separated Mg²⁺ and Cl⁻ ion pair showing the categorization of one H₂O molecule in the W₁₂ subpopulation: the O atom in the first coordination shell of Mg²⁺ and both H atoms in the second coordination shell of Cl⁻.

same subpopulation have the same behavior. We have previously demonstrated the robustness of this method to determine the distribution of the water molecules into different water subpopulations for MgCl₂(aq).⁴⁷ However, alkaline ions, such as Na⁺ and K⁺, have much faster water exchange dynamics than Mg²⁺ and Ca²⁺.⁴⁸ Other classification methods based on order parameters⁴⁹ rather than distance criteria could be appropriate for labile cations and conditions of higher concentration.

B. Equilibrium constant k and K parameters

The stepwise hydration process to form the fully hydrated cation $\left[C(H_2O)_h\right]^{z+}$ is given by the following equation:

$$\left[C(H_2O)_{h-1}\right]^{z+} + H_2O \Longrightarrow \left[C(H_2O)_h\right]^{z+}, \quad -\Delta G_n, \ K_n = k^{n-1}K,$$
(15)

where *h* is the hydration number, which represents the maximum number of water molecules influenced by the cation, *n* is the maximum degree of hydration, and K_n is the equilibrium constant of the stepwise process that is related to the free energy of removing the *h*th water molecule from $C(H_2O)_h$ by $\Delta G_n = RT \ln K_n$. Notice that for a cation ion with one or more stably coordinated water molecules, $n \le h$ because the number of water molecules for the initial hydration step is not equal to 1. The hydration of the fully hydrated cation

$$\left[C(H_2O)_h\right]^{z_+} + H_2O \Longrightarrow \left[C(H_2O)_{h+1}\right]^{z_+},$$

- $\Delta G_s, \ K_{n+1} = k^n K = K_W,$ (16)

will have an equilibrium constant equivalent to the hydration of a water molecule in pure bulk water (K_W), which is related to the free energy needed for removing one bulk-like water molecule from the fully hydrated cluster, $\Delta G_s = RT \ln(K_s)$. Therefore, the electrolyte-specific parameter k, which is the standard free energy change for successive step water attachment in the S&R hydration model, $RT \ln(k)$, is given by

$$k = \frac{K_W}{K_n}.$$
 (17)

Based on the assumption that water molecules in the same subpopulation have equal ability to bind the ion–water cluster, the electrolyte-specific parameter k can be determined from the ion-affected subpopulation with the lowest absolute value, $\Delta\Delta G$, which corresponds to the last step of the hydration process,

$$-RT \ln(k) = \min |\Delta \Delta G| = |\Delta G_n - \Delta G_s|.$$
(18)

The value of ΔG_n for each water subpopulation in an aqueous electrolyte has been computed using a computational protocol based on the BAR method, as detailed in Sec. II A. This methodology has been used to determine the *k* parameter of aqueous MgCl₂ solutions for concentrations ranging from 0.1 to 2.8 mol kg⁻¹ (Fig. 2). The method assumes that we can separate the description of the influence of the cation and anion on the water molecules. Figure 2(a) reports the average values of the free energy for the subpopulation of MgCl₂(aq). Since the effect of water in the first shell of a magnesium ion is much greater than that of a chloride ion, we can neglect the

influence of water molecules from anions when these are in the first shell of cations (W_{1x}) . Similarly, since the free energy values of the subpopulations W_{B2} and W_{BB} are similar for all the concentrations, the effect of water molecules in the second shell of the anion (W_{B2}) is small and can be neglected. As the free energy value of W₂₁ is larger than that of W₂₂, the chloride and magnesium ions have an overlapping effect on the water molecules in W₂₁. To separate the effect of Mg²⁺ and Cl⁻ on the water molecules, we generated two standard lines, the orange and light blue dashed lines, respectively, in Fig. 2(a), each representing the effect of Mg²⁺ and Cl⁻ separately. The orange dashed line (W_{BB} subpopulation) represents the free energy change of removing a bulk-like water molecule. We use this line to compare the free energy values of the water molecules in the subpopulations without the effect of chloride: W1x, W22, and W2B. These subpopulations are under the influence of Mg^{2+} . The sky-blue dashed line represents the free energy change of removing a water molecule in the first coordinate shell of Cl- obtained from MD simulations of one isolated chloride ion in the pure water system. We use this line to compare the free energy change of water molecules belonging to subpopulations that could be affected by the chloride only: W₂₁ and W_{B1}.

