UNIVERSITY OF EAST ANGLIA

DOCTORAL THESIS

Modelling Volatility in Energy Markets

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Declaration of Authorship

I, Wenxue WANG, declare that this thesis titled, “Modelling Volatility in Energy Markets ” and the work presented in it are my own. I confirm that:

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- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
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Abstract

Oil price uncertainty has a significant impact on economic growth and financial market performance, and understanding the drivers of oil price dynamics is vital for the global economy. It makes volatility modelling in crude oil pricing an essential topic for academics and practitioners. Geopolitical events in OPEC countries disrupting oil supply has long been the main driver of oil volatility, while the U.S. shale revolution has led to new dynamics on the supply side of the global oil market. The volatility transmission mechanism between crude oil and other assets in the investment market plays a crucial role in shaping international investments and economic policies. This thesis examines the channels of oil price dynamics and the volatility transmission mechanism between oil and other financial assets.

Chapter 2 compares the performance of GARCH models, stochastic volatility models, and OVX implied volatility index regarding out-of-sample forecasting accuracy in oil futures prices. To do so, the dataset of West Texas Intermediate (WTI) oil active in the U.S. markets and the Brent Crude dominating the European market for the period 2004-2015 is adopted, which includes the steep price drop in 2014. GJR-GARCH model suggests that leverage effect exists, while the stochastic volatility models are used to examine series dependence and heavy-tailed distribution in oil return series. The most important finding for this chapter is the detection of over-fitting in the GJR-GARCH model and stochastic volatility models.

Chapter 3 examines whether the shale revolution has dampened the role of geopolitical risk in oil price volatility. Using the Structural Break Threshold Vector Autoregressive (SBT-VAR) framework proposed by Galvão (2006), this chapter identifies threshold effects and a structural break in April 2014. Furthermore, this study extends the framework of Galvão (2006) to a structural SBT-VAR system by allowing for conditional heteroskedasticity. Notably, impulse responses of oil price and (co)variance to the shock of geopolitical risk are compared over 170 periods, including before and after the shale oil revolution. We find that the impulse response functions of oil prices to a unit structural geopolitical risk shock have become smoother after the breakpoint in 2014 compared with those before the break. The main finding as we are expecting initially and intuitively is that the covariance response between

geopolitical risk and oil price reduce with shale production shock compared to without. However, composing one extra unit of shale production shock makes the volatility response of oil prices to a geopolitical risk shock reaching a higher level.

Chapter 4 examines volatility spillovers and dynamic correlations between crude oil exchange-traded fund (ETF), various renewable energy ETFs, and the S&P 500 ETF by using multivariate GARCH-in-mean specifications. We find that the conditional volatility of the nuclear ETF has a significant positive effect on the oil ETF return, oppositely the volatility of the S&P 500 ETF negatively spillovers to the return of the oil ETF. The most important finding is that the long-term persistent volatility spillover from the S&P 500 ETF to renewable energy ETFs is significantly negative. Another result reveals that the dynamic correlations concurrently decrease before the financial crises (in 2008 and 2011 respectively) and then dramatically increase in the post-crisis period. Evidence shows that the dynamic correlation between oil ETF and S&P 500 ETF has always been positive since U.S. net imports of crude oil and petroleum products gradually decrease from 2005 onwards.

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List of Abbreviations

ADF	**A**ugmented **D**ickey-**F**uller
AGARCH	**A**symmetric **G**eneralized **A**utoregressive **C**onditional **H**eteroscedasticity
AIC	**A**kaike **I**nformation **C**riterion
ARCH	**A**utoregressive **C**onditional **H**eteroscedasticity
ARFIMA	**A**utoregressive **F**ractionally**I**ntegrated **M**oving **A**verage
ARIMA	**A**utoregressive **I**ntegrated **M**oving **A**verage
ARMA	**A**utoregressive **M**oving **A**verage
BEKK-GARCH	**B**aba-**E**ngle-**K**raft-**K**roner GARCH
BIC	**B**ayesian **I**nformation **C**riterion
BLM	**B**ounded **L**agrange **M**ultiplier
BWald	**B**ounded **W**ald
CAPM	**C**apital **A**sset **P**ricing **M**odel
CBOE	**C**hicago **B**oard **O**ption **E**xchange
CCC-GARCH	**C**onstant **C**onditional **C**orrelation
CME	**C**hicago **M**ercantile **E**xchange Group
DCC-GARCH	**D**ynamic **C**onditional **C**orrelation
DOE	U.S. **D**epartment **o**f **E**nergy
EGARCH	**E**xponential **G**eneralized **A**utoregressive **C**onditional **H**eteroscedasticity
EIA	U.S. **E**nergy **I**nformation **A**dministration
ETF	**E**xchange-**T**raded **F**und
FPF	**F**inal **P**rediction **E**rror
GARCH	**G**eneralized **A**utoregressive **C**onditional **H**eteroscedasticity
GIRF	**G**eneralized **I**mpulse **R**esponse **F**unction
GJR-GARCH	**G**losten-**J**agannathan-**R**unkle GARCH
GMLE	**G**aussian **Q**uasi-**M**aximum **L**ikelihood Function
GO-GARCH	**G**eneralize **O**rthogonal GARCH
GPR	**G**eopolitical **R**isk
HMAE	**M**ean **A**bsolute**E**rror adjusted for Heteroskedasticity Function
HMSE	**M**ean **S**quare**E**rror adjusted for Heteroskedasticity Function
HQ	**H**annan-**Q**uinn
ICE	**I**ntercontinental **E**xchange

LL	**L**ogarithmic **L**oss Function
LM	**L**agrange **M**ultiplier
LR	**L**ikelihood **R**atio
MA	**M**oving **A**verage
MAE	**M**ean **A**bsolute **E**rror Function
MCMC	**M**arkov **C**hain **M**onte **C**arlo Method
ML	**M**aximum **L**ikelihood
MSE	**M**ean **S**quare**E**rror Function
NASDAQ	**N**ational **A**ssociation of **S**ecurities **D**ealers **A**utomated **Q**uotations
NYMEX	**N**ew **Y**ork **M**ercantile **E**xchange
O-GARCH	**O**rthogonal GARCH
OLS	**O**rdinary **L**east **S**quare
OPEC	**O**rganization of **P**etroleum **E**xporting **C**ountries
OVX	CBOE Crude Oil Volatility Index
SBTVAR	**S**tructural **B**reak **T**hreshold **V**ector **A**utoregressive
SBVAR	**S**tructural **B**reak **V**ector **A**utoregressive
S-SBTVAR	**S**tructural-**S**tructural **B**reak **T**hreshold **V**ector **A**utoregressive
TARCH	**T**reshold **A**utoregressive **C**onditional **H**eteroscedasticity
TVAR	**T**hreshold **V**ector **A**utoregressive
VIX	CBOE Volatility Index
VIRF	**V**ariance **I**mpulse **R**esponse **F**unction
W	**W**ald
WTI	**W**est **T**exas **I**ntermediate

Chapter 1

Introduction

1.1 Overview

The theme of this thesis is the market for oil. It is a highly important market because oil has been one of the most important drivers of economic growth for the past 200 years. It is also important because it is a fossil fuel, and its continued extraction is causing ever more severe environmental problems. For this reason, we are not only interested in the oil market. We are also interested in the market for renewable energy resources. Moreover, the oil market activities impact on the wider financial market in many ways, eventually shaping final investment decisions.

The purpose of this introductory chapter is firstly to provide some historical and institutional background that motivates the research. Then we provide an outline of the three main chapters and identify the links between them.

1.2 Background

Crude oil is a mixture of hydrocarbons formed from plants and animals that lived millions of years ago. It is considered one kind of fossil fuel, and also as a form of non-renewable energy. It cannot be used directly. However, it has many useful constituents. A barrel of crude oil is, through the refining process, converted into the following petroleum products in roughly the following proportions: liquid petroleum gases (LPG), naphtha and gasoline (50%); diesel fuel, heating oil, jet fuel, kerosene (40%); residual fuel oil (10%).

We are most interested in the price of crude oil. The oil price has shown huge variation in recent decades. Much of this variation is easily explained in terms of economic and geopolitical factors. For the U.S. market before 1973, the price of oil was regulated by the government. However, the growing domestic demand for oil put a strain on the U.S. oil market. The U.S. became increasingly dependent on oil imports from the Middle East. It started the modern era of oil markets. Hamilton (2003b) concluded that a negative shock to the supply of crude oil caused the oil price shock of 1973/74. But Baumeister and Kilian (2016a) argued that the oil crisis of 1973/74 was driven more by increased demand for oil rather than by reductions in oil supply. The next major oil crisis was in 1979/80, which Hamilton (2003b) is attributed to the supply reduction caused by the Iranian Revolution. Kilian and Murphy (2014) clarified the viewpoint that the Iranian Revolution impacted on the oil price by affecting oil price expectations rather than the flow of oil production. They estimated that one third of the cumulative price increase was related to a surge in inventory demand in anticipation of future oil shortages. The remaining two-thirds of the cumulative oil price increase was associated with the cumulative effects of demand shocks. There was a systematic decline in the price of oil at the beginning of the 1980s, which was caused partly by the shift in global monetary policy (rising interest rates).

Meanwhile, having experienced the 1973 crisis, numerous non-OPEC countries, such as Mexico, Norway, and the United Kingdom, found ways of becoming oil producers themselves. As a result, the global market share of OPEC decreased from 53% in 1973 to 43% in 1980, and then to 28% in 1985. This increasing world oil production dragged the oil price downward. In 1990, geopolitical events, including Iraq's invasion of Kuwait, disrupted the supply of crude oil, leading to a sharp increase in the oil price. However, it recovered to a low price level in 1991 as a result of low oil inventory demand (Kilian, 2008). In 1997, the Asian financial crisis casued the oil price to weaken further. In 1999, increasing demand for oil inventories brought the oil price upward (Kilian and Murphy, 2014). In 2002/03, geopolitical events - this time civil unrest in Venezuela and the second Iraq War - further stimulated the oil market recovery. The surge continued until 2008, as a result of increasing demand and the expanding global economy, and an all-time high price of $145.93 was reached in June 2008. Following that, the 2008 global financial crisis sent the oil price plummeting, reaching a low of $30.28 in December of the same year. After that low point, there was a gradual recovery of the oil price until 2014, settling at a price above $100.

In late 2014, there was another sharp drop, this time caused by the shale oil boom in the U.S.. Interestingly, ever since that drop in price, the price has stayed reasonably close to $50. The reason for this is again associated with the shale market. There were reports [1] that whenever the oil price fell below $50, shale production became unprofitable and was severely reduced, removing the downward pressure on price. When the price rose above $50, shale production resumed and put downward pressure on the price once again. Hence the presence of shale production has fulfilled a price-stabilising role in this period.

At the time of completing this thesis (28 September 2019), the oil price is $55.91.

1.3 Crude Oil Benchmarks

In the last section, we discussed the movement of the oil price in recent decades, relating these movements to economic and geopolitical factors. However, it is important that there is actually more than one oil price.

Crude oil benchmarks are used to provide a price reference for the crude oil market. There are essentially two benchmarks: West Texas Intermediate (WTI hereafter); and Brent Blend (Brent hereafter). In the last section, all quoted oil prices were WTI prices.

WTI represents the benchmark price in the U.S. market. The oil represented by this benchmark is also known as Texas light sweet. It is produced by blending several U.S. local crude oil streams. While drilling for oil takes place in many US states, most of the refineries are located in the Midwest and Gulf Coast regions. For the past three decades, the delivery point for WTI crude oil (also crude contracts) is in Cushing, Oklahoma, which is a major trading hub. Cushing has a large number of intersecting pipelines and storage facilities. It provides convenient access for refiners and suppliers, either inbound or outbound, from or to any location in the U.S.. The advantages of WTI as a benchmark are that it has high liquidity, high trading volume, and high transparency. WTI is the underlying commodity of oil futures contracts traded on the New York Mercantile Exchange (NYMEX), managed and owned by the Chicago Mercantile Exchange (CME) Group.

[1] Carlson D., "Oil prices plummet amid continued oversupply, with no end in sight", *The Guardian*, 6 August 2016

Brent crude oil is the other leading benchmark in the crude oil market worldwide (two-thirds of all oil traded globally belongs to Brent), although mainly in Europe. One reason for the popularity of Brent is that it is water-borne, which means it is easier to transport than WTI which is land-locked. Brent crude oil is extracted from the North Sea and is also as known as “Brent Blend", “London Brent", or “Brent Petroleum". The main transaction point for Brent is the London-based International Petroleum Exchange, which since 2001 has been a subsidiary of the Intercontinental Exchange (ICE), also based in London. Here, options and futures on oil-related commodities are traded.

We are particularly interested in the difference between the WTI price and the Brent price. There are a number of reasons why the two prices might be different. The first is a difference in the quality (composition) of the crude oil. Specifically, WTI is “sweeter” (lower sulphur content) and “lighter” (lower density) than Brent. This essentially means that WTI is easier to refine and process. The second possible reason for the price difference is the extraction location. Less expensive delivery of the product clearly results in a lower final price. Crude oil extracted from the sea has a clear advantage in transportation over land-based oil which relies on the capacity of pipelines. For this Brent has lower transportation costs than land-based oil such as WTI.

Both of the reasons mentioned above would be expected to give rise to the WTI price being higher than the Brent price. However, for most of the last decade at least, the Brent price has been higher.

On the basis of no-arbitrage arguments, the Brent price should exceed the WTI price by the cost of transporting WTI from Cushing to the Brent trade hub. Since Brent is more easily traded than WTI, there is little chance for speculators attempting to profit from price spread.

Another reason for the WTI price being lower is the shale oil boom, which began around 2011. This had the effect of boosting supply in the U.S., but not in the rest of the world, because of the oil export ban that had been in place since 1977. The oil export ban was lifted in 2015, and unsurprisingly this coincided with a convergence of the two benchmarks.

1.4 Shale Oil

The U.S. found the most significant oil shale reserve in the world, but in early 1980 the shale oil extraction was in the experimental stage. And its development was limited by the high cost of the industrial process. Therefore, shale oil had no competitive advantage against conventional oil, which cost less than the shale oil to be extracted. Since 2000, the combination of techniques of horizontal drilling and hydraulic fracturing have led to a new era for shale oil - the shale revolution. In this time, shale oil production has rapidly increased and now plays an essential role in the U.S. energy market.

The term “shale oil" is widely used in oil research. Actually in the United States, the oil and natural gas, extracted from low-permeability formations, including that associated with shale formations, as typically referred to as “tight oil production” rather than shale oil production. In the chapter, we still use “shale oil production” in line with other researchers.

1.5 Oil and the Financial Market

Investment in the crude oil market is continually significant and can be achieved by purchasing oil company stocks, oil exchange-traded fund (ETF), oil futures, and so on. And ETF is a more and more popular way to gain substantial exposure to oil. The advantages of oil-related investment are summarised as diversification, profit potential, and tax advantages, whereas the disadvantages are volatility, liquidity, commissions, and complexity. Therefore, we advise investors who are interested in the oil investment that market investigation and careful and comprehensive consideration should be carried out.

1.6 Oil and the Environment

As regards the adverse effect of relying on oil, climate change threaten human living environment is the firmly one. To combat climate change, the Paris Agreement was reached worldwide at the end of 2015. International investment in renewable energy sources has increased by roughly 240 billion U.S. dollars in alternative energy section in the years 2000 to 2016. And the U.S. proposed the Clean Power Plan in 2014. As a solution for the U.S. environmental problems, renewable energy is dramatically expanded, then rapidly developed and applied widely. The consumption of U.S. renewable energy has doubled between 2000 and 2017 (U.S. Energy Information Administration (EIA)), getting 11% of total U.S. energy consumption in 2017. Nuclear energy, as an alternative to clean and sustainable energy, provides about 20% of total U.S. electricity (EIA & U.S. Department of Energy (DOE)).

1.7 The Motivation of the Research and the Connection between Each Chapter

Oil price uncertainty has a significant impact on economic growth and financial market performance, and understanding the drivers of oil price dynamics is vital for the global economy. It makes volatility modelling in crude oil pricing an essential topic for academics and practitioners. Geopolitical events in OPEC countries disrupting oil supply has long been the main driver of oil volatility, while the U.S. shale revolution has led to new dynamics on the supply side of the global oil market. The volatility transmission mechanism between crude oil and other assets in the investment market plays a crucial role in shaping international investments and economic policies. This thesis examines the channels of oil price dynamics and the volatility transmission mechanism between oil and other financial assets.

Chapter 2 aims to investigate volatility estimation and volatility forecasting of Brent and WTI futures prices. Volatility is a critical issue in such fields as risk management, asset allocation, and trading on futures volatility. However, volatility cannot be observed directly. Forecasting volatility is crucial for futures investment, and it is also a measure of the potential losses of assets. Besides, volatility can impact on the macroeconomy. Sadorsky (1999) demonstrated that the increase in oil price is linked to inflation and economic recession, which would affect interest rates, exchange rates, and further investment. Therefore, researchers make many efforts to point out how volatility preforms and many empirical works focus on volatility estimation and forecasting.

Kat and Heynen (1994) found evidence that the stochastic volatility model provides the best predictions for stock indices, whereas, for currencies, the

GARCH(1,1) model is the best. Chan and Grant (2016) compared a variety of GARCH and stochastic volatility models for estimating nine series of energy prices, and concluded the stochastic volatility model is favorable to their GARCH counterpart. Moreover, the stochastic volatility model with moving average innovations is the best model for all series. Wei (2012) provided consistent results in volatility forecasting, that the stochastic volatility model outperforms GARCH-type model. Lehar, Scheicher, and Schittenkopf (2002) argued that the GARCH model dominates both stochastic volatility and the benchmark Black-Scholes model in forecasting intraday FTSE 100 option prices. However, there is limited research focusing on the OVX index, GARCH-type models, and stochastic volatility models contrast and comparison in terms of Brent and WTI futures market. Moreover, previous studies focus on model variance, exploring instead of capturing data series stylish behaviour. To my understanding about a good volatility model, fitting the data and forecasting accuracy is essential. *Chapter 2* emphasizes the features of the data series in a specific period, and tests the forecast ability during the time of oil price decline. Further, we consider the over-fitting problem, which is ignored by other researchers.

The choice of models in *Chapter 2* is motivated by the following considerations. The volatility of financial time series is generally not constant over time (heteroscedasticity), and it shows long persistence (volatility clustering). Specifically, crude oil futures prices are characterized by time-varying volatility. The Autoregressive conditional heteroscedasticity (ARCH) model and the generalized autoregressive conditional heteroscedasticity (GARCH) model are able to model financial data series using conditional variances. The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model is constructed by adding a leverage effect term in the GARCH model. The leverage effect term shows that good news (positive shock) and bad news (negative shock) in the market have different effects on volatility. GARCH-type models can provide accurate out-of-sample one-step forecasts. Meanwhile, the stochastic volatility model is also a popular method to model volatility. In the advanced stochastic volatility model, 'moving average process' (MA) is added to the return measure equation. The reason is there is long persistence in financial asset return, generally. Concerning the superior stochastic volatility model, the Student's t-distributed error is employed instead of the normal distributed error. The reason is the distributions of financial data always show leptokurtosis and heavy tail characteristics. Additionally, the CBOE Crude Oil Volatility (OVX) index is chosen as a measure for market volatility forecasting because it can represent the market's expectation of stock market volatility. In the forecast part, the proxies of real variance are generated by squared return. The forecasting variance is extracted from the different measurement discussed above. We apply the forecasting variance and the proxy variance into six loss functions to evaluate the forecast ability, and further confirm the optimal forecasting model.

Moreover, from *Chapter 2*, we known Brent-WTI price spread widening in 2011, and the oil price collapse in 2014 have triggered my interest to investigate the volatility dynamics in *Chapter 3*. There are some detailed explanations following concerning the data features mentioned above.

Crude oil can not be consumed unless it is refined or transformed. Therefore, the quality of crude oil is assessed through two main criteria: density (API degree) and sulphur content, which is closely related to the cost of the refining process and the quality and quantity of crude oil production. WTI, with less sulphurous and being light, is superior to Brent. Theoretically, WTI should attract a premium relative to Brent. But the oil price varies not only on substance (quality spread) but also locations (location spread). And Fattouh (2010) provided evidence that the price differentials between crude oils are stationary from 1997 to 2008. However, after 2010 WTI has traded below Brent. The main reason is the Shale Revolution has expanded the crude oil supply in the U.S., causing the price of WTI to decline. And the U.S. has a ban on oil exports, which means WTI is stuck in the local market. It also can be a reason for its price decrease. Moreover, fears to the closure of the Suez Canal and potential disruption in Brent supplies widened the Brent-WTI spread in 2011. *Chapter 2* investigates the difference of volatility dynamics in terms of Brent and WTI under this crucial period.

Another feature in this period applied in *Chapter 2* is the crude oil price collapse in 2014. Baumeister and Kilian (2016b) provided quantitative evidence that negative demand shock and positive supply shock contributed to the price decline. And the remaining oil price decline is accounted for by a shock to oil price expectations (lowering the demand for oil inventories) lowed and a shock to the demand for oil (weakening economy). In *Chapter 2*, we assess the forecasting ability of different volatility measures through this turbulent time of the crude oil market.

Based on the research finding of *Chapter 2*, we have the intuition that there is a structural break roughly located in 2014/15. And we intend to find an appropriate model being able to give definitive evidence for that. Meanwhile, the shale oil revolution impressed me that it has started the U.S oil independent pathway. Further we am wondering if the shale oil production boom has changed the oil dynamics in the U.S. market. It is how we gradually pick up some pieces from *chapter 2*, and that motivates me to explore more.

Not surprisingly in *chapter 3*, we investigate whether the shale revolution has dampened the role of geopolitical risk in oil price volatility. Ideally, a constant threshold and a break in April 2014 are supported by the data, when the Galvão (2006)'s Structural Break Threshold Vector Autoregressive (SBT-VAR) model is applied. Furthermore, this chapter extends Galvão (2006)'s framework to a structural SBT-VAR system by allowing for conditional heteroskedasticity. The main finding as we are expecting initially and intuitively is that the covariance response between geopolitical risk and oil price reduce with shale production shock compared to without. However, composing one extra unit of shale production shock makes the volatility response of oil prices to a geopolitical risk shock reaching a higher level.

During the time of spending on the first two studies, we have learned from the previous literature that the oil price is not only affecting the macroeconomy through several channels, but also the oil price dynamics and financial market performance have tight connections. Also, how people efficiently approach oil investment in the financial market excites me. After a comprehensive investigation in the financial market, ETF investment holds appeal for me. In the intervening period, renewable energy applications and its bright future draw my attention as well. When we go deeper into the research on energy markets, we have a stronger feeling that humans can not just focus on the economic growth. Environmental sustainability and benefits are increasingly crucial. Then we finalized my research objectives of *chapter 4*. This chapter examines volatility spillovers and dynamic correlations between crude oil exchange-traded fund (ETF), various renewable energy ETFs, and the S&P 500 ETF by using multivariate GARCH-in-mean specifications. The multivariate GARCH model is also recommended in the section of further research in *chapter 2*.

1.8 Contribution of this Study

This thesis attempts to provide a comprehensive perspective on econometric modelling in the oil market. In the long-run, oil price drivers and the channels to oil price dynamics are investigated. Also, the short-run performance of the oil market is explored. In particular, the volatility spillovers between the oil ETF and other ETF markets are probed. This study not only focuses on the crude oil physical market (research object: crude oil spot price), but also the financial market (research objects: crude oil futures price and crude oil ETF price). This thesis provides a broad review for this research field, which is outstanding in previous crude oil studies.

In *chapter 2*, we discuss the in-sample estimation results and compare the out-of-sample forecast abilities in terms of OVX index, GARCH-type models, and stochastic volatility models by using Brent and WTI crude oil futures daily return. *chapter 2* contributes in several aspects as the following discussion. Previous literature rarely focuses on comparing and contrast of volatility extracted from different models. To fill the gap, we examine the adequacy of OVX, GARCH-type models, and stochastic volatility models in describing and forecasting the crude oil futures volatility behaviour. Previous works generally adopted one or two loss functions. Whereas, we include six loss functions in this chapter, which provide more elaborate evidence concerning the predictability of volatility. It provides sufficient guidance for optimal model choices. The next contribution is benefiting from the valuable market information in the period. One feature is about the price difference between Brent and WTI. After 2010, it shows divergence in prices of Brent and WTI. Moreover, there was a massive oil price turbulence in 2014. Therefore, the data, including this period, is worth being researched, and this chapter contributes to extracting useful information from the valuable data. The second one is there is a significant price collapse in crude oil futures market commencing 2014. The majority of previous works just focused on the 2008 crisis, but this work can explore model performance in forecasting the severe price decline of crude oil since 2014 as well. It is a crucial and sensitive time in crude oil volatility research. The most important contribution in this chapter is the detection of over-fitting in the GJR-GARCH model and stochastic volatility models.

In *chapter 3*, we investigate whether the shale revolution has dampened the role of geopolitical risk in oil price volatility. The first contribution is the structural model is identified. Specifically, the reduced form Structural Break Threshold Vector Autoregressive (SBT-VAR) model is extended to a structural SBT-VAR model, and the structural innovations by allowing for conditional heteroskedasticity is identified. Compared with the conventional reduced form VAR and TVAR models, an SBT-VAR with a constant threshold and a break in April 2014 are supported by the data. The second contribution is analyzing the conditional (co)variance impulse response concerning two distinct shock scenarios, one with only a geopolitical risk shock, the other with a simultaneous shale production shock and a geopolitical risk shock. Finally, The volatility responses are due to the identified contemporaneous relationships amongst geopolitical risk, shale production, and oil prices, and are conditional on volatilities at the points in time. With the extra unit shale production shock, we find that the volatility response of oil prices to a geopolitical risk shock is higher, but the response is less correlated with the geopolitical risk factor.

In *chapter 4*, We mainly contribute in two aspects. The first one is to provide evidence for volatility spillovers between crude oil ETF, renewable energy ETFs, and the S&P 500 ETF in the U.S. ETF markets. Volatility, as a proxy of risk, is significantly meaningful and crucial not only for researchers but also for the practitioners, such as policymakers, portfolio managers, investors, consumers, and producers. However, volatility modelling rarely has been focused on the energy ETF markets. We focus on the volatility spillovers and further risk management by constructing hedging strategy and portfolio weights in crude oil, renewable energy ETFs, and S&P 500 ETF. And we identify the oil ETF uncertainty has a negative and significant effect on the S&P 500 ETF return. Two factors drive the volatility of energy ETFs. One is relative to the energy market, and the other is the stock market. The dynamic correlation results tell us that renewable energy ETF is in tandem with the S&P 500 ETF. It is valuable to investigate in ETF volatilities because it provides risk management implication for practice. The second contribution is applying the GARCH-in-mean model to analyze the volatility spillover effect of energy ETFs on the mean level of the S&P 500 ETF, and vice versa. The previous energy market research mainly focuses on the volatility spillover in the variance level. They rarely investigate the volatility effect on asset returns. And BEKK GARCH-in-mean is generally applied. Our advantage is empirically using DCC and CCC VARMA-GARCH-in-mean model to explore the volatility spillover and conditional dynamic correlation together.

The most interesting finding is negative volatility spillovers in long persistence between renewable energy ETF and the S&P 500 ETF, which breaks the initial Bollerslev's positive striction to all the elements in the covariance matrix.

1.9 The Structure of the Thesis

The thesis is organized in five chapters, and the rest of the thesis is structured as follows:

Chapter 2: Forecasting Volatility of Crude Oil Future Returns: Empirical Evidence from OVX, GARCH-type Models, and Stochastic Volatility Models

This chapter compares the performance of GARCH models, stochastic volatility models, and OVX implied volatility index regarding out-of-sample forecasting accuracy in oil futures prices. To do so, the dataset of West Texas Intermediate (WTI) oil active in the U.S. markets and the Brent Crude dominating the European market for the period 2004-2015 is adopted, which includes the steep price drop in 2014. GJR-GARCH model suggests that leverage effect exists, while the stochastic volatility models are used to examine series dependence and heavy-tailed distribution in oil return series. The most important finding for this chapter is the detection of over-fitting in the GJR-GARCH model and stochastic volatility models.

Chapter 3: The Shale Revolution, Geopolitical Risk, and Oil Price Volatility

This chapter examines whether the shale revolution has dampened the role of geopolitical risk in oil price volatility. Using the Structural Break Threshold Vector Autoregressive (SBT-VAR) framework proposed by Galvão (2006), this chapter identifies the threshold effect and a structural break in April

2014. Furthermore, this study extends Galvão (2006)'s framework to a structural SBT-VAR system by allowing for conditional heteroskedasticity. Notably, impulse responses of oil price and (co)variance to the shock of geopolitical risk are compared over 170 periods, including before and after the shale oil revolution. We find that the impulse response functions of oil prices to a unit structural geopolitical risk shock have become smoother after the breakpoint in 2014 compared with those before the break. The main finding as we are expecting initially and intuitively is that the covariance response between geopolitical risk and oil price reduce with shale production shock compared to without. However, composing one extra unit of shale production shock makes the volatility response of oil prices to a geopolitical risk shock reaching a higher level.

Chapter 4: Volatility Spillovers in the Crude Oil ETF, S&P 500 ETF, and Renewable Energy ETF (Exchange-Traded Fund)

This chapter examines volatility spillovers and dynamic correlations between crude oil exchange-traded fund (ETF), various renewable energy ETFs, and the S&P 500 ETF by using multivariate GARCH-in-mean specifications. We find that the conditional volatility of the nuclear ETF has a significant positive effect on the oil ETF return, oppositely the volatility of the S&P 500 ETF negatively spillovers to the return of the oil ETF. The most important finding that is the long-term persistent volatility spillover from the S&P 500 ETF to renewable energy ETFs is significant negative. Another result reveals that the dynamic correlations concurrently decrease before the financial crises (in 2008 and 2011 respectively) and then dramatically increase in the post-crisis period. Evidence shows that the dynamic correlation between oil ETF and S&P 500 ETF has always been positive since U.S. net imports of crude oil and petroleum products gradually decrease from 2005 onwards.

Chapter 2

Forecasting Volatility of Crude Oil Futures Returns

Evidence from OVX, GARCH-type Models, and Stochastic Volatility Models

2.1 Introduction

The extensive use of crude oil has undoubtedly been one of the most critical drivers of the growth of the world economy over the past 100 years. At the same time, economic globalization also promoted the prosperity of the crude oil market. The oil crisis in the 1970s struck global oil markets and resulted in large fluctuations in crude oil prices. As a consequence, crude oil futures emerged, and the trade volumes had rapid growth since then. By now, the crude oil futures has become an essential component in futures markets. It is also a significant indicator in the financial market, and investors generally reference it as a market barometer. Not just to the investors, but also researchers and policymakers, crude oil futures is quite a current topic. Brent and WTI are the benchmarks in the crude oil market, and they are the research objectives in my work.

The crude oil can not be consumed unless it is refined or transformed. Therefore, the quality of crude oil is related to two main criteria: density (API degree) and sulphur content. The reason is it is closely related to the cost of the refining process and the quality and quantity of crude oil production. WTI, with less sulphurous and being light, is superior to Brent. Theoretically, WTI should be paid for with a premium to Brent. But oil price varies not only on substance (quality spread) but also locations (location spread). And

Fattouh (2010) provided evidence that the price differentials between crude oils are stationary from 1997 to 2008. However, WTI has traded below Brent commencing 2010. The main reason is the Shale Revolution has boomed the crude oil supply in the U.S., then the price of WTI declined. And the U.S. has a ban on oil exports, which means WTI is stuck in the local market. It also can be a reason for its price decrease. On the other hand, fears of the closure of the Suez Canal and potential disruption in Brent supplies widened the Brent-WTI spread in 2011. This chapter investigates the difference in volatility dynamics in terms of Brent and WTI under this crucial period.

