A further education student teacher's mathematical and pedagogical knowledge seen in the teaching of circumference and area of the circle

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Students get confused about the concepts of area and circumference of circles as teaching emphasizes memorizing formulas rather than understanding concepts. In this paper, I report findings of the analysis of an episode of a lesson on 'Area and circumference of the circle' taught online by a student teacher to a Further Education class. The analysis employed the Knowledge Quartet – a framework for the analysis of mathematics teaching, with a focus on teacher knowledge. Findings show the student teacher's Foundation is strong in some areas and less strong in other areas. Transformation and Connection were observed to be not strong but some good examples of recognition of conceptual appropriateness were observed. No sign of contingency was observed as students either did not contribute or their contributions were directed to the teacher only making it difficult to observe whether any teacher action was a result of the students' contributions.

Keywords: foundation; transformation; connection; contingency.

Introduction

According to the English National Curriculum: Mathematics, *Area and Circumference of the Circle* is taught at Key Stage 3 [11 – 13-year-olds] (DfE, 2021). Area and perimeter of geometrical shapes are essential parts of mathematics curriculum because of their applicability in daily lives' activities such as painting and tiling, and they are needed to introduce many other mathematical ideas (Rujeki & Putri, 2018). However, students experience difficulties with these topics due to many factors; for example, using the formula for circumference when finding area and vice versa. Also, learning experiences provided in schools give more focus on memorizing formulae, rather than understanding concepts (Rujeki & Putri, 2018). In this paper, I present the analysis of an episode of a lesson on *Area and circumference of the circle* taught by a student teacher, Job, to a Further Education (FE) class (16- to 19-year-old students). I am interested in the mathematical and pedagogical knowledge of Job which surfaces during the act of teaching. I draw on the Knowledge Quartet (KQ) (Rowland et al., 2009) to analyse the episode. I briefly describe the context in which this study is situated. This is followed by an overview of the KQ and the methodology of the study before getting into a detailed analysis of the episode.

Context of the study

Job is a 25-year-old mathematics in-service student teacher in an FE College. Job has been working for three years as a Learning Support Assistant (LSA) before being employed as a mathematics teacher. He started training the year he was employed as a teacher. This lesson was observed when Job was still in the first year of a two-year

PGCE course. Due to the COVID-19 pandemic, teachers were forced into virtual teaching and learning; something everyone in education had to learn ‘fast’. Job was caught up in a situation where he had to learn to teach online and learn to teach FE students, whose characteristics are unique.

Many students on vocational courses can be disengaged from and have negative attitudes towards learning mathematics. This can be caused by multiple factors, including negative prior experiences with learning, peer pressure and lack of confidence (Greatbatch & Tate, 2018). Noyes and Dalby (2020) propose there is a need to teach basic concepts and processes before progressing to other work to avoid over-reliance on memorisation of routines at the expense of understanding. However, developing understanding of fundamental concepts and processes is not compatible with the aims or pace of a one-year FE revision course and this presents teachers with a dilemma (Noyes & Dalby, 2020).

The Knowledge Quartet (KQ)

The KQ is a framework for the analysis of mathematics teaching, with a focus on teacher knowledge, and in this sense, it is also a tool for organising the complexity of mathematics classrooms (Rowland et al., 2015). The KQ has four dimensions which are *foundation*, *transformation*, *connection*, and *contingency*.

Foundation, the first dimension, supports the other three dimensions and it involves theoretical background; the kind of knowledge acquired at school, or in teacher education, irrespective of whether it is being put to purposeful use (Rowland et al., 2009). The conceptualisation of this category includes teachers’ beliefs, knowledge and understanding (Rowland et al., 2009). The other three dimensions are descriptions of situations that may arise during a mathematics lesson. They are not types of knowledge (Weston, 2013), as in the other theoretical frameworks; for example, Mathematical Knowledge for Teaching (MKT) (Ball et al., 2008). Transformation deals with how well teachers are able to transform what they know in ways that make foundation knowledge accessible and appropriate to students (Rowland et al., 2009). Transformation is anchored on Shulman’s (1986) observation that the knowledge base for teaching is distinguished by the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful. Connection is concerned with the decisions about sequencing and connectivity so that the lesson hangs together and relates to the context of previous lessons and the pupils’ knowledge and such decisions will typically follow from the ability to anticipate what is complex and what is conceptually appropriate for an individual or group of pupils (Shulman, 1986). Contingency involves unplanned examples in lessons, students’ unexpected ideas, the use of unpredictable opportunities at the time of teaching, and deviation from the lesson agenda in response to an unplanned opportunity as mathematics teaching rarely proceeds according to plan, if ever (Rowland, et al., 2015).