The variation of min $|\Delta\Delta G|$ of electrolyte solutions as a function of concentration is summarized in Fig. 2(b), where the blue- and orange-colored bar charts represent the chloride- and magnesiumaffected water subpopulations, respectively. For molality lower than 0.6 mol kg⁻¹, the value of min $|\Delta\Delta G|$ comes from the W_{B1} subpopulation, which corresponds to water molecules beyond the second coordinate shell of the cation but in the first coordinate shell of the anion. These water molecules can be considered non-bulk-like. The values of the *k* parameter are between 0.8 and 1. For more concentrated solutions, min $|\Delta\Delta G|$ is due to the W₂₂ subpopulation and the equilibrium constant parameter *k* shows a constant trend around 0.7. In comparison, the value of *k* obtained from the application of the S & R model to MgCl₂(aq) (fitting code with the e04fyf routine from the NAG Library) is 0.9 (Table S1 of the supplementary material).

The other equilibrium constant parameter, K_1 , is associated with the first degree of hydration (n = 1). Mg²⁺ is a strongly hydrated ion with six water molecules stably coordinated to the ion.⁴⁵ Consequently, these water molecules will not participate in the stepwise hydration process. The DFT calculations were conducted to determine the free energies of the formation of the pseudo-first hydration process,

$$Mg^{2+}(H_2O)_6 + H_2O \Longrightarrow Mg^{2+}(H_2O)_7 \qquad K_1 = K,$$
 (19)

with the seventh water molecule in the second coordinate shell binding onto the magnesium–water cluster. We calculated G_X^* , the total Gibbs free energy of species (X = Mg²⁺(H₂O)₆, Mg²⁺(H₂O)₇, or H₂O), and further got the Gibbs free energy change of the initial hydration process (Table S2 of the supplementary material). According to the equation $\Delta G = -RTln(k)$, the equilibrium constant parameter *K* of MgCl₂ is 2.72.

C. Hydration numbers and hydration stages

We present the determination of the hydration number h of an electrolyte solution using two methods based on the calculation



of the following water-specific properties: the free energy of water removal and the water dipole reorientation dynamics.

1. Free energy-based method

By inserting the values k and K determined in Sec. III B into Eq. (3), we can obtain the equilibrium constant K_i $(1 \le i \le n)$ of each degree of stepwise hydration. Figure. 3 reports the Gibbs free

FIG. 2. Results from free energy calculations of MgCl₂(aq) with concentrations from 0.1 to 2.8 mol kg⁻¹. (a) Average free energy change of removing a water molecule from each subpopulation. The orange dashed line (W_{BB}) is the free energy change of removing a bulk-like water molecule. The sky-blue dashed line is the free energy change of removing a water molecule in the first coordinate shell of CI- obtained from simulations of an isolated ion in water. (b) Values of min $|\Delta \Delta G|$ and k as a function of concentration. The blue bars are min $|\Delta \Delta G|$ values obtained from the difference of ΔG_n from the chlorideaffected subpopulation (W_{B1}) and ΔG_s from the sky-blue dashed line. The orange bars are $\Delta \Delta G$ obtained from the difference between ΔG_n from the Mgaffected subpopulations (W₂₂) and $\Delta \tilde{G}_s$ from the W_{BB} subpopulation.

energy change for the formation of the next hydration stages (ΔG_i) of aqueous MgCl₂ solutions. Reactions with a negative ΔG (dark blue) occur spontaneously and those with a positive ΔG (orange color) are unfavorable and require the energy input to take place. When ΔG is zero (white color), the system has reached the maximum hydration stages. At low concentrations, our predicted value of *h* for MgCl₂(aq) is 14, close to the infinite dilution hydration number 14.5 deduced from isothermal compressibility.⁴⁶ The hydration number







parameter is not constant and decreases to 9 for higher concentrations (Fig. 3).