Another feature in this period applied in this chapter is the crude oil price collapse in 2014. Baumeister and Kilian (2016b) provided quantitative evidence that negative demand shock and positive supply shock contribute to the price decline. And the remaining oil price decline is accounted for by a shock to oil price expectations (lowing the demand for oil inventories) lowed and a shock to the demand for oil (weakening economy). In this chapter, we assess the forecasting ability of different volatility measures through this storm time of the crude oil market.

This chapter aims to investigate volatility estimation and volatility forecasting of Brent and WTI futures prices. Volatility is a critical issue in such fields as risk management, asset allocation, and taking bets on futures volatility. However, volatility cannot be observed directly. Forecasting volatility is crucial for futures investment, and it is also a measure of the potential losses of assets. Besides, volatility can impact on the macroeconomy. Sadorsky (1999) demonstrated that the increase in oil price is linked to inflation and economic recession, which would affect interest rates, exchange rates, and further investment. Therefore, researchers make many efforts to point out how volatility preforms, and many empirical works focus on volatility estimation and forecasting.

Kat and Heynen (1994) found evidence that the stochastic volatility model provides the best predictions for stock indices, whereas, for currencies, the GARCH(1,1) model is the best. Chan and Grant (2016) compared a variety of GARCH and stochastic volatility models for estimating nine series of energy prices, and concluded the stochastic volatility model is favorable to their GARCH counterpart. Moreover, the stochastic volatility model with moving average innovations is the best model for all series. Wei (2012) provided the consistent results in volatility forecasting that the stochastic volatility model outperforms the GARCH-type model. Lehar, Scheicher, and Schittenkopf (2002) argued that the GARCH model dominates both stochastic volatility and the benchmark Black-Scholes model in forecasting intraday FTSE 100 option prices. However, there is limited research focusing on the OVX, GARCH-type, and stochastic volatility model contrast and comparison in terms of Brent and WTI futures market. Furthermore, previous studies focus on model variance, exploring instead of capturing data series stylish behaviour. To my understanding of a good volatility model, fitting the data and forecasting accuracy is essential. This chapter emphasizes the features of the data series in a specific period. And test the forecast ability in price decline time. Further, we consider the over-fitting problem, which is ignored by other researchers.

The volatility in financial time series is not constant over time (heteroscedasticity), and it shows long persistence (volatility clustering). Accurately, crude oil futures prices are characterized by time-varying volatility. The Autoregressive conditional heteroscedasticity (ARCH) model and the generalized autoregressive conditional heteroscedasticity (GARCH) model have performed well in modelling financial data series via adopting conditional variances. The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model is constructed by adding a leverage effect term in the GARCH model. The leverage effect term shows that good news (positive shock) and bad news (negative shock) in the market have different effects on volatility. GARCH-type models can provide accurate out-of-sample one-step forecasts. Meanwhile, the stochastic volatility model is also a popular method to model volatility. In the advanced stochastic volatility model, the 'moving average process' (MA) is added to the return measure equation because there is a long persistence in financial asset return, generally. Concerning the superior stochastic volatility model, the Student's t-distributed error is employed instead of a normally distributed error, because the distributions of financial data always show leptokurtosis and heavy tail characteristics. Additionally, the CBOE Crude Oil Volatility (OVX) index is chosen as a measure for market volatility forecasting because it can represent the market's expectation of stock market volatility. In the forecast part, the proxies of real variance are generated by squared returns. We use six loss functions to evaluate the forecastability of different forecasting models, and furtherly confirm the optimal forecasting model.

Motivated by the previous discussion, this chapter adopts crude oil futures prices to address several research questions. Firstly, we compare and contrast the forecasting volatility of Brent and WTI during the same period when the same model applied. Although Brent and WTI are both the benchmarks in crude oil markets, they are active in different markets, i.e., Europe market and America market, respectively. It is interesting to figure out whether they have consistent responses or not. The results show that Brent and WTI indeed have distinct performances in volatility estimation. WTI is more sensitive to the market spur than Brent, and the fluctuations of WTI last longer than Brent's. On the other hand, there is a strong correlation between WTI and Brent. The movements of Brent and WTI are driven by the same dynamics (Klein, 2018), and the increase in oil price contributes to global market factors (Kang, Kang, and Yoon, 2009).

As previously discussed, secondly, this chapter investigates the difference of volatility dynamics in terms of Brent and WTI, under this crucial period. Another feature in this period applied in this chapter is the crude oil price collapse in 2014. In this chapter, we assess the forecasting ability of different volatility measures through this storm time of the crude oil market. It is the initial time to research crude oil volatility in this aspect.

Thirdly, we compare the more flexible GARCH variant (GJR-GARCH) against the standard GARCH in terms of in-sample estimation and out-of-sample forecasting. Also, the performance of the GARCH model and ARCH model is constructed as well. Generally, models, including more information, usually have stronger predictive power. GJR-GARCH model includes an innovation term compared with the GARCH model, which can indicate the leverage effect. Accurately, it can detect if there are different effects on volatility from upward returns and downward returns. The leverage effect usually is significant in financial data series and provides more accurate predictive results. Similarly, compared to the ARCH model, the GARCH model has an additional term of last period's conditional variance standing for the volatility clustering. Results in this work support that the GJR-GARCH model has superiority because it can extract more information from data. And the leverage effect does exist in crude oil volatility modelling. In the volatility forecasting of the GARCH-type models for Brent, the GJR-GARCH model is the best. By contrast, for WTI, the ARCH model exhibits the most accurate forecast ability.

Fourthly, we test if the stochastic volatility model added an error persistency process (MA(1)) in crude oil return series better than the plain stochastic volatility model. Further, whether Student's t-distribution is a more appropriate error distribution description for MA(1) process, compared with normal distribution? Comparing stochastic volatility models, respectively, with the MA(1) process with normal distribution and MA(1) process with Student's t-distribution can answer this question. Based on the results of stochastic volatility modelling, Student's t-distributed error is also able to provide more accurate estimation results in both Brent and WTI returns. As regards the forecasting of stochastic volatility models, the evidence for the best model for Brent is mixed. In contrast, the plain stochastic volatility model is the best for WTI.

Moreover, the out-of-sample forecasting performance of OVX, GARCH-type models and stochastic volatility models are compared and contrasted with loss function values. GARCH-type models are employed in this chapter because they representatively hold the prior predictive power in the conditional variances. As an alternative method for GARCH-type models, the stochastic volatility models can also predict the time-varying volatility. The differences in the volatility forecast abilities between GARCH-type models and stochastic volatility models will be explored in this chapter. The result answers that GARCH-type models perform better than the stochastic volatility models either in Brent or WTI crude oil futures market. OVX index can provide the optimal forecast in the volatility of Brent futures. By contrast, for WTI, the ARCH model exhibits the most accurate forecast ability.

Finally, to identify the over-fitting problem, the in-sample forecasting and out-of-sample forecasting are compared. The conclusion is the stochastic volatility models suffer from over-fitting. For the other models, the results are mixed. The GJR-GARCH model appears to suffer from over-fitting in one case, but not the other.

With the aforementioned considerations in mind, this study contributes to several aspects as the following discussion. Previous literature rarely focuses on comparing and contrast volatility extracted from different models. To fill the gap, we examine the adequacy of OVX, GARCH-type models, and stochastic volatility models in describing and forecasting crude oil futures volatility behavior. Previous works generally adopted one or two loss functions. Whereas, we include six loss functions in this chapter, which provide more elaborate evidence concerning the predictability of volatility. It gives sufficient guidance for optimal model selection. The next contribution is benefiting from the valuable market information in the period applied in this chapter. One feature is about the price difference between Brent and WTI. After 2010, it shows a divergence in prices of Brent and WTI. Moreover, there was a massive oil price turbulence in 2014. Therefore, the data, including this period, is worth being researched, and this chapter contributes to extracting useful information from the valuable data. The second one is there is a significant price collapse in the crude oil futures market commencing 2014. The majority of previous works just focused on the 2008 crisis. Still, this work can explore model performance in forecasting the severe price decline of crude oil since 2014 as well. It is a crucial and sensitive time in oil volatility research.

The rest of this chapter proceeds as follows. *Section 2* introduces the previous work done by others. *Section 3* briefly presents the data sets we analyze. *Section 4* introduces the basic models and the empirical identification strategy. *Section 5* discusses the primary analyses of the GARCH-type models and stochastic volatility models. Meanwhile, this part presents the results of estimation and forecasting. *Section 6* concludes.

2.2 Literature Review

2.2.1 Different Measures of Volatility

Risk is unobserved, so several kinds of volatility or variances (the square of the volatility) are used to be a representation of this kind of uncertainty in assets. Among different methods measuring the volatility, the most straightforward way is historical volatility calculation. It is well known as the standard deviation (volatility) of a data series. However, it is only used for some fundamental financial problems because of its unpredictability and fixed indicators. The market has unpredictably changed over time, so it is not appropriate or accurate to forecast volatility by historical volatility.

Secondly, implied volatility calculated by options is applied broadly in volatility research, and the VIX index is used as a popular measure for it. Martens and Zein (2002) showed that historically, high-frequent data used by the

GARCH model has superior forecasting capability rather than implied volatility index. The GARCH model typically performs well in the in-sample estimate rather than an out-sample forecast. Comparing with implied volatility, GARCH-type models have more accurate predictive ability. The reason might be that the structure breaking can cause volatility persistence (Agnolucci, 2009). It is robust support for the GARCH model choice of volatility prediction.

Thirdly, realized volatility plays a crucial role in predictive works as the proxy of actual volatility. Szakmary et al. (2003) found the evidence from 35 futures markets, which implied volatility holds optimal unbiased predictive ability comparing historical fluctuations. In their paper, realized volatility provided a standard for comparing-predictive capabilities of implied volatility and historical volatility. Realized volatility is also used to construct models. Corsi (2009) adopted the Heterogeneous Autoregressive model of realized volatility (HAR-RV) in volatility estimation of US dollar to Swiss Franc rate. Haugom et al. (2014a) added implied volatility and other market variables in the HAR-RV model. The result exhibited more accurate predictive ability by adding implied volatility and other variables combinational.

Despite that, conditional variances predicted by GARCH-type models and the volatility extracted in stochastic volatility models are respectively widely adopted in the financial market. The CBOE Crude Oil Volatility (OVX) index is calculated from the option contracts, which can represent the market's expectation of stock market volatility. However, there is limited literature on the comparison of the forecast abilities of the three measures of the volatility. To fill the gap, this chapter examines the volatility behavior in the crude oil market by using OVX, GARCH-type models, and stochastic volatility models.

2.2.2 Previous Research about GARCH-type Models and Stochastic Volatility Models

GARCH-type models are widely used to estimate the volatility of the financial asset and show outstanding performance. It is essential to discuss the importance and history of GARCH-type models here. Starting with the autoregressive moving average (ARMA) model, it holds the stationary property, which is defined such that the variances are constant, and it is uncorrelated with time (Diebold, Kilian, and Nerlove, 2006). These kinds of models perform well on the modelling of a stationary time series with constant volatility. However, an issue will arise when the time series processes are concerned with asset prices in the financial market. The argument is that usually, the variances of this kind of data are time-varying, so the stationary property for the ARMA model is no longer well satisfied. Breaking new ground, Engle (1982) provided the ARCH model in econometrics, firstly, which is superior compared to the ARIMA model in modelling the inflation rate of the UK. Time-changing conditional variances characterize the ARCH model, and the time-changing conditional variances of the ARCH model were the first to model changing variances (Engle, 1982). On this basis, the GARCH model was produced in 1986. Specifically, autoregression of the conditional variances is added to the variances equations. Meanwhile, the method of ARMA quotation is adopted. In the GARCH model, not only is the time-varying uncertainty of the risk expressed in the explanatory variable but also lag(s) of the conditional variances(s) has better explanatory power. There is longer memory representing in the GARCH model due to the conditional variances depending on previous periods (Bollerslev, 1986). Therefore, the GARCH model experientially supports volatility analyses of financial time series. It explains volatility characteristics more accurately, in terms of long memory and time-varying.

There are a large number of extending models based on linear GARCH models, such as non-linear GARCH models and nonparametric GARCH models. There are some discussions concerning the evaluation of GARCH-type models. Non-linear models developed by Agnolucci (2009) perform better than linear models. Meanwhile, it solves the problem in the previous paper of Agnolucci (2009), that the standardization of function affects the volatility forecasting (Wei, Wang, and Huang, 2010). There are similar viewpoints for component generalized autoregressive conditional heteroskedastic (CGARCH) model and fractionally integrated generalized autoregressive conditional heteroskedastic (FIGARCH) model. They are both performing better in the prediction of the data in long memory and persistence. Data persistence is always a crucial characteristic to model (Kang, Kang, and Yoon, 2009). However, the GARCH model can not explain the symmetric effects on volatility. GJR-GARCH model proposed by Glosten, Jagannathan, and Runkle (1993) includes the leverage effect. It is also used in this chapter as an alternative of GARCH model. It is a drawback in this model because, in reality, there are asymmetric effects from positive shocks and negative shocks. There is skewed distribution in oil returns, and inequality is hard to achieve in the real world. In this situation, the Quadratic GARCH model is a better alternative to the traditional GARCH model (Franses and Van Dijk, 1996). Gokcan (2000) forecasted the volatility in the stock market by linear GARCH model and non-linear GARCH models. EGARCH model is focused on among the non-linear GARCH models, which extend Gokcan's work, including EGARCH and GJR-GARCH models from Franses and Van Dijk (1996). In Franses and Van Dijk (1996)'s paper, the GJR-GARCH model was the representative of the non-linear GARCH model.

Researchers also focus on the investigation of some specific relationships between volatility and other macro variables. Multivariate GARCH models are popular in this field. Agnolucci (2009) suggested that a combination of macroeconomic variables and predictive results are both necessary in volatility analysis. Similarly, Gospodinov and Jamali (2015) tested the dynamic response of stock market volatility to monetary policy. They also point out that Federal actions have a significant effect on stock volatility by employing the vector autoregressive GARCH (VAR-GARCH) model. Multivariate volatility models are also recommended to explore the relationship between commodities for constructing an investment strategy portfolio. Volatility estimation is an indispensable element for a diversified portfolio application. Markowitz's approach can minimize risk under a given level of expected returns, and it has become a standard method to control the risk and effectively allocate assets (Markowitz, 1968) efficiently. When there is a negative correlation of volatilities responding to different asset returns, it suggests the diversification can reduce the unsystematic risk. A paper concluded that the Constant Conditional Correlation (CCC)GARCH model, Dynamic Conditional Correlation (DCC) GARCH model, and Baba-Engle-Kraft-Kroner (BEKK) GARCH model, suggested storing more Brent futures than Brent spot to reduce risk (Chang, McAleer, and Tansuchat, 2011).

Besides conditional volatility, stochastic volatility estimated by the stochastic volatility model can also obtain time-varying volatility. This model can be estimated by Markov Chain Monte Carlo (MCMC) methods in the context of Bayesian inference. According to Shephard (2005), the original stochastic volatility model is messy, but several researchers have made contributions in this field (Clark, 1973; Tauchen and Pitts, 1983; Engle, 1982). Modern stochastic volatility models apply continuous-time, and old papers usually adopt discrete time. There are still some innovations in the stochastic volatility model that Shiraya and Takahashi (2015) develop, including a new formula for option pricing in a local stochastic volatility model with jumps. Another application of a stochastic volatility model is the organization of herding behaviour. Babalos, Stavroyiannis, and Gupta (2015) examined the herding behaviours by the evidence from the stochastic volatility model.

Other types of stochastic volatility models are also employed in volatility forecasting. The stochastic volatility diffusion model was used in foreign currency option pricing, and it showed optimal forecasting ability (Melino and Turnbull, 1990). Ozturk and Richard (2015) applied a stochastic volatility model with leverage effect to analyze S&P 500 stock returns of 24 companies from six different sections. It pointed out that financial and energy companies are remarkably different from others. However, it still is demonstrated that there are some connections between these two markets. The stochastic volatility diffusion model was then applied in stock price distribution (Stein and Stein, 1991). Adding moving average terms in the plain stochastic volatility model can improve the predictive power in inflation (Chan, 2013). Applications of stochastic volatility and GARCH model were used for estimation in S&P 500 index daily return and US/Canadian dollar exchange rate, which indicated the GARCH model with the Student's t-distribution held the best performance (Gerlach and Tuyl, 2006). A paper applied several stochastic volatility models to detect if heavy-tail and series dependence exist in exchange rates and silver spot price (Chan and Hsiao, 2013). And this chapter follows the steps of Chan and Hsiao (2013) to analyze the stochastic volatility in the crude oil market.

2.2.3 Empirical Works on Futures Volatility of Crude Oil

Initially, research mainly focuses on volatility modelling of the stock prices and exchange rates. The literature on volatility study has often contributed to researches relative to energy commodities futures, metal commodities futures, and agriculture commodities futures in recent years. Bracker and Smith (1999) applied GARCH, exponential generalized autoregressive conditional heteroskedastic (EGARCH), asymmetric generalized autoregressive conditional heteroskedastic (AGARCH), and GJR-GARCH model in volatility estimation of the copper futures market. Further, Multivariate volatility models which Efimova and Serletis (2014) applied in investigating spillovers and interactions among the U.S. oil market, is at the cutting edge of the natural gas market and electricity markets. At the same time, there are also relationships between volatility and macroeconomic variables. Guo and Kliesen (2005) found that the volatility of the New York Mercantile Exchange (NYMEX) crude oil future has a significant effect on macroeconomic variables. At the same time, terrorist attacks and military conflict can affect volatility. The forecasting ability of the GARCH model is introduced by Marzo and Zagaglia (2010) in the energy market.

Either modelling or forecasting volatility is a significantly crucial topic in financial research. There are still a large number of papers that focus on the oil options market in recent decades. One is about the forecast abilities of long memory autoregressive fractionally integrated moving average (ARFIMA) model, short memory ARMA model, GARCH model, and option implied volatility model (Pong et al., 2004). Fan et al. (2008) studied the 'Value at Risk' of WTI and Brent using a generalized error distribution GARCH-type model. And this HAR-RV-IV-EX model is an improvement of Corsi (2009)'s HAR-RV model in 2009 employed in the crude oil futures market. Agnolucci (2009) demonstrated that GARCH-type models perform better than implied volatility obtained from Black-Scholes formula when involving oil price. Wei, Wang, and Huang (2010) examined Brent and WTI volatility using GARCH-type models and capture long memory asymmetric volatility in the crude oil market. Besides the GARCH model and implied volatility, there are other methods of comparisons. One recent research of Chinese oil futures volatility contained three models: ARCH-type models, stochastic volatility model, and realized volatility model (Wei, 2012). Musaddiq (2012) concluded that GJR-GARCH (1, 2) is the best predictive model in oil futures volatility. Implied volatility, realized volatility, and other explanatory market variables are embedded in the HAR-RV model to forecast the WTI futures volatility (Haugom et al., 2014a). Therefore, GARCH-type models are adopted in this chapter.

Concerning the application of volatility in the crude oil futures market, it is an increasingly valuable research topic. The reason for it is that time-varying volatilities in the financial market usually are used for oil derivative pricing, risk management, and portfolio allocation. This chapter can provide practical suggestions for further volatility research and investment practice. Cong et al. (2008) examined the interactive relationship between the oil price and the Chinese stock market. A paper showed oil volatility has a significant impact on macroeconomic indicators in the Thai economy (Rafiq, Salim, and Bloch, 2009). Du, Cindy, and Hayes (2011) researched the linkage between crude oil volatility and agricultural commodity market. Chang, McAleer, and Tansuchat (2011) generated an optimal portfolio in crude oil spot and futures returns, using a multivariate GARCH model. Arouri, Jouini, and Nguyen (2011) explored portfolio strategy in crude oil and the stock market. They also applied several multivariate models. Sadorsky (2012) developed the research of crude oil and stock volatility, which focused on the stock prices are especially from clear energy companies and technology companies.

2.3 Data and Descriptive Statistics

2.3.1 Background of Brent and WTI

West Texas Intermediate (WTI) crude oil futures were generally recognized as a global benchmark for crude oil pricing, which holds the largest trade volume in the worldwide crude oil futures markets. However, WTI futures prices became lower than the futures price of Brent crude oil (from the North Sea) at the end of 2010, and this trend intensified in 2011 because of Egypt's political instability. Since then, Brent futures have been gradually regarded as another benchmark in the crude oil futures market. WTI and Brent currently have provided the standard and reference in the oil pricing process, and they have made oil trades easier. Therefore, the prices of WTI and Brent are involved in this study.

Specifically, WTI is the underlying commodity of the Chicago Mercantile Exchange Group's oil futures contracts. The delivery point for WTI crude oil is Cushing, Oklahoma, which is a major trading hub and has been the delivery place for over three decades. Cushing has plenty of intersecting pipelines and storage facilities. It provides convenient access for refiners and suppliers, at the same time either inbound to Cushing from all directions or outbound through dozens of pipelines. The apparent advantages of WTI are that they have the most liquidity, most customers, and most transparency. On the other hand, Brent crude oil is also a significant benchmark in the crude oil market worldwide. Brent crude oil is extracted from the North Sea and is also known as Brent Blend, London Brent, and Brent Petroleum. The primary transaction point for Brent is the International Petroleum Exchange.

Brent usually is active in Europe, while America has the highest transaction volume of WTI. Therefore, it is one of the reasons why the macro events have initially and persistently different effects on Brent and WTI.

2.3.2 Descriptive Statistics

OVX index, calculated from the option contracts, provides the market expectation of investors. The OVX is specifically the name for crude oil VIX index. VIX is a measure of the implied volatility index collected from the Chicago Board Option Exchange (CBOE), which is calculated by S&P 500 index options. Investors also call the index the fear index or the fear gauge. Occasionally it has become a proxy for market volatility because it can represent the market's expectation of stock market volatility. The VIX index is quoted in percentage points and represents the annual standard deviation to the expected movement. OVX measures explicitly the market's expectation of near-term volatility of crude oil prices by applying the same methodology as VIX generation. To implement the OVX index, I transfer the data formation from the annual standard deviations in percentages to daily variance.

Brent and WTI, as primary benchmarks, provide price reference in the whole crude oil market, then numerous studies choose them as research objective. The selection in length and frequency of time series data capably affect the performance of modelling. The research concluded that the relatively sufficient data length for GARCH-type model is around ten years if daily prices are applicable. Therefore, daily prices of Brent and WTI crude oil near month futures from January 1, 2004, to June 22, 2015, are involved in this research. The unit of future oil price is dollar per barrel (US/bbl.), and it is available in the DataStream database.

TABLE 2.1: *Summary of data group specification*

Data Groups	Start time	End time
Whole group data of price	01.01.2004	22.06.2015
Whole group data of return	02.01.2004	22.06.2015
Subgroup data for Estimation	02.01.2004	31.12.2013
Subgroup data for Forecast	01.01.2014	22.06.2015

The whole data set includes 2992 observations on daily returns from January 2, 2004, to June 22, 2015. I divide it into two subsamples. From January

2, 2004, to December 31, 2013, is used for in-sample estimation and the remaining part (from January 1, 2014, to June 22, 2015) is for the out-of-sample forecasting. We can see a detailed summary in *Table 2.1*.

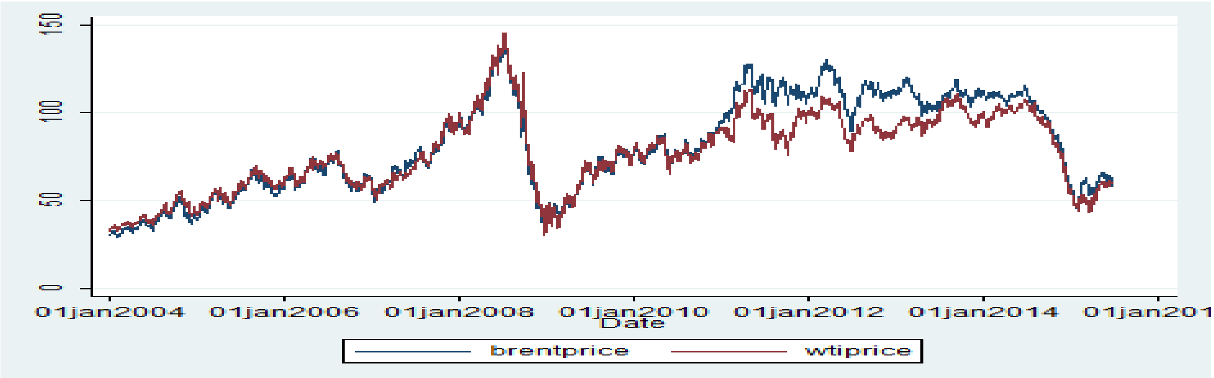

FIGURE 2.1: *Price trend of Brent and WTI volume against time*

Figure 2.1 in the Appendix displays the historical prices of Brent and WTI over the last decade. At the beginning of 2008, the financial crisis knocked 80% off the crude oil market, from nearly 150 dollars per barrel (the highest price point) downward to 30 dollars per barrel. Crude oil price did not rebound until 2009. The two branches of oil prices showed distinct trends from 2011 to 2014. In the period, Brent WTI spread diverged, and the Brent price surpassed the WTI price. During this period, Brent reached a relatively high level of roughly 125 dollars per barrel in the middle of 2011, while WTI reached 110 dollars per barrel. The reason why the Brent price was higher than the WTI's is that the crude oil in North America was oversupplied. After that, Brent and WTI dramatically dropped under 50 dollars per barrel again, because robust global production exceeded demand in the fourth quarter of 2014. Specifically, the output of global crude oil increased. On the other hand, substitutes of crude oil, such as shale gas, decreased the demand for crude oil. At the beginning of 2015, both of them increased to roughly 65 dollars per barrel. It results in Brent WTI spread to be converging from 2014.

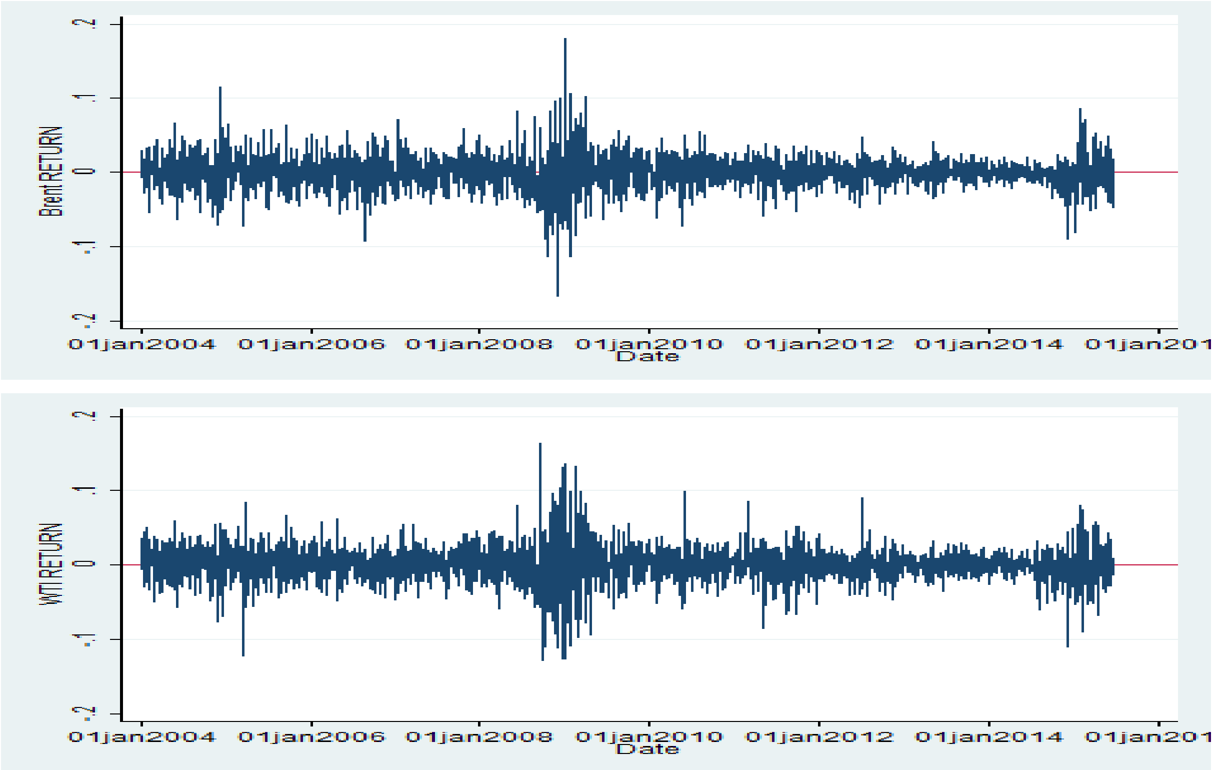

FIGURE 2.2: *Time Path of Brent and WTI return*

Volatility clustering results in volatility persistence, and this is popular in financial assets. To be more explicitly, volatility clustering refers to large changes in observations tend to be followed by large changes, while small changes follow small changes. We can see the high level of volatility persistence in *Figure 2.2*. And it indicates that WTI is more volatility persistent than Brent. Moreover, WTI's price is more sensitive to market fluctuations. For example, WTI had an earlier response to the 2008 financial crisis. As a result, it is more difficult and taking a long time for WTI than Brent to recover from market turbulence.

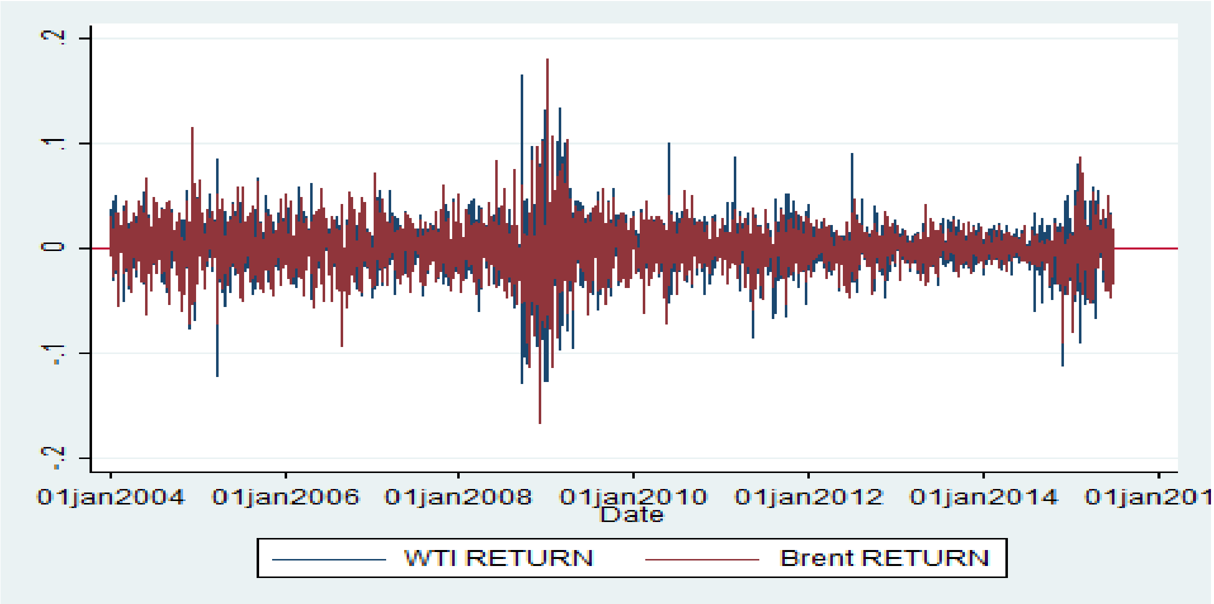

FIGURE 2.3: *Price trend of Brent and WTI volume against time*

Figure 2.3 contrasts the return series of Brent and WTI. The returns of Brent and WTI kept to about 0.05 dollars per barrel most time in the period. During the crisis period, it nearly achieved 0.2 dollars per barrel. Most of the time, WTI returns exceed Brent returns, either on the upward or on the downward.

