Running to keep up with the lecturer or gradual de-ritualization? Biology students' engagement with construction and data interpretation graphing routines in mathematical modelling tasks

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Through a commognitive lens, we examine twelve first-semester biology students' engagement with graphing routines as they work in groups, during four sessions of Mathematical Modelling (MM). We trace the students' meta-level learning, particularly as they fluctuate between deploying graphs for mere illustration of data and as sense-making tools. We account for student activity in relation to precedent events in their experiences of graphing and as fluid, if not always productive, interplay between ritualised and exploratory engagement with graph construction and interpretation routines. The students' construal of the task situations is marked by efforts to keep up with lecturer expectations which allow for changing degrees of student agency but do not factor in the influence of precedent events. Our analysis has pedagogical implications for the way MM problems are formulated and also foregrounds the capacity of the commognitive framework to trace de-ritualization and meta-level learning in students' MM activity.

mathematics in biology; mathematical modelling; commognition; meta-level learning; graph construction and interpretation routines; de-ritualization

1. Engaging Biology students with Mathematics through Mathematical Modelling

Although much university mathematics teaching is aimed at students specialising in fields other than mathematics, this aspect of university mathematics education is still relatively under-researched (Biza, Giraldo, Hochmuth, Khakbaz, & Rasmussen, 2016). Within Biology, the increased importance of mathematical methods has placed new demands on the education of future biologists (e.g., Brewer & Smith, 2010) and university biology students' engagement with Mathematical Modelling (MM) activities has been found to contribute to more positive attitudes towards, and improved competence in, both biology and mathematics (e.g., Chiel, McManus & Shaw, 2010). Our project explores this conjecture further in a context in which Biology students engage with a suite of MM activities. Where in a previous paper (Viirman & Nardi, 2019) we employed commognitive analyses (Sfard, 2008; Nardi, Ryve, Stadler & Viirman, 2014) to investigate students' assumption building, a key element of MM activity, here we focus on another: graph construction and interpretation, typically seen as a ubiquitous and quintessential part of scientific literacy (Angra & Gardner, 2017). While our work is informed by past studies which document student difficulties with graphing and differences between students' graphing reasoning and that of experts, such as Angra and Gardner's (2017), our analyses focus on how students interpret, and tackle, graphing elements in MM problems, particularly in the light of prior graphing experiences.

2. Graphing in university students' mathematical activity

Much research on graphing in university students' mathematical activity originates in science education research and deals mainly with graph interpretation. Glazer's (2011) review, for example, emphasizes how prior knowledge exerts a strong influence on graph interpretation and identifies student difficulties with interpreting complex graphs that depict several relationships in one diagram. He also agrees with Potgieter, Harding and Engelbrecht (2007) who argue that mathematics courses, particularly for non-mathematics specialists, need to put more emphasis on graph construction and interpretation “not only for the sake of deeper conceptual understanding within mathematics itself, but especially for understanding within applied fields” (p. 214). Although students engage with graphing as far back as in elementary school, “refocusing and scaffolding their data handling and graphing activities in the context of their undergraduate learning experiences is needed” (Angra & Gardner, 2017, p. 12). Amongst studies that concern graphing in university Biology education (e.g., Harsh & Schmitt-Harsh, 2016; Picone, Rhode, Hyatt & Parshall, 2007), for example, Picone et al. (2007) report that students struggled with identifying dependent and independent variables in complex graphs, and with discerning “general trends amid statistical ‘noise’ in data.” (p. 6)

Mathematics education research on university students’ graphing activity has largely focused on the mathematics of change, and here time as a variable occupies the main bulk of graphing experiences the students are exposed to. Already in the 1990s, Leinhardt, Zaslavsky and Stein (1990) pointed out that “variables that are not time dependent are rare” (p. 23), that “students deal much more competently with graphs of functions in which one of the variables is time or time-dependent” (p. 28) and that students seem to have difficulty “grasping concepts that arise from variables not actually shown on the graph” (p. 42). A number of studies (e.g., Paoletti & Moore, 2017; Patterson & McGraw, 2018) have investigated students’ parametric reasoning and the implicit role of time in covariational tasks, largely with a focus on graphing. Patterson and McGraw (2018), for instance, exemplify how students’ inclination to use time as the independent variable leads to difficulty with tasks where they are asked to graph other types of time-dependent relationships such as relative positions.

Much research (as reviewed, e.g., by Bowen, Roth & McGinn, 1999) views graphing as a cognitive ability. With Bowen and colleagues, we, instead, see graphing as a mathematical practice that students engage with, in the context of Biology, and we focus on the discursive activity that such engagement implies. Bowen et al. (1999) found that (second-year) biology students lacked the specific linguistic resources that would allow them to avoid two kinds of ambiguity when developing shared graph interpretations: first, interpreting notions as equal in meaning although distinguished in scientific discourse (e.g. a model and a statistical representation of data); and, second, using “different terms interchangeably that have different uses and meanings in scientific discourse” (p. 1033), such as talking about change in population, population density and population size as if they are one and the same. The students in the Bowen et al. study also used fewer resources when interpreting graphs, compared to experts. They tried to connect the graphs to the few relevant previous experiences they had, whereas experts had a wealth of examples to build their interpretations on. In the study by Angra and Gardner (2017), who also identified expert-novice differences in graphing, the experts drew on previous experience more: for instance, they typically spent more time reflecting on, and treating the data, before constructing the graphs, and based their choice of graphs on the purpose of the problem at hand to a larger extent.

On a related note, Bowen and Roth (2005) showed how prospective secondary science teachers, working on a task involving constructing and interpreting a scatterplot of plant growth in relation to light levels, were reticent about opting for graphical analysis and struggled with data displaying random variation. Their struggle was attributed to how “traditional schooling emphasizes particular beliefs in the mathematical nature of the universe” (p. 1063). In the same vein, Harsh and Schmitt-Harsh (2016) argue that “the generation and analysis of complex data may be seen as a means to enhance students' advanced graphing skills" (p. 50) and stress the importance of exposing science students to "messy" (p. 50) and complex data.

Our project, which engages biology students with what Harsh and Schmitt-Harsh (2016) label as "messiness" of MM situations, aims to trace students' activity in relation to the construction and interpretation of graphs over the course of said engagement. While we bear in mind that much of prior research focuses on student difficulties with graphing, our take is decidedly non-deficit: we focus on the students' mathematical practices as they engage with MM activities and we discern learning that occurs in the course of said engagement through a discursive lens – specifically, the theory of commognition (Sfard, 2008; Nardi et al, 2014; Viirman & Nardi, 2019) which sees learning evidenced as change in discursive activity.

