

From student scribbles to institutional script: Towards a commognitive research and reform programme for university mathematics education

Elena Nardi¹, Irene Biza¹, Bruna Moustapha-Corrêa², Evi Papadaki¹, Athina Thoma¹

¹*University of East Anglia*, ²*Universidade Federal do Estado do Rio de Janeiro*

Commognitive studies offer a nuanced lens on datasets that evidence micro-level accounts of mathematical experience – and are now starting to explore the theory’s capacity to support the design, tracing and dissecting of discursive shifts in medium/long term interventions. Here, we focus on two university mathematics education (UME) examples of such interventions. The Norway-based study engaged biology students with biology-themed Mathematical Modelling activities to challenge deficit narratives about the role of mathematics in their discipline and about their mathematical competence and confidence. The Brazil-based study engaged teachers with activities which feature mathematical practices from the past and in today’s mathematics classrooms to trigger changes in teachers’ narratives about how mathematics comes to be and how its emergence can be negotiated in the mathematics classroom. We show how the discursive shifts orchestrated by these interventions generate new narratives about mathematics and its pedagogy, de-ritualised participation in mathematical routines and, ultimately, meta-level learning.

Keywords: commognition; university mathematics education; discursive shifts; interventions; reform

Commognitive theory in UME research: the story so far

Commognitive theory (Sfard, 2008) – a discursive approach to the study of learning and teaching mathematics that sees constructing and conveying mathematical meaning as acts of communication conducted in accordance with shared rules within a community – has been gaining ground as a theoretical framework for university mathematics education (UME) studies in recent years (Nardi et al., 2014). University, as an institution, and university mathematics, can be seen as a world governed by new rules, a new discourse that may intrigue as well as alienate newcomers (Sfard, 2014). What commognitive research has achieved so far is to offer nuanced, micro-level accounts of mathematical experience. Commognitive accounts of teaching and learning tend to be fluid – non-binary, non-deficit – small-scale, snapshot dissections of communication (verbal, written and gestural) between learners and between learners and teachers. However, studies of discursive shifts that tread beyond snapshot accounts, are more longitudinal and hint at the potential of commognitive theory as an evaluation tool of pedagogical interventions are starting to appear.

Some such studies have been carried out at pre-university level. For example, Antonini et al. (2020) focus on Grade 10 students’ written discourse about their experiences in a dynamic interactive digital environment as they learn about functions. They introduce the commognitive construct of Dynamic Interactive Mediators and then use it to analyse students’ written productions. Their analysis offers rich descriptive accounts of the temporal and dynamic dimensions of the students’ experience. While this study is an evaluation of one lesson, the proposed tool seems to have capacity for

more longitudinal use. Another example is Ng's (2016) study which compares calculus students' communication in two classroom environments, static and dynamic, and explores the role of paper-based and digital mediations of thinking about calculus. The analysis argues for a multimodal view of communication that captures the use of gestures, and dragging, for communicating dynamic and temporal mathematical relationships. As in Antonini et al. (2020), the proposed take on the data analysis promises capacity for more longitudinal use.

So far, within UME, commognitive studies have largely focused on learners' mathematical discourse as they engage with tasks, on their own or with others, teachers' discourses, mathematical and pedagogical, as well as learner-teacher interactions. For example, Thoma and Nardi (2018) analyse unresolved commognitive conflicts observed in students' examination scripts. They make the case that such conflicts are central to the students' transition from school to university mathematics as they learn to distinguish the discourses in different mathematical domains and that lecturers' meta-level awareness of these types of conflict may assist with facilitating student transition more effectively.

Reflecting on these commognitive conflicts is at the heart of the approach also taken by Biza et al. (2018) whose work focuses on mathematics teacher education courses and professional development sessions: in these, participants are invited to engage with fictional, yet data grounded classroom situations ("mathtasks") in which said commognitive conflicts are present. Analysis of the situations as well as participants' written responses and discussions is structured around four characteristics – consistency (between stated endorsed narratives and intended practice), specificity (to the classroom situation), reification of mathematics education research discourse and reification of mathematical discourse. This analysis allows snapshot evaluations of the impact that particular course or session contents may have on participants' discourses.