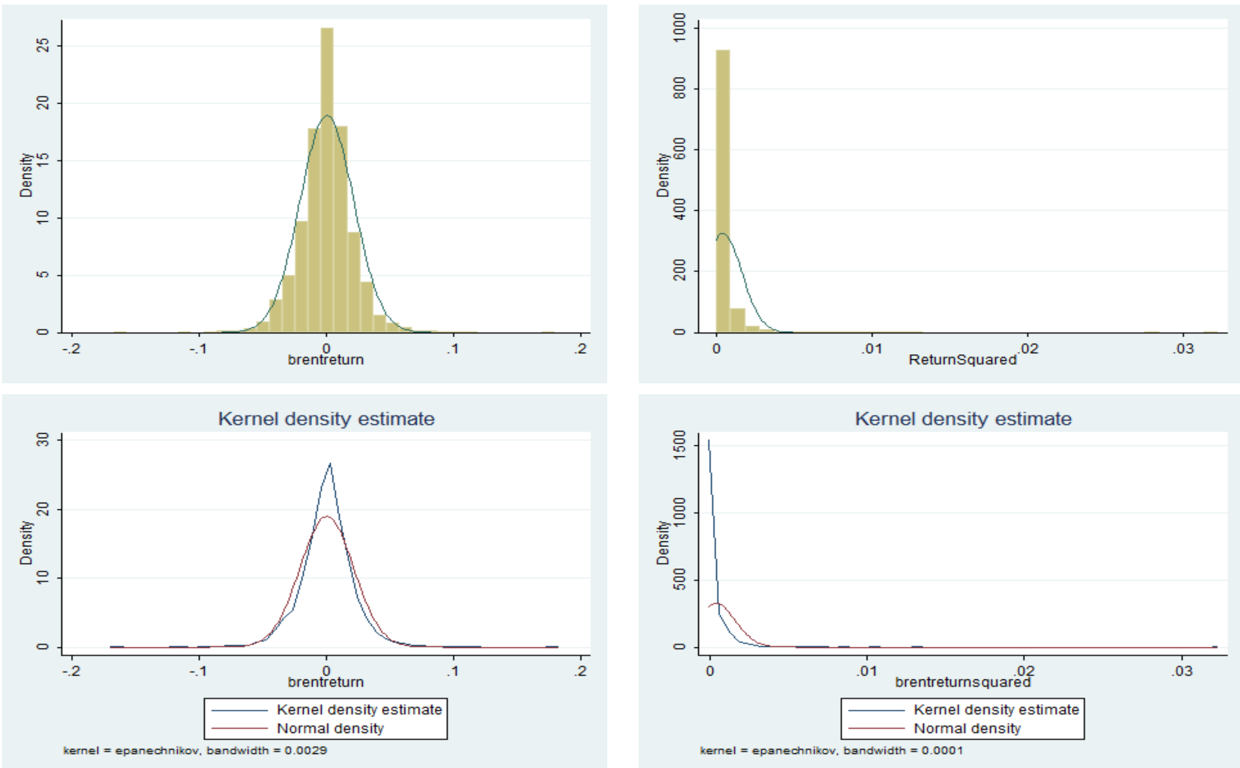

FIGURE 2.4: *Distributions of returns and squared returns in Brent*

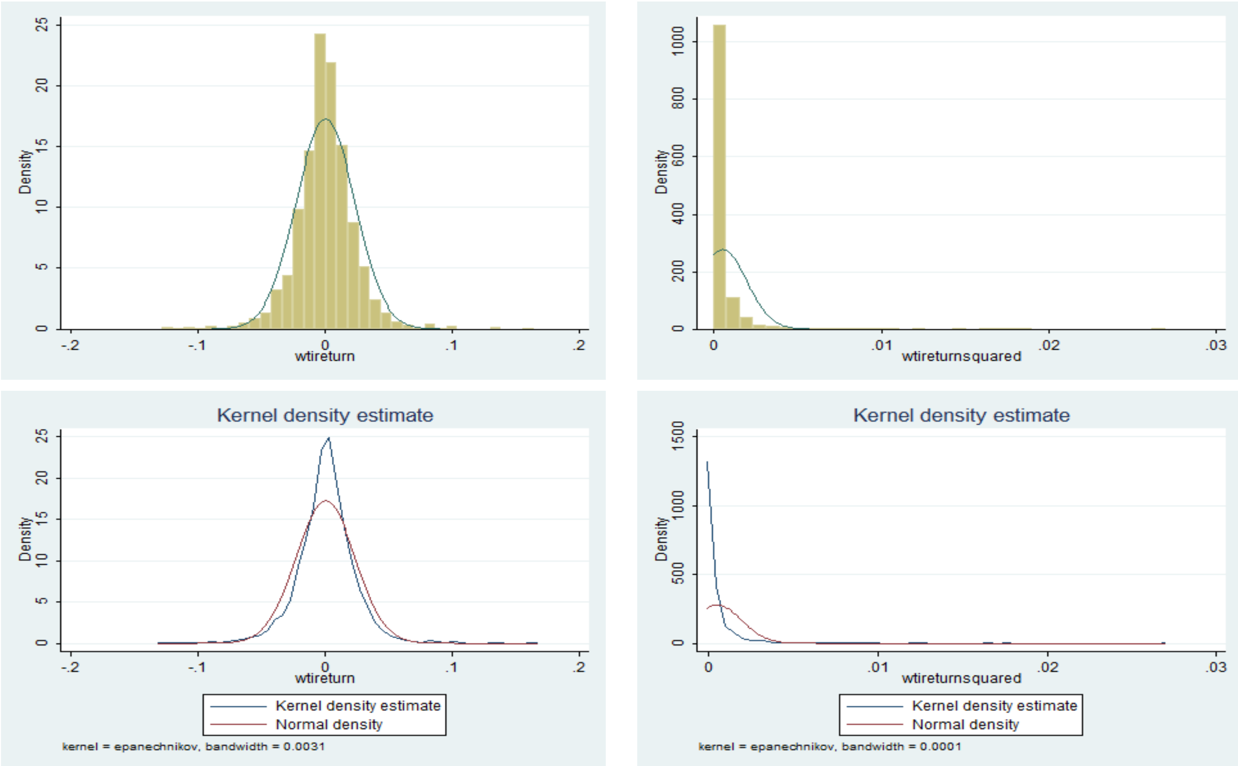

FIGURE 2.5: *Distributions of returns and squared returns in WTI*

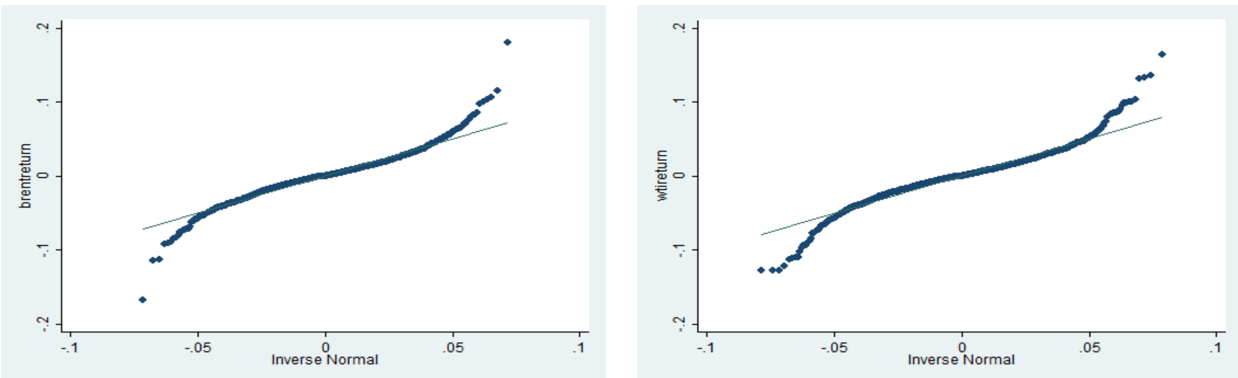

FIGURE 2.6: *Quantile-Quantile (Q-Q) plot with quantiles of normal distribution and return distribution respectively for Brent and WTI*

In *Figure 2.4* and *Figure 2.5*, Brent returns have a higher kurtosis coefficient than WTI. It demonstrates there is a higher probability that more Brent daily returns than WTI's located at the tails in distribution. Based on *Figure 2.6*, we conclude that the tails' distribution of Brent return and WTI return are fatter than the normal distribution tails. (Bradley and Taqqu, 2003) concludes the distribution has characteristics displayed by stylized fact, which is a fat-tailed distribution rather than a normal distribution.

TABLE 2.2: *Descriptive statistics for Brent and WTI returns and squared returns*

	number of observations	Mean(%)	Standard deviation (%)	Maximum	Minimum	Skewness	Kurtosis
Brent Return	2992	0.02232	2.10898	0.17969	-0.16709	0.02937	8.62275
WTI Return	2992	0.02049	2.31476	0.16413	-0.12826	-0.03523	8.20420
Brent Return^2	2992	0.04447	0.12279	0.03228	0	12.54675	257.235
WTI Return^2	2992	0.05357	0.14378	0.02694	0	7.96592	92.0579

Table 2.2 concludes the statistic results. There are 2992 observations in each return series. The means of daily prices are quite small, while the standard deviations are large. The mean of Brent (0.00022) is higher than WTI's (0.0002049), whereas the corresponding standard deviation (0.021) is lower than WTI's (0.023). It is consistent with the previous return plots. The annual volatility is around $0.333(0.021 * \sqrt{252})$ for Brent and $0.365(0.023 * \sqrt{252})$ for WTI. The absolute values of maximum and minimum of Brent returns are higher than WTI's, which means the range of Brent returns is larger than WTI. It provides consistent evidence again that the distribution of Brent return has a fatter tail than WTI's. The skewness of normal distribution is 0, and the kurtosis is 3. The skewness of Brent is positive (skewed to the right), and WTI is negative (skewed to the left). Negative skewness means that the lower tail of the distribution is fatter than the upper tail (Cashin and McDermott, 2002). Under this circumstance, the mean of WTI return is smaller than the median of WTI return, and the median is smaller than the mode. The kurtosis of crude oil futures return is relatively higher than 3 (the kurtosis of standard normal distribution). The peak value of Brent is larger than WTI's. Then we confirm that the leptokurtosis and heavy tails are the statistic characteristics for Brent and WTI futures returns.

TABLE 2.3: *Preliminary tests for Brent and WTI returns*

	Jarque-Bera	Q(4)	Q(20)	ADF	P-P	ARCH test
Brent Return	3941.819**	4.0543	24.4579	−57.724**	−57.744**	51.762**
WTI return	3377.062**	14.7586**	55.1940**	−57.24**	−57.242**	144.435**
Brent Squared Return	8136438**	208.8708**	1188.1045**	−47.876**	−50.257**	0.263
WTI Square Return	1020415**	551.6950**	2616.7092**	−43.663**	−48.432**	41.503**

1

** *indicates rejection at the 1% significance level*
* *indicates rejection at the 5% significance level*

Table 2.3 presents the results of preliminary tests for Brent returns and WTI returns. The Jarque-Bera test (normal distribution test) at the 1% level strictly rejects normality. The Ljung-Box Q-statistic rejects the hypothesis of no serial autocorrelation at 1% significance level up to 20^{th} order except Brent returns (Wei, Wang, and Huang, 2010). Augmented Dickey-Fuller (ADF) test and Phillips-Perron test both conclude rejections of unit root at 1% significant level. Thus, all the variables are stationary and they can be directly employed in estimation without any transformations. ARCH effect holds the null hypothesis (H0) which is no ARCH effect and alternative hypothesis (H1) is that ARCH(1) disturbance exist. And the result supports that the return series have the ARCH effect.

2.4 Methodology Frameworks

2.4.1 GARCH-Type Models

Starting with asset price modelling, x_t denotes the futures price, and it is modelled with its own lagged price x_{t-1} and a drift term α. Asset return is denoted by y_t and it is independent identically distributed. The return series is stationary, so it can be used to model without any transformation. $ln\frac{x_t}{x_{t-1}}$ is the formula to calculate the daily return, which also is as known as the rate of increase in price (Bentes, 2015). In asset volatility modelling, asset returns are the input variable. As previously discussed, volatility can be understood as the uncertainty of price changes. $E(\mu)$ is the expectation value of the mean, which is constant. Volatility is used to measure the uncertainty ($E(\varepsilon_t)$). The asset returns normally is a mean-reverting process, which implies that it is changing over time but around the same average value-μ (Dokuchaev, 2007).

$$x_t = \alpha + x_{t-1} + \delta_t \tag{2.1}$$
$$\Delta \ln(x_t) = y_t = \mu + \varepsilon_t \tag{2.2}$$
$$E(\Delta \ln(x_t)) = E(\mu + \varepsilon_t) \tag{2.3}$$
$$= E(\mu) + E(\varepsilon_t) \tag{2.4}$$

$$y_t = c_y + u_t \tag{2.5}$$
$$Where, u_t \sim N(0, \sigma_t^2) \tag{2.6}$$

Autoregressive Moving Average (ARMA) model is one of the most general frameworks to catch the characteristics of the return series. In this conventional econometric model, homoscedasticity is the classic assumption. Specifically, the variance of the disturbance term is assumed to be constant ($V(u_t) = \sigma_t^2$). Nevertheless, the stylish fact of the return plot demonstrates that the economic time series exhibit periods of unusually large volatility followed by periods of relative tranquility. The characteristic goes against the assumption of homoscedasticity. Therefore, the ARCH model proposed by Engle (1982) and GARCH model developed by Bollerslev (1986) assume volatility with persistency and time-varying characteristics. In other words, heteroscedasticity of financial asset returns in the GARCH model does not need to suffer from the stationary constraint (Engle, 2001).

There is no evidence supporting the autoregressive process in crude oil return series according to the ADF test result. Therefore, I impose the mean term and an unpredictable error term u_t in the return equation, which is following the works of Musaddiq (2012) and Wei (2012). In the empirical literature, order one is the most popular choice for ARCH effect term, GARCH effect term, and leverage effect term. And I also follow this step. The maximum likelihood estimation method is used to estimate GARCH-type models.

Engle (1982) introduced ARCH model to estimate the variance of United Kingdom Inflation. In ARCH model, time-varying variances (conditional heteroscedasticity) are generated ($var(u_t) = \sigma_t^2$). The standard ARCH(1) model is given by

$$\sigma_t^2 = var(u_t/u_{t-1}) \tag{2.7}$$
$$\sigma_t^2 = c_\sigma + au_{t-1}^2 \tag{2.8}$$

Where σ_t^2 denotes the variance of u_t conditional on the value of u_{t-1}. It means the variances in current period depend on the error term of the previous period. Conditional variances demonstrate that variance in the current period (σ_t^2) cannot be known until the previous period's variance (σ_{t-1}^2) is known.

When data series exhibits heteroskedasticity and volatility clustering, GARCH(1,1) model has better fitting performance. Based on Engle's work, Bollerslev (1986) put forward the GARCH model. In this specification, the conditional variance is defined as:

$$\sigma_t^2 = c_\sigma + au_{t-1}^2 + b\sigma_{t-1}^2 \tag{2.9}$$

The assumption for GARCH-type model is σ_t^2 is a nonnegative function. Furthermore, c_σ, a and b are positive and $a + b < 1$ (Bollerslev, 1986). GARCH model allows both autoregressive and moving average components in the heteroscedastic variance. GARCH model is suggested to provide a relatively accurate estimation for conditional variances compared to the ARCH model because the variance in the last period increase the additional explanation ability for volatility modelling. The variances that come out at this period always depend on last period's result. Therefore, GARCH-type models are also expected to provide more accurate out-of-sample one-step forecast than the ARCH model.

Although the GARCH(1,1) model deal with heteroskedasticity and volatility clustering properly, it is not able to explain the leverage effect in financial time series data. To fill this gap, the GJR-GARCH model was proposed by Glosten, Jagannathan, and Runkle (1993). Basically, it allows bad news (negative shocks) and good news (positive shocks) have different effects on volatility. The size of u_t is used to measure the different information of news. It is constructed to model the asymmetric ARCH effect resulting from a weaker influence of positive market shocks ($u_{t-1} \geq 0$). It can capture the asymmetric leverage effect. It encourages researchers to consider psychology expectation of the investors. The GJR-GARCH model is defined as follow:

$$\sigma_t^2 = c_\sigma + au_{t-1}^2 + b\sigma_{t-1}^2 + \lambda u_{t-1}^2 \gamma_{t-1} \tag{2.10}$$

$$Where,\ \gamma_{t-1} = 1, if\ u_{t-1} > 0, \tag{2.11}$$

$$and\ \gamma_{t-1} = 0, if\ u_{t-1} \leq 0 \tag{2.12}$$

It is easy to understand the advantage of the GJR-GARCH model in a stock example. One widespread phenomenon in the financial market is a significant negative relationship between current return and futures volatility in financial assets. And the explanation is the negative shock to the stock price reduces the value of a firm's equity relative to its debt. It means the current return decreases. Therefore, the leverage (debt-to-equity) ratio rises, and the riskiness of stockholders also rise. In other words, asset volatility increases. Then it seems that negative shocks on return have large potential impact on volatility than those positive shocks. Furthermore, GJR-GARCH model counts the asymmetric leverage volatility effect.

Overall, ARCH effect(s) have bursts of volatility followed by recovery, and GARCH effect(s) exhibits persistence in volatility clusters (Bollerslev, 1986). GJR-GARCH term(s) represents the impact of negative market shock. GARCH model and GJR-GARCH model are extending frameworks on the basis of the ARCH model, and they are expected to exhibit optimal estimate and forecasting abilities, which is evidenced by (Kang, Kang, and Yoon, 2009).

2.4.2 Stochastic Volatility Models

As an alternative to GARCH models, the stochastic volatility model is applied by voluminous previous works to identify time-varying volatility. The main feature of GARCH-type models is that the conditional variance of return can be estimated if previous returns are observed. The definition of return in the GARCH model is explicit, but in the stochastic volatility model, it is indirect via the structure of the model (Shephard, 2005). Taylor (1982) defined that ε_t decides the sigh of return and μ_t determines volatility clustering and fat tails in the marginal distribution (Shephard, 2005).

Financial time series data generally exhibit properties departing from classic assumptions of series independence and normality in economic time series. Chan and Hsiao (2013) applied a variety of highly flexible stochastic volatility models to estimate the volatilities of exchange rates and silver spot prices. Stochastic volatility models in the chapter are used to capture the prominent features in financial data series regarding volatility clustering, heavy-tailedness, and serial dependence. Based on the data description, daily return series of crude oil is observed volatility clustering, leptokurtosis, and serial dependence. In this chapter, I follow the same steps to apply the stochastic volatility models to analyze the volatility of Brent and WTI crude oil futures returns.

The plain vanilla stochastic volatility model is the standard stochastic volatility model in variety stochastic volatility model. The assumption of the measurement equation is constant mean and series independent errors. With regards to the next two kinds of models, more explanatory indicators which can simulate more complicated markets are added to these models. It makes a closer modelling to the real market. This model can be estimated by Markov Chain Monte Carlo (MCMC) methods in the context of a Bayesian inference. There are 20000 draws from the posterior distribution using Gibbs sampler, after a burn-in period of 1000.

$$y_t = \mu_y + u_t \tag{2.13}$$

$$u_t = exp(\frac{h_t}{2})\varepsilon_t \tag{2.14}$$

$$h_t = \mu_h + \phi_h(h_{t-1} - \mu_h) + \zeta_t \tag{2.15}$$

$$Where,\ \varepsilon_t \sim N(0,1),\ \zeta_t \sim N(0,{\sigma_h}^2) \tag{2.16}$$

In the return measurement equation, μ_y denotes the constant conditional mean. The conditional variance of y_t is $Var(y_t \mid h_t) = exp(h_t)$, and h_t is called log-volatility. ε_t is independent of ζ_t in any leads and lags. I assume $\mid \phi_h \mid < 1$ and the states h_t are initialized with $h_1 \sim N(\mu_h, \sigma_h^2/(1-\phi_h^2))$. For identification, the invertibility condition is imposed-the roots of the characteristic polynomial associated with MA coefficients $\psi = (\psi_1, \ldots, \psi_q)'$ are all outside the unit circle.

The assumption of the model specification is prior distribution for μ_h, ϕ_h and σ_h^2 are independent.We consider the independent prior distribution below:

$$p(\mu_h, \phi_h, \sigma_h^2) = p(\mu_h)p(\phi_h)p(\sigma_h^2) \tag{2.17}$$

$$\mu_h \sim N(\mu_{h0}, V_{\mu_h}), \phi_h \sim N(\phi_{h0}, V_{\phi_h})1(\mid \phi_h \mid < 1), \sigma_h^2 \sim IG(\nu_h, S_h) \tag{2.18}$$

Plain stochastic volatility model assumes the errors in the measurement equation are serially independent given the log-volatilities. In this case, errors exhibit persistency under the real market circumstance. Stochastic volatility model with an MA(1) process for normally distributed error is an extension model of the standard stochastic volatility model, which assumes moving average errors in the return equation. Here, u_t shows the market serially dependency.

$$y_t = \mu_y + u_t \tag{2.19}$$

$$u_t = exp(\frac{h_t}{2})\varepsilon_t + \psi_1 exp(\frac{h_{t-1}}{2})\varepsilon_{t-1} + \ldots + \psi_q exp(\frac{h_{t-q}}{2})\varepsilon_{t-q} \tag{2.20}$$

$$h_t = \mu_h + \phi_h(h_{t-1} - \mu_h) + \zeta_t \tag{2.21}$$

$$Where,\ \varepsilon_t \sim N(0, 1), \zeta_t \sim N(0, {\sigma_h}^2) \tag{2.22}$$

ε_t and ζ_t are independent of each other. As before, we assume $\mid \phi_h \mid < 1$ and the states h_t are initialized with $h_1 \sim N(\mu_h, \sigma_h^2/(1 - \phi_h^2))$. Under the moving average variant, the conditional variance of y_t is given by

$$Var(y_t|\mu, \boldsymbol{\psi}, \boldsymbol{h}) = exp(h_t) + \psi_1^2 exp(h_{t-1}) + \ldots + \psi_q^2 exp(h_{t-q}) \tag{2.23}$$

The conditional variance through two channels to be time-varying. The first one is moving average of the $q + 1$ most recent variances $exp(h_t), \ldots, exp(h_{t-q})$. And the second one is the log-volatilities h_t is in a stationary AR(1) process. The difference from the standard stochastic volatility model is that y_t is no longer serially independent (even after conditioning on the log-volatilities). The conditional autocovariances are given by

$$Cov(y_t, y_{t-j} \mid \mu, \psi, h) = \sum_{i=0}^{q-j} \psi_{i+j}\psi_i exp(h_{t-i}) \tag{2.24}$$

where $j = 1, \ldots, q$ and 0 for $j > q$, and $\phi_0 = 1$. The autocovariances of y_t are also time-varying due to the presence of the time-varying log-volatility.

To complete the model spesification, we assume independent prior distributions for μ, ψ, μ_h, ϕ_h and σ_h^2. We assume a multivariate normal prior with support in the region where the invertibility conditions on ψ hold. For other model parameters, we assume the following independent prior distributions

$$p(\mu, \psi, \mu_h, \phi_h, \sigma_h^2) = p(\mu)p(\psi)p(\mu_h)p(\phi_h)p(\sigma_h^2) \tag{2.25}$$

$$\mu \sim N(\mu_0, V_\mu), \mu_h \sim N(\mu_{h0}, V_{\mu_h}), \phi_h \sim N(\phi_{h0}, V_{\phi_h})1(\mid \phi_h \mid < 1), \sigma_h^2 \sim IG(\nu_h, S_h) \tag{2.26}$$

The normal distribution is inappropriate to characterise the presence of outliers, as the financial assets returns always have the characteristics of high-kurtosis and fat-tails. Therefore, Student's t-distribution is applied to address this issue.

$$y_t = \mu_y + u_t \tag{2.27}$$

$$u_t = exp(\frac{h_t}{2})\lambda_t^{1/2}\varepsilon_t + \psi_1 exp(\frac{h_{t-1}}{2})\lambda_{t-1}^{1/2}\varepsilon_{t-1} + \ldots + \psi_q exp(\frac{h_{t-q}}{2})\lambda_{t-q}^{1/2}\varepsilon_{t-q} \tag{2.28}$$

$$h_t = \mu_h + \phi_h(h_{t-1} - \mu_h) + \zeta_t \tag{2.29}$$

$$Where, (\lambda_t|\nu) \sim inverse\ gamma(\nu/2, \nu/2), \varepsilon_t \sim N(0,1), \zeta_t \sim N(0, {\sigma_h}^2) \tag{2.30}$$

ε_t, ζ_t and λ_t are independent of each other. As before, we assume $\mid \phi_h \mid < 1$ and the states h_t are initialized with $h_1 \sim N(\mu_h, \sigma_h^2/(1-\phi_h^2))$. $\lambda_t^{1/2}\varepsilon_t$ has a standard Student's t-distribution with degree of freedom parameter ν. Koop, Poirier, and Tobias (2007), Nakajima and Omori (2009), and Wang, Chan, and Choy (2011) adopt the consistent stochastic volatility spesification with Student's t-distributed errors. The Student's t-distribution can be written as a mixture of normal distribution.

To complete the model specification, we assume independent prior distributions for μ, ν, ψ, μ_h, ϕ_h and σ_h^2. ν follows a uniform distribution with mean $\bar{\nu}/2$ and support $(0, \bar{\nu})$. For other model parameters, we assume the following independent prior distributions

$$p(\mu, \nu, \psi, \mu_h, \phi_h, \sigma_h^2) = p(\mu)p(\nu)p(\psi)p(\mu_h)p(\phi_h)p(\sigma_h^2) \tag{2.31}$$

$$\mu \sim N(\mu_0, V_\mu), \nu \sim U(0, \bar{\nu}), \mu_h \sim N(\mu_{h0}, V_{\mu_h}), \tag{2.32}$$

$$\phi_h \sim N(\phi_{h0}, V_{\phi_h})1(\mid \phi_h \mid < 1), \sigma_h^2 \sim IG(\nu_h, S_h) \tag{2.33}$$

2.4.3 Forecasting Methodology and Evaluation Methodology

The rolling window method is adopted to obtain the out-of-sample forecasting volatility, which is following the studies of Lv (2018), Sévi (2014), Haugom et al. (2014b). As aforementioned, the forecast evaluation spans the period from January 1, 2014, to June 22, 2015. The rolling window length is 2608. For each next period forecasting, the estimation subsample is rolled forward by adding one new day in and dropping the most distant day off. The forecasting processes gain time-varying parameters and do not overlap, and the sample size remains fixed. I also present the loss function values based on the in-sample forecast to identify over-fitting.

Andersen, Bollerslev, and Lange (1999) concludes that it is an effective and efficient way to evaluate predictive capacity by using loss functions. However, Lopez et al. (2001) discuss that there is not a most appropriate evaluation of volatility forecast. Therefore, I adopt six loss functions in this chapter rather than a single selection.Following the works of Kang, Kang, and Yoon (2009), Wei, Wang, and Huang (2010), Sadorsky (2006). Squared returns is calculated to apply in the loss functions as the proxy variances and denoted as σ_t^2 hereafter. The forecasting variance (squared volatility) is indicated by $\tilde{\sigma}_t^2$. N is the number of forecasting data. In principle, the smaller value obtained using loss functions, the better forecasting accuracy it is achieving.

Following the steps of Patton (2011), we evaluate the forecast ability mainly reference the MSE value and GMLE value, as we use imperfect proxy variance when compare volatility forecasts.

Mean Square Error Function

$$MSE = \frac{1}{N}\sum_{1}^{N}\left(\sigma_t^2 - \tilde{\sigma}_t^2\right)^2 \tag{2.34}$$

Mean Absolute Error Function

$$MAE = \frac{1}{N}\sum_{1}^{N}\left|\left(\sigma_t^2 - \tilde{\sigma}_t^2\right)\right| \tag{2.35}$$

Mean Squared Error adjusted for Heteroskedasticity Function

$$HMSE = \frac{1}{N}\sum_{1}^{N}\left(1 - \frac{\sigma_t^2}{\tilde{\sigma}_t^2}\right)^2 \tag{2.36}$$

Mean Absolute Error adjusted for Heteroskedasticity Function

$$HMAE = \frac{1}{N}\sum_{1}^{N}\left|1 - \frac{\sigma_t^2}{\tilde{\sigma}_t^2}\right| \tag{2.37}$$

Logarithmic Loss Function

$$LL = \frac{1}{N}\sum_{1}^{N}\left(\ln\left(\sigma_t^2\right) - \ln\left(\tilde{\sigma}_t^2\right)\right)^2 \tag{2.38}$$

Gaussian Quasi-Maximum Likelihood Function

$$GMLE = \frac{1}{N}\sum_{1}^{N}\left(\ln\left(\tilde{\sigma}_t^2\right) + \frac{\sigma_t^2}{\tilde{\sigma}_t^2}\right) \tag{2.39}$$

Where, $\tilde{\sigma}_t^2$ is the forecast variance and σ_t^2 is proxy variance.

MSE is the average squared difference between the proxy of actual variances and the corresponding predictive variances. Squared returns are used to be the proxy variances. With the same principle, MAE is the absolute value of the average difference. Generally, other extensional models are all based on these two loss functions. HMSE is the squared difference between one and the ratio of proxy variances and estimation variances. Koopman, Jungbacker, and Hol (2005) apply MSE and MAE to measure the forecasting ability of GARCH-family models and implied-type models in the financial market. And Liu and Wan (2012) examines the predictive abilities of GARCH-class models by using loss functions.

Instead of squared value, HMAE operates absolute value. Penalize variance asymmetrically forecasts the results in the Logarithmic loss function, which was employed by Pagan and Schwert (1990). It needs to be acknowledged that the function will be inaccurate when the actual variance is too low because the function will reach a huge value. HMSE (Bollerslev and Ghysels, 1996) is a better transformation of Logarithmic loss function. Gaussian quasi-maximum likelihood function was suggested by Bollerslev, Engle, and Nelson (1994) to employ in estimating GARCH models.