3. Conceptualising Biology students' graphing activity in MM as engagement with routines

In the commognitive analyses we present in this paper, we trace students' discursive activity in relation to graphing – specifically, the construction and interpretation of graphs – over the course of engagement with MM activities. The discursive activity we mostly focus on is the students' engagement with graphing *routines*. According to the commognitive perspective, discourse is distinguished by a community's *word use, visual mediators, endorsed narratives* and *routines* (Sfard, 2008, p. 133–135). Specifically, a *routine* is a set of meta-rules that describe a repeated discursive action. Per Sfard, *routines* can be *explorations, deeds* or *rituals*. *Explorations* are *routines* "whose performance counts as completed when an endorsable narrative is produced or substantiated" (p. 224), for example conjecturing the trend in a dataset from observing a graph. *Deeds* are *routines* that involve practical action, resulting in change in objects, either primary or discursive (p. 236), such as plotting a dataset to construct a graph of a function. Some *routines* "begin their life as neither *deeds* nor *explorations* but as *rituals*, that is, as sequences of discursive actions whose primary goal […] is neither the production of an endorsed narrative nor a change in objects, but creating and sustaining a bond with other people" (p. 241). For Sfard, *rituals* are a "natural, mostly inevitable, stage in routine development" (p. 245).

Our analysis draws on recent theoretical developments (Lavie, Steiner & Sfard, 2019) concerning the operationalization of *routine*, where "a routine performed in a given task situation by a given person is the task, as seen by the performer, together with the procedure she executed to perform the task" (p. 161). Key here is situatedness: "routine will no longer be an abstract free-floating construct, but it will be tied to a particular task situation and to a particular person" (p. 161). A *task situation* denotes "any setting in which a person considers herself bound to act" (p. 159), such as the invitation to participate in a group MM activity in class. A *task* "as understood by a person in a given task situation, is the set of all the characteristics of the precedent events that she considers as requiring replication" (p. 161). *Precedent events* are "past situations which [the learner] interprets as sufficiently similar to the present one to justify repeating what was done then, whether it was done by herself or by another person." (p.160). Prior experiences, at school as well as at university, of what variables to choose when constructing a graph and what type of graph to construct in order to generate a meaningful interpretation about how these variables relate to each other are examples of precedent events particularly relevant to our analysis.

We are also particularly interested in how students select which of these precedent events are of significance in the current task situation. The notion of *precedent-search-space* (PSS) is quite handy for this purpose:

> "A default PSS is often created before any specific task situation is set, the moment we enter a certain environment, such as a school or supermarket. This default PSS is predicated on the unarticulated assumption that precedents for whatever happens in this setting should come from the same discursive, material, institutional, and historical context." (p.160)

Which "elements of the PSS will be chosen as precedents of a specific task situation" (p.160) is particularly important to us: our participants are invited to draw on graph construction and interpretation experiences lived in their past in distinctly different task situations, most likely quite removed from a MM or a biological context. Their choice from the PSS will be made using *precedent identifiers*, that is,

> "according to those features of the current task situation that a person considers as sufficient to view a task situation from the past as a precedent (those characteristics that define PSS are necessary in a precedent; when then taken together with precedent identifiers, they become sufficient)." (p.160)

We anticipate that "[d]ifferent participants faced with the same task situation may be using different precedent identifiers" (p.160), and we are also keen to observe which precedent identifiers prevail as the students work on the MM activities in groups.

We concur with Lavie et al. (2019) that "investigating learning is tantamount to answering the question of how routines emerge and how they later evolve" (p. 162). In our study overall, we seek evidence that may signal said evolution in the students' engagement with MM routines. In a previous paper (Viirman & Nardi, 2019), we studied one quintessential element of the students' MM activity: assumption building. There, we traced attempts starting from ritualized engagement in the shape of "guesswork" and evolving into more productively exploratory formulations. We also identified persistent commognitive conflicts in the students' activity, both intra-mathematical (concerning what makes a task "mathematical") and extra-mathematical (concerning what makes a solution to a problem biologically plausible). In the analyses in that paper, we paid attention to the interplay between ritualized and exploratory engagement in the students' cross-disciplinary discursive activity, thus "moving away from commonly used binaries and taking a more fluid perspective on the students' activity" (p.249).

We see this more fluid view of "the apparent dichotomy of the ritual-exploration dyad – where rituals morph into explorations" (p.249) – the potentially "gradual and slow" process of "de-ritualization" (Sfard, 2020, p. 99) – as exploring hitherto underused interpretive potential in the commognitive framework. We explore "what fuels or obstructs processes of de-ritualization" (p.99). Of Lavie et al.'s (2019) "desirable characteristics of routine" (flexibility, bondedness, applicability, performer's agentivity, objectification of the discourse, and substantiability), in this paper we present evidence particularly of applicability ("considering the range of task situations for which its performances so far are likely to constitute precedents.", p. 169), performer agentivity (a performer's extent of freedom "to decide what should be done and when", p. 170) and substantiability (how students respond "to the request for a substantiation of what [they] did", p. 171)

In sum, here we bring this fluid take on ritualised and exploratory engagement with MM routines – hitherto applied on the parts of the students' activity that relate to assumption building routines (Viirman & Nardi, 2017; 2019) – to an analysis of students' engagement with graphing routines, (graph construction and interpretation, Viirman & Nardi, 2018). We discern student learning both at the *object level* ("growth in the number and complexity of endorsed narratives and routines", Sfard, 2008, p. 300) – such as practising particular plotting techniques – and at the meta-level ("express[ing] itself in the change in the metarules of the discourse", p. 300) – such as rethinking through which rules meaning can be made out of a graph in order to inform decisions that bring a mathematical model into being. We are particularly interested in the latter: our evidencing of students' meta-level learning focuses specifically on how the students' engagement with graphing routines manifests itself as gradual de-ritualization as well as faces off with their lecturer's expectations. In our account, their prior experiences with graphing feature as highly influential precedent events and we take particular note of which of these events act as precedent identifiers.

4. Context, participants, data collection MM activities and data analysis

4.1 Context, participants and data collection

The MM sessions took place in a large, research-intensive Norwegian university where biology students take a compulsory mathematics course in the first semester of their studies, designed for students from about twenty different natural science programmes. The course covers differential and integral calculus and some linear algebra. Here, we draw on data from four three-hour sessions with a group of twelve students (volunteers from a cohort of approximately 100). All students on the Biology programme have taken mathematics up to the R1 or S2[1] levels and the twelve volunteers reflect the demographic makeup, regarding gender and educational background, of the whole cohort. To the best of our knowledge, the students had no prior experience of mathematical modelling and their prior experience of biology is the elementary introduction that constitutes part of the Norwegian school curriculum and what they had so far covered in the first-semester biology course running in parallel with the mathematics course. In the Norwegian upper secondary mathematics curriculum (Utdanningsdirektoratet, 2006), graphs are mentioned under the heading of "functions". The few examples present in the curriculum document concern graphs of continuous functions and graph construction is mentioned in connection with derivatives (p. 5).