Also, with the focus squarely on instances of student mathematical discourse are the studies by Kim et al. (2012), Schüler-Meyer (2020) and Biza (2021). Kim et al. (2012) present a comparison of English and Korean speaking university students' discourses on infinity. They observe that in Korean, unlike in English, there is a disconnection between colloquial and mathematical discourses on infinity. This difference between the two languages is associated with different ways in which the students grow towards canonical mathematical discourse. Schüler-Meyer (2020) casts a commognitive lens on how a guided intervention in an upper secondary classroom focused on defining the convergence of a sequence may facilitate a transition from experiential to abstract realisations of convergence. Biza (2021) investigates how experiences with one mathematical topic, tangent lines, which is present in multiple mathematical domains, leave their marks on students' subsequent work with this topic. She introduces the notion of the *discursive footprint* of tangents, traces this in prior curricular experiences (e.g., textbooks) of the students with the topic and then identifies manifestations of this footprint in undergraduate mathematics students' responses to a questionnaire about tangents. The discursive footprint is shown as a pedagogically potent descriptor, and evaluator, of students' work on a specific mathematical topic (in this study: tangency).

In the studies cited so far, there are hints – but far from fully-fledged realisations – of the potential that commognitive analyses carry to offer a nuanced evaluative lens on data. We now present two studies, in which one or more of us have been recently involved, and which aim to build a little further on this capacity of the commognitive framework as a tool to evaluate longitudinal pedagogical interventions.

Example Study I: Mathematical Modelling activities in a biology course

Viirman and Nardi (2021) explore the conjecture that integration of mathematics and biology through Mathematical Modelling (MM) activities may improve biology students' mathematical experience and appreciation of the role that mathematics can play in their discipline. Through a commognitive lens, their study examines twelve first-semester biology students' engagement with mathematical routines that are essential to MM, such as assumption building and graphing, as they work in groups on a suite of MM problems, during four sessions of two to three hours each, spread over a university semester. The analysis aims at tracing the students' ritualised and exploratory participation in the MM activities as well as the commognitive conflicts they experience as they engage with colloquial and literate (mathematical and biological) discourses. Taking a largely qualitative, narrative approach, analysis consists of sets of episodes which mark milestones in the students' engagement.

Aiming to deploy the fluidity of the commognitive lens, the students' learning is traced through evidence of discursive shifts, particularly concerning MM routines as well as narratives about what mathematics is and what its role may be within biology. In parallel, their lecturers' interjections are seen as facilitating such shifts (and doing so with varying degrees of success). In these sets of episodes, the students are seen identifying, formulating and revising the assumptions that underpin their mathematical models in the making. They are seen questioning what constitutes legitimate mathematical practice, particularly within a biology context. Their meaning-making efforts include constructing and interpreting graphs. These efforts fluctuate between deploying graphs as mere illustrations of the data and as a means of gaining insights that may benefit the construction of their mathematical models.

Over the period of the intervention, the de-ritualisation of the students' practices – the transition from ritualised to exploratory engagement with MM – is also traced. The question whether ritualised engagement is merely compliance with lecturer expectations or a stepping stone towards participation with more agency also emerges from the analysis. Student activity is accounted for in relation to prior experiences of graphing routines – and, in the case of assumption building routines, the sparsity of such experiences – and as fluid, if not always productive, interplay between ritualised and exploratory engagement with such routines. The analysis concludes that the students' work is marked by their efforts to keep up with the expectations of the lecturer, whose MM task design and task formulations allow for changing degrees of student agency but do not factor in the influence of the students' prior experiences.

In a spirit similar to that of Coles and Sinclair (2019) – whose fieldwork also spans a relatively substantial period of time and also aims at tracing discursive shifts in the course of a school-level pedagogical intervention – Viirman and Nardi (2021) argue against the dichotomising between rituals and explorations and illustrate how some initially ritualised participation may eventually facilitate exploratory engagement. Their Biology students start to mathematise through ritualised engagement first. The moment they start to wonder whether what they do is “mathematics”, is perhaps the moment they cross the “threshold of the discourse” (Coles and Sinclair, 2019, p.181). The analysis has pedagogical implications for the way MM problems are formulated and also foregrounds the capacity of the commognitive framework to trace de-ritualisation and meta-level learning in students' MM activity. We see the conclusions of these commognitive analyses as evaluative of the interventions reported in these studies and therefore as carrying the capacity to influence future interventions directly.

One recommendation for future interventions that comes out of the analyses by Viirman and Nardi (2021) is challenging the pedagogical deficit that may arise from the “tacitness of metarules” (Sfard, 2008, p.59). This concern is shared also by Thoma and Nardi (2018) as well as Güçler (2016) who engaged pre-service teachers’ with activities that foreground such tacit metarules (in the context of studying the object of function) and concluded in favour of courses that privilege such activities. Heyd-Metzuyanim et al. (2019) make a similar case, in the context of a professional development course which focused on teaching practices that may generate productive discussions and accountable talk in the mathematics classroom. An apt tool in the enterprise of foregrounding hitherto tacit metarules in order to orchestrate teachers’, and learners’, discursive shifts – and one which is directly relevant to our second Example Study (Moustapha-Corrêa et al., 2021) – is, according to Kjeldsen and Blomhøj (2012), history of mathematics.