Methodology

This report is part of a longitudinal study which seeks to gain an insight into the role of mentoring in the mathematical and pedagogical development of mathematics student teachers in FE colleges. I take the socio-constructivist perspective of ontology and the interpretivist perspective of epistemology as I believe knowledge is produced through interaction of social actors – teachers, mentors and students – and the interpretation of such interactions. The methodology of my study is qualitative;

established on an interpretative research methodology that values the participant's views and reflections and looks for meanings within the participant's environment (Merriam & Tisdell, 2016; Stake, 2010). Data were collected online through lesson and mentor meeting observations, interviews with mentors and student teachers, student teachers' responses to mathtasks[1] and mentor reports on student teachers. However, the focus of the research in this paper is on Job, a student teacher's observed lesson and does not include the data from other methods.

After gaining entry to the college and getting consent from Job and his students, Job invited me into the lesson which I recorded. I analysed the whole lesson, but due to space limitations, I report analysis of an episode from the lesson. In the next section, I provide a detailed analysis of the mathematical and pedagogical knowledge of Job as seen through the lenses of the Knowledge Quartet (KQ).

Teaching circumference and area of the circle: As seen through the Knowledge Quartet lens

Job displayed **CIRCLES – AREA & PERIMETER** on the first slide of his lesson and said "Today we are going to learn about area and circumference of the circle. Before we get too much into it, here is a starter based on what we did last week, which was angles." Job displayed the starter (see Figure 1). The starter connects with the previous topic, which was about angles. After 5 minutes, Job worked out the example explaining "Always remember by heart, angles in a triangle add up to 180 degrees." This is anticipation of complexity as students might forget this fact, while 'learning by heart' might be interpreted as an encouragement for memorization. Job explained how to get the third angle in the triangle by subtracting; 180 – 120 – 35 = 25. He could have connected procedures by also explaining that 120 + 35 = 155, then180 – 155 = 25. Job could also have explained the theorem: one exterior angle is equal to two interior opposite angles in a triangle and connect it to the other procedures. Job then said, "Again that 180 we need to learn. 180 inside a triangle and 180 on a straight line."

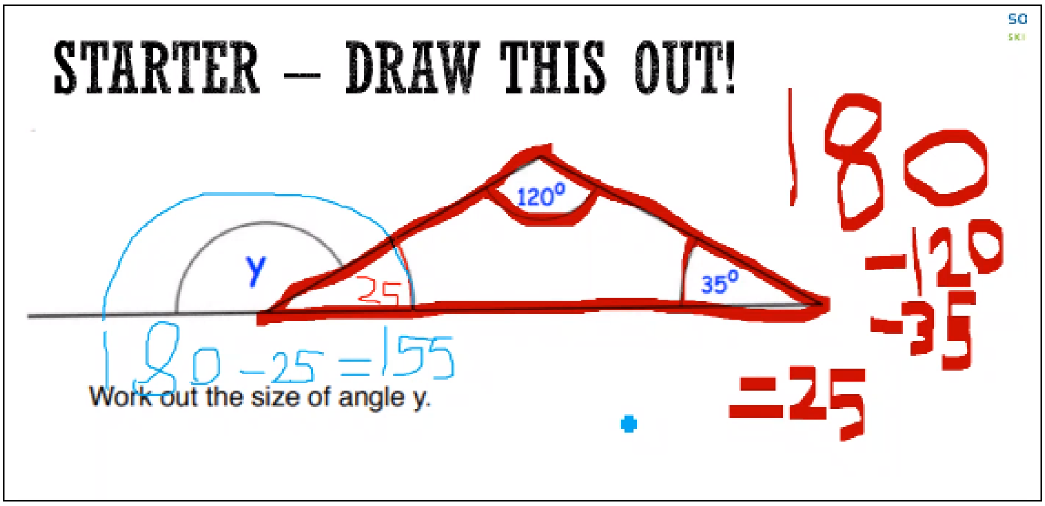

Figure 1: Starter question with Job's working

[1] Mathtasks can be described as classroom situation-specific incidents which are hypothetical but grounded on learning and teaching issues that previous research and experience have highlighted as seminal and are likely to occur in actual practice (Biza et al., 2007).