The research team comprised three mathematics education researchers (the first author is one of the three) and one research mathematician with extensive experience of MM and teaching applied mathematics. None had extensive prior experience with teaching MM. The sessions took place concurrently with, but without connection to, the students' mandatory first-semester mathematics course. All sessions consisted of brief lectures introducing various aspects of MM, followed by group work. The sessions were designed to showcase how MM can be used in a biological setting and were designed primarily by the mathematician. The rest of the team attended and documented the sessions. There were three groups, labelled A, B and C. Students in the groups are labelled A1, A2, etc. The teaching was conducted in English, but most student group work and student contributions to group discussions were in Norwegian. An overview of the four sessions follows.

4.2 Overview of the MM sessions

[1] In Norwegian upper secondary school, R1 is the first course of two for the natural science programme, whereas S2 is the second course of two for the social sciences programme. Content includes elementary differential calculus, but not, for instance, integrals or differential equations.

The first session began with an introduction to MM. Most of the session was on the *Roadkill Rabbits* problem, where students were asked to estimate the population density of rabbits based on observations of traffic intensity and the number of roadkill rabbits. The bulk of the second session concerned the modelling of change, and focussed on a problem concerning *Yeast Growth*[2] in a petri dish where the students were provided with authentic growth data. The students were expected to use this to find an approximately linear relation between the change and the amount of biomass, estimate the proportionality constant, and conclude that this rate of growth cannot continue indefinitely. Then, with additional data provided, they were expected to conclude that the growth decreases and the amount of biomass stabilizes at the carrying capacity of the petri dish, in this case 665. The students were then given a suggested non-linear model and were expected to check the validity of the model by finding the proportionality constant. Finally, they were expected to use the model to generate values that could be compared with the actual data.

The third session began with a further problem on modelling change, this time concerned with the decay in the body of *Digoxin*, a drug used to treat heart disease. In part (a), the students were expected to find a linear relationship between the change and the amount of digoxin remaining, and estimate the proportionality constant from the graph. In part (b), they were expected to construct a model of the form $a_{n+1} = 0.5a_n + 0.1$ and then, in part (c), use this model with different initial conditions to realize that, in all cases, an equilibrium of 0.2mg will eventually be reached, leading to a recommended initial dose of 0.2mg.

After short lectures on non-linear models and modelling using geometric similarity, the *Terror Bird* problem was then introduced. Here, students were asked to estimate the weight of an extinct species of bird given the circumference of its fossilized femur. To this end, they were provided with a table of femur circumference and body weight in various present-day bird species (Table 1, Section 5.2). To solve the problem, the students were expected to build on a few basic assumptions: that the Terror Birds were geometrically similar to present day birds, that the volume of the bird was proportional to the cube of any characteristic length measurement, and that the bird had constant weight density, so that a similar proportionality holds also for the weight. These assumptions lead to a model of the form $W = kl^3$, where W is the weight and l the femur circumference. Line fitting on a plot of the weights against the cubes of the femur circumferences leads to a reasonable estimate of the proportionality constant, and thus to an estimate of the weight of the Terror Bird.

Finally, the bulk of the fourth and final session was devoted to a problem concerning the dynamics of interaction between two populations, *Rabbits and Foxes*.

4.3 Method of data analysis: tracing students' engagement with graphing routines

Student and lecturer activity during the sessions was video and audio recorded, and then transcribed. Written material produced by the students was also collected. As working with condensed accounts makes potential patterns in discursive activity more easily discernible, the first author, who had been present at all four sessions, first produced descriptive accounts of these. Both authors then scrutinized these accounts for evidence of students' engagement with graphing routines, cataloguing episodes where one or more students engaged with routines

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[2] See *Supplementary Materials* for a precise formulation of the *Yeast Growth* problem, as well as the *Digoxin* and *Terror Bird* problems mentioned below.

actually engaged in, such as graph construction, as well as routines talked about but not performed, such as data collection through observation or sampling. More detail on how we catalogued episodes is in (Viirman & Nardi, 2019, p. 240-241).

While working with the descriptive accounts, we noticed how the students' engagement with graphing routines fluctuated between graph construction and interpretation and manifested evidence of how the students' spaces of precedents influenced their graphing activity. We also noticed how their engagement seemed to vary according to what they thought their lecturer's expectations were. Subsequently, the first author returned to the raw data and produced preliminary analytical accounts of the sessions with a focus on the students' graphing activity. These accounts included data excerpts of all episodes (24) relevant to the theme of graphing, together with a preliminary analysis, including, for instance, the previously produced observations regarding the students' graphing activity. Using these analytical accounts, we then selected significant or illustrative instances that we see as most clearly evidencing, on one hand, a mismatch between lecturer expectations and student engagement and, on the other hand, a gradual de-ritualization of the students' graphing activity.

5. Data and findings: Mere illustration and producing new insights in the students' engagement with graphing routines

Three of the problems, *Yeast Growth*, *Digoxin* and *Terror Bird*, included graphing as part of the lecturer's intended solution. In the first two, this was explicit in the problem formulation. However, both the types of graphs and the way in which they were to be used differed from what the students were used to: the students were required to rethink both the 'how' and the 'when' of graphing – namely, to engage in meta-level learning. Already in the introductory lecture during the first session, the lecturer emphasised graphs as tools for solving the equations underpinning mathematical models, stating that "we may have, for instance, a graphical solution – when we are good at describing the situation graphically, we can make our predictions just based on the graphs". The problems called for the use of graphical methods for both analyzing and solving. However, they also required the students to interpret the notions of "graph" and "model" in ways unfamiliar to them. To handle the situations in which they found themselves they had to transcend their PSS of graphing and modelling, moving from exploratory engagement in the old discourse to initially ritualized engagement in the new discourse. In what follows, we see examples of how this expected change in discourse was sometimes successful, leading to meta-level learning. We also see examples of how the students' engagement with graphing routines turned into a race to keep up with the lecturer who introduced new challenges just as the students were beginning to master the old ones.

5.1 *Yeast Growth* and *Digoxin:* persistent graphing routines with time as variable; graphing as mere illustration

Yeast Growth deals with discrete, rather than continuous, measurements. This made calculations easier, as there was no need for differential equations, but it also made the setting less familiar, since the students' previous experiences with graphing from upper secondary school mainly concerned the graphing of continuous functions. Students were explicitly expected to do graphing by hand and they were provided with millimeter paper. After introducing the task, the lecturer gave these instructions:

> Lecturer: So, you look at your data, and you make a graph. What you need to do is to plot the data and try to make a prediction; whether you see something which gives you an idea

> of how you can model the situation using a simple difference equation similar to the one I was using in the example last time: $p_{n+1} - p_n =$ coefficient times p_n. You assume that you have only proportional growth, so that what you have in an hour is some coefficient times what you have now. The key thing is to see what would be your coefficient, and this you need to take from the data in the table. The graph is a means to make the patterns in the data more visible.