Example Study II: History of mathematics and mathtasks in a teacher PD course

As Kjeldsen and Blomhøj (2012) note, “history might have a profound role to play for learning mathematics by providing a self-evident (if not indispensable) strategy for revealing meta-discursive rules in mathematics and turning them into explicit objects of reflection” (p.327). Their analysis of undergraduate and postgraduate students’ reflections as they engage with primary sources offers empirical support for their claim – as do the interventions designed and implemented by Bernardes and Roque (2018). These interventions were a direct source of inspiration for Moustapha-Corrêa et al. (2021) who deployed a combination of history-focussed tasks and mathtasks (Biza et al., 2018) as the basis for an intervention (thirteen, three-hour sessions) in an in-service mathematics teacher education and professional development post-graduate course in Brazil. At the heart of the intervention is the conjecture that in engaging with discussions of different mathematical practices – from the past and from today’s mathematics classrooms – teachers will problematise what mathematics is, how it develops and how it is/can be learned and taught.

Indeed, as the teachers engage with the history-focused tasks, they become aware that the metarules according to which the mathematics of the past was shaped are often distinct from the metarules governing mathematics today. They are starting to acknowledge the value of highlighting and debating changes in metarules in the classroom – for example, across mathematical domains such as algebra and geometry, across elementary and advanced mathematics, etc.. Analyses of datasets collected over the period of the intervention – which include teachers’ portfolios of reflections, interviews, written responses to the tasks and recorded class discussions – trace and evidence the discursive shifts experienced by the participants.

For example, in Moustapha-Corrêa et al. (2021), one set of episodes evidences participants’ engagement with primary sources (Galileo’s, Euler’s, Dirichlet’s and, finally, Bourbaki’s, realisations of the mathematical object of function) and with a mathtask that exposes learners’ ambivalence about the difference between x as an unknown and as a variable. As the episodes unfold, the participants’ narratives about functions, variables, unknowns (object-level) but also, crucially, about mathematics as an ever-changing discipline (metalevel) keep shifting – and, with them, their narratives on mathematics pedagogy. As in Viirman and Nardi (2021), commognitive analysis functions as an evaluation of whether the intervention achieved its discourse-shifting aims and, thus, has implications for the way such courses are designed and delivered.

Towards a commognitive research and reform programme for UME

Here, we focused on two studies, both consisting of the design, implementation and evaluation of interventions embedded in established courses (a biology undergraduate course in Norway; and, an in-service mathematics teacher education and professional development post-graduate course in Brazil). We presented how the discursive shifts orchestrated by these medium-term interventions generate new narratives about mathematics and its pedagogy, de-ritualised participation in mathematical routines and, ultimately, meta-level learning.

Our overall proposition is that there is a potentiality in commognitive UME research that is yet to be more fully tapped into: generating theoretically robust evaluations of UME longitudinal pedagogical interventions and thus informing a reform agenda. Our examples may suggest a shift in the grain size of commognitive research that we see as a first step towards a much-needed, larger research agenda.

But, what do we mean by “larger”? Do we mean studies of a larger data corpus, but of the same ilk, as those we have seen so far? Or, rather, do we mean longitudinal studies of the institutional, curricular, pedagogical, social and cultural discourses – the larger institutional scripts so-to-speak that codetermine what students and teachers do and say when they engage with mathematics (and, by “say” we also mean write, gesture, as well as any other mode of conveying mathematical meaning to others and / or self)?

And, why would such studies matter? They do, because time is apt for “change research” (Reinholz et al., 2020) in UME and, if “change research” is to take off, it needs to do so with the rigour that strong theoretical lenses can provide. Within UME, Sfard (2014) observed how a “discursive vision offered by the commognitive framework [may] impact our understanding of the learning and teaching of university mathematics” – while at face value not offering directly “innovative idea[s] about how [the] deeply entrenched practices [of university mathematics teaching] could be changed” (p.202). We propose that small but non-negligible steps are being taken in deploying this vision towards “helpful and sustainable pedagogical innovation” (p.202).