We tested the same methodology on the aqueous CaCl₂ solutions with concentrations from 0.1 to 2.0 mol kg⁻¹ (Fig. 4). The variation of min $|\Delta\Delta G|$ as a function of the concentration and the values of k are summarized in Fig. 4(a). The water molecules that belong to the first and second coordination shells of Ca²⁺ change during the simulation, with the coordination number varying between six and seven.⁴⁸ The calcium ion is not as strongly hydrated as the magnesium ion, and water exchanges between the first and second hydration shell Ca²⁺ could be observed throughout the simulation compared with none of such events around Mg²⁺.⁴⁷ The water molecules under the effect of chloride ions in the CaCl₂ solutions do not show a different behavior compared with that under the effect of a single Cl⁻ in the pure water system. For dilute solutions (up to 0.3 mol kg^{-1}), the last hydration process occurs on the water molecule belonging to the W₂₂ or W_{2B} subpopulation. For higher concentrations, this process always occurs in the W₂₁ subpopulations. The reason for this trend is that at low concentrations (lower than 0.5 mol kg⁻¹), a small number of water molecules from the W₂₂ subpopulation are non-bulk-like and most of the water molecules are bulk-like. Consequently, the average result in W₂₂ is close to the reference W_{BB} subpopulation. When the concentration is higher, most of the water molecules in the W₂₁ subpopulation are non-bulklike, so the average result of water in W_{21} is relatively biased to W_{1X} . DFT calculations were also used to calculate the Gibbs free energy change of the formation of next hydration degree. There are only a small number of water molecules stably "binding" to calcium ions at the initial hydration stage ion-water cluster. The initial hydration process is

$$Ca^{2+}(H_2O)_1 + H_2O \Longrightarrow Ca^{2+}(H_2O)_2$$
 $K_1 = K$, (20)

where the equilibrium constant parameter K of $CaCl_2$ is 3.43 (Table S3 of the supplementary material). The Gibbs free energy change of the formation of the next hydration stages (ΔG_i) calculated with the values of k and K parameters is given in Fig. 4(b). There is some disagreement between previously reported values of the hydration number of the calcium ion. Zavitsas reported a hydration number of 12.0 from freezing point depression measurements and 6.7 from boiling point elevations.¹⁰ Moreover, the value of h obtained by applying the S&R model to CaCl₂(aq) is in the range of 4 to $8.^{18}$ The *h* value obtained from our free energy based atomistic simulation method shows dependence on the concentration and suggests a value of 9 for dilute solutions and 4 for solutions with concentrations around 0.5 mol kg^{-1} or higher. In Fig. 4(b), there is a significant increase in the hydration number for CaCl₂(aq) below 0.5 mol kg⁻¹. For these solutions, the last hydration process occurs on the water molecule belonging to the W_{22} or W_{2B} subpopulation (the difference of free energy is between subpopulations W₂₂ and W_{BB}). For higher concentrations, the last hydration process always occurs in the W₂₁ subpopulations (the difference of free energy is between subpopulations W_{21} and Cl^- in pure water). For concentrations lower than 0.5 mol kg⁻¹, a small number of water molecules from the W₂₂ subpopulation are non-bulk-like and most of the water molecules are bulk-like. Consequently, the average result in W₂₂ is close to the reference W_{BB} subpopulation. When the concentration is higher, most of the water molecules in the W₂₁ subpopulation are non-bulk-like, so the average result of water in W₂₁ is relatively biased to W_{1X}. For CaCl₂(aq), the subpopulation method might be sensitive to water exchanges between



FIG. 4. Results of CaCl₂(aq) from 0.1 to 2.0 mol kg⁻¹. (a) Variation of min $|\Delta\Delta G|$ and *k* with the solution concentration. The orange bars were obtained from the difference between ΔG_n of the Mg²⁺ affected subpopulations labeled in the figure and ΔG_s of the W_{BB} subpopulation. (b) Gibbs free energy change of previous hydration stage calcium water cluster to this hydration stage calcium water cluster in a CaCl₂ aqueous solution.

the first and second hydration shell Ca^{2+} occurring during the MD simulation.

2. Orientational dynamics-based method

The second method to determine the hydration number parameter relies on the orientational dynamics of water molecules.

Rotational motion plays a crucial role in the breaking and making of hydrogen (H) bonds (more strongly H-bonded molecules reorient more slowly) and can be computed from the first-order Legendre time correlation function, $^{48-50}$

$$C_1(t) = \langle \vec{\mu}(0) \cdot \vec{\mu}(t) \rangle / \vec{\mu}(0)^2,$$
(21)

ARTICLE

where $\vec{\mu}(0)$ and $\vec{\mu}(t)$ are the unit vectors defining the orientation of the dipole moment of H₂O at times 0 and *t*, respectively. The application of this method to MgCl₂(aq) is reported in Fig. 5. The $C_1(t)$ function starts at 1 and decays asymptotically to zero because of the random and isotropic orientation of the water molecules in the solution [Fig. 5(a)]. The early stage of fast loss of correlation is caused by librational motion, whereas the long term decay is due to reorientational motion and can be fit by a bi-exponential function, $a \exp(-t/\tau_1) + b \exp(-t/\tau_2)$.³⁴ The overall time associated with this process, τ_{reor} , is given by the sum of fitting parameters $\tau_1 + \tau_2$.