He explained that $y = 180^\circ - 25^\circ = 155^\circ$. This starter showed Job's overt subject knowledge of angles in a triangle and in a straight line.

After the starter, Job displayed the learning objectives: 1) To calculate the area and perimeter of circles and 2) To recognize the different parts of circles. The sequencing of the objectives might have started with parts of the circle as understanding of parts of the circle leads to understanding of area and perimeter. Besides the display, Job started explaining parts of the circle – chord, arc, tangent, segment, sector, emphasizing radius, diameter, and circumference. Job explained, "The radius is from the centre. That's in any direction. Diameter just cut always straight through the circle, but it has to go through the middle and the circumference is the entire outside of it." Job showed recognition of conceptual appropriateness as students need to understand these parts before learning circumference and area. Job also showed secure subject knowledge of parts of the circle although there are some mathematical inaccuracies, which might be caused by not being thorough on use of terminology. After explaining radius, diameter and circumference, Job said the following:

> We're gonna try to look at calculating the area and the perimeter of circles. So, the rules of calculating area and circumference so for the circumference of a circle we say pi d. That symbol means pi not like you're gonna go and eat. So, all I am asking you to do is to work out the circumference, which is the outside of that circle. You take its diameter, which is the entire length from one side to the other, and you would times that by pi. Now if you don't have a scientific calculator. We just shorten pi down says to 3.14. OK, so it is a lot longer than that. It goes on forever however you either press pi button on a calculator or you're going to put 3.14.

The definition of circumference and diameter might be problematic to students as they might confuse diameter with chord and think of circumference as not belonging to the circle but 'outside' like compliment elements in Venn diagrams. The word 'outside' is not mathematically precise as the circumference 'is' the circle; it is not 'outside' the circle. Job showed understanding of the use of π; however, it is not clear whether he knows the conceptual meaning.

Job paused then continued, "So for the area. It's pi times radius squared. You would square it before you times it by pi. You don't do pi times the radius, then square, you do pi times the radius squared." The explanation of how to use the formula $A = \pi r^2$ shows good anticipation of complexity as students might square πr. Job's explanation of area and circumference of circle by using the formulae shows good subject knowledge, though it sounds rather procedural.

Job gave the students an exercise (see Figure 2) without any demonstration saying, "OK. Do circumference questions, so we have the first set, just green second is amber, and the third set is red. So, the first one is asking for the length of the circumference." By giving an exercise without showing students an example, Job might have wanted the student to use their knowledge acquired at school or he might have explained this before. However, Job could have shown good teacher demonstration by explaining at least one example before giving students the work. Although the examples were differentiated from easy to difficult, some questions (A3, B4 and C2) asked students to find the diameter while the rest were asking students to find the circumference. This might be a result of not selecting examples carefully or more likely reliance on textbooks and internet resources without checking their suitability for one's students. The conceptual appropriateness of question C4 (22/7 – π) is not clear. After some time, Job displayed the answers and asked students to mark their work.

After three minutes, Job said, "Anyone who gets a question wrong without knowing why it is wrong let me know."