From the lecturer's actions during the whole-class follow-up, it can be concluded that his preferred solution involved plotting the change in amount of yeast against the amount, fitting a straight line to the graph thus constructed, and calculating its slope to find the proportionality coefficient. From the lecturer's perspective, this is what the problem formulation (see Supplementary Materials) and the spoken instruction tells the students. Examining the written and spoken instructions more closely, however, gives further indication of the precedent events envisaged by the lecturer as relevant to the problem. The students' interpretation of which precedent events may be relevant appear to be different. For example, when the lecturer says that "you can model the situation using a simple difference equation similar to the one I was using in the example last time", this suggests that he was offering the students an opportunity to participate ritually in the new discourse, through engaging in "thoughtful imitation" (Sfard, 2008, p. 251). However, as we see in what follows, the students did not take up this offer; instead, they opted for explorative, but unproductive, engagement with established routines. Moreover, although the lecturer explicitly stated that the graph should be used to make predictions about the properties of the model, the statement that the coefficient needs to be taken from the data in the table, while the graph is there "to make the patterns in the data more visible", suggests that the role of the graph is to illustrate the data. That neither the problem formulation nor the spoken instructions explicitly stated that the graph should be used to find the value of the coefficient k_1 supports this interpretation, which was repeatedly seen in the students' work during the sessions. For the lecturer, the graph is a problem-solving tool, containing crucial qualitative *and* quantitative information about the model. As we shall see, for the students this was not necessarily the case.

As discussed in (Viirman & Nardi, 2017), the students' engagement with the first part of the task was strongly influenced by precedent graphing events. For example, the presence of time – a variable prevalent in mathematics and science graphing activities in secondary school – as one of the quantities in the table of data appears to serve as a precedent identifier; all three groups used time as the independent variable. Furthermore, all three groups explicitly stated that the growth would be exponential – we note that exponential functions are familiar from secondary school even though we note the novelty of dealing with an exponential function where the base of the exponent is not given. However, despite the lecturer explicitly stating that there is proportional growth, all three groups appeared to take the fact that the change in amount could be derived from the amount as indication that the change was of secondary importance to the amount: "…we should do a graph of how much there is, instead of the change." (A3) Group C went even further: "'Change in biomass', that's completely uninteresting. It's completely irrelevant." (C2) We interpret this as an influence of precedent events: the students' prior experience with graphing in connection to exponential functions has mainly, and merely, involved graphing exponential growth as a function of time.

Having drawn their graphs, again, precedents came into play. Despite the discrete set of data provided in the problem, all three groups connected the points on their discrete graphs with curves, making the graphs continuous (as in Leinhardt et al., 1990, p 34). The groups all seemed aware that they were expected to fit a line to their graphs: "And then we should make [the

Norwegian word is “lage”, akin to “put” or “do” and signifying various types of action] a line, or make a linear function.” (A1) “It should be, like, the best straight line. That is, close to the points.” (B3) However, since they had all drawn amount-time graphs, clearly displaying exponential growth, this was not possible. In conclusion, although the problem required the students to use graphing in ways novel to them, and the spoken instructions of the lecturer invited them to do so through ritual participation, their previous experiences with graphing led them to engaging, in a largely exploratory manner, with graphing routines that were familiar to them from upper secondary school. However, since these routines were unsuitable to the task at hand, their exploratory engagement, trying to adapt their graphing activity to the present situation, turned out to be unproductive.

Realizing that all groups had made the same mistake when graphing, for the next part of the problem, the lecturer emphasised that they were supposed to plot change against amount, not against time. That is, he explicitly addressed the problem with over-reliance on time as the independent variable, and implicitly indicated that the students were expected to mimic what he had previously done. Under time pressure, he then skipped the intended second part of the problem and, instead, explained the reasoning behind it to the students: “Then you need to validate this model, and this is the next step.” This is a new type of activity, presumably novel to the students, and was not exemplified by the lecturer. He handed out a second table of data which extended several hours beyond that already provided: “How good is this, if you start reflecting about [the model] in the long run? What do you say? If you kept going with the experiment further, would it be a good model or not?” When the students did not respond, the lecturer explained further (we note the lecturer’s effort here with this example but we also note that the students did not necessarily notice this example as intended by the lecturer):

> Lecturer: “The difference between the previous value and the one you have now is proportional to what you have now with some coefficient. (…) Is it a good model? (…) If you leave the yeast in the petri dish forever? Will you get an amount which will fill the room?”

The students responded that you clearly would not, since there is not enough space or nutrient. Indeed, in the new table of data, the increment decreased as the amount of yeast approached the carrying capacity of the dish. Hence, the adjusted model needed an added component, flattening it after a certain level was reached. The lecturer handed out a time-amount graph (Figure 1), and introduced the new, adjusted model. This was no longer linear: instead of Δp_n being proportional to p_n, it was now proportional to $p_n(665 - p_n)$ (665 being the carrying capacity). This added complexity, both computationally and in terms of relating empirical data (Δp_n) to data generated by the model. We note that what was actually expected of the students was to plot change against the outcome of the revised model. This is more than just mimicking, and hence requires more than just reflective imitation. In this episode, we see a new type of task being introduced, requiring new forms of discursive activity, namely that of validating a model. Although the lecturer at least implicitly invited the students to mimic his previous activity, the additional complexity he introduced made this difficult, and the example he gave of what validation entails did not enable students’ ritualized participation. Instead, they were required to cope with the added complexity on their own, and the three groups did this very differently.

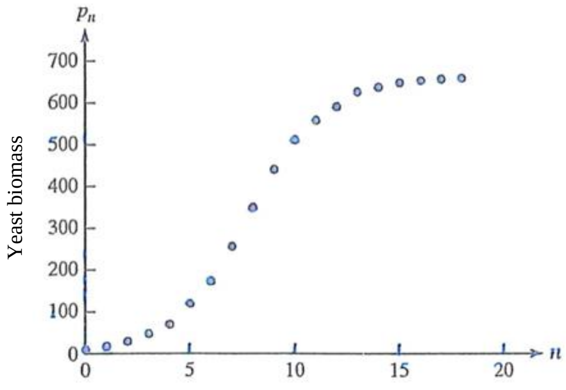

Figure 1: *Yeast Growth*; a yeast biomass versus time graph handed out by the lecturer.