Logarithmic loss function penalizes the inaccurate forecast variances because when real variances are small, its logarithm value will get far away from zero. It exaggerates the gap between proxy values and predictive ones. Gaussian quasi-maximum likelihood (GMLE) function was implicitly recommended by Bollerslev, Engle, and Nelson (1994) and it was also mentioned again in Lopez and Walter (2000). GMLE index holding a lower value indicates that the model has better estimation ability (Huang, Wang, and Yao, 2008). MSE is the average squared difference between the proxy of actual variances and the corresponding predictive variances. Squared returns are used to be the proxy variances. With the same principle, MAE is the absolute value of the average difference. Generally, other extensional models are all based on these two loss functions. HMSE is the squared difference between one and the ratio of proxy variances and estimation variances. Koopman, Jungbacker, and Hol (2005) apply MSE and MAE to measure the forecasting ability of GARCH-family models and implied-type models in the financial market. And Liu and Wan (2012) examines the predictive abilities of GARCH-class models by using loss functions.

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2.5 Empirical Results

2.5.1 Estimation Results for Different Volatility Models

In this section, I discuss the in-sample estimation results and compare the out-of-sample forecast abilities in terms of OVX index, GARCH-type models, and stochastic volatility models. Daily returns of Brent and WTI crude oil futures are employed, from 2^{nd} January 2004 to 31^{th} December 2013, is applied in GARCH-type models and stochastic volatility models. There are 2608 observations in total. The mean of Brent daily return is 0.02232%, and WTI return is 0.02049%, which is smaller than the constant in return measurement equations in GARCH-type models and stochastic volatility models. I use a continuous and an error term to model the return measurement equation. I do not include the AR(1) autoregressive process in the return measurement equation, because the return series is stationary and directly applicable.

2.5.1.1 GARCH Models Estimation Results

Table 2.4 presents the in-sample estimation results for the three GARCH-type specifications discussed in Section 4. In the ARCH(1) model, the ARCH effect parameter a is strongly significant at 1% level in each group. The parameter implies when the last period's squared residual (a_{t-1}^2) changes one unit, the conditional variance of Brent in this period will roughly change 10% of it (a_{t-1}^2). For WTI, the change is relatively higher getting 26%. The squared errors of Brent exhibit high autocorrelation than WTI. Also, ARCH term detects the feature well.

Correspondingly in the GARCH(1,1) model, the ARCH effect parameter of Brent (0.037) is also smaller than WTI's (0.055). Additionally, the GARCH effect is 0.95 of Brent and 0.93 of WTI. It strongly shows series autocorrelation in conditional variances. The estimation results of the GARCH term indicate there is a significant and high level of volatility persistence in Brent and WTI futures market. Moreover, the effect is slightly stronger in Brent than it in WTI. In GARCH specification, both GARCH effect and ARCH effect in Brent and WTI estimations are strongly significant at 1% level.

TABLE 2.4: *In-sample estimation results of GARCH-type models*

	Brent			WTI		
Parameter	ARCH(1)	GARCH(1,1)	GJR-GARCH(1,1,1)	ARCH(1)	GARCH(1,1)	GJR-GARCH(1,1,1)
Mean equation						
c_y	8.15E-04	7.03E-04	4.23E-04	1.09E-03	6.96E-04	3.87E-04
	(0.049)	(0.043)	(0.223)	(0.006)	(0.048)	(0.284)
Variance equation						
c_σ	4.15E-04	1.21E-06	1.33E-06	4.05E-04	4.04E-06	3.89E-06
	(0.000)	(0.000)	(0.015)	(0.000)	(0.001)	(0.001)
a	0.1013	0.0376	0.0579	0.2632	0.0545	0.0753
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
b		0.9599	0.9625		0.9375	0.9420
		(0.031)	(0.000)		(0.000)	(0.000)
λ			-0.0470			-0.0505
			(0.000)			(0.000)
Log-likelihood	6336.696	6598.716	6613.514	6202.461	6473.645	6483.654
AIC	-12667.39	-13189.43	-13217.03	-12398.92	-12939.29	-12957.31
BIC	-12649.79	-13165.97	13187.7	-12381.32	-12915.83	-12927.98
Observations	2608	2608	2608	2608	2608	2608
Degree of freedom	3	4	5	3	4	5

Turning to the estimation result for Brent and WTI by using GJR-GARCH(1,1,1) model. ARCH effect parameter of a_{t-1}^2 is roughly 0.058 for Brent and 0.075 for WTI. Brent still exhibits a slightly stronger GARCH effect (0.96) than WTI (0.94). Particularly, the TARCH term in this model represents the leverage effect. The coefficient of the TARCH term is expected to be negative in the analysis of financial assets. It implies there are smaller effects on the conditional variance (σ_t^2) if there is a positive shock on the market in the last period than those negative shocks. In other words, a negative effect will be on conditional variances (σ_t^2) if there is a decline in return in the last period. Larger return decrease in the last period will trigger higher conditional variance this period. There will be a total effect of 0.011 (0.058 minuses 0.047) of last period's squared residual on this period's variance if there is a non-negative shock ($a_{t-1} \geq 0$) on the Brent crude oil market. Respectively, the effect increases to around 0.058 if there is a negative shock from the market. Concerning WTI, there will be a total effect of 0.025 (0.075 minus 0.05) of last period's squared residual on this period's variance if there is increasing return in the last period. Respectively, the effect turns to around 0.075 if there is a decreased return in the last period. The negative impact is nearly triple as the positive impacts in WTI. TARCH term can distinguish the upwards and downwards of returns in the crude oil market, which has distinct influences in the conditional variances. It demonstrates that a strongly significant leverage effect exists in Brent and WTI volatility modelling. All the parameters are strongly significant at 1% level. What is of interest, is that if either positive effect or negative effect comes from the market, conditional variances of WTI are more sensitive than Brent ones.

In conclusion, all the effects of GARCH-type models are strongly significant at 1% level except for the GARCH effect being significant at 5% level in the GARCH(1,1) model of Brent estimation. In the ARCH model, the ARCH effect in WTI is twice as much as Brent's. However, ARCH effect parameters are lower than the corresponding ones in the GARCH model and GJR-GARCH model, which is the same situation in Brent and WTI. The ARCH effect is stronger in WTI modelling than Brent's. Conversely, Brent exhibits stronger ARCH and GARCH effect than WTI. It means the last period's squared error term and conditional variance have more influence on this period's conditional variance in Brent than WTI. In the GJR-GARCH model, the ARCH effect and GARCH effect exhibits a similar characteristic as previously discussed concerning TARCH. Larger leverage effect exists in WTI model than Brent model, which implies downside price change will have more impact on WTI than Brent. Notably, negative shocks will increase more volatility in WTI futures price than Brent counterpart. In a nutshell, the leverage effect in the WTI futures market is stronger than the Brent market. We can also say that the investors holding WTI are more sensitive to the negative news (negative shocks in the market) than the Brent futures holders.

I use the information criterion of AIC and BIC to test the preferred model, as the ARCH(1) model is nested in the GARCH(1,1) model and the GARCH(1,1) is nested in the GJR-GARCH(1,1,1) model. The selection principle is the minimum value (more negative) means the best goodness of fit of the model. Both information criterion in *table 2.4* support that the GJR-GARCH(1,1,1) model performed the best.

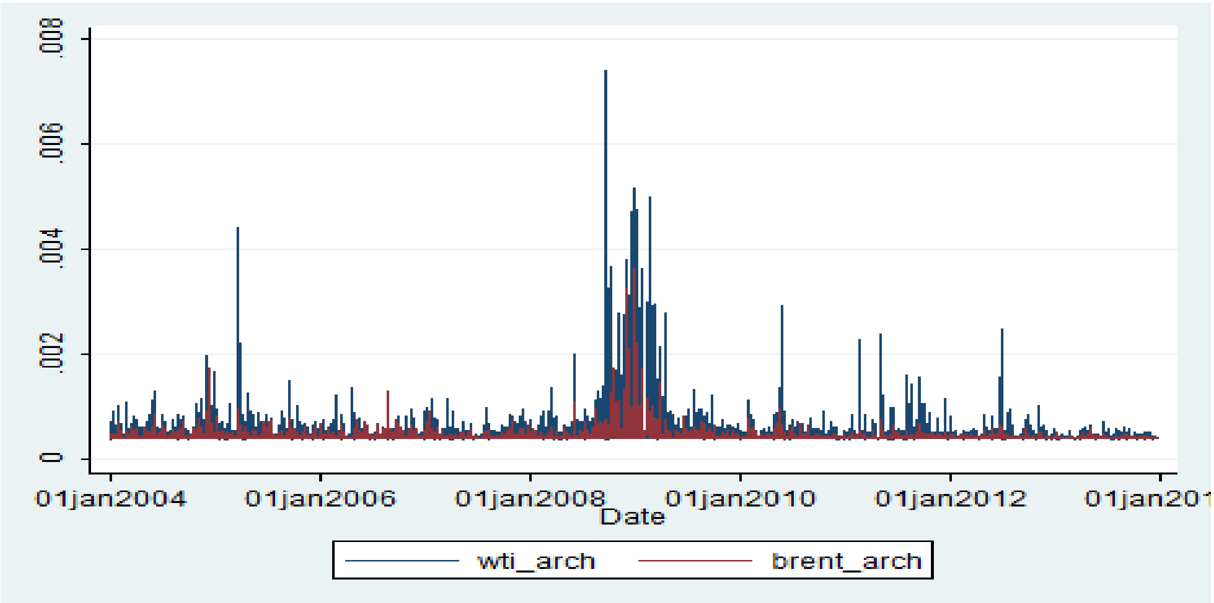

FIGURE 2.7: *Conditional variances from ARCH(1) model for WTI and Brent*

According to *Figure 2.7*, Brent and WTI have similar patterns for the conditional variances extracted from the ARCH(1, 1) model. It illustrates WTI has higher conditional variances than Brent's in the same period. Rationally, the variances show a rapid increase during the financial crisis in 2008. What is most impressive, is that the conditional variance of WTI always has an earlier reaction to stimulations, which is even stronger and more persistent than Brent. For example, in April 2008, the WTI variance reached the highest point of nearly 0.08 in the recent decade. It should be the earliest reaction to the crisis. After one month, there is a similar increasing trend of variances for Brent. There is some volatility clustering of WTI standing during the period, but it does not correspondingly appear in Brent's variance. It also means compared to Brent, WTI is more sensitive to market changes.

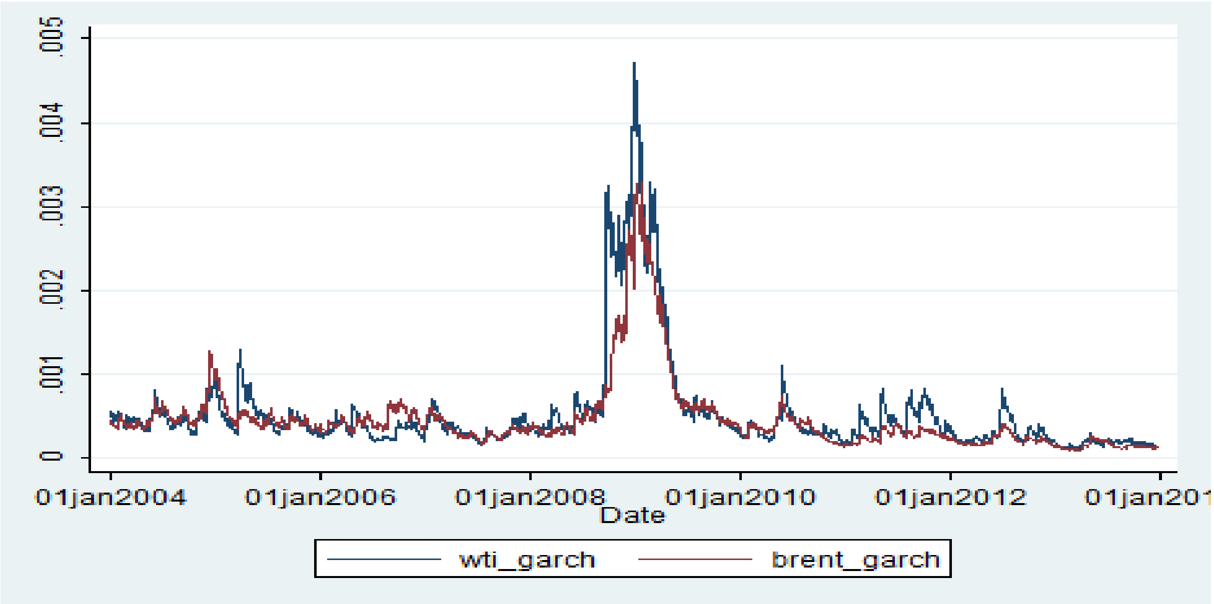

FIGURE 2.8: *Conditional variances from GARCH (1, 1) model for WTI and Brent*

Based on *Figure 2.8*, it shows the broadly same changes in both Brent and WTI figures. When the financial crisis in 2008 hits, WTI had an earlier response to the vulnerable market. During the crisis period, the conditional variance of WTI also shows higher volatility persistence and more turbulence than Brent. The highest variance level in WTI is nearly 0.005, larger than the 0.0035-the highest level of Brent in January 2009. The conditional variance of WTI still tends to be larger and keeps longer than Brent's.

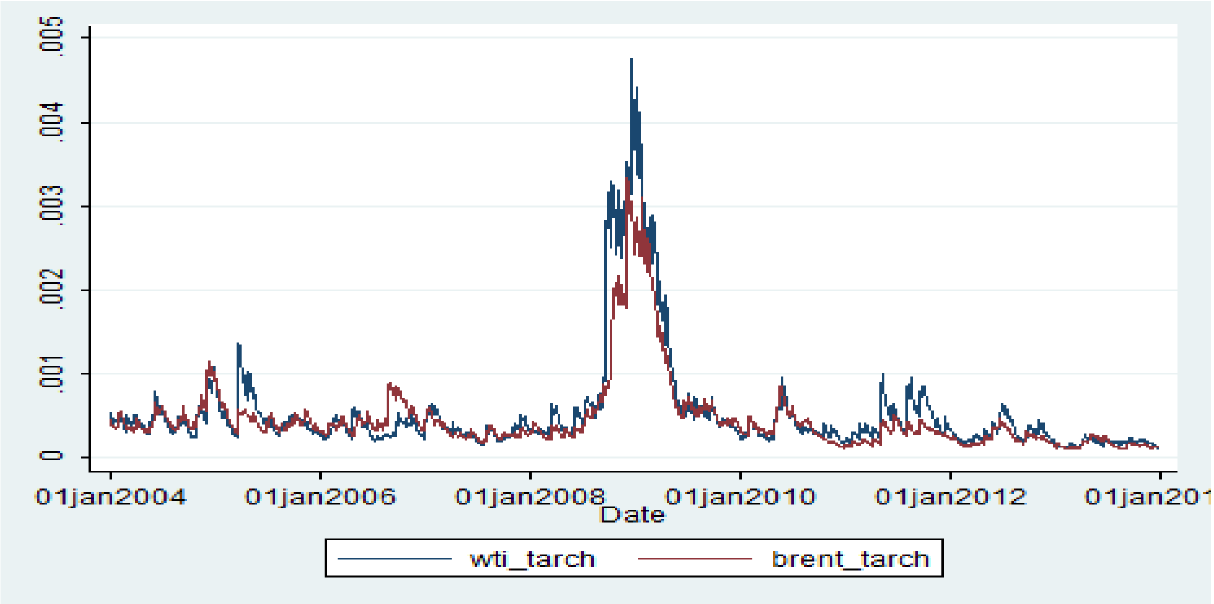

FIGURE 2.9: *Conditional variances from GJR-GARCH (1, 1, 1) model for WTI and Brent*

The *Figure 2.9* shows that the conditional variance of WTI and Brent extracted from the GJR-GARCH(1,1,1) model exhibit similar patterns as well. At the early years of the whole period, it explores several unexpected changes. Roughly in the first half of 2005, WTI shows an obvious higher upward than Brent. After it, at the end of 2006, in contrast, Brent illustrates bigger volatility than WTI. But, the conditional variances of WTI usually are higher than Brent, especially during the 2008 crisis. The highest point of WTI is nearly 0.005, and the corresponding one for Brent is around 0.0035. WTI is the first one to react to the crisis and the last one to recover from the crisis. WTI variances are more persistent than Brent variances.

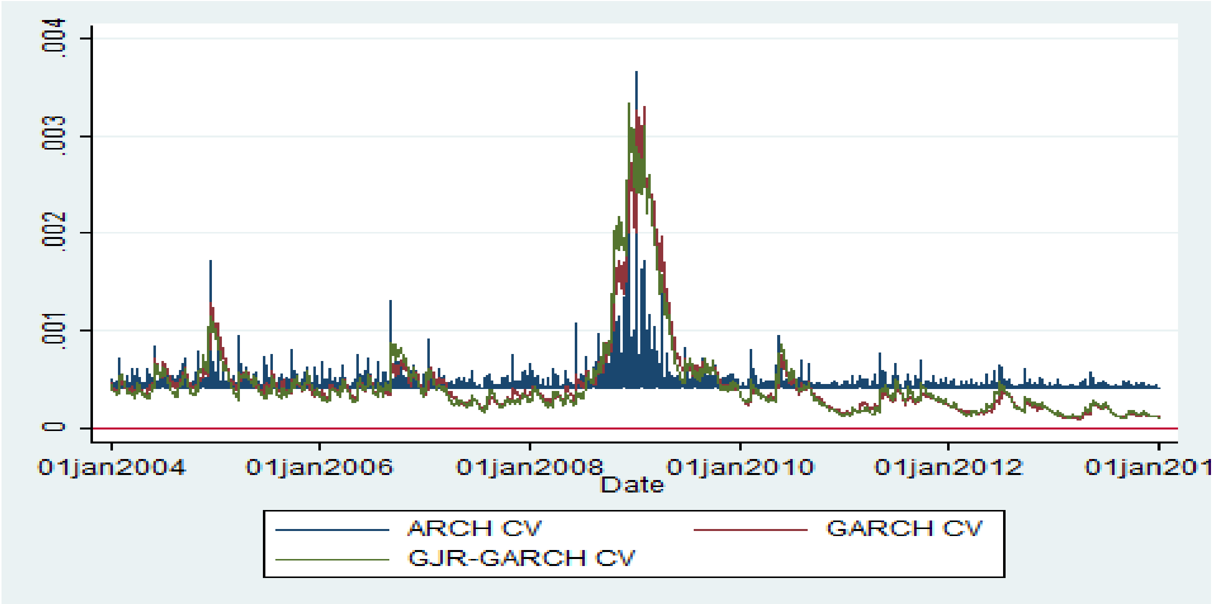

FIGURE 2.10: *In-Sample predict in one-step for Brent crude oil future*

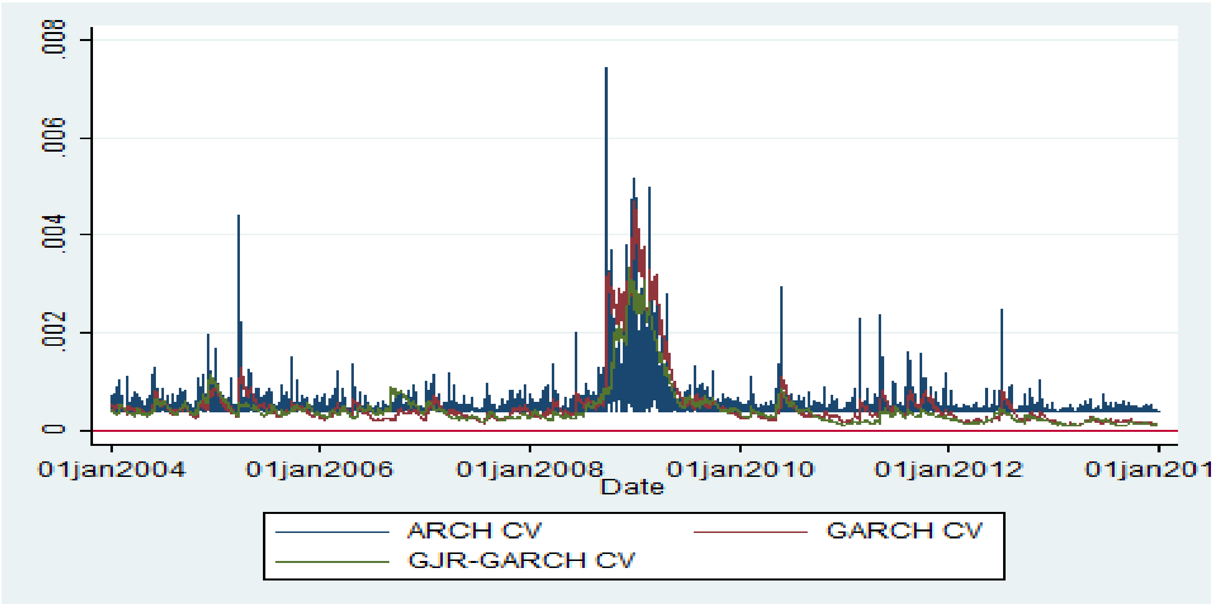

FIGURE 2.11: *In-Sample predict in one-step for WTI crude oil future*

Figure 2.10 shows the one-step in-sample forecast of Brent conditional variances, which are responding extracted from the ARCH(1), the GARCH(1,1) and the GJR-GARCH(1,1,1) model. The conditional variance from the ARCH model exhibits more spikes than the GARCH model and the GJR-GARCH model, but less volatility persistence is predicted by the ARCH model. It seems like oscillation takes more time to recover in the GARCH model and GJR-GARCH model. In general GARCH and GJR-GARCH model provide very similar tendencies, except the GJR-GARCH model, predict higher variances at the beginning time of the crisis.

Based on the combining conditional variances in WTI (*Figure 2.11*), the high spikes still present in the conditional variance extracted from the ARCH model. Much larger conditional variances are predicted of WTI than Brent. What is the most attractive sign, is a spike of 0.008, which shows the first response of crude oil change to the crisis by ARCH model. GARCH model usually illustrates slight larger volatility than the GJR-GARCH model during crisis time. The crisis impact lasts from the middle of 2008 to the interim of the next year. GARCH model predicts the highest point of conditional variances being nearly 0.045, and the homologous one in GJR-GARCH model is just above 0.003.

2.5.1.2 Stochastic Volatility Models Estimation Results

TABLE 2.5: *In-sample estimation results of Stochastic Volatility models*

	Brent			WTI		
Parameter	Plain SV	SV MA Normal	SV MA Student's t	Plain SV	SV MA Normal	SV MA Student's t
μ_y	8.69E-04	8.75E-04	8.76E-04	7.74E-04	7.76E-04	7.77E-04
μ_h	-7.6047	-7.5986	-7.6514	-7.5428	-7.5205	-7.5332
ϕ_h	0.9882	0.9880	0.9880	0.9885	0.9888	0.9889
σ_h^2	0.0081	0.0082	0.0078	0.0095	0.0092	0.0088
ψ		0.0051	0.0058		-0.0363	-0.0349
ν			44.9383			44.5402

Table 2.5 shows the estimation results for the stochastic volatility models. Firstly, I compare the estimation results of the plain stochastic volatility model in terms of Brent and WTI futures returns. The average daily Brent return is estimated to be 8.69E-04. The posterior mean of the AR(1) coefficient of the state equation is 0.9882, which indicates a relatively high level of persistence. It is also the explanation for volatility clustering in the Brent futures market. The expectation of WTI daily return is 7.74E-04, which is lower than Brent (8.69E-04). Conversely, the mean of log-volatility of WTI is higher. The persistency of log-volatility of WTI is nearly the same as Brent's. The variance of log volatility is higher in WTI than Brent, which means WTI volatility

changes in a larger range than Brent. In other words, there is larger uncertainty in WTI investment compared to Brent.

Secondly, the model we discuss here is an extension of the plain stochastic model presented the last part, in series dependency of error terms in the measurement equation. This model allows the errors for persistence via moving average process with order 1 (MA(1)). The market is not assumed as effective as the plain stochastic volatility modelling. The mean of Brent daily return is 8.75E-04 and also high persistency exists in log-volatility. The special aspect of the extended model is the MA(1) process of error terms obtained returns. Not only does this period's error term affect this period's return, but also the last period's error term has an effect by multiplying 0.0051. The 90% credible interval of last period error term's coefficient is (-0.03135, 0.04203), which means it can be zero because of zero included. There is also a probability that the errors are series independent.

Unlike the plain stochastic models of Brent and WTI, the mean of WTI return is lower than Brent, but the persistence is slightly higher. What stays the same is that the variance of log-volatility is still more significant than Brent. The most valuable result is the property of the MA process. Last term error holds the negative effect on WTI but positive effect on Brent. It means that the previous term error has opposing direction effects on last term return and this term return. It is weakly capable of lower turbulence and stronger recovery than the plain model in WTI. Moreover, the differences between two adjacent daily volatilities became smaller. The reason is a slightly negative effect (-0.0363) from the last term error on this term's, so the difference between the two continuous days' returns shrinks in WTI compared to Brent. In this model, we can also see that 0.98882 is a considerably high persistency of log-volatility.

Another classic and conventional assumption is a normal distribution, which ignores the presence of outliers. In financial series data, the heavy-tailed and higher top distribution is capable of explaining more extreme values. Student's t-distribution is one of the most useful and efficient distributions to express leptokurtosis.

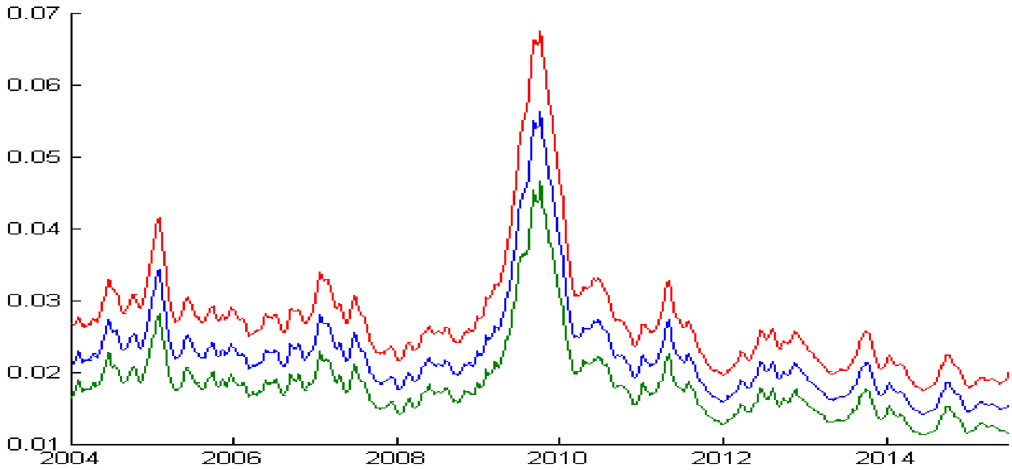

FIGURE 2.12: *Posterior means (blue line) and 90% credible intervals (red line and green line) of the time-varying standard deviation* $exp\left(\frac{h_t}{2}\right)$ *based on the Brent crude oil future daily returns in plain stochastic volatility model*

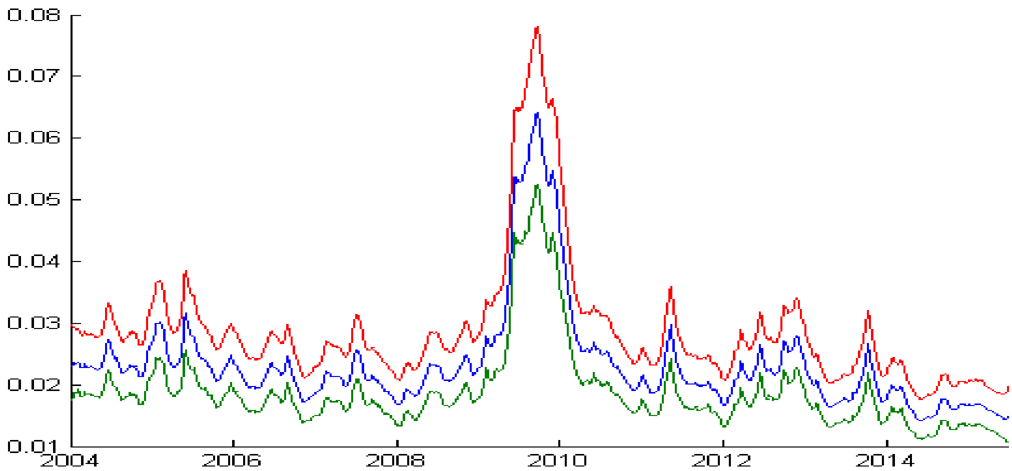

FIGURE 2.13: *Posterior means (blue line) and 90% credible intervals (red line and green line) of the time-varying standard deviation* $exp\left(\frac{h_t}{2}\right)$ *based on the WTI crude oil future daily returns in plain stochastic volatility model*

Figure 2.12 shows 90% credible intervals (red line and green line) of the time-varying standard deviation $exp(\frac{h_t}{2})$ based on the Brent return in the plain stochastic volatility model. Volatilities always change. Before 2008 the highest point of the fluctuation reached more than 0.04 in 2005. The highest volatility peaked 0.07 during the global financial crisis in 2008. There is a dramatic drop after 2010, which was even lower than before the crisis but more persistent.

More fluctuations exist in WTI than Brent in the corresponding timings from *Figure 2.13*. The highest point of WTI is 0.08 during the 2008 crisis, and the volatility goes up in 2009 sharply and goes down violently at the beginning of 2010. WTI exhibits a higher level of volatility than Brent at the same periods. What is surprising is that the oscillations became weaker after 2014.

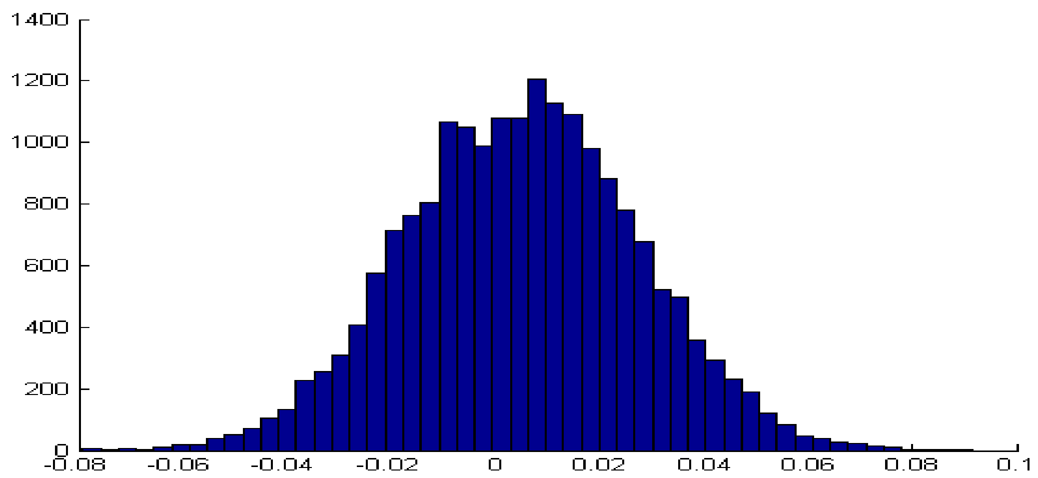

FIGURE 2.14: *density of ψ estimated of $p(\psi|y_t)$ based on the Brent crude oil future daily returns in Stochastic volatility model with MA(1) process for normally distributed error*

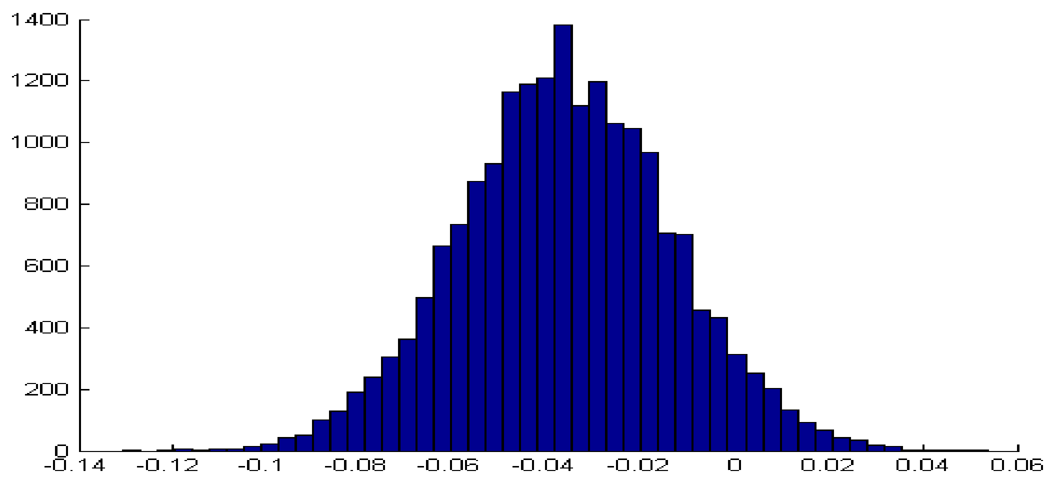

FIGURE 2.15: *density of ψ estimated of $p(\psi|y_t)$ based on the WTI crude oil future daily returns in Stochastic volatility model with MA(1) process for normally distributed error*

According to *Figure 2.14*, the distribution of the parameters is concentrated between -0.08 and 0.1, which is virtually mass around 0. It is consistent with the estimation result of 0.005052. There is a limited effect of last period' errors in Brent return modelling. The MA(1) process does not significantly exist in Brent return.