For each subpopulation, the orientation time correlation function has been computed by tracking the dipole vectors of the water molecules belonging to that specific subpopulation. In Fig. 5(b), we



FIG. 5. Determination of the hydration numbers of aqueous MgCl₂ solutions from water reorientation dynamics. (a) Orientational time correlation function $C_1(t)$ of the W₂₁ subpopulation in 0.61 mol kg⁻¹ MgCl₂(aq). (b) Retardation factor computed as the ratio between the reorientation relaxation time of the subpopulation and bulk water used to determine slow reorienting subpopulations in 0.61 mol kg⁻¹ MgCl₂(aq). (lnset) Number of water molecules per Mg–Cl ion pairing among the subpopulations. (c) The number of water molecules from the non-bulk-like water subpopulation, corresponding to the hydration number, during each step of the MD simulation of 0.61 mol kg⁻¹ MgCl₂(aq). (d) Distribution of different hydration stages, C_i/C , existing in the MgCl₂ solutions.

define the retardation factor as the ratio between the relaxation times of a specific subpopulation, W_{XX} , and bulk water, W_{bulk} ,

$$f_{W_{XX}} = \frac{\tau_{W_{XX}}}{\tau_{W_{bulk}}}.$$
 (22)

A slow relaxation dynamic should be observed for water molecules that are in non-bulk-like subpopulations. The statistical approach used to differentiate between bulk-like and hydration water subpopulations are based on the empirical 68-95-99.7 rule. For a subpopulation W_{XX} to be classified as non-bulk-like, the retardation factor must lie outside 3σ of the mean value of bulk-like water, that is, the value of $f_{W_{XX}}$ should be larger than the 3σ deviation. This subpopulation analysis has been conducted at each time step using four (non-overlapping) simulation blocks each lasting 5 ps. The water molecules that are in the second coordination shell of Mg^{2+} (subpopulations W_{2X}) are outside the 2σ deviation and are classified as non-bulk-like water. A much slower relaxation dynamic compared to bulk ($5 \le f_{W_{XX}} \le 6$) is observed for water molecules that are in the first coordination shell of Mg^{2+} (subpopulations W_{1X}). The chloride ion does not have a significant effect on the reorientation dynamics of water: subpopulations such as W_{1B} with water molecules in the first coordination shell of Cl⁻ and outside the second coordination shell of Mg^{2+} have a reorientation relaxation time that is very close to that of bulk water. These results confirm the long- and short-range effects of Mg²⁺ and Cl⁻ on the reorientation water dynamics, respectively. The inset figure of Fig. 5(b) reports the distribution of water molecules per Mg-Cl ion pairing among different subpopulations. It is also possible to evaluate the number of water molecules that are in the bulk (free water) or coordinated to Mg^{2+} or Cl^{-} .

Figure 5(c) shows the number of slow-orienting (non-bulklike) water molecules [belonging to the subpopulations in the red shadow in Fig. 5(b)] of 0.61 mol kg⁻¹ MgCl₂(aq) during the simulation time. The frequency of each hydration number during the simulation corresponds to the distribution of hydration stages, C_i/C , in the S & R's stepwise hydration model.¹⁸ The different hydration degrees that exist in the solutions with concentrations ranging from 0.1 to 2.8 mol kg⁻¹ are reported in Fig. 5(d). The weighted average value of the number of water molecules in these non-bulk-like subpopulations is the average hydration number. This figure shows a trend of the hydration number to lower values as the concentration increases.

3. Theory and experiment comparison

Figure 6 compares the hydration numbers of MgCl₂(aq) as a function of concentration obtained from the hydration models of S & R and Balomenos, computed from the MD simulations using the free energy and orientational dynamics methods, and determined experimentally from concentration-dependent THz dielectric relaxation (DR) spectroscopy measurements.⁴⁷ The hydration number measured from THz-DR spectroscopy corresponds to the average number of moles of water molecules per mole of dissolved salt that no longer participate in bulk-like reorientation dynamics.⁴⁸ This molecular definition of the hydration number pertains to "irrotationally bound" water molecules (the ones tightly bound to the solute). The values from Schönert's model are not listed in



FIG. 6. Comparison of the hydration number calculated from thermodynamic stepwise hydration models (S & R 1973 and Balomenos), the free energy- and water reorientation dynamics-based simulation methods, and THz experiments.