Group A realized that they were supposed to use graphical reasoning but they were perhaps not aware of the invitation to participate ritually – to mimic what the lecturer did: "Does this mean that we should plot them [the values in the table] in, or…?" (A1). Indeed, rather than engaging ritually with the new graphing activity, they quickly abandoned any attempt at graphing, opting for estimating the proportionality constant numerically by computing the ratio at specific points. Possibly, the added complexity of not only having to construct a graph of a type they were unfamiliar with, but also on data calculated from the adjusted model – this was another novelty for the students – made the offer for ritual participation even harder to take up. It became more difficult for the students to build on graphing precedents, causing them to, instead, interpret the task as computational. They were indeed working with the formula for the adjusted model, but, as they only did pointwise calculations, they struggled with finding one single coefficient fitting all data points. After briefly considering averaging the value at several points, they instead opted for using two different constants – one for the early part of the growth, and another for the late part, where the yeast is near its carrying capacity:

> A1: It will never fit completely with all, but it fits somewhat (…) Maybe it only works where it [the graph] flattens out. It worked here [at $n = 10$], that's where it started. But since it runs like this [traces the S-shape of the graph (Figure 1) with her pen], you will never get the same value here [where the graph flattens] and here [where it is steepest]. So, the average will go through here [traces an "average" slope of the graph with her pen]. And this is the slope of that line [indicating the tangent of the graph at $n = 10$], it's the tangent, in a way. And it's much flatter here than here, and therefore it will never fit. (…) I don't know how we shall find one [value] that fits for all.

Here, we see A1's explorative engagement, using the graph and her own gestures as visual mediators supporting quite elaborate reasoning about the behavior of the yeast over time, and also connected the value of the constant to the slope of the graph. However, this did not solve the problem, since the value of the constant she was seeking was not the slope of *this* graph, but of the graph that the problem formulation instructed them to construct and which they opted *not* to construct. The lecturer expected them to engage ritually in a new type of graphing task, but, since the way towards this was not paved well enough, the students rather opted for explorative engagement with a computational task. Engaging with the graphing expected by the lecturer would have required them to adjust both the how of the routine – perform calculations on the data before plotting – and the when – find numerical information (the value of the coefficient) required for solving the task, rather than just gathering qualitative information. By not doing so, an opportunity for meta-level learning is missed.

Group B, on the other hand, took up the lecturer's offer for ritualized engagement, and constructed a change vs amount-graph similarly to what he had done. However, their imitation was not thoughtful – they did not adapt the procedure used by the lecturer to fit the current situation by performing calculations on the data and using the adjusted model. Rather, they worked with the original, that is, plotting Δp_n against p_n, not against $p_n(665 - p_n)$. Thus, they ended up with a graph the first part of which displayed the linear growth they were expected to find in the first part of the problem, and which then rapidly decreased towards zero as the yeast reached the carrying capacity of the dish. Rather than discussing the mathematical behaviour of the models, they mainly engaged in discerning biological meaning, debating whether the decrease in Δp_n should be interpreted as signifying a drastic decrease in the amount of yeast (a "population crash") or a stabilization of the population as the carrying capacity was reached. The interested reader may find a more elaborate account of this group's (unrelated to graphing) work in Viirman & Nardi (2017, p. 2278).

Finally, following the cue from the lecturer, group C, after some initial confusion, seized the opportunity for ritual participation, and started working on computing the values they needed to be able to construct the expected graph. However, their participation was hampered by computational difficulties, and they never actually constructed the graph. Still, out of the three groups, only they adapted their established graphing routine to the new situation:

> C4: And that, is that your k_2?
> C2: No, I have to plot that and plot… If you plot that one and that one [pointing at the values in the table and the values he has computed], then you will get k_2 on a line. In theory. I think.

In this exchange, C2 addresses both the "how" of the graph construction routine using the adjusted model and the "when" of using it for estimation purposes. The uncertainty he expressed about his interpretation indicates ritual participation, but we still interpret this as a step towards the productive participation in the new discourse.

Yeast Growth was designed with a follow-up problem, *Digoxin*. This was introduced at the beginning of the third session. It also concerned the modelling of change, but, instead of growth, it dealt with the decay of a drug in the human body. No additional difficulty was introduced, and indeed the students performed quite well (we present this evidence in Viirman & Nardi, 2018). Here, we mention merely those aspects of the students' work pertinent to our present theme.

First, in all three groups, there were students for whom the influence of precedent identifiers was so strong that they persisted in wanting to use time as the independent variable in their graphs, despite the problem formulation. In two of the groups, this again led to exploratory but non-productive engagement in the old discourse, here taking the form of constructing innovative but mathematically unsound graphs (for an example, see Figure 2), plotting amount and change against time in the same graph. This persistence led to conflict within the groups, and groups A and C, who handled this conflict well, also successfully handled the problem.

Second, even though all groups constructed appropriate graphs, only group A actually used it to solve the problem, by estimating the slope and thus finding the coefficient. The other two groups used the graphs as mere illustrations and, in the case of group C, as a source of qualitative information about the situation. For the quantitative work, they invented ad hoc methods for calculating the coefficient directly from the data, similarly to how Group A handled the second part of *Yeast Growth*. This worked, since the data in *Digoxin* behaved nicely. This was not the case though for the data in *Terror Bird*.

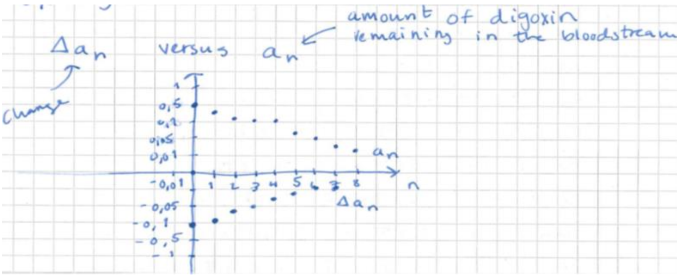

Figure 2: *Digoxin*; graph constructed by student B2

5.2 *Terror Bird*: Calculation routines substituting aborted graphing routines

The short lecture on modelling using geometric similarity preceding the *Terror Bird* problem, introduced the idea of weight as proportional to volume, and thus to length cubed, using the example of measuring fish. The lecturer explicitly stated that the model was found through data plotting:

> Lecturer: You may see that, more or less, for the same size fish it's more or less giving the same weight. (…) What we see is that if we graph the weight against the cube of the length (…) then we expect it to be a straight line as before because it should be proportional (…) and it is indeed looking like a straight line, so we accept it as a working model.