Based on *Figure 2.15*, the range of MA parameters is from -0.13 to 0.05, which also include zero. However, the main part of the parameters is not centered on 0. It is obvious that the MA process is recommended in the WTI volatility process rather than Brent. Most of the coefficients are located in the left zero areas, which indicate that the slight negative effects (-0.0363) are found in last period errors. The obvious difference between Brent and WTI is the sign of the previous period's error term's effect on this period's return. In Brent result, it is an insignificant positive effect, while it is a negative effect in WTI estimation, and it is more significant than the Brent result. In conclusion, the last period's errors have a significant negative effect on WTI but a weak positive effect on Brent. Adding the MA(1) process should be a more useful explanation in WTI than Brent.

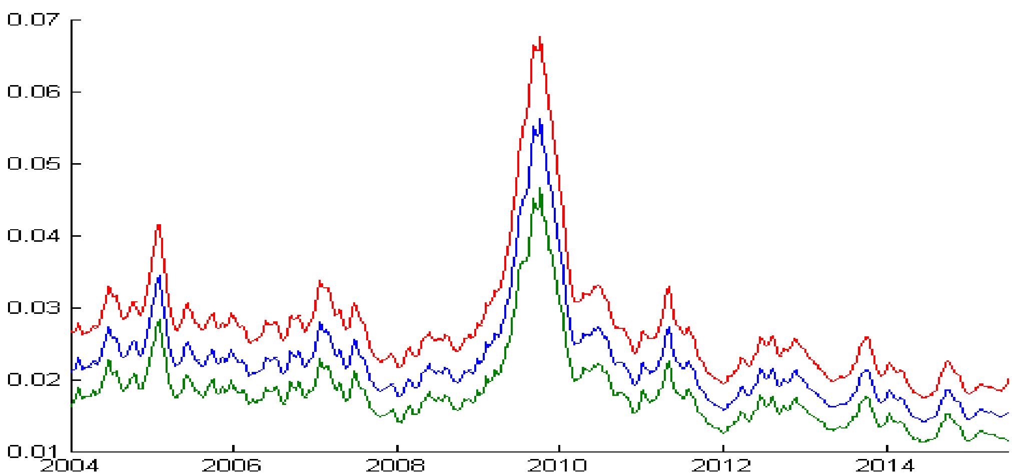

FIGURE 2.16: *Posterior means (blue line) and 90% credible intervals (red line and green line) of the time-varying standard deviation exp* $\left(\frac{y_t}{2}\right)$ *based on the Brent crude oil future daily returns in Stochastic volatility model with MA(1) process for normally distributed error*

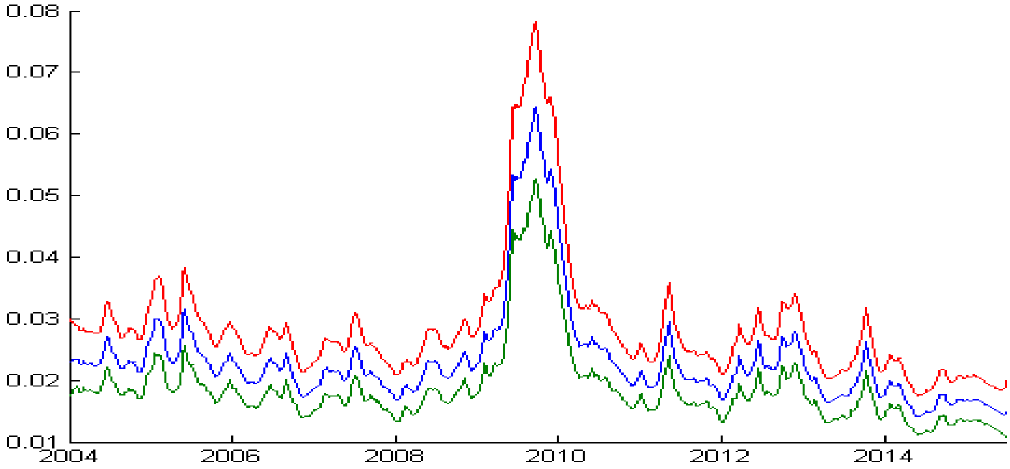

FIGURE 2.17: *Posterior means (blue line) and 90% credible intervals (red line and green line) of the time-varying standard deviation* $exp\left(\frac{y_t}{2}\right)$ *based on the WTI crude oil future daily returns in Stochastic volatility model with MA(1) process for normally distributed error*

Figure 2.17 of WTI is very similar as the result from the plain stochastic volatility model and it is consistent with these two models' comparison of Brent.

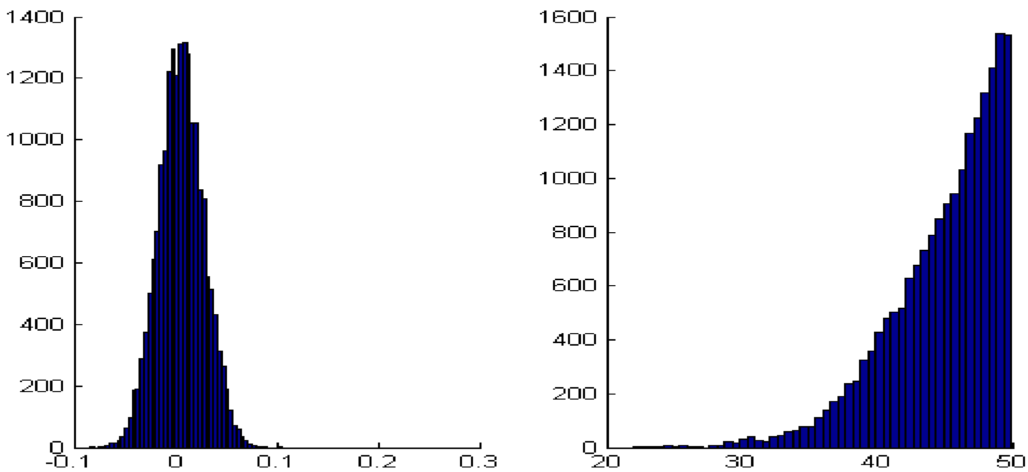

FIGURE 2.18: *density of* ψ *estimated of* $p\left(\psi|y_t\right)$ *(left panel) and density of* v *(right panel) estimated of* $p\left(v|y_t\right)$ *based on the Brent crude oil future daily returns in Stochastic volatility model with MA(1) Student's t-distributed error*

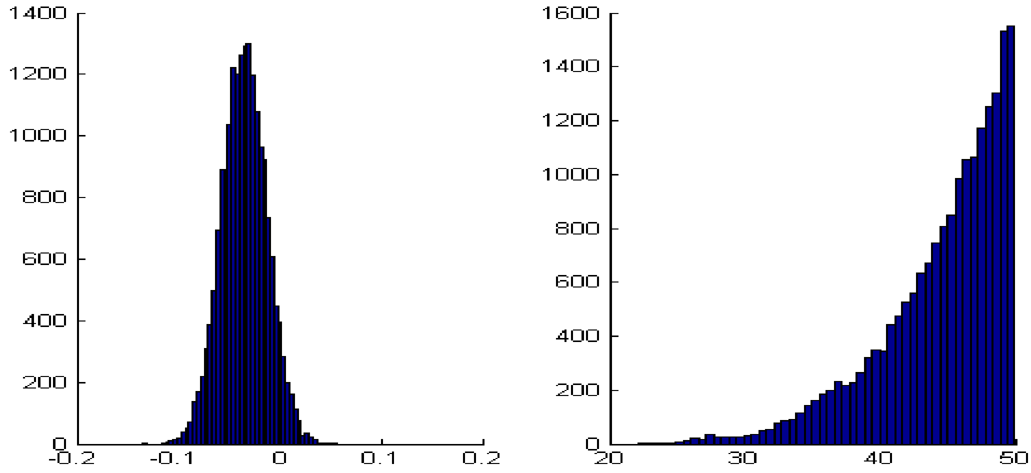

FIGURE 2.19: *density of ψ which estimated of $p(\psi|y_t)$ (left panel) and density of v (right panel) estimated of $p(v|y_t)$) based on the WTI crude oil future daily returns in Stochastic volatility model with MA(1) process for Student's t-distributed error*

According to *Figure 2.18*, the estimation result is similar to those obtained in the previous models. The range of the percentage of the last period's error-parameter is also not useful here, which include zero. It is consistent with the result in the MA(1) process for the normally distributed error. The density of v is concentrated between 40 and 50, which indicates it is a proper application of Student's t-distributed error here.

According to *Figure 2.19*, the mass of ψ distribute to the left of zero. Therefore there is evidence for the MA(1) process in WTI return series. It is a consistent result of the normally distributed error in MA(1) process. The sign of the effect of last period's error is also negative in WTI, and Brent return correspondingly generate the negative sign. Concerning the v distribution, the concentrated values are far away from zero, so Student's t-distributed error is an efficient assumption for the WTI modelling. By comparing the parameters in inverse gamma distribution, Brent return (44.9383) is more leptokurtic than WTI return (44.54021). It is consistent with the Kernel density estimation results in Section 3. In conclusion, plain stochastic volatility model and stochastic volatility model with an MA(1) process for normally distributed error perform similarly in Brent estimation. MA(1) process is not effective in Brent return, but it gains some efficiency in WTI estimation. Stochastic volatility model with MA(1) Student's t-distributed error is the optimal model for both Brent and WTI, mainly because the Student's t-distribution provides an appropriate fitting.

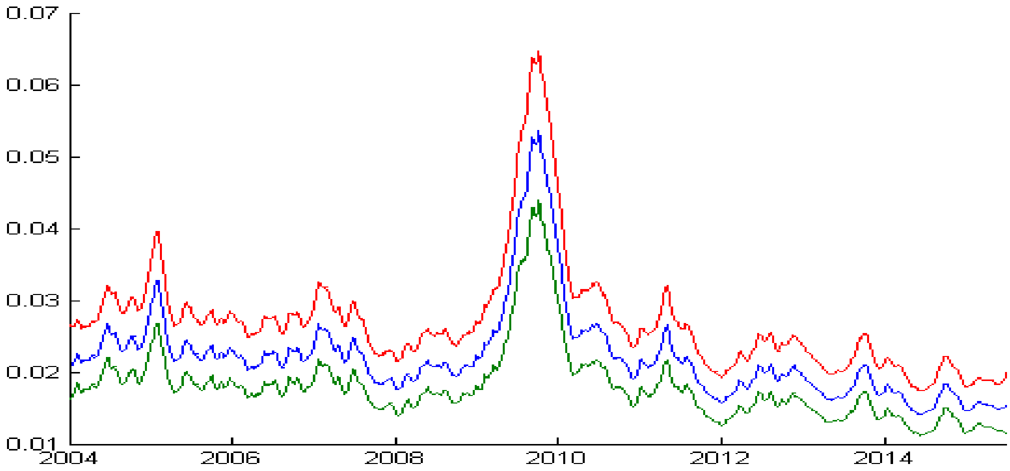

FIGURE 2.20: *Posterior means (blue line) and 90% credible intervals (red line and green line) of the time-varying standard deviation* $exp\left(\frac{y_t}{2}\right)$ *based on the Brent crude oil future daily returns in Stochastic volatility model with MA(1) process for Student's t-distributed error*

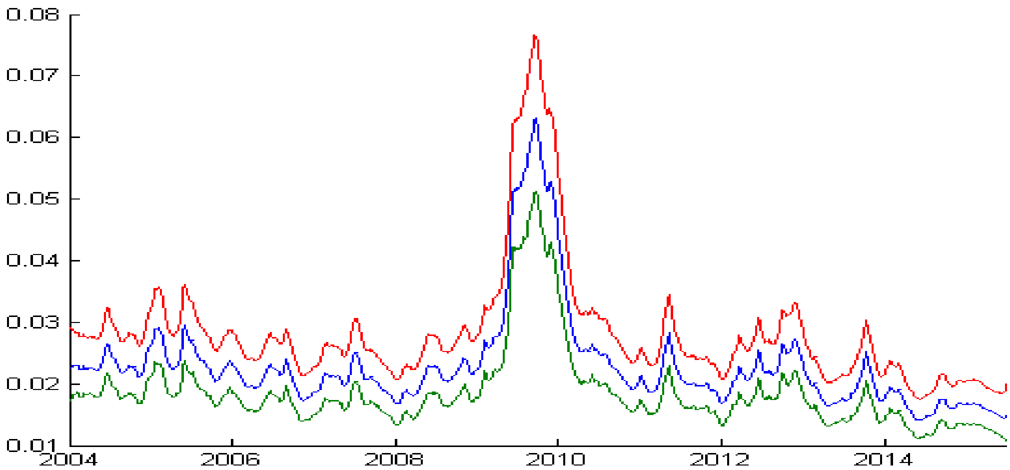

FIGURE 2.21: *Posterior means (blue line) and 90% credible intervals (red line and green line) of the time-varying standard deviation* $exp\left(\frac{y_t}{2}\right)$ *based on the WTI crude oil future daily returns in Stochastic volatility model with MA(1) process for Student's t-distributed error*

Mean returns of Brent are always bigger than WTI in the three models, which is also consistent with the GARCH model and GJR-GARCH model. Therefore, the log-volatility of Brent holds a more significant mean than WTI. Turbulences of WTI show stronger persistence than Brent. It makes sense that

WTI is active in the American area, while Brent is normally traded in European countries. The macro events' effects are larger under more open economic circumstance (such as in America). MA(1) process is useful in WTI return modelling, but not in Brent one. Brent return has a more significant leptokurtosis than WTI. It is also a consistent result as data preparation work, and the detailed results can be seen in *Figure 2.4*, *Figure 2.5*, and *Figure 2.6* in Section 3.

2.5.2 Forecast Results and Detection of Over-fitting

I use the squared return series as the proxy of the real variance in the crude oil market. The squared return is convenient to apply in predictive ability evaluation. OVX index is the implied volatility index collected from the CBOE platform. It represents the market expectation of crude oil futures. Moreover, GARCH volatility and stochastic forecasting volatility are constructed using the rolling forecasting methodology discussed in Section 4.

Table 2.7 presents the value of loss function for the OVX index, GARCH-type models, and stochastic volatility models in out-of-sample forecasting. At first, we focus on the forecast evaluation on the Brent oil futures. The MSE, HMAE, HMSE, and GMLE each rank the OVX index the most accurate forecasting method. The MAE, MSE, LL, and GMLE select the GARCH-type models as a better forecasting model than the stochastic volatility model. Furtherly, the GJR-GARCH model exhibits more superior forecasting ability than the ARCH and GARCH counterparts. The GJR-GARCH model incorporates more information than ARCH and GARCH models, and it fit the stylish fact of Brent futures very well. I suppose it is the reason for the GJR-GARCH model producing the smaller forecasting errors than the ARCH model and GARCH model. The results from the stochastic volatility models, the model with moving average order 1 process and normal distributed errors has the worst forecasting ability, but evidence regarding the best forecasting model in the stochastic volatility subgroup is mixed. However, we have some evidence (MAE, LL, and GMLE) that the stochastic volatility model imposed the student's t-distributed errors have better forecasting performance than the counterparts with normal distributed errors.

TABLE 2.6: *In-sample forecasting evaluation across competing models*

		MAE($\times 10^4$)	R	,MSE($\times 10^6$)	R	HMAE	R	HMSE	R	LL	R	GMLE	R
	ARCH	$5.2327^{\dagger}$	6	$1.5987^{\dagger}$	6	$1.1368^{\dagger}$	6	$8.0848^{\dagger}$	6	$7.7875^{\dagger}$	6	$-6.6965^{\dagger}$	6
	GARCH	4.9342	2	1.4587	5	1.0466	5	3.1473	5	6.8638	2	-6.8979	5
Brent	GJR-GARCH	4.9066^{*}	1	1.4358	4	1.0416	4	2.9991	4	6.8113^{*}	1	-6.9090	4
	PLAIN SV	5.1860	4	1.3624	2	0.8714	2	1.3716	2	7.4601	4	-6.9258	3
	SV MA NORMAL	5.1861	5	1.3618^{*}	1	0.8708^{*}	1	1.3675^{*}	1	7.4607	5	-6.9262	2
	SV MA STUDENT'S T	5.1066	3	1.3690	3	0.8847	3	1.4795	3	7.3875	3	-6.9284^{*}	1
	ARCH	$6.2330^{\dagger}$	6	$2.0954^{\dagger}$	6	$1.1327^{\dagger}$	6	$6.5141^{\dagger}$	6	$7.3272^{\dagger}$	6	$-6.5943^{\dagger}$	6
	GARCH	5.7050	2	1.8083	5	1.0423	5	4.2785	4	6.3918	2	-6.8023	5
WTI	GJR-GARCH	5.6600^{*}	1	1.7988	4	1.0393	4	4.4649	5	6.3551^{*}	1	-6.8097	4
	PLAIN SV	5.8140	5	1.6620^{*}	1	0.8596^{*}	1	1.3912^{*}	1	6.8379	4	-6.8565	2
	SV MA NORMAL	5.8131	4	1.6636	2	0.8612	2	1.4033	2	6.8398	5	-6.8554	3
	SV MA STUDENT'S T	5.7386	3	1.6744	3	0.8757	3	1.5410	3	6.7683	3	-6.8568^{*}	1

2

* *indicates the best fitting model*
† *indicates the worst fitting model*
Column R indicates the ranking of goodness of fit

TABLE 2.7: *Out-of-sample forecasting evaluation across competing models*

		MAE($\times 10^4$)	R	MSE($\times 10^7$)	R	HMAE	R	HMSE	R	LL	R	GMLE	R
	OVX	3.9958	3	5.0250*	1	0.8226*	1	1.0007*	1	36.3950†	7	−7.5088*	1
	ARCH	4.6464	4	6.9477	4	1.0995	6	4.0835	5	10.3691	4	-7.0348	4
	GARCH	3.4526*	1	6.3645	3	1.1737†	7	7.3008†	7	6.7006*	1	-7.4580	3
Brent	GJR-GARCH	3.5672	2	6.3155	2	1.0739	5	4.6751	6	6.9601	2	-7.4957	2
	Plain SV	6.2640	6	8.4142	5	1.0220	2	2.6197	2	12.1600	5	-6.8700	6
	SV MA NORMAL	6.3856†	7	8.6815†	7	1.0623	4	3.4279	4	12.1899	6	−6.8394†	7
	SV MA STUDENT'S T	6.1623	5	8.4691	6	1.0613	3	2.7414	3	11.9015	3	-6.8776	5
	OVX	4.5640	2	9.9164	4	1.1484	6	6.4389†	7	13.6945†	7	-6.9904	4
	ARCH	4.3075*	1	5.8209*	1	0.7477*	1	0.6765*	1	8.9825	3	-7.0764	3
	GARCH	4.6193	3	9.2540	3	0.9311	3	1.7877	3	7.2915*	1	−7.1706*	1
WTI	GJR-GARCH	4.7024	4	9.2127	2	0.9042	2	1.5138	2	7.4304	2	-7.1670	2
	Plain SV	6.4440	5	11.3949	5	1.0530	4	2.4897	4	10.6657	5	-6.6749	5
	SV MA Normal	6.6623†	7	12.3995†	7	1.1235	5	4.1115	5	10.6758	6	-6.6025	6
	SV MA Student's t	6.5401	6	12.2780	6	1.1714†	7	5.1473	6	10.6329	4	−6.5841†	7

3

* *indicates the best fitting model*
† *indicates the worst fitting model*
Column R indicates the ranking of goodness of fit

Turning to the forecasting evaluation results for WTI futures, each value of the loss function supports the GARCH-type models are superior to stochastic volatility models in volatility forecasting. Notably, loss function values of MAE, MSE, HMAE, and HMSE support the ARCH model as the best model, while LL and GMLE rank the GARCH model the toppest. Superisely, GJR-GARCH model is not the best one in WTI futures volatility forecasting. In the results of the stochastic volatility subgroup, plain stochastic volatility model is superior, which means the moving average process and Student's t-distribution are not expectingly effective in stochastic volatility forecasting.

In *Table 2.6*, we see that the best models for fitting in-sample are GJR-GARCH model and stochastic volatility type models. However, out-of-sample, the best models seems to be simpler models, such as ARCH. This strongly suggests that the most complex models, GJR-GARCH model, and stochastic volatility type models, suffer from over-fitting.

2.6 Conclusion

In this chapter, I investigate the performance of volatility modelling and forecasting in crude oil futures markets. In particular, I compare the OVX index, the GARCH-type models, and stochastic volatility models concerning estimation and forecasting abilities in crude oil futures markets (Brent and WTI). As alternatives to the ARCH model, the GARCH model is the most popular one, and the GJR-GARCH model incorporates the information of the leverage effect. On the other hand, the plain stochastic volatility model and another two advanced stochastic volatility models are adopted to identify the moving average process of order 1 in return series, and Student's t-distribution is also verified.

According to the findings presented in this chapter, many conclusions can be drawn from the previous analysis. First of all, I find that the volatility responses of Brent and WTI to the market stimulation are distinct. Specifically, WTI is generally more sensitive than Brent to the market changes, and its fluctuation also keeps a little longer than Brent. Further, the investors holding WTI are more vulnerable to the negative news (negative shocks in the market) than the Brent futures holders. Based on the volatility patterns extracted from the GARCH-type models and stochastic volatility models, WTI has a quicker and stronger response to the 2008 financial crisis compared with Brent. The variance of WTI is still more persistent than Brent's. The reason is that WTI and Brent operate in different areas. WTI is a considerably common commodity in the American market, while Brent takes over the European market for more than forty years. The American futures market is the most changeable and unpredicted market overall, globally. Moreover, in an open market, it is easier to trigger a financial crisis and more challenging to recover. Macro events, for instance, wars, systematically cause more instability in America than Europe. Because of the high degree of freedom of the American market, investors are more sensitive, and the government tends to make delayed policy decisions. Therefore it is reasonable for WTI's quick response to the market stimulation. Briefly, WTI is always the first reaction to market stimulation, and it also needs more time to calm down after the financial crisis than Brent.

In terms of estimation of the GARCH-type models, we find the significant evidence that the model extracting more information from the data, can provide more effective estimation results. Accurately, I identify the Brent and WTI futures returns have the financial data characteristic of the leverage effect. It is the downward returns of crude oil that tends to have more influence on volatility than the upward returns. It is a reasonable interpretation for market investors' psychology that investors tend to be more worried about price decline than the price increase. The leverage effect indeed exists in Brent, and WTI futures, which is also demonstrated by Kristoufek (2014) but go against the adverse leverage effect of WTI in Aboura and Chevallier (2013)'s study. As mentioned before, the WTI futures have a stronger leverage effect than Brent futures.

Concerning the estimation of stochastic volatility models, the most important result is that the moving average process is not useful in Brent crude oil volatility modelling, while it is effectively helpful in WTI volatility analysis. It means that stochastic volatility modelling of Brent returns a constant mean process rather than a moving average with order one. WTI comparatively holds more significant financial characteristics than Brent. It is also a reason why WTI shows a more and stronger cluster than Brent, which is also consistent with the GARCH-type model estimation results in this chapter. In the third stochastic volatility model estimation, Student's t-distributed error is also able to provide more accurate estimation results in both Brent and WTI returns, which is generally closer to real financial market circumstance than the normal distribution described. Therefore, crude oil futures are one common form of financial derivative, provides the evidence for the characteristic of departing from normality.

In terms of the volatility forecast, I mainly conclude that GARCH-type models perform better than the stochastic volatility models either in Brent or WTI crude oil futures market. OVX index can provide the optimal forecast in the volatility of Brent futures. In the volatility forecasting of the GARCH-type models for Brent, the GJR-GARCH model is the best. By contrast, for WTI, the ARCH model exhibits the most accurate forecast ability. As regards the forecasting of stochastic volatility models, the evidence for the best model for Brent is mixed. In contrast, the plain stochastic volatility model is the best for WTI. When I consider the over-fitting problem, I furtherly compare the in-sample and out-of-sample forecasting results. Then we conclude the stochastic volatility models suffer from over-fitting. For the other models, the results are mixed. The GJR-GARCH model appears to suffer from over-fitting in one case, but not the other.

Based on what I discussed above, there are several aspects to extend in futures research. First, GARCH-type models with Student's t-distribution is expected to provide efficient predictive ability. So it is valuable to investigate further. Second, it would be worthwhile to compare multivariate GARCH models in fitting multiple energy prices. Third, it would be attractive to construct a multivariate model by including some macroeconomic variables to the current form. I can further identify the interaction between the crude oil market and the macroeconomics. Finally, it would improve the forecasting evaluation by using high-frequency intraday data as proxy volatility.

On a final note, arbitrarily choosing a volatility model to forecast is not wise. The findings presented in this chapter provide evidence on how to select a volatility forecasting model for financial practitioners, energy economists, and policymakers. However, the data sample length and the choice for loss functions and proxy variance make evaluation vary as regards as the forecasting performance of the different models.

2.7 Appendix to Chapter 2

Table 2.8: *Posterior means, standard deviations and quantiles of model parameters Brent daily returns data*

parameter	Posterior mean	Posterior S.D.	5% percentile	95% percentile
μ	0.000869	0.000401	0.000213	0.001524
μ^y	-7.60467	0.540222	-7.91652	-7.34082
ϕ^y	0.988187	0.00422	0.981062	0.994773
$(\sigma^y)^2$	0.008075	0.001435	0.006006	0.010698

Table 2.9: *Posterior means, standard deviations and quantiles of model parameters WTI daily returns data*

parameter	Posterior mean	Posterior S.D.	5% percentile	95% percentile
μ	0.000774	0.000409	9.99E-05	0.001445
μ^y	-7.54282	0.552301	-7.88534	-7.23148
ϕ^y	0.988488	0.004118	0.981535	0.995056
$(\sigma^y)^2$	0.009508	0.001744	0.006907	0.01267

Table 2.10: *Posterior means, standard deviations and quantiles of model parameters Brent daily returns data*

parameter	Posterior mean	Posterior S.D.	5% percentile	95% percentile
μ	0.000875	0.000405	0.000213	0.001542
μ^y	-7.59857	0.613401	-7.91755	-7.34589
ϕ^y	0.987968	0.004195	0.980856	0.994483
$(\sigma^y)^2$	0.008209	0.001406	0.0061	0.010697
ψ	0.005052	0.022323	-0.03135	0.04203

Table 2.11: *Posterior means, standard deviations and quantiles of model parameters WTI daily returns data*

parameter	Posterior mean	Posterior S.D.	5% percentile	95% percentile
μ	0.000776	0.000396	0.000127	0.001424
μ^y	-7.52048	0.641916	-7.87905	-7.21052
ϕ^y	0.98882	0.004095	0.981931	0.995366
$(\sigma^y)^2$	0.009171	0.001643	0.006846	0.012208
ψ	-0.0363	0.022513	-0.0735	0.001015

Table 2.12: *Posterior means, standard deviations and quantiles of model parameters Brent daily returns data*

parameter	Posterior mean	Posterior S.D.	5% percentile	95% percentile
μ	0.000876	0.000407	0.000208	0.001542
μ^y	-7.65142	0.41852	-7.94142	-7.37782
ϕ^y	0.98802	0.004202	0.980899	0.994647
$(\sigma^y)^2$	0.007803	0.001367	0.005752	0.01024
ψ	0.005848	0.022175	-0.0305	0.043096
ν	44.9383	4.120623	37.06956	49.64792

Table 2.13: *Posterior means, standard deviations and quantiles of model parameters WTI daily returns data*

parameter	Posterior mean	Posterior S.D.	5% percentile	95% percentile
μ	0.000777	0.000398	0.000126	0.001432
μ^y	-7.5332	0.694973	-7.90652	-7.22701
ϕ^y	0.988927	0.004111	0.982087	0.995599
$(\sigma^y)^2$	0.008813	0.001627	0.006494	0.011853
ψ	-0.03489	0.022359	-0.07146	0.001989
ν	44.54021	4.659866	35.14271	49.66445

Chapter 3

The Shale Revolution, Geopolitical Risk, and Oil Price Volatility

3.1 Introduction

Understanding the price dynamics of crude oil is crucial for policymakers, business leaders, and consumers. Because the variation in oil prices is susceptible to supply shocks, it is therefore important to analyse the impacts of geopolitical risk, which has been one of the main security risks to oil supply.

Due to the advances in technology in fracking shale oil, the supply condition in the global oil market has changed. It was confirmed by the U.S. Energy Information Administration on July 16, 2018, "the U.S. oil output from seven major shale formations is expected to rise to a record 7.47 million barrels per day in August 2018". As the shale revolution continues to drive oil production, we have observed low oil prices between June 2014 and February 2016. Although the shale revolution is not solely responsible for the substantial fall in oil prices since 2014, Kilian, 2017 argue that the Brent price of crude oil was lower by $10 than it would have been in the absence of the fracking boom.

The long quote above constitutes one of the hypotheses for the consequential technology changes in the shale oil industry: "(the shale revolution) should reduce the volatility of the oil price". Thus, in this chapter, one of our objectives is to investigate the impact of geopolitical uncertainties on oil price volatility under a simultaneous shale oil production shock.

Most of oil price-related research focuses on the nonlinear relationship between oil prices and GDP growth, such as in Hamilton, 2003a. Consistent with most of the findings in the literature, Hamilton, 2003a concludes that upside risks in oil prices are more of a threat than the downside risks in real economic activities. The framework laid by Hamilton, 2009 and Kilian, 2009a lead many recent studies, such as Prest, 2018, to identify the oil shocks and improve our understanding of their historical causes and consequences. The oil price driving force has been identified from both the supply and the demand sides, see Dées et al., 2007 and Hamilton, 2009. Empirical evidence also supports that most of the historical oil price shocks were caused by physical disruptions of supply, see Hamilton, 2009. Hamilton, 2009 finds that the oil price run-up of 2007-08 was a joint effect of stagnating world production and demand. We follow a similar direction, and besides evaluating the oil price volatility responses to exogenous shocks, we focus on the nonlinear oil prices dynamics and its response to geopolitical risks amidst the shale revolution.

