Entropy generation for the blood flow in an artery with multiple stenosis having a catheter

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Abstract This article explicates the blood flow across an artery with multiple stenosis at outer wall and a thrombus at the centre. The symmetric multiple stenosis and non-symmetric multiple stenosis shapes are both considered in this study. Entropy analysis is also taken into account for a detailed study of irreversibility. The governing equations are interpreted with provided boundary conditions and exact mathematical solutions are developed. Further, these exact solutions are elucidated graphically. Streamlines show that the closed contours are symmetric in shape for symmetric multiple stenosis but non-symmetric in shape for non-symmetric multiple stenosis shape. The increase in multiple stenosis heights, increases the shear stress at wall having multiple stenosis.

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1. Introduction

The blood flow is confined across diseased arteries having stenosis. This stenosis evolves by plaque collection due to fats and oils at the walls of such arteries. In certain terrible conditions, this plaque collection can also result in multiple stenosis. In these circumstances, the flow across such arteries is confined at multiple locations due to several stenosis. In his doctoral dissertation, Ponalagusamy [1] first time studied the flow

across such tapered tubes. Mandal [2] considered the power law flow model to illustrate the time dependent flow of blood through such tapered vessels. The distinct shapes of stenosis are also incorporated in Ponalagusamy's [3] study. Vonruden et al. [4] had presented an experimental study for flow of blood across a diseased vessel with multiple stenosis. Misra et al. [5] had theoretically interpreted the blood flow as Casson fluid inside an artery having multiple stenosis. The flow of blood across an artery having multiple stenosis, considering mild case of multiple stenosis along the arterial length was studied by Sreenadh et al. [6]. Nadeem et al. [7] had mathematically explained the flow of carbon nanotube across an artery with multiple stenosis, considering case of variable viscosity.

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Peer review under responsibility of Faculty of Engineering, Alexandria University.

Latterly, many researchers have concentrated on the study of blood flow across arteries with a blood clot at their centre position. These blood clots (Thrombus) are more frequently developed in diseased arteries already having tapered walls. Thus, an artery with multiple stenosis gets more chances to have a thrombus as well. This is the worst scenario that almost blocks the blood flow and has deadly consequences. In this situation, the flow is again improved with catheter application. This slender, vacant tube can be placed in such troubled arteries to refine the flow. The flow problem considering both models, (i.e. thrombus and stenosis) was experimentally as well as theoretically studied by Doffin et al. [8]. The macroscopic double-phase flow model for blood flow across a tapered tube by utilizing catheter was mathematically interpreted by Srivastava et al. [9]. Nadeem et al. [10] had mathematically studied the blood flow across a catheterized artery with a mild stenosis. The surgical method used for the placement of moving catheter across a stenosed artery was explained by Mekheimer et al. [11]. The couple stress model of fluid for the mathematical interpretation of blood flow across a diseased tapered artery by making use of a catheter was presented by Reddy [12].

The transfer of heat procedure for blood flow across a tapered artery, considered two phase model, with catheter application was studied by Garcia [13]. Tashtoush et al. [14] had interpreted the heat transfer mechanism of blood flow across a diseased artery with multiple stenosis. The study of temperature for blood flow across diseased tapered arteries, considering various fluid models are presented in the studies [15–18]. Mekheimer et al. [19] had mathematically interpreted the irreversibility of blood flow across a stenosed tube with catheter application by considering entropy generation study case. Some recent bio-mathematical models are also referred as [20–23].

The previously obtained literature work is thoroughly studied, and it is disclosed that the blood flow across a diseased artery having multiple stenosis at outer wall and a thrombus at the centre is not yet explored with application of a catheter. This work explains the blood flow across an artery with multiple stenosis at outer wall and a blood clot at the centre. This confined flow across the diseased artery is refined by using a catheter. The symmetric and non-symmetric multiple stenosis shapes are also considered in this study. This blood flow study includes a Newtonian viscous model. The viscous effects are also incorporated in energy equation by considering viscous dissipation. Entropy analysis is also considered for a detailed study of irreversibility. The governing flow problem equations are interpreted with provided boundary conditions and exact mathematical solutions are developed. Further, these exact solutions are elucidated graphically. Streamlines show that the closed contours are symmetric in shape for symmetric multiple stenosis but non-symmetric in shape for non-symmetric multiple stenosis shape.