The lecturer then introduced *Terror Bird*. This problem requires use of geometric similarity, assuming proportionality between weight and the cube of femur circumference. This requirement was only implicitly stated, however: "You know that the circumference is that much, and we expect you to predict the weight of this terror bird." Here, we see again an offer for ritual participation through thoughtful imitation, but the offer is not made explicit. Moreover, the problem again introduces additional difficulty, here in the form of a messy dataset. The relationship between weights and femur measurements in the table is not functional (the same femur circumference may correspond to different weights), and it is not ordered in any obvious sense (see Table 1). Hence, unlike the function graphs of the previous two problems, it requires constructing a scatterplot (Figure 3) (see also Bowen & Roth, 2005). This scatterplot can then be used for line fitting, similarly to the previous problems.

Femur circumference (cm)	**Body weight (kg)**
0.7943	0.0832
0.7079	0.0912
1.000	0.1413
1.1220	0.1479
1.6982	0.2455
1.2023	0.2818
1.9953	0.7943
2.2387	2.5199
2.5119	1.4125
2.5119	0.8913
3.1623	1.9953

3.5481	4.2658
4.4668	6.3096
5.8884	11.2202
6.7608	19.95
15.136	141.25
15.85	158.4893

Table 1: *Terror Bird*; femur circumference and body weight of birds (adapted from Giordano, Fox & Horton, 2013)

Group B struggled even with the basic premise of the task, spending quite some time discussing the meaning and relevance of the data they had been given from a biological point of view. They did not appear aware of the offer for ritual participation; nothing in their exchanges indicates that they connected the problem with the lecturer's fish example, and there was no consideration of graphing as a tool for addressing the problem. Instead, the group began performing calculations on the data, struggling with the arithmetic, and mostly aiming at reaching a result that seemed reasonable to them from a biological point of view. For instance, using one of the values for the constant *k* that they had calculated, they estimated the weight at 22.6 kg. B4 dismissed this: "My cat weighs more than that!" B3 agreed: "When you have one with 16 cm weighing 160 kilos, that's where you see that this is wrong." For group B, biological and mathematical narratives remained separate, and evidence of meta-level learning concerning graphing or modelling was not discerned. Such evidence, albeit of a different character, was discerned in the work of groups A and C.

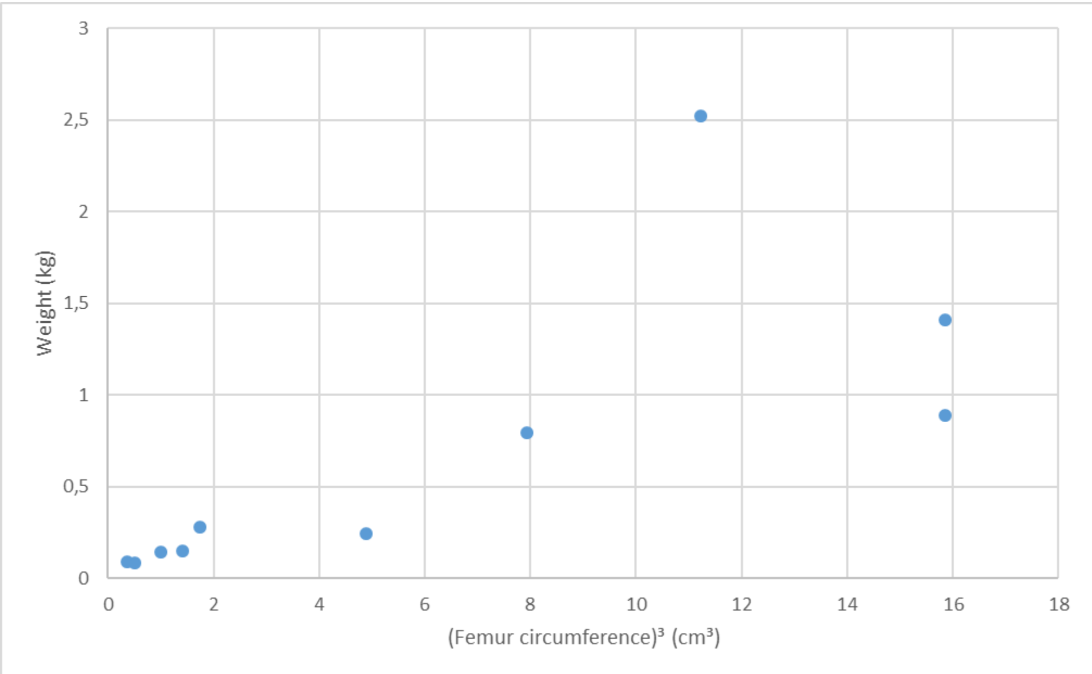

Figure 3: *Terror Bird*; Weight versus (femur circumference)³
(reconstructed by the authors: to make the characteristics of the graph more easily discernible the scatterplot shows the first ten entries in the table)

Of the three groups, group A is the one that managed to make sense of the relationship between weight and length. Still, even for this group, connecting weight with volume, and hence with length cubed, was not immediate. They began by examining the entries in the table, noting that the weights increase much faster than the circumferences:

A1: So it increases, maybe not exponentially, but at least by a power of two.
A3: At least to the second or third.

At this point, there was no consideration yet of the real-world origin of the data. Instead, the conclusion was drawn purely from the numerical relationship between the figures in the table. As soon as they started discussing the graph, they readily noticed the messiness of the dataset:

A2: Here the weight goes up, and then it goes down again, and then it goes up again.
A3: What?
(…)
A1: You, look! [pointing at the two “2.5119” entries in the table] These are the same!
A2: What?
A1: These two are the same.
(…)
A1: [noticing that the circumferences are not in increasing order] These are not in order!

The students kept noticing what they saw as “strange” features of the dataset. For instance, the difference in weight for the two equal femur measurements is relatively large: “It will be a very strange graph.” (A1) The notion of the scatterplot did not seem familiar. Rather, previous experience of graphing functional relationships seemingly took precedence, and, rather than trying to adapt their established graphing routines to the present situation, the group decided to dismiss graphing as a solution strategy. Instead, they started discussing what the values in the table actually mean in biological terms:

A1: If we look at this big bird here, then this [points at final entry in the table] is 15 cm, but the whole bird weighs 158 kg. And what weighs is this part here.
A2: Yes, the body.
A1: And it is like a ball.
A3: Yes, more or less.
A4: The head is pretty heavy too.
(…)
A1: Never mind the head. If we just try to do what he said. Circumference to the third times some constant or other.