Fig. 6 because they are similar to those from the Balomenos model and only available up to a molality of 1 mol kg^{-1} . The stepwise thermodynamic models cannot give hydration number parameters that change with concentration and are close to the experiment only at high concentrations. In comparison, the hydration numbers computed from atomistic properties of water molecules are concentration-dependent and close to the THz-DR experimental results.

Figure 7 compares the experimental¹ and predicted values of the osmotic coefficients, ϕ , of MgCl₂(aq). We computed the osmotic coefficients from the original S & R and Balomenos models, in which the hydration number h was obtained by fitting to experimental measurements, and the modified version of these two models, where h has been determined in this study either from the free energybased or water orientational dynamics-based methods. When the value of hydration number differs vastly from the original hydration number data fitting in the models, the osmotic coefficient obtained by the formula in the model will have a large gap with the results from experiments. This difference of hydration number between fitting results and computational method results has a more noticeable impact on the osmotic coefficient as the concentration increases. For example, the osmotic coefficient from Balomenos reparameterized with the orientational dynamics-based method deviates significantly from the experimental data when the concentration is higher than 1.0 mol kg⁻¹. The osmotic coefficient from the S&R 1973 model and the Balomenos reparameterized with the h values from the free energy-based method shows a good agreement with the experiments.

D. A reparameterized Stokes and Robinson model

In Secs. III B and III C, we have computed the parameters k, K, and h. Moreover, we have related these parameters to



FIG. 7. Experimental¹ and predicted osmotic coefficients, ϕ , of MgCl₂ (aq). Theoretical values obtained from the original and reparameterized S&R model and Balomenos models using the free energy-based method and orientational dynamics-based method.

molecular processes occurring in the MgCl₂ solutions. Here, we show a simple modification of the S & R model that considers these parameters and the molecular-scale behavior of the ions in the solution. The original S & R 1973 hydration model assumes the zeroth degree of hydration of a solute in a solution to be the solute ion itself. However, this could not be a realistic picture for most ions and, especially, strongly hydrated cations. For example, the hydrated Mg²⁺ has a very stable minimum corresponding to sixfold coordination with water, $Mg(H_2O)_6^{2+}$, and the five-coordinated intermediate, $Mg(H_2O)_5^{2+}$, is inaccessible at 300 K due to the very high activation barrier between the six- and five-coordinated configurations of Mg²⁺.⁴⁵ Consequently, the water exchange is drastically retarded in its first solvation shell.^{48,50} It is possible to assume that i, the degree of hydration of the hydrated ion cluster, starts from 0 [Mg²⁺ and six water clusters, $Mg(H_2O)_6^{2+}$] to a maximum hydration stage n $[Mg^{2+} and h number of water clusters, Mg(H_2O)_h^{2+}]$. The partial molar volumes related to the degree of hydration are given by the following expression:

$$\overline{V}_i = \overline{V}_C + (6+i)\overline{V}_W \quad (0 \le i \le n), \tag{23}$$

where \overline{V}_C is the partial molar volume of solute and \overline{V}_W is the partial molar volume of water. Therefore, the relationship between hydration stage *n* and hydration number *h* is

$$h = 6 + n. \tag{24}$$

By inserting Eq. (24) into Eq. (6), we obtain

$$\ln a_{A} = \ln \left[1 - c (\overline{V}_{C} + h \overline{V}_{W}) \right] + c \left[\overline{V}_{C} + (h - v) \overline{V}_{W} \right] \\ + \left[\kappa^{3} / (24\pi N_{A}) \right] \overline{V}_{W} S(\kappa a).$$
(25)

We calculated the values of the concentration ratios c_i/c using the equations

$$c_i/c_{i-1} = (K_i/Y)a_A,$$

$$c_i = c_0 \cdot (K/Y)^i \cdot k^{i(i-1)/2}a_A^i,$$
(26)

where $lnY = c(\overline{V}_C + h\overline{V}_W - v\overline{V}_W)$. The fitting code used to reproduce the hydration number, equilibrium constants distribution of different hydration degree concentrations, and osmotic coefficient data from the S & R model were based on the e04fyf routine from the NAG Library.⁵¹

In Eq. (26), c_i/c represents the existing hydration stages in the solutions. It has been calculated by quantifying the cooperative effect of ions on the water reorientation dynamics of different water subpopulations. As shown in Fig. 8, the hydration stages of MgCl₂ solutions from the original S & R 1973 model and the one obtained using the values of the parameters obtained from atomistic simulations show different behaviors. The modified model shows a higher number of accessible hydration stages and leads to a range of hydration number values, from 7.7 to 9.3, compared to the original model, from 8.6 to 7.8. This behavior reflects the number of water molecules per ion in solution with a slow reorienting behavior and links the microscopic dynamic equilibrium to the macroscopic affected water shell. Therefore, the modified model reflects the underlying process controlling the hydration process



FIG. 8. Ratio C_i/C of existing hydration stages and the hydration number from the fitting code of the (a) original Stokes and Robinson 1973 and (b) reparameterized models.