2. Mathematical model

The blood flow across an artery with multiple stenosis at outer wall and a thrombus at the centre is examined. This confined flow across the diseased artery is refined by using a catheter. The symmetric multiple stenosis and non-symmetric multiple stenosis shapes are both considered in this study (see Fig. 1).

The geometries of exterior boundary $\bar{\eta}(z)$, i.e. (wall having multiple stenosis) and inner boundary $\bar{\epsilon}(z)$, i.e. (wall having a thrombus at the centre) are provided in dimensional form [24,25].

$$\bar{\eta}(z) = \begin{cases} R\left[1 - K\left\{s_l^{n-1}(\bar{z} - d_l) - (\bar{z} - d_l)^n\right\}\right], & d_l \leqslant \bar{z} \leqslant d_l + s_l \\ R \end{cases}$$
(1)

$$\bar{\epsilon}(z) = \begin{cases} R[c + f_1(\bar{z})], & d_2 \leqslant \bar{z} \leqslant d_2 + s_2 \\ cR & otherwise \end{cases}.$$
(2)

The clot's shape depends on choice of $f_1(\bar{z})$. Further, K is defined as
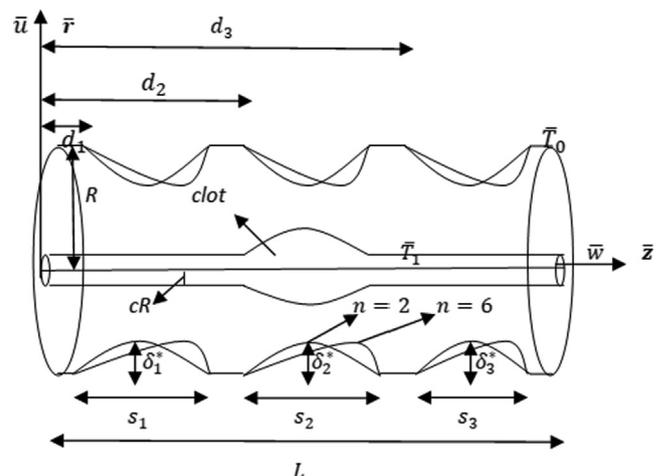
Fig. 1 Geometry of flow Problem.

$$K = \frac{\delta_l^*}{Rs_l^n} \frac{n^{n/n-1}}{n-1}. \tag{3}$$

Here the utmost height attained by stenosis is given by δ_l^* at location $z = d_l + \frac{s_l}{n^{1/n-1}}$. The position of each stenosis is measured by d_l and $l = 1, 2, 3$. The stenosis shape is dependent on the value of n and it has symmetric shape for $n = 2$ but for $n = 6$, it has non-symmetric shape. The governing mathematical equations are provided below in their dimensional form [26].

$$\frac{1}{\bar{r}} \frac{\partial(\bar{r}\,\bar{u})}{\partial \bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0, \tag{4}$$

$$\rho \left[\bar{u} \frac{\partial \bar{u}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} \right] = -\frac{\partial \bar{p}}{\partial \bar{r}} + \mu \left[\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} - \frac{\bar{u}}{\bar{r}^2} \right]. \tag{5}$$

$$\rho \left[\bar{u} \frac{\partial \bar{w}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} \right] = -\frac{\partial \bar{p}}{\partial \bar{z}} + \mu \left[\frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{w}}{\partial \bar{r}} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right]. \tag{6}$$

$$\rho C_p \left[\bar{u} \frac{\partial \bar{T}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{T}}{\partial \bar{z}} \right] = k \left[\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} \right]$$
$$+ \mu \left[2 \left\{ \left(\frac{\partial \bar{u}}{\partial \bar{r}} \right)^2 + \left(\frac{\bar{u}}{\bar{r}} \right)^2 + \left(\frac{\partial \bar{w}}{\partial \bar{z}} \right)^2 \right\} + \left(\frac{\partial \bar{w}}{\partial \bar{r}} + \frac{\partial \bar{u}}{\partial \bar{z}} \right)^2 \right]. \tag{7}$$