With respect to econometric modelling, the seminal chapter by Sims, 1980 proposes to use the Vector Autoregressive (VAR) to conduct macroeconomic analysis. For instance, a bivariate VAR model is employed in Kilian and Vigfusson, 2011 to study how GDP responds to asymmetric oil price changes. The baseline VAR model is specified as follows:

$$\boldsymbol{y_t} = \boldsymbol{\Gamma_0} + \boldsymbol{\Gamma_1 y_{t-1}} + \ldots + \boldsymbol{\Gamma_p y_{t-p}} + \boldsymbol{u_t}, \tag{3.1}$$

where $\boldsymbol{y_t} = (y_{1t}, \ldots, y_{kt})^T$ and $\boldsymbol{y_t} \in \mathbb{R}^{k\times 1}$. The vector of intercepts $\boldsymbol{\Gamma_0} \in \mathbb{R}^{k\times 1}$ and the coefficients are squared matrix $\boldsymbol{\Gamma_i} \in \mathbb{R}^{k\times k}$ with $i = 1, \ldots, p$. Therefore, eq. (3.1) can be summarized as

$$\boldsymbol{y_t} = \boldsymbol{\Gamma X_t} + \boldsymbol{u_t}, \tag{3.2}$$

denoting $\boldsymbol{\Gamma} = (\boldsymbol{\Gamma_0}, \boldsymbol{\Gamma_1}, \ldots, \boldsymbol{\Gamma_p})$, and $\boldsymbol{X_t} = (1, \boldsymbol{y_{t-1}}, \boldsymbol{y_{t-2}}, \ldots, \boldsymbol{y_{t-p}})^T$ where $\boldsymbol{\Gamma} \in \mathbb{R}^{k\times(kp+1)}$, and $\boldsymbol{X_t} \in \mathbb{R}^{(kp+1)\times 1}$. The error term $\boldsymbol{u_t} \overset{i.i.d}{\sim} MN(\boldsymbol{0}, \boldsymbol{\Sigma_u})$, where $\boldsymbol{\Sigma_u} \in \mathbb{R}^{k\times k}$ and $MN(\boldsymbol{0}, \boldsymbol{\Sigma})$ denotes a multivariate normal distribution with mean $\boldsymbol{0}$ and a constant covariance matrix $\boldsymbol{\Sigma}$.

The baseline VAR model has been generalized in many different dimensions to capture nonlinear dynamics in macroeconomic variables. For example, a time-varying-parameter VAR (TVP-VAR) (Koop and Korobilis, 2013) and a smooth-transition VAR (Hubrich and Teräsvirta, 2013) are proposed to address different types of nonlinearities. To answer our question, whether the response of oil price to geopolitical risk has changed under the shale revolution, we apply a generalization of the baseline VAR model - a structural break threshold VAR (SBT-VAR) model proposed by Galvão, 2006.

In order to quantify the dependent variable $\boldsymbol{y_t}$'s response to a specific exogenous shock, mutually independent structural shocks have to be identified. Because of the covariance $\boldsymbol{\Sigma}_u$ specification in a reduced form VAR system, a hypothetical shock, therefore, cannot be isolated from other error terms. Hence, it is impossible to provide a clear interpretation of the impulse response function using the reduced form VAR. Therefore, the reduced form VAR has been extended to a structural VAR (SVAR) in the literature. There are many ways to identify the structural shocks in SVAR, such as by imposing different identification restrictions. An SVAR model is specified as follows:

$$\boldsymbol{A_0 y_t} = \boldsymbol{A_1 X_t} + \boldsymbol{\epsilon_t}, \tag{3.3}$$

where $\boldsymbol{\epsilon_t}$ are serially and mutually uncorrelated structural innovations. The reduced form VAR in eq. (3.2) and SVAR in eq. (3.3) can be linked with $\boldsymbol{u_t} = \boldsymbol{A_0}^{-1}\boldsymbol{\epsilon_t}$ and $\boldsymbol{\Sigma_u} = \left(\boldsymbol{A_0}^{-1}\right) \boldsymbol{\Sigma_\epsilon} \left(\boldsymbol{A_0}^{-1}\right)^T$, where $\boldsymbol{u_t}$ and $\boldsymbol{\Sigma_u}$ can be achieved by estimation and treated as observables. From eq. (3.2) and eq. (3.3), $\boldsymbol{\Gamma}$ in VAR has the form $\boldsymbol{\Gamma} = \boldsymbol{A_0}^{-1}\boldsymbol{A_1}$, where $\boldsymbol{A_1}$ is a reflection of the feedback dynamics in SVAR.

Identifying the structural shocks has been a focus in the literature. One obvious solution to identification is to use the Cholesky decomposition, i.e. $\boldsymbol{\Sigma_\epsilon}$ is normalized to unity, then $\boldsymbol{\Sigma_u}$

$$\boldsymbol{\Sigma}_u = \left(\boldsymbol{A_0}^{-1}\right)\left(\boldsymbol{A_0}^{-1}\right)^T. \tag{3.4}$$

By Cholesky decomposition, $\boldsymbol{A_0}^{-1}$ is the lower triangular. Thus, the statistical innovation $\boldsymbol{u}_t$ depends recursively on the mutually uncorrelated structural innovation $\boldsymbol{\epsilon}_t$.

The use of Cholesky decomposition, i.e., by imposing exclusion restrictions on $\boldsymbol{A_0}$, relies on the ordering of the variables. Therefore, justifications of the ordering have to be made after consulting economic theory. The exclusion restriction on the impact effects of structural shocks has been applied in Sims, 1980 and Kilian, 2009a. In Blanchard and Quah, 1989, identification is achieved by restrictions on the long-run effects. For instance, the restriction is imposed on the aggregate demand shock, assuming the aggregate demand shock has no long-run impact on the GNP. Restrictions, such as on the signs of the responses of specific variables to a shock, were used for identification in Uhlig, 2005. Interested readers can find a comprehensive review of the various identification methods in Kilian and Lütkepohl, 2017. Because of the drawbacks of identifications through Cholesky decomposition and sign restriction methods, Bouakez, Essid, and Normandin, 2013, Bouakez, Chihi, and Normandin, 2014, Lütkepohl and Netšunajev, 2014, Lütkepohl and Netšunajev, 2017 and Elder and Serletis, 2010a propose to use a GARCH specification for the structural innovation to identify the SVAR model. In this chapter, we utilized the identification procedure by allowing for heteroskedasticity in the structural innovations, and illustrate that using an exclusion restriction on A_0 may arrive at very different inferences of the impulse response functions.

With respect to empirical applications in the literature, SVAR has been utilized to analyse the oil prices' responses to different measures of geopolitical risks. For instance, Coleman, 2012 finds that the frequency of fatal terrorist attacks in the Middle East and the U.S. troop numbers in the Middle East explains a significant amount of variation in crude oil prices. Chen et al., 2016 find a significant and positive casual effect of OPEC political risk on Brent crude oil prices. Rather than using dummy variables and focusing on a specific geopolitical event as in Noguera-Santaella, 2016, we apply a geopolitical risk index, constructed by Caldara and Iacoviello, 2018, to a structural break threshold VAR (SBT-VAR) model. With the SBT-VAR model specification, the unknown breakpoint and threshold can be estimated using maximum likelihood. Then, the reduced form SBT-VAR model is generalized to its structural form, which is identified through a GARCH specification in the structural innovations as in Bouakez, Essid, and Normandin, 2013 and Lütkepohl and Netšunajev, 2017. Finally, we analyse the impulse response functions of the oil prices to geopolitical shocks. Further, we compare the volatility impulse responses of oil prices to geopolitical shocks under two distinct scenarios. In the first scenario, oil price volatilities respond to a simultaneous shale production shock and a geopolitical risk shock. In the second scenario, the oil price volatilities respond to a hypothetical shock from only one source, i.e., the geopolitical risk.

The chapter is organized as follows: Section 2 presents the model. Section 3 illustrations the empirical results. Section 4 concludes.

3.2 Methodology

3.2.1 Reduced Form SBT-VAR and Structural SBT-VAR

In order to identify the changing dynamics in the oil price, first, we apply a reduced form Structural Break Threshold VAR (SBT-VAR) model proposed by Galvão (2006). The SBT-VAR is a generalization of the baseline VAR model, which is specified in eq. (3.2).

The reduced form SBT-VAR is specified as

$$\begin{aligned} \boldsymbol{y_t} = & \left[\boldsymbol{\Gamma_1 X_t} I_{1,t-d_1}(r_1) + \boldsymbol{\Gamma_2 X_t}\left(1 - I_{1,t-d_1}(r_1)\right)\right] I_t(\tau) + \\ & \left[\boldsymbol{\Gamma_3 X_t} I_{2,t-d_2}(r_2) + \boldsymbol{\Gamma_4 X_t}\left(1 - I_{2,t-d_2}(r_2)\right)\right] (1 - I_t(\tau)) + \boldsymbol{u_t}, \end{aligned} \tag{3.5}$$

where $I(\cdot)$ is an indicator function. Denote the threshold as r_i, the delay parameter as d_i, the break-point as τ, and the transition variable as z, then, the threshold and break indicator functions are defined as $I_{i,t-d_i}(r_i) = 1\left(z_{t-d_i} \leq r_i\right)$, and $I_t(\tau) = 1(t \leq \tau)$. The error term (or statistical innovation), $\boldsymbol{u_t}$, follows a multivariate normal, $\boldsymbol{u_t} \overset{i.i.d}{\sim} MN(\boldsymbol{0}, \boldsymbol{\Sigma_u})$, and $\boldsymbol{\Sigma_u} \in \mathbb{R}^{k \times k}$.

As mentioned before, a reduced form model is not sufficient for the impulse responses analysis because the correlation of the statistical innovations makes it impossible to disentangle the marginal effects of exogenous shocks. Therefore, in order to analyse how the oil prices respond to the structural shocks of geopolitical risks before and after the shale revolution, we are motivated to identify the mutually independent structural shocks in a regime-changing system.

Nonlinear structural models, incorporating thresholds and breaks are proposed in Baum and Koester, 2011 and Galvão and Marcellino, 2013. We extend the reduced form SBT-VAR to its structural form and denote the structural model as SBT-SVAR hereafter. The SBT-SVAR is specified as follows:

$$\begin{aligned} \boldsymbol{A_0 y_t} = & \left[\boldsymbol{A_1 X_t} I_{1,t-d_1}(r_1) + \boldsymbol{A_2 X_t}\left(1 - I_{1,t-d_1}(r_1)\right)\right] I_t(\tau) + \\ & \left[\boldsymbol{A_3 X_t} I_{2,t-d_2}(r_2) + \boldsymbol{A_4 X_t}\left(1 - I_{2,t-d_2}(r_2)\right)\right] (1 - I_t(\tau)) + \boldsymbol{\epsilon_t}, \end{aligned} \tag{3.6}$$

where $\boldsymbol{u_t} = \boldsymbol{A_0}^{-1}\boldsymbol{\epsilon_t}$ or $\boldsymbol{A_0}\boldsymbol{u_t} = \boldsymbol{\epsilon_t}$. The coefficient matrix $\boldsymbol{\Gamma}_i$ can be found from $\boldsymbol{\Gamma}_i = \boldsymbol{A_0}^{-1}\boldsymbol{A}_i$ with $i = 1, 2, 3, 4$.

The (co)variance of the error term of the model in the reduced form $\boldsymbol{\Sigma_u}$ in eq. (3.5) and the unconditional variance of structural shocks $\boldsymbol{\Sigma_\epsilon}$ in eq. (3.6) can be linked by $\boldsymbol{\Sigma_u} = \left(\boldsymbol{A_0}^{-1}\right)\boldsymbol{\Sigma_\epsilon}\left(\boldsymbol{A_0}^{-1}\right)^T$. We impose normalization on the $\boldsymbol{\Sigma_\epsilon}$. Then, $\boldsymbol{\Sigma_\epsilon}$ is normalized to an identify matrix, $\boldsymbol{I_k}$ with $\boldsymbol{I_k} \in \mathbb{R}^{k \times k}$, and $\boldsymbol{\Sigma_u}$ is

$$\boldsymbol{\Sigma_u} = \left(\boldsymbol{A_0}^{-1}\right)\left(\boldsymbol{A_0}^{-1}\right)^T, \tag{3.7}$$

or in the form of the precision matrix

$$\boldsymbol{\Sigma_u}^{-1} = \boldsymbol{A_0}^T\boldsymbol{A_0}. \tag{3.8}$$

$\boldsymbol{A_0}$ is the upper-triangular matrix by applying a Cholesky factorization.

Similarly to the case with a simple structural VAR in eq. (3.3), restricting $\mathbf{A_0}$ as a lower triangular matrix and using the Cholesky decomposition offers a straightforward solution to identify the structural shocks in a structural SBT-VAR (SBT-SVAR) model in eq. (3.6). However, using the Cholesky decomposition implies restrictions on the direction of contemporaneous effects of structural shocks.

For instance, suppose the order of the variables is fixed and the structural error is $\boldsymbol{\epsilon_t} = \left(\boldsymbol{\epsilon_{t,GPR}}, \boldsymbol{\epsilon_{t,ShaleP}}, \boldsymbol{\epsilon_{t,OilP}}\right)^T$, where $\boldsymbol{\epsilon_{t,GPR}}$, $\boldsymbol{\epsilon_{t,ShaleP}}$, and $\boldsymbol{\epsilon_{t,OilP}}$ are the orthogonal mutually independent structural shocks to geopolitical risk, to shale oil production, and to oil prices at time t, respectively. A lower triangular restriction in $\mathbf{A_0}^{-1}$ implies that a structural shock in geopolitical risk has an instantaneous impact on shale oil production, the oil price, not vice versa. Similarly, the shale oil production shocks, $\boldsymbol{\epsilon_{t,ShaleP}}$, instantaneously affect the oil price, but not vice versa. These assumptions on the contemporaneous relationship amongst the variables appear to be too restrictive and unrealistic.

Next section demonstrates another flexible method for identification - allowing for heteroskedastic structural errors. The identification method is applied to the STB-SVAR model in our empirical applications.

3.2.2 GARCH Structural Errors

Rather than using the Cholesky decomposition method to identify eq. (3.6), we can exploit the conditional heteroskedasticity, i.e. $\boldsymbol{\Sigma}_{\epsilon,t}$, in the structural shocks $\boldsymbol{\epsilon}_t$, to identify more unrestricted elements in $\boldsymbol{A}_0$. The SVAR with GARCH (structural) innovations has been proposed in Lütkepohl and Netšunajev, 2017, Bouakez, Essid, and Normandin, 2013, Bouakez, Chihi, and Normandin, 2014, and Sentana and Fiorentini, 2001. A SVAR with GARCH-m type innovations is proposed in Elder and Serletis, 2010a.

In the context of analysing the responses of oil prices and oil price volatilities to geopolitical risk shocks under the shale production, a flexible $\boldsymbol{A}_0$ is needed. A flexible $\boldsymbol{A}_0$ requires a relaxation of restrictions on the contemporaneous relationship amongst the variables. In other words, a flexible $\boldsymbol{A}_0$ allows for impact effects of an oil price shock ($\epsilon_{t,OilP}$) on shale oil production and geopolitical risk. For instance, the low oil prices since 2014 did not help with the recent Venezuela crisis[1] and it is reasonable to believe that the oil price shock has had impact effects on geopolitical risk in Venezuela.

By utilizing the estimated statistical innovations $\hat{\boldsymbol{u}}_t$ from the SBT-VAR model in eq. (3.5) and a GARCH specification of heteroskedasticity in $\boldsymbol{\Sigma}_{\epsilon,t}$, we can identify more unrestricted elements in $\boldsymbol{A}_0$ in a more realistic setting. Recall that the statistical innovation $\boldsymbol{u}_t$ and the structural innovations $\boldsymbol{\epsilon}_t$ are linked by $\boldsymbol{A}_0$, where

$$\boldsymbol{u}_t = \boldsymbol{A}_0^{-1}\boldsymbol{\epsilon}_t, \tag{3.9}$$

or $\boldsymbol{A}_0\boldsymbol{u}_t = \boldsymbol{\epsilon}_t$, and the unconditional statistical innovation is $\boldsymbol{\Sigma}_u = \left(\boldsymbol{A}_0^{-1}\right)\boldsymbol{\Sigma}_\epsilon\left(\boldsymbol{A}_0^{-1}\right)^T$. For convenience, the unconditional variance of the structural innovations are normalized to unity, i.e. $E\left(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^T\right) = \boldsymbol{I}_k$ with $\boldsymbol{I}_k \in \mathbb{R}^{k\times k}$. Denote the information set up to t is $\mathcal{F}_t$, $E_{t-1}(\cdot) \equiv E(\cdot \mid \mathcal{F}_{t-1})$, the heteroskadastic (co)variance of the statistical innovation and structural innovation conditional on the historical information are $\boldsymbol{\Sigma}_{u,t} = E_{t-1}\left(\boldsymbol{u}_t\boldsymbol{u}_t^T\right)$ and $\boldsymbol{\Sigma}_{\epsilon,t} = E_{t-1}\left(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^T\right)$.

[1]"Its oil revenues account for about 95% of its export earnings. But when the oil price plummeted in 2014, Venezuela was faced with a shortfall of foreign currency." in *How Venezuela's crisis developed and drove out millions of people, BBC, Aug 22, 2018* https://www.bbc.co.uk/news/world-latin-america-36319877

In Lütkepohl and Milunovich, 2016, a GARCH(1,1) process is specified for the conditional variance of the structural innovations:

$$\boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{t}} = (\boldsymbol{I_k} - \Delta_1 - \Delta_2) + \Delta_1 \circ \left(\boldsymbol{\epsilon_{t-1}\epsilon_{t-1}}^T\right) + \Delta_2 \circ \boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{t-1}}, \tag{3.10}$$

where Δ_1 and Δ_2 are diagonal matrices, and "$\circ$" denotes the Hadamard product operator. If Δ_1 and Δ_2 are null, then $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{t}}$ is constant. Whereas, if Δ_1 and Δ_2 are positive semi-definite, then $(\boldsymbol{I_k} - \Delta_1 - \Delta_2)$ is positive definite, which indicates that at least one of the structural innovations follow a GARCH(1,1) process. Therefore, the GARCH(1,1) specification for an individual conditional structural variance is

$$\sigma^2_{m,t|t-1} = (1 - \gamma_m - \delta_m) + \gamma_m \epsilon^2_{m,t-1} + \delta_m \sigma^2_{m,t-1|t-2}, \quad m = 1, \ldots, k. \tag{3.11}$$

We follow a two-step procedure, see Bouakez, Essid, and Normandin, 2013 and Bouakez, Chihi, and Normandin, 2014, to estimate the ARCH coefficients Δ_1 and GARCH coefficients Δ_2 by maximizing the likelihood function as follows:

$$\log L \approx -T \log \left|\det\left(\mathbf{A_0}\right)\right| - \frac{1}{2}\sum_{t=1}^{T} \log \left|\det\left(\boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{t}}\right)\right| - \frac{1}{2}\sum_{t=1}^{T} \boldsymbol{\epsilon_t}^T \boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{t}}^{-1} \boldsymbol{\epsilon_t}, \tag{3.12}$$

where the initialization $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon},\boldsymbol{0}} = \left(\boldsymbol{\epsilon_0\epsilon_0}^T\right) = \boldsymbol{I_k}$. This initialization is consistent with the intercept term $\boldsymbol{I_k}$ in eq. (3.10).

The first step requires extracting the estimated statistical innovations $\hat{\boldsymbol{u}}_{\boldsymbol{t}}$ from eq. (3.5). The reduced form SBT-VAR(p) in eq. (3.5) is estimated using maximum likelihood (ML)[2], where the order of p is pre-selected by estimating the baseline VAR in eq. (3.1) using AIC and BIC. Please refer to Galvão, 2006 for discussions regarding the estimation procedure with the reduced form SBT-VAR model. Then, the $T \times k$ matrix of statistical innovation $\hat{u}_t$ can be treated as observables in the second step.

[2]The ML estimation is achieved by $\hat{r}_1, \hat{r}_2, \hat{\tau} = \min\limits_{\substack{L \le r_1 \le r_U \\ r_L \le r_2 \le r_U \\ \tau_L \le \tau \le \tau_U}} \log\left(det\left(\hat{\boldsymbol{\Sigma}}\left(r_1, r_2, \tau\right)\right)\right)$

The second step involves estimating the structural parameters, i.e. the non-zero elements in $\boldsymbol{A_0}$, as well as Δ_1 and Δ_2. As pointed in Lütkepohl and Netšunajev, 2017, this GARCH type structural error resembles the Generalized Orthogonal GARCH (GO-GARCH) model proposed in Weide, 2002. As pointed out in Lanne and Saikkonen, 2007, the GO-GARCH is a special case of factor-GARCH model. This GO-GARCH representation in the statistical innovation not only helps us to identify $\boldsymbol{A_0}$ but also offers us a convenient form for the (co)variance impulse response analysis in the next step.

3.2.3 Generalized Impulse Response Functions with SBT-SVAR

In Hamilton, 1994, p.92, p.327, the impulse response functions reflect how the perturbing shock spreads across time. Evaluation of the dynamic consequence of structural shocks is a particular interest of policy makers. Denote h as the periods succeeding one unit structural shock $\boldsymbol{\xi}_{j,t}$ on variable j at time t, given the information available up to t as $\mathcal{F}_{t-1}$, the impulse response functions, IR in a linear covariance stationary VAR(p) system can be calculated by

$$IR\left(\boldsymbol{h}, \boldsymbol{\xi}_{j,t}, \mathcal{F}_{t-1}\right) = \frac{\partial \boldsymbol{y}_{t+h}}{\partial \boldsymbol{\xi}_{j,t}} \tag{3.13}$$

using the Wold representation of a VAR(p).

However, in a nonlinear system, such as the SBT-SVAR model specified in eq. (3.6), eq. (3.13) can no longer be used to calculate the impulse response functions. In the nonlinear SBT-SVAR model, the variable responses to a shock not only depend on the estimated delay variable d_i, but also depend on the history preceding the shock. Moreover, the perturbing shock might trigger a regime switch if the threshold variable z goes above the threshold r_i.

To evaluate the impact effects of structural shocks in a non-linear SBT-SVAR system, we apply the Generalized Impulse Response Functions (GIRF) proposed by (Koop, Pesaran, and Potter, 1996), which requires a h-step ahead forecasting of the conditional mean of $\boldsymbol{y}_{t+h}$, in response to a one-unit structural shock, $E\left[\boldsymbol{y}_{t+h} \mid \boldsymbol{\xi}_t, \mathcal{F}_{t-1}\right]$, and a h-step ahead forecasting of $\boldsymbol{y}_{t+h}$ only conditional on the history, $E\left[\boldsymbol{y}_{t+h} \mid \mathcal{F}_{t-1}\right]$. The GIRF is then

$$GIRF\left(\boldsymbol{h}, \boldsymbol{\xi}_t, \mathcal{F}_{t-1}\right) = E\left[\boldsymbol{y}_{t+h} \mid \boldsymbol{\xi}_t, \mathcal{F}_{t-1}\right] - E\left[\boldsymbol{y}_{t+h} \mid \mathcal{F}_{t-1}\right]. \tag{3.14}$$

3.2.4 Variance Impulse Response Functions with GARCH Structural Errors

Besides evaluating the impact effects of structural shocks on the conditional means by using the GIRF suggested by Koop, Pesaran, and Potter, 1996, we are also interested in tracing the dynamic responses of the conditional (co)variances $\boldsymbol{\Sigma}_{u,t}$ to perturbing structural shocks. In particular, we want to compare the dynamic responses of the conditional covariance between oil prices and geopolitical risk under two distinct scenarios - one with a simultaneously perturbing shale production structural shock and one without.

In the spirit of Koop, Pesaran, and Potter, 1996, Hafner and Herwartz, 2006 propose a variance impulse response function (VIRF). Denote $\boldsymbol{\Sigma}_{u,t}$ as the initial conditional variance preceding the strucutral shock $\boldsymbol{\xi}_t$, the general expression for VIRF is

$$V_{t+h}\left(\boldsymbol{\xi}_t\right) = E[\operatorname{vech}\left(\boldsymbol{\Sigma}_{u,t+h}\right) \mid \boldsymbol{\xi}_t, \mathcal{F}_{t-1}] - E[\operatorname{vech}\left(\boldsymbol{\Sigma}_{u,t+h}\right) \mid \mathcal{F}_{t-1}]. \tag{3.15}$$

The VIRF calculates the differences between the expectation of volatility conditional on a perturbing shock $\boldsymbol{\xi}_t$ and the history $\mathcal{F}_{t-1}$, and the expectation of volatility only conditional on the history. Hafner and Herwartz, 2006 consider a vec representation of multivariate GARCH(1,1) specified as

$$\operatorname{vech}(\boldsymbol{\Sigma}_{u,t}) = \boldsymbol{W} + \tilde{\boldsymbol{A}}_1 \operatorname{vech}\left(\boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}{}^T\right) + \tilde{\boldsymbol{B}}_1 \operatorname{vech}(\boldsymbol{\Sigma}_{u,t-1}). \tag{3.16}$$

Using eq. (3.15) and using VARMA representation of GARCH model, the general analytic expression for VIRF in Hafner and Herwartz, 2006 is as follows:

$$V_{t+h}\left(\boldsymbol{\xi}_t\right) = \boldsymbol{\phi}_h \boldsymbol{D}_k^+ \left(\boldsymbol{\Sigma}_{u,t}{}^{1/2} \otimes \boldsymbol{\Sigma}_{u,t}{}^{1/2}\right) \boldsymbol{D}_k \operatorname{vech}\left(\boldsymbol{\xi}_t \boldsymbol{\xi}_t{}^T - \boldsymbol{I}_k\right), \tag{3.17}$$

where $\boldsymbol{\phi}_h = \left(\tilde{\boldsymbol{A}}_1 + \tilde{\boldsymbol{B}}_1\right)^{h-1} \tilde{\boldsymbol{A}}_1$, $\boldsymbol{D}_m$ denotes the duplication matrix defined by the property $\boldsymbol{vec}(\boldsymbol{Z}) = \boldsymbol{D}_m \operatorname{vech}(Z)$ for any symmetric $(m \times m)$ matrix $\boldsymbol{Z}$, and $\boldsymbol{D}_m^+$ denotes its Moore-Penrose inverse.

In the previous section, we propose to parameterize the heteroskedastic statistical innovations $\boldsymbol{u}_t$ with a GO-GARCH model. The analytic expression of

VIRF in eq. (3.17) offers an obvious solution to analyse the dynamic impact effects of a structural shock on the conditional (co)variances.

In order to apply eq. (3.17), we have to make a connection between the vec GARCH and the GO-GARCH(1,1) representations. In Weide, 2002 and Bauwens, Laurent, and Rombouts, 2006b, the GO-GARCH model is a generalization of the Orthogonal-GARCH model, which is also a special case of the factor GARCH models. Thus, the GO-GARCH model is nested in the general BEKK model (Engle and Kroner, 1995a), where its properties follow from those of the BEKK model. Hence, we first transform the GO-GARCH into the BEKK, and then into its vec GARCH representation. See Appendix (3.5.1) for the transformation from a GO-GARCH model to a vec GARCH. Finally, the VIRF in eq. (3.17) can be calculated using the estimated $\widehat{\boldsymbol{A}_0}^{-1}$ from the identified SBT-SVAR model.

The conditional moment profile framework proposed in Gallant, Rossi, and Tauchen, 1993 is similar to the VIRF proposed in Hafner and Herwartz, 2006. A comparison of the conditional moment profile of volatility to the baseline profile, we refer to it as conditional volatility profile hereafter, is analogous to VIRF. In Gallant, Rossi, and Tauchen, 1993, the shocks are interpreted as a direct perturbation on $\boldsymbol{y}_t$. Therefore, the statistical innovation $\boldsymbol{u}_t$ can be viewed as an impulse or shock adding on the contemporaneous $\boldsymbol{y}_t$.

The analytic expressions of the conditional volatility profile are also given in Hafner and Herwartz, 2006 based on the types of shock and baseline. Suppose the baseline $\boldsymbol{u}_0$ is fixed to $\boldsymbol{0}$, a shock is fixed at $\boldsymbol{\delta}_t$, the conditional volatility profile is denoted as $\boldsymbol{v}_{t+h}\left(\boldsymbol{\delta}_t\right)$,

$$\boldsymbol{v}_{t+h}\left(\boldsymbol{\delta}_t\right) = E[\text{vech}\left(\boldsymbol{\Sigma}_{u,t+h}\right) \mid \boldsymbol{u}_t = \boldsymbol{u}_0 + \boldsymbol{\delta}_t, \mathcal{F}_{t-1}] - E[\text{vech}\left(\boldsymbol{\Sigma}_{u,t+h}\right) \mid \boldsymbol{u}_t = \boldsymbol{u}_0, \mathcal{F}_{t-1}]. \tag{3.18}$$

We skip the derivations in Hafner and Herwartz, 2006, but only demonstrate the analytic expression of VIRF from Hafner and Herwartz, 2006 achieved using the VMA representation,

$$
\begin{aligned}
v_{t+h}\left(\boldsymbol{\delta}_t\right) &= \boldsymbol{\phi}_h\left[\operatorname{vech}\left((\boldsymbol{u}_0+\boldsymbol{\delta}_t)(\boldsymbol{u}_0+\boldsymbol{\delta}_t)^T\right)-\operatorname{vech}\left(\boldsymbol{u}_0\boldsymbol{u}_0{}^T\right)\right] \\
&= \boldsymbol{\phi}_h\boldsymbol{D}_k^+\operatorname{vec}\left(\boldsymbol{\delta}_t\boldsymbol{\delta}_t{}^T+2\boldsymbol{\delta}_t\boldsymbol{u}_0{}^T\right).
\end{aligned} \tag{3.19}
$$

Given the baseline $\boldsymbol{u}_0$ is fixed to $\mathbf{0}$, the conditional volatility profile simplifies to

$$
v_{t+h}\left(\boldsymbol{\delta}_t\right) = \boldsymbol{\phi}_h\operatorname{vech}\left(\boldsymbol{\delta}_t\boldsymbol{\delta}_t{}^T\right). \tag{3.20}
$$

Suppose a fixed shock with size $\boldsymbol{\delta}_t = \boldsymbol{\Sigma}_{\boldsymbol{u},t}{}^{1/2}\boldsymbol{\xi}_t$, the conditional volatility profile is then

$$
\begin{aligned}
v_{t+h}\left(\boldsymbol{\Sigma}_{\boldsymbol{u},t}{}^{1/2}\boldsymbol{\xi}_t\right) &= \phi_h\operatorname{vech}\left(\boldsymbol{\Sigma}_{\boldsymbol{u},t}{}^{1/2}\boldsymbol{\xi}_t\boldsymbol{\xi}_t{}^T\boldsymbol{\Sigma}_{\boldsymbol{u},t}{}^{1/2}\right) \\
&= V_{t+h}\left(\boldsymbol{\xi}_t\right)+\boldsymbol{\phi}_h\operatorname{vech}\left(\boldsymbol{\Sigma}_{\boldsymbol{u},t}{}^{1/2}\right).
\end{aligned} \tag{3.21}
$$

Comparing eq. (3.15) and eq. (3.21), there is a clear connection between the VIRF and the conditional volatility profile. However, the interpretations of the perturbing shocks and the initial conditions are different. Gallant, Rossi, and Tauchen, 1993 argue for a representative impulse response sequence, that is either using an "average" initial condition or taking the average of many impulse-response sequences conditional on many different initial conditions draws drawn from their marginal density. Regarding the perturbing shocks, Gallant, Rossi, and Tauchen, 1993 experiment with different sizes of shocks in a bivariate system. Hafner and Herwartz, 2006 argue that the choices of innitial condition (baseline) and shock could be arbitary. Hafner and Herwartz, 2006 give a comprehensive discussion of the four different resulting $v_{t+h}\left(\boldsymbol{\delta}_t\right)$ due to the permutations of (either fixed or random) baselines and shocks.