We interpret this as an instance of “thoughtful imitation”: the students engaged in modelling discourse reasoning about simplifying assumptions concerning the “geometry” of birds. In doing so, they mimicked what the lecturer did for the fish earlier (A1: “If we just try to do what he said.”), but at the same time, they reasoned about how that method could be modified to fit the current situation. We argue that, in modifying the model, the students showed signs of de-ritualization, in terms of applicability, agentivity, and, substantiability – as in the exchange above where A1 navigates into biological narratives to substantiate the modified model. The group then attempted a computational approach similar to the one they had been using in *Yeast Growth*. On the face of it, given the messy dataset, it is not quite clear that this strategy would simplify matters, but, possibly, the students felt confident in using a strategy that had served them well previously. Indeed, they did end up with a reasonable estimate for the weight.

Group C, on the other hand, were clearly intent on taking up the offer from the lecturer, and aimed at using graphical methods from the outset:

C1: Measured at 21 cm, that isn't given here [points at the table], so we need to find the body weight from…
C2: We set it up with such a line there [indicating an increasing straight line with his hand].

Still, like group A, they were initially baffled by the messy dataset:

C4: Look here, there are two individuals with exactly the same [femur circumference], and one weighs half of the other.
C3: Wow.
(...)
C4: This doesn't make sense. Look here, some of them have smaller thighbones than the others and still they weigh twice as much.
C3: This one has just as large a thighbone as the other, but there is a very large difference in weight.
C4: Look here. These two, the same number, but widely different weights.
C2: Yes, that's why we need to find a line.
C4: What? To find the most correct curve?
C1: The best possible.

Their surprised reactions suggest that data of this type was unfamiliar. However, as this exchange indicates, they handled the situation differently from group A. Instead of allowing the unfamiliar properties of the dataset to turn them away from graphing, they saw the graph as a necessary tool to handle the messiness. However, again in contrast to group A, they did not consider that they needed to plot weight against circumference *cubed,* and worked instead with the circumferences only:

C1: They said something about weight being proportional to...
C2: Yes it should be...
C4: But it's not proportional, we can see that here [pointing at the table]. Look at the two towards the end, there is a little less than triple the femur circumference, but still we have seven, eight times the weight.

They observed a reasonable proportionality for the birds with small femur circumferences, but, for the larger birds, the weight grew much faster than the femur circumference. The group struggled with this for a while, and asked a member of the research team for help. With scaffolding support from the researcher, they realized that weight depends upon volume, but this still did not help them to proceed with the problem. In the end, realizing that they were pressed for time, the researcher spelled it out:

Researcher: how do you move from a length measure to a volume measure?
C2: You set it up in the power of three?
R: Yes.
C2: Yeah. So the…
R: So if you got length measures here…
C2: Yeah. Yeah, we set it up in the power of three.
R: Yeah, not graphing that to that, but if you graph that to the power of three to that, you might get a straight line.

As seen in *Yeast Growth* (5.1), when provided with a ready-made model, Group C used it for graphing purposes, even if it involved performing calculations on data. Judging from the exchange above, the prior experience with the provided ready-made model did not feature in the students' PSS in this case, where the model was not explicitly given. In other words, performing operations on data based on a given model does not appear to have prepared them for constructing and graphically verifying a model based on geometrical considerations. The thoughtful imitation we saw in group A concerning the construction of the model did not occur here. On the other hand, where group A did not adapt their established graphing routines to the requirements of *Terror Bird*, group C displayed a clear awareness of how graphing and line fitting might help them handle the messy data. In their work, we detect signs of de-ritualization, for instance concerning applicability where, despite the unfamiliar character of the required graph, they still argue for adapting line fitting techniques (C2: "Yes, that's why we need to find a line.") The exchanges above also indicate increased agentivity, where the students proceed without much need for external validation, up to the point where they run into computational difficulties and need to consult the researchers. In sum, we consider these instances of de-ritualization as indicative of meta-level learning concerning graphing.

6. Students' graphing activity as running to keep up with varying lecturer expectations

Through an analysis of the students' engagement with graphing routines in the three MM problems, we have discerned indications of meta-level learning evidenced through gradual de-ritualization of graphing and modelling routines, influenced by precedent events concerning the construction and interpretation of graphs as well as narratives concerning what models and modelling are. However, we have also seen how activities, seemingly intended by the lecturer as offers for ritual participation in a new discourse, were interpreted by the students in the established discourse, leading to exploratory but unproductive participation. The introduction of gradual challenges, intended by the lecturer as a progression, apparently interfered with the students' use of what they had learnt about graphing from previous problems: they were always one step behind the lecturer, running to keep up. Not unlike the "novices and experts" in the study by Angra and Gardner (2017, p.11), the lecturer saw the graph construction as straightforward and focused on graph interpretation, while the students invested more effort in graph construction.

In the first part of *Yeast Growth*, the presence of time in the data served as a precedent identifier seemingly triggering the construction of amount-time graphs instead of change-amount. In the second part, the students appear more receptive to invitation for ritual participation, apparently aware of the need for change-vs-amount-graphs. However, when additionally expected to perform certain calculations so that they could work on a model of logistic growth, the students seemed to conflate the model and the empirical data. This is in resonance with the discursive ambiguity described by Bowen et al. (1999), where "students' discourse did not allow them to distinguish between a model and a statistical description of actual data" (p. 1033). By contrast, *Digoxin* required engagement with the graphing routines already established in *Yeast Growth*, in a different context but without added complexity, and indeed the students' work on this task was mostly successful. Still, contrary to the lecturer's expectations, they mostly used graphs merely as illustrations of the data and developed ad hoc, numerical, highly process-oriented routines to estimate the growth rate.

In *Terror Bird*, additional complexity came from the expectation to work on a dataset quite different from what the students were used to: their PSS for graphing, including also the previous problems in the sessions, involved functional relationships between quantities. Now, they had to deal with a dataset suitable for presenting through a scatterplot, rather than a function graph. Groups A and B opted out of engaging with graphing altogether: instead, they engaged in ad hoc calculations with more or less successful results – much like the participants in Bowen and Roth (2005). Group C, on the other hand, although challenged by the messy dataset, seemed aware of the kind of graph they were expected to construct, and of how to use it to solve the problem. Contrary to the students in Bowen and Roth's (2005) study, they had only minor difficulty with handling the randomness in the data. However, it was only through substantial prompting that they overcame calculation obstacles before plotting and realised the need to work with the cube of the circumference.