The non-dimensional variables are

$$r = \frac{\bar{r}}{R}, z = \frac{\bar{z}}{s_l}, w = \frac{\bar{w}}{u_0}, u = \frac{L\bar{u}}{u_0 \delta_l^*}, p = \frac{R^2 \bar{p}}{u_0 s_l \mu_f}, \theta = \frac{\bar{T} - \bar{T}_0}{\bar{T}_1 - \bar{T}_0},$$

$$\epsilon(z) = \frac{\bar{\epsilon}(z)}{R}, \eta(z) = \frac{\bar{\eta}(z)}{R}, h_l = \frac{d_l}{s_l}, \delta_l = \frac{\delta_l^*}{R},$$

$$B_r = \frac{\mu_f u_0^2}{k_f(\bar{T}_1 - \bar{T}_0)}, S_{G_0} = \frac{k_f(\bar{T}_1 - \bar{T}_0)^2}{\bar{T}_0^2 R^2}, \theta_0 = \frac{\bar{T}_0}{(\bar{T}_1 - \bar{T}_0)}, \tag{8}$$

The case of a mild stenosis is considered by applying these assumptions

$$\delta_l = \frac{\delta_l^*}{R} \ll 1, \frac{Rn^{1/n-1}}{s_l} \overset{\circ}{\sim} 1. \tag{9}$$

The non-dimensional variables provided in Eq. (8) are utilized in Eqs. (4)–(7) and then the assumptions given in Eq. (9) are used to obtain these dimensionless governing equations.

$$\frac{\partial p}{\partial r} = 0. \tag{10}$$

$$\frac{\partial p}{\partial z} = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r}. \tag{11}$$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + B_r \left(\frac{\partial w}{\partial r} \right)^2 = 0. \tag{12}$$

The relevant dimensionless boundary conditions are

$$w = 0, at\, r = \epsilon(z), and\, w = 0, at\, r = \eta(z). \tag{13}$$

$$\theta = 1, at\, r = \epsilon(z), and\, \theta = 0, at\, r = \eta(z). \tag{14}$$

The geometry of outer boundary $\eta(z)$, i.e. (wall having multiple stenosis) and inner boundary $\epsilon(z)$, i.e. (wall having blood clot at centre) is provided in non-dimensional form. The thrombus model is expressed by suitably selecting $f_1(\bar{z})$ [25].

$$\eta(z) = \begin{cases} 1 - \delta_l \frac{n^{n/n-1}}{n-1} [(z - h_l) - (z - h_l)^n], h_l \le z \le h_l + 1 \\ 1 \qquad\qquad\qquad\qquad otherwise \end{cases}. \tag{15}$$

$$\epsilon(z) = \begin{cases} c + \sigma e^{-\pi^2(z - z_d - 0.5)^2}, h_2 \le z \le h_2 + 1 \\ c \qquad\qquad\qquad\qquad otherwise \end{cases}. \tag{16}$$

3. Exact solution

The velocity profile is evaluated exactly by solving Eq. (11) with conditions mentioned in Eq. (13).

$$w = \frac{\left[\frac{\partial p}{\partial z} \{ (-\epsilon^2 + \eta^2) Log(r) + (r^2 - \eta^2) Log(\epsilon) + (-r^2 + \epsilon^2) Log(\eta) \} \right]}{4(Log(\epsilon) - Log(\eta))}. \tag{17}$$

The rate of flow for volume is evaluated as

$$Q = \int_\epsilon^\eta rwdr. \tag{18}$$

The calculated expression for pressure gradient is

$$\frac{dp}{dz} = \frac{16Q(Log(\epsilon) - Log(\eta))}{(\epsilon^2 - \eta^2)[-\epsilon^2 + \eta^2 + (\epsilon^2 + \eta^2)(Log(\epsilon) - Log(\eta))]}. \tag{19}$$

The result interpreted for wall shear stress is

$$\tau_w = -\frac{\partial w}{\partial r} \Big|_{r=\eta} = \frac{-\frac{\partial p}{\partial z} \left[\frac{-\epsilon^2 + \eta^2}{\eta} + 2\eta(Log(\epsilon) - Log(\eta)) \right]}{4(Log(\epsilon) - Log(\eta))}. \tag{20}$$