J. Chem. Phys. **156**, 024502 (2022); doi: 10.1063/5.0074970 Published under an exclusive license by AIP Publishing

ARTICLE



FIG. 9. The comparison of the osmotic coefficient from experiments, the original S & R 1973 model, and the modified model with the physical-meaning parameter *h*.

without affecting the ability of the model to calculate the osmotic coefficient.

Figure 9 shows that using parameters based upon computed microscopic properties, the reparameterized S&R (1973) model can predict well the osmotic coefficients. This result could also be because the stepwise hydration-equilibrium model of S & R (1973) provides a simple but still realistic description of the processes controlling the thermodynamic properties of aqueous electrolyte solutions. Moreover, a detailed comparison of the raw data shows a minor but still quantifiable improvement of our reparameterized model compared with the original S & R (Table S4 of the supplementary material). It is worth noting that the force field used to describe the ion-water and ion-water interactions was verified to scattering patterns at only a few concentrations,³⁰ which is not compelling enough to assume good agreement over the entire concentration range. However, the reparameterized S & R (1973) model still provides an excellent predictive power of the osmotic coefficient. This result could suggest the accuracy of the ECC force field. On the other hand, it also affirms the correctness of the thermodynamic hydration model on the underlying ion hydration and ion-ion processes.

IV. CONCLUSIONS

Thus far, thermodynamic models used to estimate the activity coefficients of electrolyte solutions contain empirical parameters determined by fitting the models to experimental data. In this study, we have conducted atomistic simulations to show the link between the parameters of stepwise hydration models and the molecular-level ion-hydration processes in electrolyte solutions. We have developed computational procedures to determine the concentrationdependent values of the k, K, and h parameters used in the Stokes and Robinson (1973) and Balomenos stepwise hydration models and reparameterized such models to compute the osmotic coefficients of $MgCl_2$ solutions.

- We have implemented a simplified subpopulation water category methodology to describe the behavior of water in the hydration shell of ions and quantify the cation–anion mixture effect. Using this classification method, we have computed the hydration free energy and reorientation dynamics of the water molecules in the subpopulations of MgCl₂(aq) and CaCl₂(aq).
- Based on the assumptions that water molecules belonging to the same subpopulation have equal ability to bind to the ion-water cluster and do not exchange between different subpopulations, we have determined the electrolyte-specific parameter *k* from the ion-affected subpopulation with the lowest absolute value of the free energy of water removal.
- The equilibrium parameter, *K*₁, associated with the first degree of hydration, was computed from density functional calculations of the free energy of hydration of the hydrated ionic clusters in solution.
- The hydration number *h* was determined by considering whether the reorientation time of the water subpopulations is retarded with respect to bulk-like behavior.
- We used the computed values of the parameters *k*, *K*, and *h* to reparameterize the hydration models of Stokes & Robinson (1973) and Balomenos and compute osmotic coefficients of aqueous MgCl₂ solutions as a function of concentration. The osmotic coefficients obtained from the reparameterizations of the Stokes and Robinson (1973) and Balomenos show a generally good agreement with the experiments.
- We have implemented a version of the Stokes and Robinson (1973) model to describe aqueous MgCl₂(aq) where the highly hydrated character of Mg²⁺ is part of the model. This reparameterized model can predict well the osmotic coefficients and leads to some improvements compared with the original model.

Our work represents an attempt to parameterize aqueous hydration models using first principles molecular-scale properties computed from atomistic simulations, rather than fitting the models to experiments. Such an approach, if found to be generally applicable to a range of electrolyte solutions and able to incorporate a treatment of chemical equilibrium between solvent species, would remove some of the dependence on experimental thermodynamic measurements and pave the way to develop aqueous thermodynamic models capable of estimating the values of solute and solvent activities for a wide range of compositions and concentrations.