Hafner and Herwartz, 2006 compare the conditional volatility profile in eq. (3.18) with VIRF in eq. (3.15), focusing on the impact effects of two specific events on the conditional volatilities.

To calculate the conditional volatility profile $v_{t+h}\left(\boldsymbol{\delta}_t\right)$ in (3.18), Hafner and Herwartz, 2006 set the baseline as zero, i.e. $\boldsymbol{u}_0 = \mathbf{0}$. The estimated residual

$\hat{\boldsymbol{u}}_t$ on the event day t is considered to be the perturbing shock $\boldsymbol{\delta}_t$, i.e. $\boldsymbol{\delta}_t = \hat{\boldsymbol{u}}_t$. Given the interpretation of shocks is different in the VIRF framework than in the conditional volatility profile, to calculate $V_{t+h}(\boldsymbol{\xi}_t)$ in eq. (3.15), a standardized estimated residual is taken as the shock, i.e. $\boldsymbol{\xi}_t = \hat{\boldsymbol{\epsilon}}_t$. Hence, using eq. (3.9) we can estimate the shock $\hat{\boldsymbol{\epsilon}}_t$ on day t using the estimated residual $\hat{\boldsymbol{u}}_t$ and the estimated volatility state $\boldsymbol{\Sigma}_{u,t}$. In this way, the identified structural shock $\hat{\boldsymbol{\epsilon}}_t$ in Hafner and Herwartz, 2006 is interpreted as a materialised shock, which reflects the information in independent news.

In our case, we neither focus on the impact effects of a historical shock $\boldsymbol{\xi}_t$ on a particular day, treating $\boldsymbol{\xi}_t = \hat{\boldsymbol{\epsilon}}_t$ as proposed in Hafner and Herwartz, 2006, nor try to directly inflict different contemporaneously related $\boldsymbol{u}_t$ on $\boldsymbol{y}_t$, treating $\boldsymbol{\delta}_t = \hat{\boldsymbol{u}}_t$ as proposed in Gallant, Rossi, and Tauchen, 1993, we arbitrarily choose fixed values of hypothetical $\boldsymbol{\xi}_t$, and investigate the conditional (co)variances respond to these hypothetical orthogonal shocks $\boldsymbol{\xi}_t$, given different points in time with a heteroskedastic $\boldsymbol{\Sigma}_{u,t}$.

Suppose the dependent variable $\boldsymbol{y}_t$ is ordered as $\boldsymbol{y}_t = (GPR_t, 100 \times \Delta \log ShaleP_t, 100 \times \Delta \log OilP_t)^T$, where $100 \times \Delta \log ShaleP_t$ calculates the percentage change in the shale production and $100 \times \Delta \log OilP_t$ calculates the oil returns. Let a hypothetical structural shock $\boldsymbol{\xi}_t$ at time t as $\boldsymbol{\xi}_t = \boldsymbol{\epsilon}_t = (1,0,0)^T$, the order of the variables indicates that there is only one unit hypothetical structural shock imposed on the geopolitical risk at time t. Similarly, let another type of unit hypothetical structural shock $\boldsymbol{\xi}_t^*$ be imposed on both geopolitical risk and shale production simultaneously, i.e. $\boldsymbol{\xi}_t^* = \boldsymbol{\epsilon}_t^* = (1,1,0)^T$. Denoting the VIRF with shock $\boldsymbol{\xi}_t^*$ for h periods ahead as V_{t+h}^*, and VIRF with shock $\boldsymbol{\xi}_t$ as V_{t+h}, using eq. (3.17),

$$V_{t+h}^*(\boldsymbol{\epsilon}_t^*) - V_{t+h}(\boldsymbol{\epsilon}_t) = \boldsymbol{\phi}_h \boldsymbol{D}_k^+ \left(\boldsymbol{\Sigma}_{u,t}{}^{1/2} \otimes \boldsymbol{\Sigma}_{u,t}{}^{1/2}\right) \boldsymbol{D}_k \operatorname{vech}\left(\boldsymbol{\xi}_t^* \boldsymbol{\xi}_t^{*T} - \boldsymbol{\xi}_t \boldsymbol{\xi}_t^T\right) \tag{3.22}$$

calculates the differences in VIRF under two different circumstances given a perturbing geopolitical shock, i.e. one under a simultaneous shale shock, whereas the other without.

We now look at the differences in variance responses in the two scenarios using the conditional volatility profiles. Suppose the shock $\boldsymbol{\delta}_t$ directly inflicts on $\boldsymbol{y}_t$ takes the form $\boldsymbol{\delta}_t = \boldsymbol{\Sigma}_t^{1/2} \boldsymbol{\epsilon}_t$, and suppose a hypothetical shock $\boldsymbol{\epsilon}_t = (1,0,0)^T$, the corresponding conditional variance profile is denoted as $\boldsymbol{v}_{t+h}$. In another scenario, suppose the hypothetical shock consists of both a geopolitical risk shock and a shale production shock, i.e. $\boldsymbol{\epsilon}_t^* = (1,1,0)^T$, and shock δ_t^* takes the form $\delta_t^* = \boldsymbol{\Sigma}_t^{1/2}\epsilon_t^*$, denote the corresponding conditional variance profile as v_{t+h}^*. Using eq. (3.18), the difference between these two conditional (co)variance profiles under the two different direct perturbations is

$$
\begin{aligned}
v_{t+h}^*\left(\delta_t^*\right) - v_{t+h}\left(\delta_t\right) &= \boldsymbol{\phi}_{t+h}\left(\operatorname{vech}\left(\boldsymbol{\Sigma}_{u,t}^{1/2}\epsilon_t^*\epsilon_t^{*T}\boldsymbol{\Sigma}_{u,t}^{1/2}\right) - \operatorname{vech}\left(\boldsymbol{\Sigma}_{u,t}^{1/2}\epsilon_t\epsilon_t^T\boldsymbol{\Sigma}_{u,t}^{1/2}\right)\right) \\
&= V_{t+h}^*\left(\boldsymbol{\epsilon}_t^*\right) - V_{t+h}\left(\boldsymbol{\epsilon}_t\right)
\end{aligned}
\tag{3.23}
$$

Therefore, using a fixed baseline, the impact differences between the two fixed hypothetical shocks $\boldsymbol{\epsilon}_t^*$ and $\boldsymbol{\epsilon}_t$ has the same analytical expression in the conditional volatility profile as that in the VIRF framework.

As seen from eq. (3.17), the three functions, $V_{t+h}^*\left(\boldsymbol{\epsilon}_t^*\right) - V_{t+h}\left(\boldsymbol{\epsilon}_t\right)$, $V_{t+h}^*\left(\boldsymbol{\epsilon}_t^*\right)$ and $V_{t+h}\left(\boldsymbol{\epsilon}_t\right)$, share the same rate of decay, which is determined by $\boldsymbol{\phi}_h$. Also, because we fix the size of shocks in $\boldsymbol{\epsilon}_t^*$ and $\boldsymbol{\epsilon}_t$, $\operatorname{vech}\left(\boldsymbol{\epsilon}_t^*\boldsymbol{\epsilon}_t^{*T}\right) - \operatorname{vech}\left(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^T\right)$ is also fixed. Therefore, the volatility impulse responses difference, $V_{t+h}^*\left(\boldsymbol{\epsilon}_t^*\right) - V_{t+h}\left(\boldsymbol{\epsilon}_t\right)$, will mainly depend on the conditional variance $\boldsymbol{\Sigma}_{u,t}$ at that point in time. It also depends on $\widehat{\boldsymbol{A}_0}^{-1}$, which is also reflected in the decay parameter $\boldsymbol{\phi}_h$.

In order to evaluate the impact effects of shale revolution, we analyse the differences in VIRF, $V_{t+h}^*\left(\boldsymbol{\epsilon}_t^*\right) - V_{t+h}\left(\boldsymbol{\epsilon}_t\right)$, or the differences in two conditional volatilities profiles, $v_{t+h}^*\left(\boldsymbol{\epsilon}_t^*\right) - v_{t+h}\left(\boldsymbol{\epsilon}_t\right)$, under the two types of hypothetical shocks, i.e. shock $\boldsymbol{\epsilon}_t^*$ represents that shale production is imposed with a simultaneous shock as well as geopolitical risk, whereas $\boldsymbol{\epsilon}_t$ indicates only the geopolitical risk variable is imposed with a unit shock.

3.3 Empirical results

In this section, we aim to analyse the changing dynamics of oil prices under the shale oil revolution. Also, we would like to pin down the impact effects of geopolitical risk on oil prices by allowing for simultaneous shale production shocks.

3.3.1 Data

For the measurement of geopolitical risk, we use an index constructed by Caldara and Iacoviello, 2018. Caldara and Iacoviello, 2018 follow the method proposed in Saiz and Simonsohn, 2013 and Baker, Bloom, and Davis, 2016 and conduct an automatic text search over 11 English newspapers, including 6 from the U.S., 4 British ones, and 1 Canadian newspaper. The index is constructed by counting the frequency of articles related to geopolitical risks, i.e. $GPR = No.GPRrisk\ articles/No.Total\ articles$. In Caldara and Iacoviello, 2018, geopolitics is defined as "the practice of states and organizations to control and compete for territory", whereas geopolitical risk is defined as "risks associated with wars, terrorist acts, or tensions between states that affect the normal and peaceful course of international relations". Hence, the geopolitical threat risk index constructed by Caldara and Iacoviello, 2018 relates to threats of conflicts in the world and it represents the proportion of western mainstream media coverage in English speaking countries. In fig. (3.1), the top left plots the monthly geopolitical threat risk index (GPR).

Then the GPR index is standardized using $\left(GPR_t - \overline{GPR}\right)/\sigma_{GPR}$. The monthly shale oil production (ShaleP), measured by thousands barrels per day, is collected from the U.S. Energy Information Administration. The West Texas Intermediate (WTI) crude oil spot prices, in dollars per barrel, are also collect from the U.S. Energy Information Administration. Then, the WTI prices are adjust to real prices using a monthly CPI index. The CPI index (for all urban consumers) is collected from the Bureau of Labour Statistics, [1982-84=100] [3]. The return of oil price is calculated by $R_t = 100 \times \log\left(\Delta OilP_t\right)$. Fig. (3.1) plots all original and transformed data series.

3.3.2 Estimation Results

First, the data $\boldsymbol{y_t} = \left(GPR_t, 100 \times \Delta \log ShaleP_t, 100 \times \Delta \log OilP_t\right)^T$ is fitted using five competing models, which are VAR, TVAR, SBVAR, SBTVAR with changing thresholds, and SBTVAR with a constant threshold. The geopolitical threat risk Index, GPR_t, is chosen as the threshold variable z. Table (3.1) indicates that based on the LR, FPF and AIC criterion, the optimal lag length is selected as $p = 2$ in the baseline VAR model from an Ordinary Least Square

[3] https://www.bls.gov/cpi/tables/supplemental-files/historical-cpi-u-201805.pdf

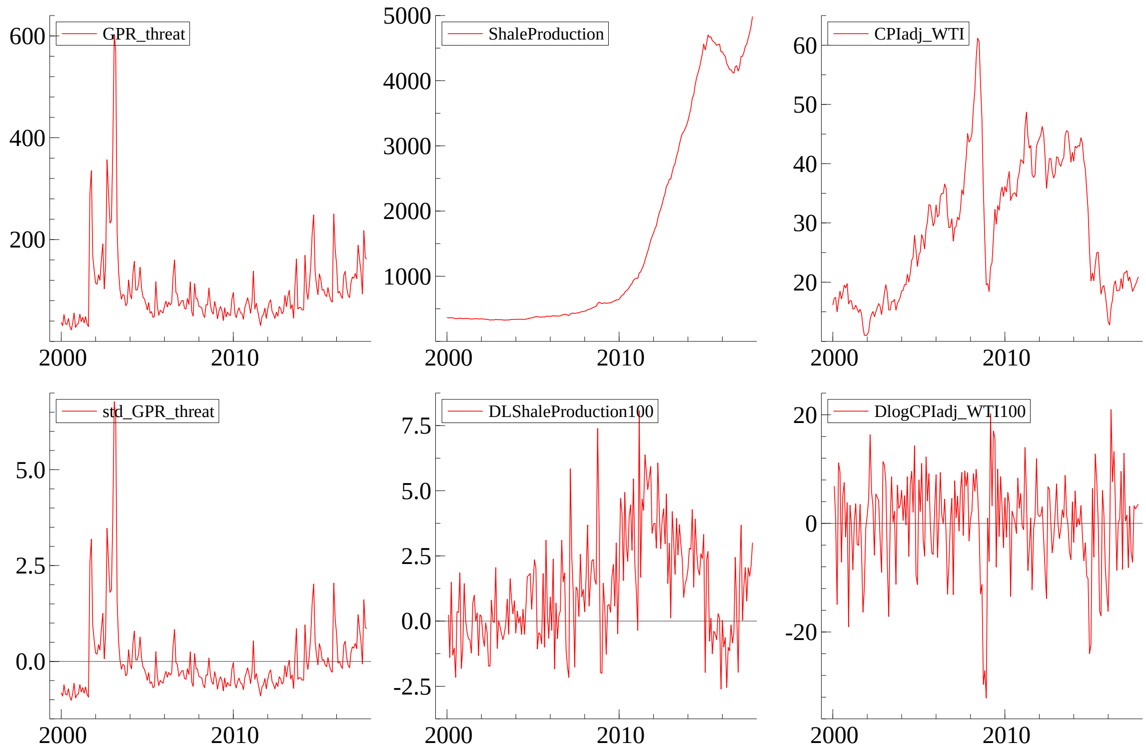

FIGURE 3.1: Plots of data series. Top panel from left to right plots: monthly GPR-threat index, monthly shale oil production (ShaleP) in thousands barrels per day, CPI adjusted monthly WTI oil price. Bottom panel from left to right plots: $GPR - threat_t$, standardized monthly GPR-threat index, $100 \times \Delta \log ShaleP_t$, monthly percentage change in shale oil production, $100 \times \Delta \log OilP_t$, monthly return on WTI

(OLS) estimation. Therefore, we pre-specify $p = 2$ in the reduced form SBT-VAR.

TABLE 3.1: Optimal lag length p selection in the baseline VAR

lag	LL	LR	df	p	FPF	AIC	HQIC	SBIC
0	-1497.62				301.937	14.223	14.243	14.271
1	-1365.41	264.41	9	0.000	93.918	13.056	13.133*	13.247*
2	-1351.3	28.227*	9	0.001	89.48*	13.008*	13.142	13.341*

TABLE 3.2: Estimation results using the reduced form models

	TVAR	SBVAR	SBTVARc	SBTVAR
$\hat{d}$	12	–	1	15, 23
$\hat{r}$	0.088	–	-0.089	-0.357
	(-4.819, 4.417)		(-12.332, -1.284)	(0.043, 2.229)
	TR1[-0.4252]			0.306
	TR2[0.0333]			(-3.345, 13.099)
$\hat{\tau}$	–	2007	2014.417	2013.75
		(2006.917, 2007.667)	(2013.483, 2014.750)	(2013.482, 2014.750)
		SB1[2007.000]		
		SB2[2011.750]		
T[189]	139	59	105	68
	50	130	43	72
			9	41
			32	8

Table (3.2) presents the estimation results using the competing models in their reduced form. Then, we follow a "specific to general" approach in Galvão, 2006, to select the model based on bounded Wald (W) and LM statistics.

Denote θ_1 under the null and θ_2 under the alternative, the W and LM statistics are

$$\begin{aligned} W(\theta_2) &= n\left(\frac{SSR(\hat{\theta}_1) - SSR(\theta_2)}{SSR(\theta_2)}\right), \\ LM(\theta_2) &= n\left(\frac{SSR(\hat{\theta}_1) - SSR(\theta_2)}{SSR(\hat{\theta}_1)}\right). \end{aligned}$$

Based on Altissimo and Corradi, 2002, BWald and BLM are the maximum values of a Wald and LM statistic over a grid of possible values for the nuisance parameter. Using the asymptotic bounds $(1/2ln(ln(T)))$, the decision rule is that the model under alternative will be selected if Bounded Wald (BWald) or Bounded LM statistic (BLM) is larger than 1. The bounded Wald statistic (BWald) is

$$BWald = \left[\frac{1}{2\ln(\ln(T))}\left[\underset{\theta_2^L \le \theta_2 \le \theta_2^U}{sup} \; W(\theta_2)\right]^{1/2}\right] > 1. \tag{3.24}$$

Interested readers can refer to the simulation study in Galvão, 2006 with respect to the ability of BWald and BLM to discriminate between different reduced form VAR specifications.

Table (3.3) presents the estimated BWald and BLM for the model selection procedure. Step 1 consists two sets of model comparisons in 1A and 1B. The baseline VAR is compared with the alternative TVAR and SBVAR models. The BWald and BLM in 1B are both larger than 1. Therefore, SBVAR is selected based on the desicion rule. As pointed out in Galvão, 2006, only if none of the alternative hypotheses are rejected using the decision rule, the VAR shall be chosen. Otherwise, if at least one of the statistics suggests rejection of the VAR, we have to proceed to step 2.

We then proceed to step 2, i.e. 2A1, 2A2, 2B1, and 2B2, which consists of four sets of model comparison. According to the statistics in step 1 (1A and 1B) in table (3.3), BWald (BLM) with SBVAR under the alternative is larger than BWald (BLM) with TVAR under the alternative, we use the statistics

TABLE 3.3: BWald and LM bounds with monthly data 2000 : 1 - 2017 : 10. Selection rule: if BWald(BLM) $>$ 1, choose model under alternative.

	H_0 VS H_A	BWald	BLM
1A	VAR : TVAR	0.834	0.818
1B	**VAR : SBVAR**	**1.116**	**1.078**
2A1	TVAR : SBTVARc	1.514	1.422
2A2	TVAR : SBTVAR	0.894	0.874
2B1	**SBVAR : SBTVARc**	**1.308**	**1.248**
2B2	SBVAR : SBTVAR	0.513	0.509
X1	TVAR : 3R-TVAR	1.171	1.127
X2	SBVAR : 2-SBVAR	1.277	1.221

in step 2B1 and 2B2 to verify whether the inclusion of a threshold improves the SBVAR using estimated SBT-VAR and SBT-VARc under the alternative. Because the statistic with SBT-VARc under the alternative (in 2B1) is larger than the statistic with SBT-VAR under the alternative (in 2B2), the inclusion of a constant threshold has to be considered. Therefore, from table (3.3), the SBT-VARc model is chosen based on the decision rule. If both statistics in 2B are smaller than 1, the SBVAR would have been chosen.

According to the estimation results in table (3.2), a break is detected in April/May, 2014. The 90% confidence interval, computed using bootstrap, for the break is between April, May 2013 and September 2014.

3.3.3 GARCH Structural Innovations and a Flexible A_0

Fig. (3.2) plots the estimated residuals $\hat{u}_t$ from the reduced form SBT-VARc model. From the histograms and a clustering patten of $\hat{u}_t$, we are incentivized to use the GO-GARCH model and allow for heteroskedasticity in the conditional (co)variances. Hence, with the GO-GARCH representation, we are able to identify ${A_0}^{-1}$ in

$$\boldsymbol{u_t} = \boldsymbol{A_0}^{-1}\boldsymbol{\epsilon_t},$$

as well as to identify the heteroskedastic structural shocks $\boldsymbol{\epsilon_t}$.

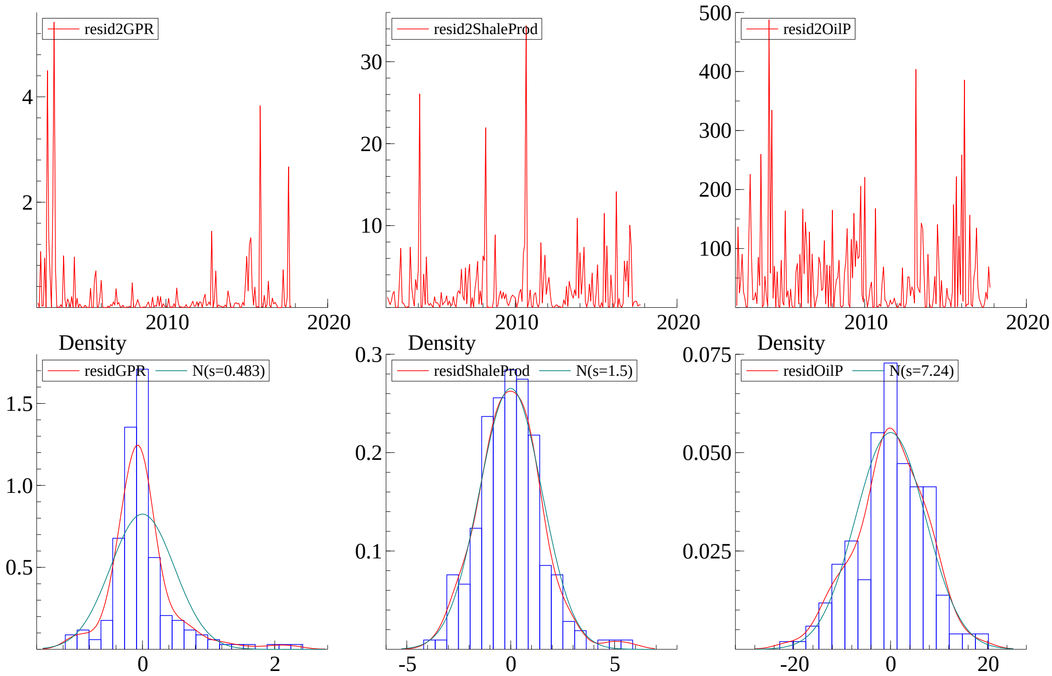

FIGURE 3.2: Heterskadsitic statistic innovation $\hat{u}_t$ and fitted distribution

Table (3.4) presents the estimated results from the GO-GARCH model in eq. (3.9) and eq. (3.11). From table (3.4), the structural shocks in the standardized geopolitical threat risk $\epsilon_{t,GPR}$ is estimated to follow a univariate GARCH(1,1) process and the structural shocks in the oil returns $\epsilon_{t,OilP}$ are estimated to follow an ARCH process.

TABLE 3.4: Identify $\mathbf{A_0}^{-1}$ with GARCH structural errors

$\widehat{A_0^{-1}}$	-0.945	0.323	-0.023
	0.129	0.126	-0.983
	-0.311	-0.947	-0.074
		coef.	t-prob
STAD GPR Threat	$\hat{\gamma}_1$	0.122	0.034
	$\hat{\delta}_1$	0.817	0.000
Δ log Shale	$\hat{\gamma}_2$	0.099	0.108
	$\hat{\delta}_2$	0.000	0.1878
Δ log OilP	$\hat{\gamma}_3$	0.320	0.011
	$\hat{\delta}_3$	-0.054	0.490

Fig. (3.3) plots the estimated conditional variance, covariance and correlation for $u_{t,GPR}$, $u_{t,ShaleP}$ and $u_{t,OilP}$. Towards the end of 2011, there is a big spike in the conditional covariance between shale production and oil prices, largely due to the spike in the shale production variance. This result coincides with the time line of the shale revolution after the successful horizontal drilling experiment in Bakken Shale Play[4].

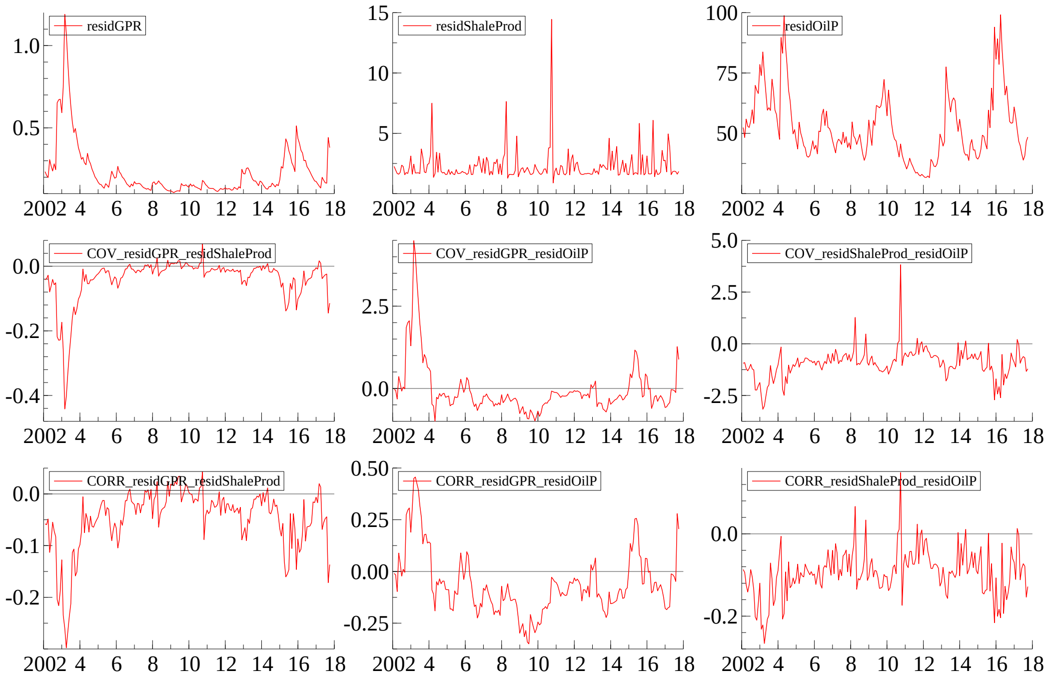

FIGURE 3.3: Estimated conditional (co)variance and correlation using GO-GARCH structural errors. Top panel plots the conditional variances, middle panel plots the conditonal covariances, and bottom panel plots the conditional correlation.

3.3.4 Generalized Impulse Responses and Variance Impulse Response Functions

Using eq. (3.14), we analyse the responses of oil returns to one-unit structural shock in the geopolitical risk index. Because the geopolitical risk variable (GPR_t) is chosen as the threshold variable, a structural shock - in combination

[4]For more information relating to the history of advances in the technology of shale production, through fracking oil from its rock formation, please see https://bakkenshale.com

with the feedback effects from other endogenous variables such as $ShaleP_t$ and $OilP_t$ - may trigger a regime switch. Therefore, the impulse response functions in the nonlinear SBT-SVARc may not appear to be smooth decaying functions as it would be in a linear baseline VAR. Moreover, according to eq. (3.5), the regime switch also depends on the estimated delay variable $\hat{d}_i$. Therefore, the impulse response functions for h step ahead also depends on the history $\mathcal{F}_{t+h-1}$.

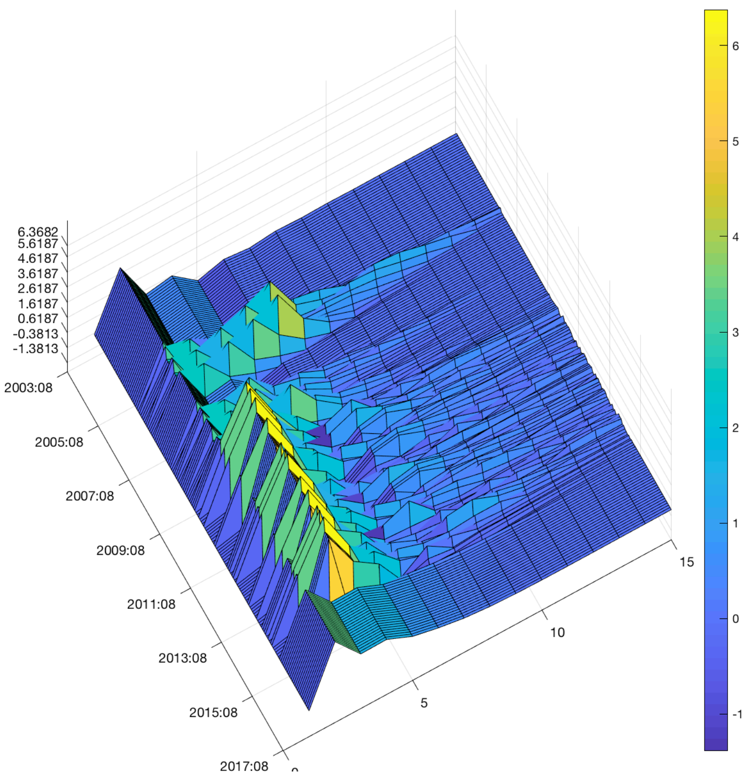

FIGURE 3.4: ΔOil price responses to shock in GPR with GO-GARCH structural errors

Using the estimated results from the reduced form SBT-VARc and $\widehat{\boldsymbol{A_0}^{-1}}$ in table (3.4), we impose a structural shock $\boldsymbol{\epsilon_t} = (1, 0, 0)^T$, with $t = 1, \ldots, 170$. That is imposing a one unit shock on geopolitical risk, where $\epsilon_{t,GPR} = 1$, starting from August 2003. We would like to see how oil returns respond to the same size of geopolitical shocks over time. The same exercise is repeated over 170 periods. Fig. (3.4) plots these 170 generalized impulse response functions of oil price returns to one - unit shock on the geopolitical risk variable ϵ_t using eq. (3.14).

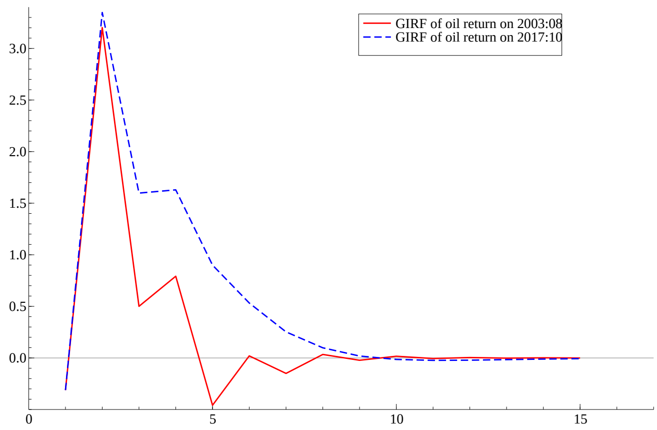

FIGURE 3.5: ΔOil price responses to shock in GPR a comparison between 2003:08 and 2017:10. Dash line plots the GIRF of ΔOil price in August 2003. Solid line plots the GIRF of ΔOil price in August 2017.

We pick the first series (August 2003) and the last series (October 2017) from fig. (3.4) and plot them together for an ad hoc comparison. Fig. (3.5) compares the oil price responses to a one unit geopolitical risk shock before and after the estimated break in 2014. Both fig. (3.4) and fig. (3.5) show that the generalized impulse response functions are smoother towards the end of the sample of 170.