The temperature profile is also evaluated exactly by interpreting Eq. (12) with conditions mentioned in Eq. (14)

$$\theta = \frac{1}{64(Log(\epsilon) - Log(\eta))^2} \left[-2B_r \left(\frac{\partial p}{\partial z} \right)^2 (\epsilon^2 - \eta^2)^2 (Log(r))^2 \right.$$

$$- B_r \left(\frac{\partial p}{\partial z} \right)^2 (r^2 - \eta^2) Log(\epsilon) \{ -4(\epsilon^2 - \eta^2) + (r^2 + \eta^2) Log(\epsilon) \}$$

$$+ \left\{ -4B_r \left(\frac{\partial p}{\partial z} \right)^2 (r^2 - \epsilon^2)(\epsilon^2 - \eta^2) \right.$$

$$+ \left(-64 + B_r \left(\frac{\partial p}{\partial z} \right)^2 (2r^4 - 3\epsilon^4 + 4\epsilon^2\eta^2 - 3\eta^4) \right) Log(\epsilon) \bigg\} Log(\eta)$$

$$+ \left(64 + B_r \left(\frac{\partial p}{\partial z} \right)^2 (-r^4 + \epsilon^4) \right) (Log(\eta))^2$$

$$+ Log(r) \left\{ -4B_r \left(\frac{\partial p}{\partial z} \right)^2 (\epsilon^2 - \eta^2)^2 + \left(64 + B_r \left(\frac{\partial p}{\partial z} \right)^2 (3\epsilon^4 - 4\epsilon^2\eta^2 + \eta^4) \right) Log(\epsilon) \right.$$

$$\left. + \left(-64 + B_r \left(\frac{\partial p}{\partial z} \right)^2 (\epsilon^4 - 4\epsilon^2\eta^2 + 3\eta^4) \right) Log(\eta) \right\} \bigg]. \tag{21}$$

4. Entropy analysis

The equation for entropy analysis in its dimensional form is provided [27].

$$S_G = \frac{k_f}{\overline{T}_0^2}\left[\left(\frac{\partial \overline{T}}{\partial \overline{r}}\right)^2 + \left(\frac{\partial \overline{T}}{\partial \overline{z}}\right)^2\right]$$

$$+ \frac{\mu_f}{T_0}\left[2\left\{\left(\frac{\partial \overline{u}}{\partial \overline{r}}\right)^2 + \left(\frac{\overline{u}}{\overline{r}}\right)^2 + \left(\frac{\partial \overline{w}}{\partial \overline{z}}\right)^2\right\} + \left(\frac{\partial \overline{w}}{\partial \overline{r}} + \frac{\partial \overline{u}}{\partial \overline{z}}\right)^2\right].$$

(22)

Now by application of non-dimensional variables mentioned in Eq. (8) and also with use of given assumptions in Eq. (9). The equation in dimensionless form for entropy analysis is

$$N_S = \frac{S_G}{S_{G_0}} = \left(\frac{\partial \theta}{\partial r}\right)^2 + \theta_0 B_r\left(\frac{\partial w}{\partial r}\right)^2.$$

(23)

Eq. (23) is sum of two terms and first term is basically entropy produced by measurable difference of temperature ($N_{S_{cond}}$) while the later term is entropy caused by viscous results ($N_{S_{visc}}$). Moreover, Bejan number appears as

$$B_e = \frac{N_{S_{cond}}}{N_{S_{cond}} + N_{S_{visc}}} = \frac{\left(\frac{\partial \theta}{\partial r}\right)^2}{\left(\frac{\partial \theta}{\partial r}\right)^2 + \theta_0 B_r\left(\frac{\partial w}{\partial r}\right)^2}.$$

(24)