SUPPLEMENTARY MATERIAL

See the supplementary material for the application of the Stokes and Robinson (1973) model to aqueous $MgCl_2$ solutions (Table S1), density functional theory calculations of the Gibbs free energy change of the initial hydration process of Mg^{2+} (Tables S2 and S3), and a comparison of experimental and calculated values of the osmotic coefficients of aqueous $MgCl_2$ solutions (Table S4).

ACKNOWLEDGMENTS

X.W. acknowledges the QMUL Principal's Studentship for funding. D.D.T. acknowledges the ACT programme (Accelerating CCS Technologies, Horizon2020 Project No. 294766), which funded the FUNMIN project. Financial contributions were made from the Department for Business, Energy & Industrial Strategy (BEIS) together with extra funding from the NERC and EPSRC research councils (United Kingdom), ADEME (FR), and MINECO-AEI (ES). We thank the UK Materials and Molecular Modelling Hub for computational resources, which is partially funded by the EPSRC No. (EP/P020194/1). Through our membership of the UK's HEC Materials Chemistry Consortium, which is funded by the EPSRC No. (EP/L000202), this work used the ARCHER UK National Supercomputing Service (http://www.archer.ac.uk). This research utilized Queen Mary's Apocrita HPC facility, supported by QMUL Research-IT (http://doi.org/10.5281/zenodo.438045).

AUTHOR DECLARATIONS

Conflict of Interest

All authors declare that they have no conflicts of interest.

Author Contributions

X.W. performed software testing, validation, formal analysis, investigation, and visualization and wrote the original draft; D.D.T. contributed to conceptualization, methodology, writing—review and editing, resources, and supervision; and S.L.C. contributed to conceptualization, methodology, writing—review and editing, resources, and supervision.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

REFERENCES

- ¹R. A. Robinson and R. H. Stokes, J. Am. Chem. Soc. 70, 1870 (1948).
- ²K. S. Pitzer, J. Chem. Phys. 77, 268 (1973).
- ³S. L. Clegg and K. S. Pitzer, J. Chem. Phys. **96**, 3513 (1992).
- ⁴S. L. Clegg and A. S. Wexler, J. Geophys. Res.: Atmos. 107(D14), 4207, https://doi.org/10.1029/2001JD000451 (2002).
- ⁵Y. Song and C. C. Chen, Ind. Eng. Chem. Res. 44, 8909 (2005).
- ⁶C. C. Chen and Y. Song, Ind. Eng. Chem. Res. 48, 5522 (2009).
- ⁷E. Balomenos, D. Panias, and I. Paspaliaris, Miner. Process. Extr. 27, 1 (2006).
- ⁸J. Mähler and I. Persson, Inorg. Chem. 51, 425 (2012).
- ⁹Z. Gong and H. Sun, J. Chem. Inf. Model. 57, 1599 (2017).
- ¹⁰A. A. Zavitsas, J. Phys. Chem. B **105**(32), 7805 (2001).
- ¹¹L. X. Dang and T.-M. Chang, J. Phys. Chem. Lett. **3**, 175 (2012).
- ¹²S. Uhlmann, T. Frauenheim, and Y. Lifshitz, Phys. Rev. Lett. 81, 641 (1998).
- ¹³T. C. Berkelbach, H.-S. Lee, and M. E. Tuckerman, Phys. Rev. Lett. **103**, 238302 (2009).

¹⁴ M. Pavlov, P. E. M. Siegbahn, and M. Sandström, J. Phys. Chem. A **102**, 219 (1998).