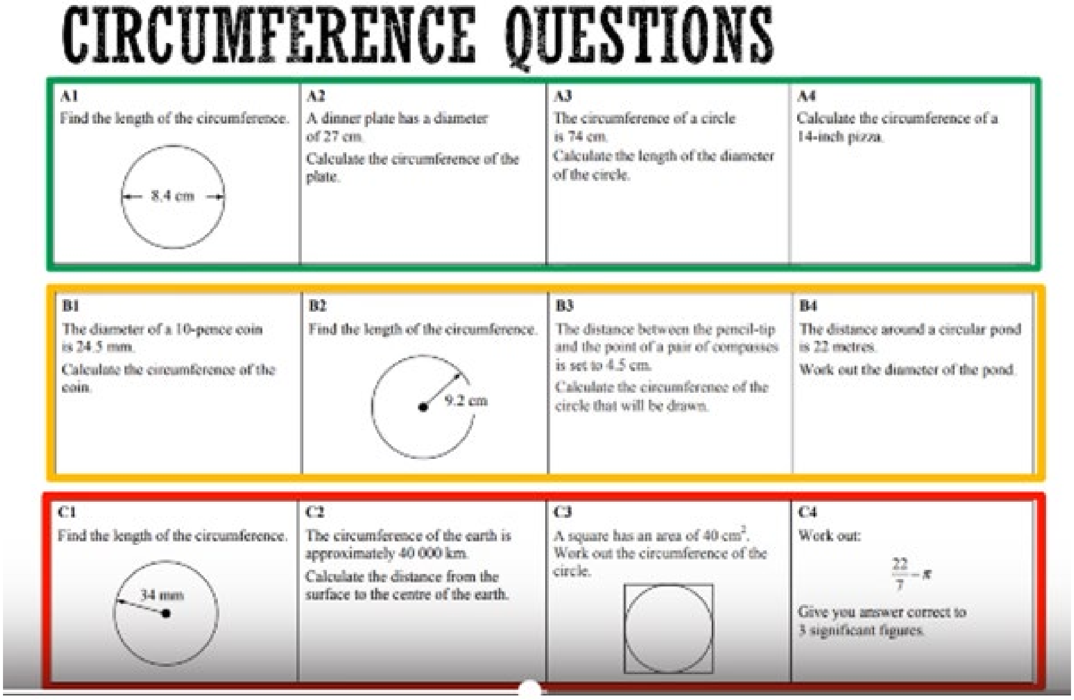

Figure 2: Questions given to the students to practice

Without any further comment, Job switched to area saying, "We are going to go over area…."

Throughout the lesson, Job used the terms perimeter and circumference interchangeably. It is not clear whether Job's use of the terms interchangeably is deliberate, not knowing the difference, or not paying attention to detail. In this case, Job could at least have made an effort to explain the meaning of perimeter and circumference and their differences. There was no connection between area and circumference mentioned.

Discussion and conclusion

I am interested in how Further Education (FE) student teachers' mathematical and pedagogical knowledge is seen through the lenses of the Knowledge Quartet (KQ). In this paper, I report the analysis of a segment of Job's lesson, during the teaching of *Area and Circumference of the Circle*. In my analysis, I notice aspects of Job's foundation knowledge. Job's overt subject knowledge was strong; for example, on angles in a triangle and parts of the circle as he correctly worked out examples and defined the parts with confidence. However, no overt subject knowledge was seen on area and circumference of the circle as Job did not work out an example besides explaining the formulae and how to use the formulae. Job's attention to the technical characteristics of the formulae especially for area shows good teacher demonstration when he explained that πr should not be squared but r should be squared before multiplying by π. Job paid little attention to use of terminology; for example, defining circumference as 'the outside of the circle' which shows lack of mathematical precision. Job did not show any teacher demonstration and choice of representation on the main topic as he did not demonstrate any example. However, good teacher demonstration was shown on explaining angles in a triangle and on a straight-line. On connection, Job showed good anticipation of complexity by explaining the correct use of the formulae for area of circle. Job's recognition of conceptual appropriateness was seen to be good as he showed that understanding parts of the circle, emphasizing diameter, radius, and circumference, is a prerequisite for understanding area and circumferences. In this segment of the lesson, Job displayed the title; *Circles- Area and Perimeter*, then went to give a starter on angles. After the starter, Job explained parts of the circle then how to calculate circumference followed by how to calculate area and this was immediately followed by an exercise on circumference. The part of the lesson lacks connectivity and does not flow smoothly. Unlike face-to-face teaching, it is difficult to observe students' contributions. In this lesson students did not talk and did not show their faces. The contributions were through the chat and there is no evidence to show any teacher action caused by student contribution, making it difficult to see any contingency action.

The analysis shows that Job's foundation was generally strong while transformation was not clearly visible, and connection had some strong and not so strong areas. No contingency was observed during this episode of the lesson.

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