5. Results and discussion

The exact results evaluated for velocity, pressure gradient, wall shear stress τ_w, temperature and entropy are graphically explained in this section. These graphs elucidate the results for both symmetric multiple stenosis, (i.e. $n = 2$) and non-symmetric multiple stenosis, (i.e. $n = 6$). In Fig. 2a, Fig. 2b, Fig. 2c and Fig. 2d, the velocity $w(r,z)$ for the governing flow problem is plotted against the radial coordinate r, for various enhancing values of dimensionless parameters of interest. Fig. 2a reveals that the velocity, in case of symmetric as well as non-symmetric shapes of stenosis, increases in the middle of channel with enhancing δ_l. It is clear from this graph that velocity increments in the middle, as the channel turns narrow with enhancing δ_l but it reduces towards the multiple stenosis wall. For both symmetric as well as non-symmetric shapes of multiple stenosis, Fig. 2b shows enhance in the velocity for increasing Q. Fig. 2c displays decrease in the velocity at wall
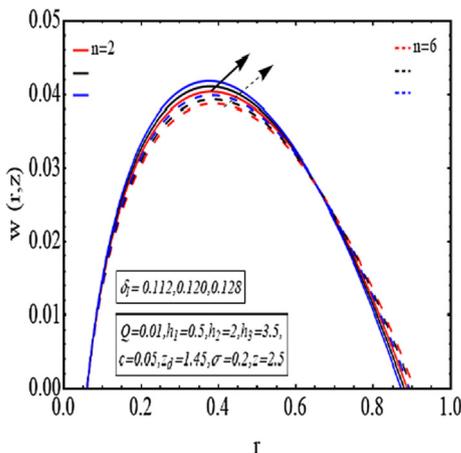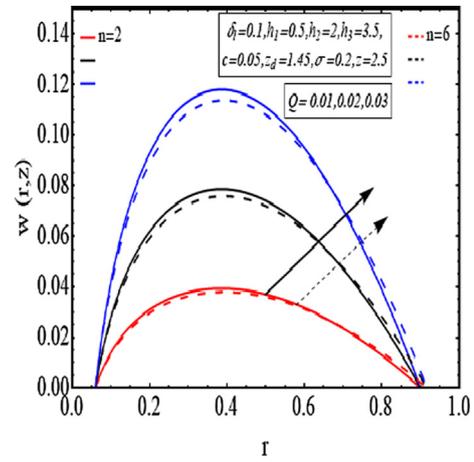
Fig. 2b   Velocity for Q.
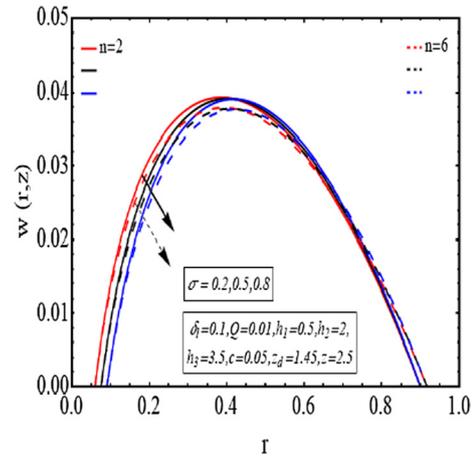
Fig. 2c   Velocity for σ.
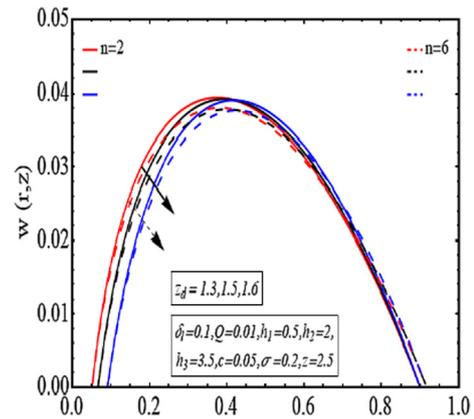
Fig. 2d   Velocity for z_d.

with clot whether stenosis has symmetric or non-symmetric shape, but velocity depicts fixed behaviour at multiple stenosis wall for increasing clot height σ. There is decrease in the velocity at the clot wall as the axial displacement z_d of clot increases but the behaviour of velocity profile is not changing near multiple stenosis wall, as shown in Fig. 2d. In all these velocity

Fig. 2a   Velocity for. δ_l.