Acknowledgements

Three of the articles we reference here (Biza; Moustapha-Corrêa et al.; and, Viirman & Nardi) are due to appear in a 2021 *Journal of Mathematical Behavior* [Special Issue entitled *Advances in commognitive research*](#). We thank our research partners Aline Bernardes, Victor Giraldo, Olov Viirman and Theodossios Zachariades warmly for numerous and ...commognitively productive collaborations over the years!

References

- Antonini, S., Baccaglini-Frank, A., & Lisarelli, G. (2020). From experiences in a dynamic environment to written narratives on functions. *Digital Experiences in Mathematics Education* 6, 1-29.
- Bernardes, A., & Roque, T. (2018). History of matrices: Commognitive conflicts and reflections on metadiscursive rules. In K. M. Clark, T. H. Kjeldsen, S. Schorcht, & C. Tzanakis (Eds.). *Mathematics, education and history. ICME-13 Monographs* (pp. 209–227). Springer.
- Biza, I. (2021). The discursive footprint of learning across mathematical domains: The case of the tangent line. *Journal of Mathematical Behavior*, [10.1016/j.jmathb.2021.100870](https://doi.org/10.1016/j.jmathb.2021.100870).

- Biza, I., Nardi, E., & Zachariades, T. (2018). Competences of mathematics teachers in diagnosing teaching situations and offering feedback to students: Specificity, consistency and reification of pedagogical and mathematical discourses. In T. Leuders, J. Leuders, & K. Philipp (Eds.), *Diagnostic Competence of Mathematics Teachers: Unpacking a complex construct in teacher education and teacher practice*, (pp. 55-78). Springer.
- Coles, A., & Sinclair, N. (2019). Ritualisation in early number work, *Educational Studies in Mathematics*, 101(2), 177-194.
- Güçler, B. (2016). Making implicit metalevel rules of the discourse on function explicit topics of reflection in the classroom to foster student learning. *Educational Studies in Mathematics*, 91(3), 375-393.
- Heyd-Metzuyanım, E., Smith, M., Bill, V., & Resnick, L. B. (2019). From ritual to explorative participation in discourse-rich instructional practices: a case study of teacher learning through professional development. *Educational Studies in Mathematics*, 101(2), 273-289.
- Kim, D. J., Ferrini-Mundy, J., & Sfard, A. (2012). How does language impact the learning of mathematics? Comparison of English and Korean speaking university students' discourses on infinity. *International Journal of Educational Research*, 51-52, 86-108.
- Kjeldsen, T. H., & Blomhøj, M. (2012). Beyond motivation: history as a method for learning meta-discursive rules in mathematics. *Educational Studies in Mathematics*, 80(3), 327-349.
- Moustapha-Corrêa, B., Bernardes, A., Giraldo, V., Biza, I., & Nardi, E. (2021). Problematizing mathematics and its pedagogy: Teachers' discursive shifts through history-focussed and classroom situation-specific tasks. *Journal of Mathematical Behavior*, [10.1016/j.jmathb.2021.100840](https://doi.org/10.1016/j.jmathb.2021.100840).
- Nardi, E., Ryve, A., Stadler, E., & Viirman, O. (2014). Commognitive analyses of the learning and teaching of mathematics at university level: The case of discursive shifts in the study of Calculus. *Research in Mathematics Education*, 16(2), 182-198.
- Ng, O. L. (2016). Comparing calculus communication across static and dynamic environments using a multimodal approach. *Digital experiences in mathematics education*, 2(2), 115-141.
- Reinholz, D.L., Rasmussen, C., & Nardi, E. (2020). Time for change (research) in undergraduate mathematics education. *International Journal for Research in Undergraduate Mathematics Education*, 6(2), 147-158.
- Sfard, A. (2008). *Thinking as communicating. Human development, the growth of discourse, and mathematizing*. Cambridge University Press.
- Sfard, A. (2014) University mathematics as a discourse – why, how, and what for? *Research in Mathematics Education*, 16(2), 199-203.
- Schüler-Meyer, A. (2020). Mathematical routines in transition: facilitating students' defining and proving of sequence convergence. *Teaching Mathematics and its Applications*, 39(4), 237-247.
- Thoma, A., & Nardi, E. (2018). Transition from school to university mathematics: Manifestations of unresolved commognitive conflict in first year students' examination scripts. *International Journal for Research in Undergraduate Mathematics Education*, 4(1), 161-180.
- Viirman, O., & Nardi, E. (2021). Running to keep up with the lecturer: Biology students' engagement with graphing routines in mathematical modelling tasks. *Journal of Mathematical Behavior*, [10.1016/j.jmathb.2021.100858](https://doi.org/10.1016/j.jmathb.2021.100858).