- ¹⁵J. C. Rasaiah and H. L. Friedman, J. Chem. Phys. 48, 2742 (1968).
- ¹⁶I. Kalcher and J. Dzubiella, J. Chem. Phys. **130**, 134507 (2009).
- ¹⁷N. Naleem, N. Bentenitis, and P. E. Smith, J. Chem. Phys. **148**, 222828 (2018).
- ¹⁸R. H. Stokes and R. A. Robinson, J. Solution Chem. 2, 173 (1973).
- ¹⁹E. Balomenos, D. Panias, and I. Paspaliaris, Fluid Phase Equilib. 243, 29 (2006).
- ²⁰H. Schönert, Z. Phys. Chem. Neue Folge **150**, 163 (1986).
- ²¹ H. Schönert, Z. Phys. Chem. Neue Folge **150**, 181 (1986).
- ²² R. H. Stokes and R. A. Robinson, *Electrolyte Solutions* (Butterworth & Co., London, 1970).
- ²³K. S. Pitzer, Acc. Chem. Res. 10, 371 (1977).
- ²⁴ B. Hess, C. Kutzner, D. van der Spoel, and E. Lindahl, J. Chem. Theory Comput. 4, 435 (2008).
- ²⁵ H. J. C. Berendsen, J. R. Grigera, and T. P. Straatsma, J. Phys. Chem. **91**, 6269 (1987).
- ²⁶E. Duboué-Dijon, P. Delcroix, H. Martinez-Seara, J. Hladílková, P. Coufal, T. Křížek, and P. Jungwirth, J. Phys. Chem. B **122**, 5640 (2018).
- ²⁷D. Laage and G. Stirnemann, J. Phys. Chem. B 123, 3312 (2019).
- ²⁸I. Leontyev and A. Stuchebrukhov, Phys. Chem. Chem. Phys. **13**, 2613 (2011).
- ²⁹T. Martinek, E. Duboué-Dijon, Š. Timr, P. E. Mason, K. Baxová, H. E. Fischer,
- B. Schmidt, E. Pluhařová, and P. Jungwirth, J. Chem. Phys. 148, 222813 (2018).
- ³⁰ P. E. Mason, E. Duboué-Dijon, H. E. Fischer, and P. Jungwirth, J. Phys. Chem.
 B 122, 3296 (2018).
- ³¹X. Zhang, P. Alvarez-Lloret, G. Chass, and D. Di Tommaso, Eur. J. Mineral. **31**, 275 (2019).
- ³²G. Bussi, D. Donadio, and M. Parrinello, J. Chem. Phys. **126**, 014101 (2007).
- ³³M. Parrinello and A. Rahman, J. Appl. Phys. **52**, 7182 (1981).
- ³⁴G. Stirnemann, E. Wernersson, P. Jungwirth, and D. Laage, J. Am. Chem. Soc. 135, 11824 (2013).
- ³⁵M. Bonomi, D. Branduardi, G. Bussi, C. Camilloni, D. Provasi, P. Raiteri, D. Donadio, F. Marinelli, F. Pietrucci, R. A. Broglia, and M. Parrinello, Comput. Phys. Commun. **180**, 1961 (2009).
- ³⁶The PLUMED consortium, Nat. Methods 16, 670 (2019).
- ³⁷M. J. Frisch, G. W. Trucks, H. B. Schlegel, G. E. Scuseria, M. A. Robb, J. R. Cheeseman, G. Scalmani, V. Barone, B. Mennucci, G. A. Petersson *et al.*, Gaussian, Wallingford, CT, 2009.
- ³⁸M. J. Frisch, G. B. Trucks, and H. B. Schlegel, Gaussian 09, Revision E.01, Gaussian, Inc., Wallingford, CT, 2013.
- ³⁹S. Grimme, J. Comput. Chem. 27, 1787 (2006).
- ⁴⁰M. D. Liptak and G. C. Shields, J. Am. Chem. Soc. **123**, 7314 (2001).
- ⁴¹J. Ho, A. Klamt, and M. L. Coote, J. Phys. Chem. A 114, 13442 (2010).
- 42 D. Di Tommaso and K. L. Watson, J. Phys. Chem. A 118, 11098 (2014).
- ⁴³ R. F. Ribeiro, A. V. Marenich, C. J. Cramer, and D. G. Truhlar, Phys. Chem. Chem. Phys. 13, 10908 (2011).
- ⁴⁴E. Tang, D. Di Tommaso, and N. H. de Leeuw, Phys. Chem. Chem. Phys. 12, 13804 (2010).
- ⁴⁵D. Toroz, F. Song, G. A. Chass, and D. Di Tommaso, CrystEngComm 23, 4881 (2021).
- ⁴⁶Y. Marcus, J. Phys. Chem. B 118, 10471 (2014).
- ⁴⁷C. K. D. Jiao, A. Grossfield, T. A. Darden, and P. Ren, J. Phys. Chem. B 110, 18553 (2006).
- ⁴⁸S. Kim, X. Wang, J. Jang, K. Eom, S. L. Clegg, G. S. Park, and D. Di Tommaso, ChemPhysChem **21**, 2334 (2020).
- ⁴⁹D. Laage and J. T. Hynes, Chem. Phys. Lett. **433**, 80 (2006).
- ⁵⁰D. Laage and J. T. Hynes, Proc. Natl. Acad. Sci. U. S. A. 104, 11167 (2007).
- ⁵¹ The NAG Library, The Numerical Algorithms Group (NAG), Oxford, United Kingdom www.nag.com.