Fig. 3a τ_w for δ_l.
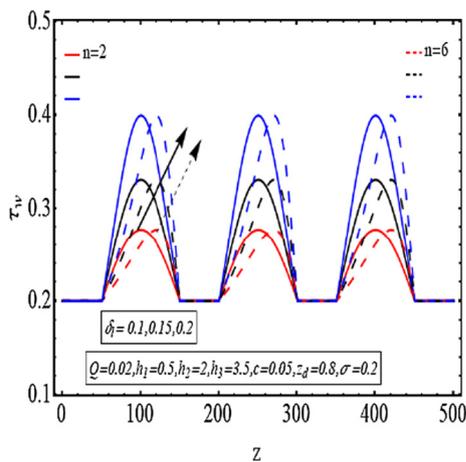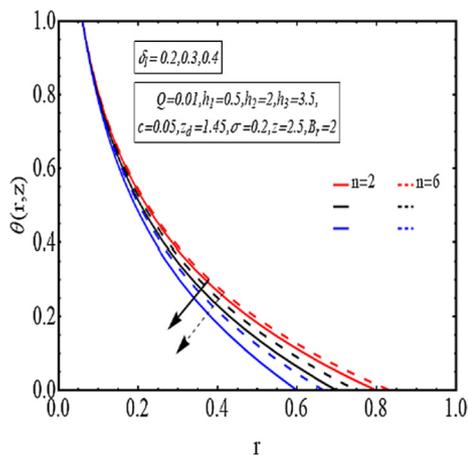
Fig. 4b Temperature for δ_l.
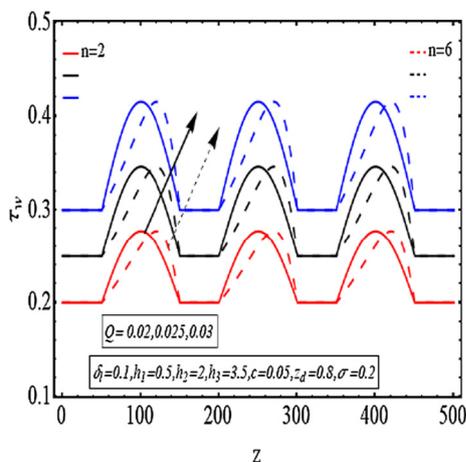
Fig. 3b τ_w for Q.
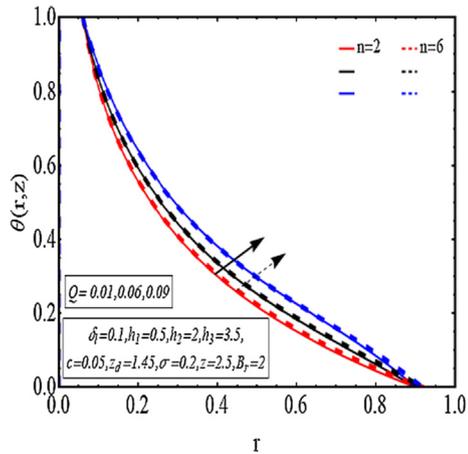
Fig. 4c Temperature for Q.
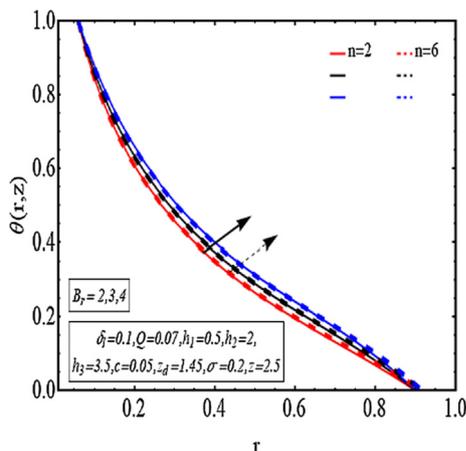
Fig. 4a Temperature for B_r.
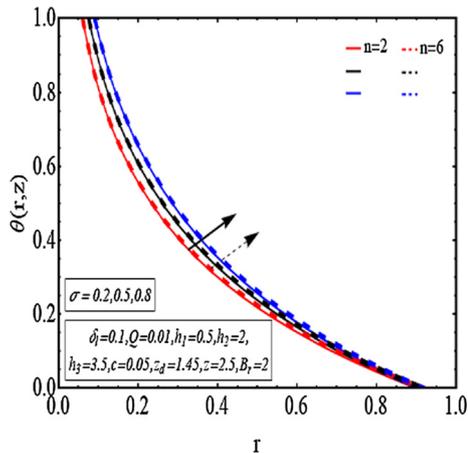
Fig. 4d Temperature for σ.

graphs, it is observed that the velocity for non-symmetric shape of multiple stenosis is lower than the symmetric shape.

In Fig. 3a, Fig. 3b, τ_w the wall shear stress against z-axis is plotted. Fig. 3a reveals that τ_w varies directly with incrementing δ_l. It is seen that the enhance in multiple stenosis heights,