Across the *Yeast Growth* and *Terror Bird* sessions, in the students' engagement with graphing, both for construction and estimation purposes, we see evidence of meta-level learning, albeit variably across the groups. Group B seemed to view graphs not as tools for reasoning (B4: "When it says 'draw a graph' then I usually skip the exercise."), but solely as illustrations of their work's results. Groups A and C, on the other hand, displayed signs of de-ritualization of their engagement with routines, particularly with regard to what Lavie et al. (2019) have labelled as applicability, performer agentivity and substantiability. Group C used graphs as sources of information about the problems consistently – if mainly qualitatively rather than quantitatively. For instance, working on *Digoxin*, they used the graph to conclude that the coefficient had to be negative (Viirman & Nardi, 2018, p. 369), but not to calculate its value. They also appeared aware of how graphing could be used to handle the unusual dataset in *Terror Bird*, even though calculation issues hindered them from succeeding fully. On the other hand, while Groups B and C reasoned largely within biological discourse, Group A displayed a clear shift in their MM discourse: at least in their work on *Terror Bird*, they engaged with the properties of the model they were constructing in a manner akin to what the lecturer had done in his lecture on modelling using geometric similarity. In taking up this offer for ritual participation, they helped pave the way for their own gradual de-ritualization of MM discourse in a manner that, for instance, Group B did not. Indeed, one of the pedagogical implications of our analyses is that lecturers not only need to be aware of the need for offering opportunities for ritualized participation in the new discourse into which students are entering – they also need to orchestrate this so that invitations for thoughtful imitation are explicit to students.

There are two other features of the students' work identified by our commognitive analyses which indicate further pedagogical implications. The first feature is the presence of time in the data as an explicit or implicit variable serving as a strong precedent identifier for its use as the independent variable in the current task situation, even though the problem formulation asks for something different. This strong influence of precedent events echoes findings by Patterson and McGraw (2018) as well as Bowen et al. (1999). School and university mathematics pedagogy may need to facilitate students' building of a more diverse PSS, with exposure to mathematical problems in which engaging with a greater range of variables is necessary. The second feature is what we labelled graphing merely as a tool for illustration, and less as a means to make meaning from data. This is in line with earlier findings, e.g., Leinhardt et al. (1990), and indicates the need for lecturers to engineer this meta-level learning – foregrounding graphing as a means not only for illustrating but, often more crucially, for interpreting data – extensively and explicitly.

In this paper, we contribute commognitive analyses of the mathematical practices of university students in a discipline other than mathematics and in the context of MM. We see studies that navigate across disciplinary discourses as highly fertile ground for commognitive research and we demonstrated this here through tracing the students' evolving discursive activity in relation to how they engage with graphing routines to solve problems in a Biology setting. While the students' work on the problems, which were designed by the lecturer with a learning trajectory in mind that would take the students towards greater agentivity (per Lavie et al., 2019) in their graphing activity, did not always go according to his expectations, we traced evidence of meta-level learning as well as evidence of mismatch between lecturer and student interpretation of the task situation. Unfamiliarity with certain routines – or persistent influence of precedent events that evoked engagement with routines that the students felt comfortable with but were not necessarily appropriate to tackle the current task situation – played out in the students' discursive activity. Our analyses point to a pedagogical need: that MM activity is designed so that it orchestrates progression and allows connections to – and builds upon – previously established routines in ways that are explicit for the students to pick up and use in new situations.

Our analyses also point to a yet to be substantially explored capacity of the commognitive framework: to trace in detail the unfolding of students' discursive activity; and, through doing so, to provide much needed, theoretically robust (Reinholz, Rasmussen & Nardi, 2020) evaluative indicators of whether, and how, pedagogical interventions – such as the one we report in this paper in the context of MM activities for students majoring in a discipline other than mathematics – work.

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Supplementary Materials

***Yeast Growth,* first subtask**: [the students were given a part of a table of data with three columns (time, amount of biomass, and change in biomass) describing the growth of a yeast culture.]

- analyze the numerical data in the table
- plot the data and analyze the graph
- suggest a simple model based on a difference equation of the form $\Delta p_n = k_1 p_n$, where p_n is the size of the biomass after n hours, $\Delta p_n \stackrel{\text{def}}{=} p_{n+1} - p_n$ is the change of biomass between two measurements, and $k_1$is a positive constant
- explain what your expectations would be regarding the predictive power of the model.

Intended **second subtask:** [the students were to be given the second part of the table]

- analyze this new data
- plot the population against time, explore the shape of the graph and state what you would expect in the long run
- calculate the expected value for “carrying capacity”.

[In the actual session (due to limitations of time), this part was skipped, and instead the students were provided with a non-linear model based on incorporating the carrying capacity.]

Actual **second subtask:**
We may estimate carrying capacity to be 665 (this value is not precise and your value may differ a bit). As the number $665 - p_n$ gets smaller and smaller as p_n approaches 665, we may adjust our simple linear model replacing it with a nonlinear model $\Delta p_n = k_2 p_n(665 - p_n)$ or alike, if you have chosen 664 or 666. Test a new model by plotting Δp_n against $p_n(665 - p_n)$ to check whether a reasonable proportionality is observed. Then, estimate the proportionality constant k_2. What is your value?

Third and final **subtask**:
Use the new model, with $k_2 = 0.00082$, to compute values and compare them with the actual data: Compute twelve values of $p_n$using the formula, starting with the initial value $p_0 = 9.6$.

Digoxin:
(**a**) For an initial dosage of 0.5mg in the bloodstream, the table shows the amount of digoxin a_n remaining in the bloodstream of a particular patient after n days, together with the change Δa_n each day. Plot Δa_n versus a_n and explore the graph. Suggest a simple model based on a difference equation of the form $\Delta a_n = k_3 a_n$, where k_3 is a positive constant. What is your choice of k_3?
(**b**) Now our objective is to consider the decay of digoxin in the blood stream to prescribe a dosage that keeps the concentration between acceptable levels so that it is both safe and effective. Design a simple linear model describing the following scenario: we prescribe a daily drug dosage of 0.1mg and know that half the digoxin remains in the system in the end of each dosage period.
(**c**) Consider three different options where the initial one-time dose of medicine received by the patient is $a_0 = 0.1$mg, 0.2mg or 0.3mg. What are your conclusions? What would you recommend if you were this patient’s GP?

Terror Bird:

The terror birds were giant, flightless, predatory birds. The terror bird known as Titanis walleri was a fleet hunter that would lie in ambush and attack from the tall grasses. These birds killed with their beaks, pinning down their prey with an inner toe claw 4 to 5 in. long and shredding their prey. The various terror birds ranged in size from 5 ft to 9 ft tall, Titanis being the largest. Because very little fossil material of Titanis has been discovered, its exact size is unclear. Your task: predict the weight of the terror bird as a function of the circumference of its femur measured as 21 cm.

To this end, the students were given a table of femur circumference and body weight in various present-day bird species.

Highlights:

- **Mathematical modelling tasks trigger student navigation across disciplines**
- **Students' graphing activity excessively privileges time as independent variable**
- **Students' graphing activity privileges data illustration rather than interpretation**
- **Commognitive analysis traces meta-level learning in students' modelling activity**
- **Lecturer awareness of student meta-level learning needs improves task design**