enhances the shear stress at wall having multiple stenosis. Further, τ_w also varies directly with increasing flow rate Q, as displayed in Fig. 3b. In Fig. 4a, Fig. 4b, Fig. 4c and Fig. 4d, the temperature against r-axis is plotted for various parameters of physical interest. For multiple stenosis having symmetric or
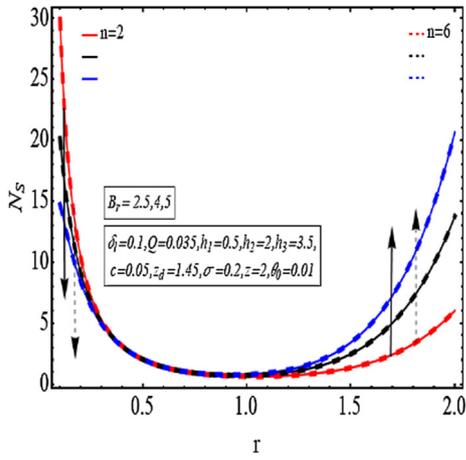
Fig. 5a   Entropy for B_r.
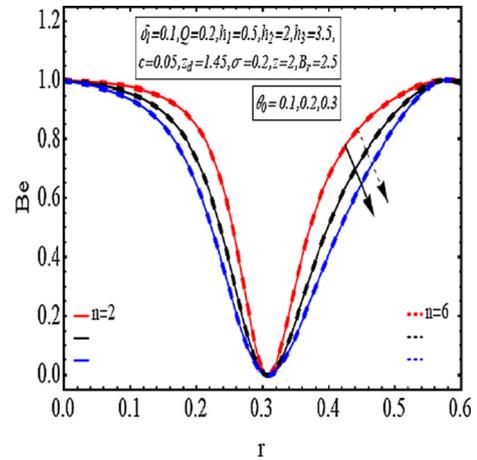
Fig. 6b   Bejan number for θ_0.
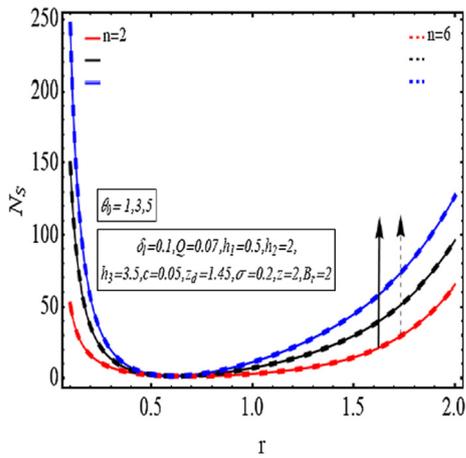
Fig. 5b   Entropy for θ_0.
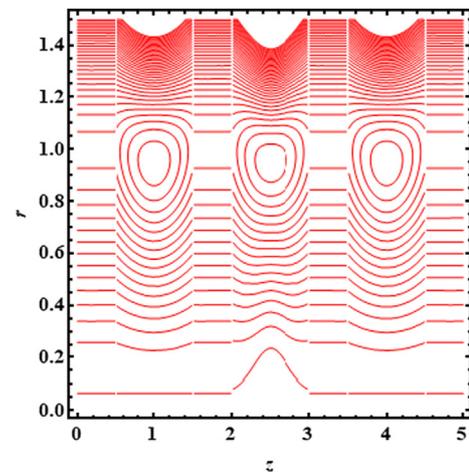
Fig. 7a   Streamlines at.$Q = 2, n = 2$.
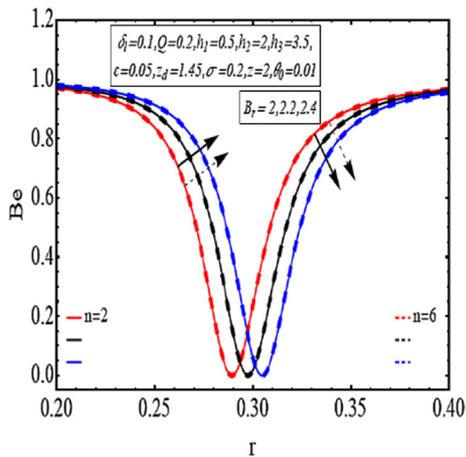
Fig. 6a   Bejan number for B_r.
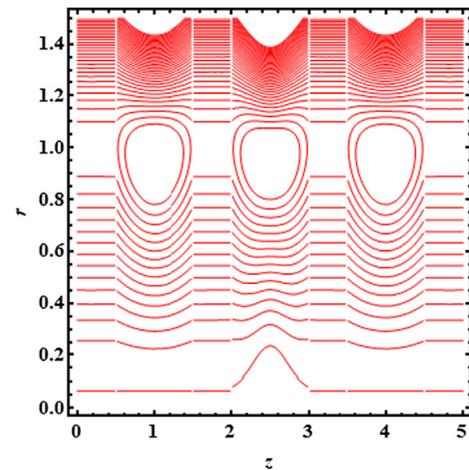
Fig. 7b   Streamlines at $Q = 2.2, n = 2$.

non-symmetric shapes, Fig. 4a shows that when the value of B_r increments, there is enhance in temperature. The temperature varies inversely for incrementing δ_l, as depicted in Fig. 4b. The temperature enhances with enhancing Q, shown in Fig. 4c. In Fig. 4d, it is revealed that temperature enhances

for increasing clot height σ. The irreversibility is discussed by plotting graphs for entropy, shown in Fig. 5a, Fig. 5b. Fig. 5a reveals that N_S behaves differently at different walls, i.e. it decreases with wall having thrombus but it enhances at
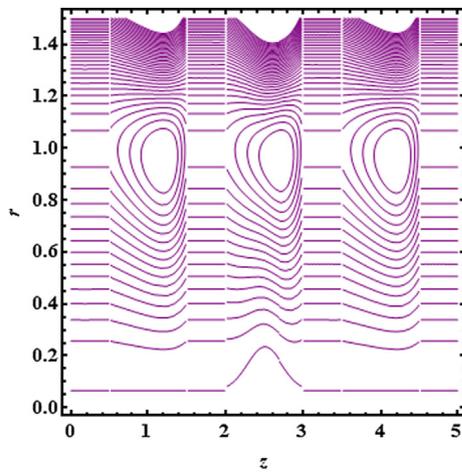
Fig. 7c Streamlines at $Q = 2, n = 6$.
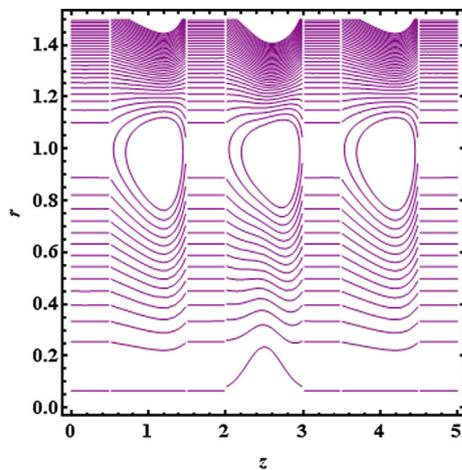
Fig. 7d Streamlines at $Q = 2.2, n = 6$.

6. Conclusions

The blood flow across an artery with multiple stenosis at outer wall and a thrombus at the centre is mathematically elucidated. This confined flow across the diseased artery is refined by using a catheter. The symmetric multiple stenosis and non-symmetric multiple stenosis shapes are also considered in this study. The key results are

- The velocity enhances in the middle, as the channel turns narrow with enhancing δ_l but it reduces towards the multiple stenosis wall.
- The velocity for non-symmetric shape of multiple stenosis is lower than the symmetric shape.
- The temperature varies inversely for incrementing δ_l.
- N_S behaves differently at different walls, i.e. it decreases with wall having thrombus but it enhances at the multiple stenosis wall for incrementing B_r.
- The closed contours are symmetric in shape for symmetric multiple stenosis but non-symmetric in shape for non-symmetric multiple stenosis shape.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

the multiple stenosis wall for incrementing B_r. The entropy varies directly with both thrombus and multiple stenosis wall for incrementing θ_0, as displayed in Fig. 5b. In Fig. 6a, Fig. 6b Bejan number is discussed with help of graphs. B_e has distinct behaviour at distinct walls, i.e. it increases with thrombus wall but drops with multiple stenosis wall for increasing values of B_r, revealed in Fig. 6a. B_e declines everywhere for increasing θ_0, depicted in Fig. 6b. In Fig. 7a, Fig. 7b, Fig. 7c and Fig. 7d the flow behaviour is seen by plotting streamlines for this problem. Fig. 7a and Fig. 7b are plotted for symmetric shape of multiple stenosis and it is clearly seen that the trapping increments in size but declines in count for increasing Q. Further, the symmetric shapes of multiple stenosis at one wall and the clot at other wall can clearly be seen. Fig. 7c and Fig. 7d are plotted for non-symmetric multiple stenosis shape and here again the trapping increments in size but declines in count for enhancing Q. Also, the non-symmetric shapes of multiple stenosis at one wall and thrombus at other wall are clearly observed. Moreover, the closed contours are symmetric in shape for symmetric multiple stenosis but non-symmetric in shape for non-symmetric multiple stenosis shape.

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