Given that the size of the hypothetical structural shocks is fixed and the identified $\widehat{\boldsymbol{A_0}}^{-1}$ does not vary with time, the smoothness in the response functions imply that the impose structural shock $\boldsymbol{\epsilon_t}$ did not induce abrupt regime switches after the break. Given the GPR index around August 2003 and October 2017 are on a similar level, see from fig. (3.1), this difference in the impulse response functions must be owing to a joint effort from the feedback coefficient matrices ($\widehat{\boldsymbol{A_3}}$ and $\widehat{\boldsymbol{A_4}}$) and identified contemporaneous relationship amongst the variables $\widehat{\boldsymbol{A_0}}^{-1}$. The shock impact effects has a smoother spread over time after the break.

Appendix (3.5.2) plots the generalized impulse response functions of oil prices to a one unit structural shock in the geopolitical risk variable using the Cholesky identification method, where $\widehat{\boldsymbol{A_0}}^{-1}$ is restricted to be the lower triangular of $\boldsymbol{\Sigma_u}$ via a Cholesky decomposition. We argue that, in line with the existing literature, the Cholesky method to identify structural models implies restrictive and unrealistic assumptions.

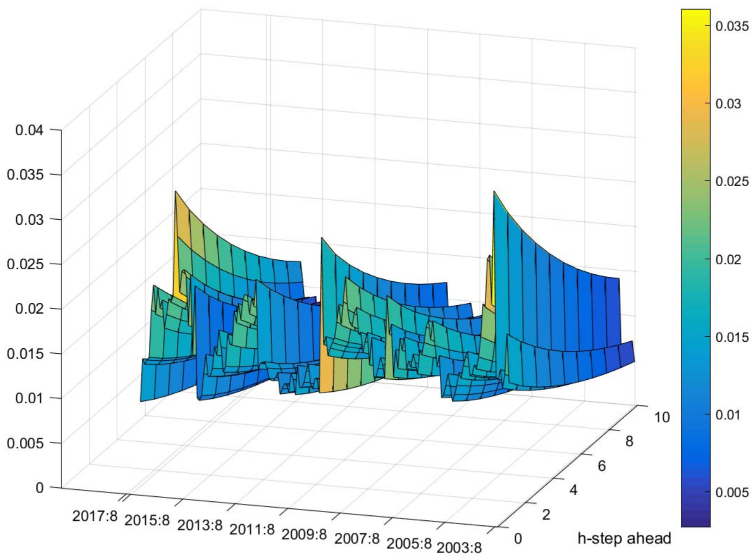

FIGURE 3.6: $V^*_{t+h}(\boldsymbol{\epsilon}^*_t) - V_{t+h}(\boldsymbol{\epsilon}_t)$ for $\sigma^2_{OilP,t}$. The imposing shock $\boldsymbol{\xi}^*_t = (1,1,0)^T$ represents a structural shock with one unit on geopolitical risk with a simultaneous unity shale production shock. Shock $\boldsymbol{\xi}_t = (1,0,0)^T$ represents only geopolitical risk variable is imposed with a unit structural shock.

Rather than focusing on the effects of a single historical event as proposed in Hafner and Herwartz, 2006, we aim to uncover the (co)variance response functions to different hypothetical orthogonal shocks at different points in time. The two hypothetical shocks we impose on the system are $\boldsymbol{\epsilon}^*_t$ and $\boldsymbol{\epsilon}_t$, where $\boldsymbol{\epsilon}^*_t = (1,1,0)^T$ represents a simultaneous unity geopolitical risk shock and a unity shale production shock, whereas $\boldsymbol{\epsilon}_t = (1,0,0)^T$ represents the hypothetical unit structural shock is only imposed on the geopolitical risk variable.

The variance impulse responses $V^*_{t+h}(\boldsymbol{\epsilon}^*_t)$ and $V_{t+h}(\boldsymbol{\epsilon}_t)$ are then calculated using eq. (3.15). Given the discussion in section (3.2.4), the difference in VIRF, $V^*_{t+h}(\boldsymbol{\epsilon}^*_t) - V_{t+h}(\boldsymbol{\epsilon}_t)$, and the difference in two conditional volatilities profiles, $v^*_{t+h}(\boldsymbol{\epsilon}^*_t) - v_{t+h}(\boldsymbol{\epsilon}_t)$, have the same analytic expression. Therefore, using eq. (3.22), the function $\boldsymbol{\phi}_h \boldsymbol{D}^+_k \left(\boldsymbol{\Sigma}^{1/2}_{u,t} \otimes \boldsymbol{\Sigma}^{1/2}_{u,t}\right) \boldsymbol{D}_k \operatorname{vech}\left(\boldsymbol{\xi}^*_t \boldsymbol{\xi}^{*T}_t - \boldsymbol{\xi}_t \boldsymbol{\xi}_t{}^T\right)$ is determined by $\boldsymbol{\phi}_h$ and the estimated time-varying (co)variance $\boldsymbol{\Sigma}_{u,t}$. Fig. (3.6) plots the volatility impulse response difference of oil returns and fig. (3.7) plots the difference of $cov(GPR_t, OilP_t)$ responses, to the two types of hypothetical shocks respectively.

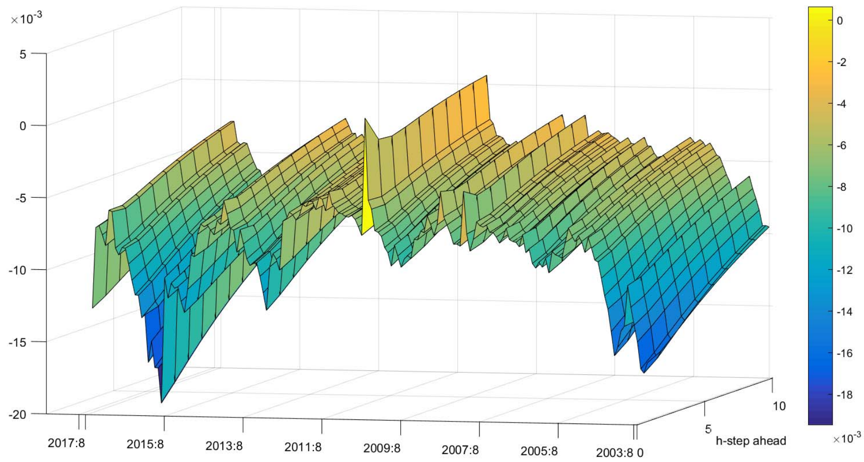

FIGURE 3.7: $V^*_{t+h}(\boldsymbol{\epsilon}^*_t) - V_{t+h}(\boldsymbol{\epsilon}_t)$ for $cov_{GPR_t,OilP_t}$. The imposing shock $\boldsymbol{\xi}^*_t = (1,1,0)^T$ represents a structural shock with one-unit on geopolitical risk with a simultaneous unity shale production shock. Shock $\boldsymbol{\xi}_t = (1,0,0)^T$

From fig. (3.6), the largest VIRF difference is at around 2003 and 2015 (0.035). The smallest VIRF difference is at around 2010(0.01), which implies that the extra unit shale oil production shock induces small positive increases on oil price volatility. In other words, the conditional volatility of oil prices is higher under a simultaneous geopolitical risk and shale oil production shock, compared with the counterpart scenario where the shock is only imposed on the geopolitical risk variable. Because the VIRF difference function, in eq. (3.22), mainly depends on the conditional (co)variance at that point in time, by examining fig. (3.3), the conditional variance $\boldsymbol{\sigma_{OilP}}$ in $\boldsymbol{\Sigma_{u,t}}$ is maximized at around 2003 and 2015 periods, and minimized around 2010, it is not surprising that a positive shale production shock does not lower the conditional volatility response in oil price to geopolitical risk shocks.

From fig. (3.7), the geopolitical risk and oil price covariance responses ($\sigma_{GPR_t,OilP_t}$) to the extra unit shale production shock ($\boldsymbol{\epsilon}^*_t = (1,1,0)^T$) is decreased by $5 \sim 20 \times 10^{-3}$ compared with the covariance responses without shale production ($\boldsymbol{\epsilon}_t = (1,0,0)^T$) in the 170 periods, apart from a small window around 2010.

The differences of $\sigma_{GPR_t,OilP_t}$, which are the 4-th elements in vech $\left(V^*_{t+h}\left(\boldsymbol{\epsilon}^*_t\right) - V_{t+h}\left(\boldsymbol{\epsilon}_t\right)\right)$, were maximized around 2015 and 2003, and minimized round 2010. The shape of the differenced (co)variance response surface over these 170 sample periods corresponds to the estimated conditional GO-GARCH (co)variance, which is plotted in fig. (3.3).

3.4 Conclusion

In this chapter, we focus on two aspects: First, the impact of geopolitical uncertainties on oil price volatility under the shale oil production. Second, oil price impulse response functions under a structural geopolitical risk shock amidst the shale revolution.

We extend a reduced form structural break threshold vector autoregressive (SBT-VAR) model to its' structural form (SBT-SVAR). Then, we allow for a flexible contemporaneous relationship amongst the variables and identify the structural innovations by allowing for heteroskedasticity. Compared with the conventional reduced form VAR and TVAR models, a SBT-VAR with a constant threshold is supported by the data. A break point in 2014 is identified. We find that the Cholesky decomposition method and fixing the order of the variables may lead to misinterpretations and over-estimations of the responses of oil price to the geopolitical risk shocks. Over a 170 sample period, the impulse response functions of oil prices to a unity structural geopolitical risk shock is smoother after the break point in 2014 compared with those before the break. We argue, given a similar level of the threshold variable, i.e. the geopolitical risk index, the identified feedback coefficient matrices in a structural SBT-VAR model help the imposed shock to smoothly spread over time after 2014.

We then analyse the (co)variance impulse response with respect to two distinct shock scenarios, one with only a geopolitical risk shock, the other with simultaneous shale production and geopolitical risk shocks. We find the conditional volatility of oil prices are higher with a shale production shock than without in the 170 sample period. Allowing for changes in the unconditional variances could be a further extension to this research.

The covariance response between geopolitical risk and oil price is reduced by $5 \sim 20 \times 10^{-3}$ with the extra unit shale production shock. The scale of the differences in the (co)variance responses over this 170 sample period depends on the identified ${\boldsymbol{A_0}}^{-1}$ in the SBT-SVAR model, as well as the GO-GARCH specification in the statistical innovations. The differences in the (co)variance responses under the two scenarios also correspond to the estimated conditional volatilities at those points in time.

In terms of the further research, firstly the impact of shale oil production on the U.S. crude oil market is valuable to study by add other crude oil market variables, such as crude oil inventory in the U.S. market. Also, it is interesting to discuss in further if the U.S. becomes more dominant in the world crude oil market by shale oil production boosting. Finally, it probably results different impose response if other structure model identify method is applied. I encourage researchers to apply a different method to redo the same analysis. It will be interesting to compare the difference and valuable to make some conclusion.

3.5 Appendix to Chapter 3

3.5.1 GO-GARCH in BEKK representation

Denote information set up to t is $\mathcal{F}_t$, $E_{t-1}(\cdot) \equiv E(\cdot \mid \mathcal{F}_{t-1})$ and the statistical innovation is

$$\boldsymbol{u_t} = {\boldsymbol{A_0}}^{-1}\boldsymbol{\epsilon_t}, \tag{3.25}$$

where $\boldsymbol{\Sigma_{\epsilon,t}} = E_{t-1}\left(\boldsymbol{\epsilon_t}{\boldsymbol{\epsilon_t}}^T\right)$ and $\boldsymbol{\Sigma_{u,t}} = E_{t-1}\left(\boldsymbol{u_t}{\boldsymbol{u_t}}^T\right)$. The unconditional variance of the structural innovations are normalized to unity, i.e. $E\left(\boldsymbol{\epsilon_t}{\boldsymbol{\epsilon_t}}^T\right) = \boldsymbol{I}$. The unconditional variance of the statistical innovation is $\boldsymbol{\Sigma_u} = \left({\boldsymbol{A_0}}^{-1}\right)\boldsymbol{\Sigma_\epsilon}\left({\boldsymbol{A_0}}^{-1}\right)^T$. The heteroskedasticity in the structural innovations can be specified with a GARCH(1,1) process as follows

$$\boldsymbol{\Sigma_{\epsilon,t}} = (\boldsymbol{I} - \Delta_1 - \Delta_2) + \Delta_1 \circ \left(\boldsymbol{\epsilon_{t-1}}{\boldsymbol{\epsilon_{t-1}}}^T\right) + \Delta_2 \circ \boldsymbol{\Sigma_{\epsilon,t-1}}, \tag{3.26}$$

where Δ_1 and Δ_2 are diagonal matrices, and "$\circ$" denotes the Hadamard product operator. If Δ_1 and Δ_2 are null, then $\boldsymbol{\Sigma_{\epsilon,t}}$ is constant. Δ_1 and Δ_2 are positive semi-definite, and $(\boldsymbol{I} - \Delta_1 - \Delta_2)$ is positive definite, which indicate that at least one structural innovation is GARCH(1,1). Therefore, the GARCH(1,1) for an individual conditional structural variance is

$$\sigma^2_{m,t|t-1} = (1-\gamma_m-\delta_m) + \gamma_m\epsilon^2_{m,t-1} + \delta_m\sigma^2_{m,t-1|t-2}, \quad m = 1,\ldots,k. \tag{3.27}$$

Therefore, the linear combination of $\boldsymbol{A_0^{-1}}$ and $\boldsymbol{\epsilon_t}$ fit in a GO-GARCH representation proposed in Weide, 2002.

$$\boldsymbol{A_0}^{-1} = \boldsymbol{P\Lambda}^{1/2}\boldsymbol{U}^T, \tag{3.28}$$

where $\boldsymbol{P}$ and $\boldsymbol{\Lambda}$ denote the matrices with the orthogonal eigenvectors and the eigenvalues of $\boldsymbol{\Sigma_u} = \left(\boldsymbol{A_0}^{-1}\right)\boldsymbol{\Sigma_\epsilon}\left(\boldsymbol{A_0}^{-1}\right)^T$, respectively. $\boldsymbol{U}$ is the orthogonal matrix of eigenvectors of $\boldsymbol{A_0}^{-1}\boldsymbol{A_0}^{-1T}$. In Weide, 2002, the matrices $\boldsymbol{P}$ and $\boldsymbol{\Lambda}$ are estimated directly by means of unconditional information, e.g. from the sample covariance matrix $\boldsymbol{\Sigma_u}$. Therefore, in the GO-GARCH model, to identify $\boldsymbol{A_0}^{-1}$, we have to identify the orthogonal matrix $\boldsymbol{U}$. Considering the GO-GARCH is nested in the more general BEKK model, we fit the GO-GARCH in the BEKK representation.

Consider the BEKK model proposed by Baba, Engle, Kraft, and Kroener in Baba et al., 1990,

$$\boldsymbol{\Sigma_{u,t}} = \boldsymbol{C} + \sum_{i=1}^{k}\boldsymbol{A_i u_{t-1} u_{t-1}}^T\boldsymbol{A_i}^T + \sum_{j=1}^{k} B_j\Sigma_{u,t-1}B_j^T, \tag{3.29}$$

matrices $\{\boldsymbol{A_{i=1}^k}\}$ and $\{\boldsymbol{B_{j=1}^k}\}$ are restricted to have identical eigenvector matrix $\boldsymbol{A_0}^{-1}$, where the eigenvalues of A_i and B_j are all zero except for the i-th and j-th one, respectively. Assume C can be decomposed as $\boldsymbol{A_0}^{-1}\boldsymbol{D_c A_0}^{-1T}$, where $\boldsymbol{D_c}$ is a positive definite diagonal matrix. Then the associate BEKK parameterization is equivalent to a GO-GARCH(1,1).

Proof. matrices $\{\boldsymbol{A_{i=1}^k}\}$ and $\{\boldsymbol{B_{j=1}^k}\}$ are restricted to have identical eigenvector matrix $\boldsymbol{A_0}^{-1}$ so that they can be diagonalized as

$$\boldsymbol{A_i} = \boldsymbol{A_0}^{-1}\boldsymbol{D_{A_i}A_0}^{-1T} \quad \boldsymbol{B_j} = \boldsymbol{A_0}^{-1}\boldsymbol{D_{B_j}A_0}^{-1T}, \tag{3.30}$$

where $\boldsymbol{D_{A_i}}$ and $\boldsymbol{D_{B_j}}$ denote diagonal eigenvalue matrices. Note all elements of $\boldsymbol{D_{A_i}}$ and $\boldsymbol{D_{B_j}}$ are zero except for the i-th and j-th elements. Therefore, denoting the only non-zero elements a_i in $\boldsymbol{D_{A_i}}$ and b_j in $\boldsymbol{D_{B_j}}$, where a_i and b_j represent the only non-zero eigenvalue of $\boldsymbol{A_i}$. By substitution we have

$$\begin{aligned}\boldsymbol{\Sigma_{u,t}} =& \boldsymbol{A_0}^{-1}\boldsymbol{D_c}\boldsymbol{A_0}^{-1T} + \sum_{i=1}^{k}\boldsymbol{A_0}^{-1}\boldsymbol{D_{A_i}}\boldsymbol{A_0}\boldsymbol{u_{t-1}}\boldsymbol{u_{t-1}}^T\boldsymbol{A_0}^T\boldsymbol{D_{A_i}}\boldsymbol{A_0}^{-1T} \\ &+\sum_{j=1}^{k}\boldsymbol{A_0}^{-1}\boldsymbol{D_{B_j}}\boldsymbol{A_0}\boldsymbol{\Sigma_{u,t-1}}\boldsymbol{A_0}^T\boldsymbol{D_{B_j}}\boldsymbol{A_0}^{-1T}\end{aligned}, \tag{3.31}$$

which can be simplified to

$$\boldsymbol{\Sigma_{u,t}} = \boldsymbol{A_0}^{-1}\left[\boldsymbol{D_c} + \sum_{i=1}^{k}\boldsymbol{D_{A_i}}\boldsymbol{A_0}\boldsymbol{u_{t-1}}\boldsymbol{u_{t-1}}^T\boldsymbol{A_0}^T\boldsymbol{D_{A_i}} + \sum_{j=1}^{k}\boldsymbol{D_{B_j}}\boldsymbol{A_0}\boldsymbol{\Sigma_{u,t-1}}\boldsymbol{A_0}^T\boldsymbol{D_{B_j}}\right]\mathbf{A_0}^{-1T}. \tag{3.32}$$

According to eq. (3.25), $\boldsymbol{u_t}$, where $\boldsymbol{u_t} = \boldsymbol{A_0}^{-1}\boldsymbol{\epsilon_t}$ or $\boldsymbol{\epsilon_t} = \boldsymbol{A_0}\boldsymbol{u_t}$, represents the unobserved components in the GO-GARCH model. Denoting $\boldsymbol{\Sigma_{\epsilon,t}} = \boldsymbol{A_0}\boldsymbol{\Sigma_{u,t}}\boldsymbol{A_0}$ as the conditional covariance of $\boldsymbol{\epsilon_t}$, by arranging eq. (3.31), we find

$$\boldsymbol{\Sigma_{\epsilon,t}} = \boldsymbol{D_c} + \sum_{i=1}^{k}\boldsymbol{D_{A_i}}\boldsymbol{\epsilon_{t-1}}\boldsymbol{\epsilon_{t-1}}^T\boldsymbol{D_{A_i}} + \sum_{j=1}^{k}\boldsymbol{D_{B_j}}\boldsymbol{\Sigma_{\epsilon,t-1}}\boldsymbol{D_{B_j}}. \tag{3.33}$$

By the properties of matrices $\boldsymbol{D_{A_i}}$ and $\boldsymbol{D_{B_j}}$, it follows that the sum can be re-written using Hadamard product as

$$\sum_{i=1}^{k}\boldsymbol{D_{A_i}}\boldsymbol{\epsilon_{t-1}}\boldsymbol{\epsilon_{t-1}}^T\boldsymbol{D_{A_i}} = \boldsymbol{D_A}\circ\boldsymbol{\epsilon_{t-1}}\boldsymbol{\epsilon_{t-1}}^T, \quad \sum_{j=1}^{k}\boldsymbol{D_{B_j}}\boldsymbol{\Sigma_{\epsilon,t-1}}\boldsymbol{D_{B_j}} = \boldsymbol{D_B}\circ\boldsymbol{\Sigma_{\epsilon,t-1}}, \tag{3.34}$$

where $\boldsymbol{D_A} = diag\,(a_1,\ldots,a_k)$ and $\boldsymbol{D_B} = diag\,(b_1,\ldots,b_k)$. Then $\boldsymbol{D_c}$, $\boldsymbol{D_A}\circ\boldsymbol{u_{t-1}}\boldsymbol{u_{t-1}}^T$, and $\boldsymbol{D_B}\circ\boldsymbol{\Sigma_{\epsilon,t-1}}$, are all diagonal, and $\boldsymbol{\Sigma_{\epsilon,t}}$, the conditional covariance matrix of $\boldsymbol{\epsilon_t}$, is also diagonal. Therefore, eq. (3.34) implies a univariate GARCH(1,1) specification for $\boldsymbol{\epsilon_t}$ as it is assumed by the GO-GARCH model. Therefore, using our GARCH(1,1) specification in eq. (3.26),

$$
\begin{aligned}
C &= A_0{}^{-1} D_c A_0{}^{-1T} \\
A_i &= A_0{}^{-1} D_{A_i} A_0{}^{-1T} \\
B_j &= A_0{}^{-1} D_{B_j} A_0{}^{-1T}.
\end{aligned}
$$

With the estimates of D_A and D_B, we can find matrices $\{D_{A_i}\}$ with $i = 1, \ldots, k$ and $\{D_{B_j}\}$ with $j = 1, \ldots, k$. After formalizing the GO-GARCH into the BEKK form, eq. (3.29) can be transformed to the corresponding vec form.

Vectorizing eq. (3.29),

$$
\operatorname{vech}\left(\Sigma_{u,t}\right) = \operatorname{vech}\left(C\right) + \sum_{i=1}^{k} \operatorname{vech}\left(A_i u_{t-1} u_{t-1}{}^T A_i{}^T\right) + \sum_{i=1}^{k} \operatorname{vech}\left(B_j \Sigma_{u,t-1} B_j{}^T\right), \tag{3.35}
$$

Denote $\otimes$ as the Kronecker product operator, recognizing $\operatorname{vec}\left(xy^T\right) = y \otimes x$ and using the product rule [5] with Kronecker product, eq. (3.35) becomes

$$
\operatorname{vec}\left(\Sigma_{u,t}\right) = \operatorname{vec}\left(C\right) + \sum_{i=1}^{k} \left(A_i \otimes A_i\right) \operatorname{vec}\left(u_{t-1} u_{t-1}{}^T\right) + \sum_{i=1}^{k} \left(B_j \otimes B_j\right) \operatorname{vec}\left(\Sigma_{u,t-1}\right). \tag{3.36}
$$

Denoting D_k as the duplication matrix, given $\operatorname{vec}(A) = D_k \operatorname{vech}(A)$, eq. (3.36) is

$$
\begin{aligned}
D_k \operatorname{vech}\left(\Sigma_{u,t}\right) = D_k \operatorname{vech}(C) &+ \sum_{i=1}^{k} \left(A_i \otimes A_i\right) D_k \operatorname{vech}\left(u_{t-1} u_{t-1}{}^T\right) \\
&+ \sum_{i=1}^{k} \left(B_j \otimes B_j\right) D_k \operatorname{vech}\left(\Sigma_{u,t-1}\right).
\end{aligned} \tag{3.37}
$$

Define the generalized inverse of D_k as $D_k^+ = \left(D_k{}^T D_k\right)^{-1} D_k{}^T$, that is a $(k \times (k+1)/2) \times (k^2)$ matrix, where D_k where $D_k^+ D_k = I_k$ Then, we can have a unique transformation from BEKK to vech as follows:

[5] The product rule is $(A \otimes B)(C \otimes D) = AC \otimes BD$, and $\operatorname{vec}[(Ax)(y^T B)] = (B^T y) \otimes (Ax) = (B^T \otimes A)(y \otimes x) = (B^T \otimes A) \operatorname{vec}(xy^T)$. $\operatorname{vec}(ABC) = (C^T \otimes A) \operatorname{vec} B$

$$\begin{aligned}\text{vech}(\boldsymbol{\Sigma}_{u,t}) = \text{vech}(\boldsymbol{C}) + \boldsymbol{D}_k^+ \left(\sum_{i=1}^{k} (\boldsymbol{A}_i \otimes \boldsymbol{A}_i)\right) \boldsymbol{D}_k \,\text{vech}\left(\boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}{}^T\right) \\ +\boldsymbol{D}_k^+ \left(\sum_{i=1}^{k} (\boldsymbol{B}_j \otimes \boldsymbol{B}_j)\right) \boldsymbol{D}_k \,\text{vech}\left(\boldsymbol{\Sigma}_{u,t-1}\right).\end{aligned}$$

Given the vech model

$$\text{vech}(\boldsymbol{\Sigma}_{u,t}) = \boldsymbol{W} + \tilde{\boldsymbol{A}}\,\text{vech}\left(\boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}{}^T\right) + \tilde{\boldsymbol{B}}\,\text{vech}(\boldsymbol{\Sigma}_{u,t-1}), \tag{3.38}$$

We have the following relations

$$\begin{aligned}\tilde{\boldsymbol{A}} &= \boldsymbol{D}_k^+ \left(\sum_{i=1}^{k} (\boldsymbol{A}_i \otimes \boldsymbol{A}_i)\right) \boldsymbol{D}_k, \\ \tilde{\boldsymbol{B}} &= \boldsymbol{D}_k^+ \left(\sum_{i=1}^{k} (\boldsymbol{B}_j \otimes \boldsymbol{B}_j)\right) \boldsymbol{D}_k.\end{aligned}$$

After fitting the GO-GARCH in a vech GARCH form, we can apply the results in Hafner and Herwartz, 2006 and calculate VIRF.

3.5.2 GIRF with a Cholesky decomposition

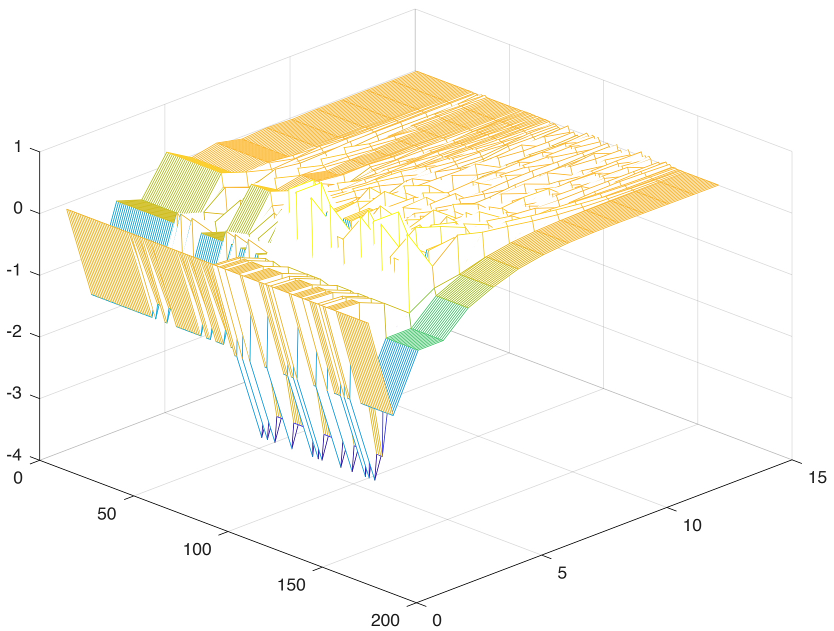

FIGURE 3.8: ΔOil price responses to a structural shock in GPR by a Cholesky decomposition.

Chapter 4

Volatility Spillovers in ETFs

Crude Oil ETF, S&P 500 ETF, and Renewable Energy ETF (Exchange-Traded Fund)

4.1 Introduction

An exchange-traded fund (ETF) is a collection of securities, which can be traded on a stock exchange. Investors can use ETFs to enter a variety of asset classes, such as stocks, bonds, and commodities. Besides flexibility, ETF having loads of advantages will be introduced in the following content. The United States Oil ETF (USO) fund is applied in this chapter as oil ETF representive, which is constructed by near-month NYMEX futures contracts on WTI crude oil. It is favorable for short-term investors. Nowadays, renewable energy has increasingly become a substitute for crude oil. Moreover, the connection between them has been extending to the financial market. Further, the price of oil has a substantial effect on the stock market, which is verified by a significant amount of previous studies. Therefore, this chapter aims to investigate the volatility spillovers in these three ETF markets.

Outstandingly, we have chosen to study the uncertainty dynamics in crude oil and renewable energies in the ETF market. The investment in energy derivatives is increasingly popular, especially via ETFs. The ETFs originally and mainly issued in the U.S. market and gain rapid expansion, which reached a high level of $5.12 trillion in June 2018 by contract with a monthly record of $44 billion in 2013 (ETFGI). And there is still enormous potential for growth in the ETF market (Financial Time, 2018). The ETF market has attracted to growth investors and gained impressively rapid development, due to loads of advantages of ETF, such as diversification, low management fee, easy trading (therefore high liquidity), and tax-efficient. Since the last decade, renewable energy ETFs have been launched and expected a bright future. Buckle et al. (2018) present ETFs play a dominant role in price discovery, comparing with stock markets and futures contracts, in three major U.S. stock markets (Dow Jones, NASDAQ, and S&P).

Nowadays, investors focus on ETF investment increasingly. And ETF is gradually traded as hedging instrument. There are several reasons making ETF popularity. The stock indice measure the stock market performance and provide information to investors to guide their investment behaviour. However, indice are not tradeable. ETF solve this problem by replication index and tracking its performance. And ETF is investable. Moreover, the underlying index component is highly transparent in the ETF. So investors are able to adjust their strategy promptly and appropriately. Compared with future, ETF also has its advantage that getting rid of maturity date.

The United States has been the second largest carbon dioxide emitter worldwide, getting more than 5 millions kiloton of fossil fuel consumption in 2017 (EIA). The environmental damage and pollution generated by energy producing and consuming processes make not only the U.S. also the world take energy issues seriously. To combat climate change, the Paris Agreement is reached worldwide at the end of 2015. International investment in renewable energy sources has increased roughly 240 billion U.S. dollars in alternative energy section in the years 2000 to 2016. And the U.S. proposed the Clean Power Plan in 2014. As a solution for the U.S. environmental problems, renewable energy is dramatically expanded, then rapidly developed and applied widely. The consumption of U.S. renewable energy has doubled between 2000 and 2017 (U.S. Energy Information Administration (EIA)), getting 11% of total U.S. energy consumption in 2017. Nuclear energy, as alternatively clean and sustainable energy, provides about 20% of the total U.S. electricity (EIA & U.S. Department of Energy (DOE)).

Policy decisions influent crude oil price. Many nations endeavour to reduce the environmental impact of the petroleum industry. On the other hand, crude oil price fluctuations have been playing a crucial role in every field of the economy. Based on the long run market activities, consumers and enterprises adjust their customer strategy according to upward oil fuel price.

Households decline consumption and switch to substitutes for oil fuel energy. As reported in Managi and Okimoto, 2013' study that there is a positive relationship between oil prices and clean energy prices. Rising oil prices also increase the producing cost, and then depress the supply of the goods. In macroeconomy, it decreases economic activities and reduces economic growth (Rahman and Serletis, 2012) (Elder and Serletis, 2010b) (Rahman and Serletis, 2011). Rising oil prices dampen the cash flow and reduce stock prices in sequence as the producing cost increase. So the stock market also reacts to oil price changing. Oil price uncertainty is considered an important driver of stock price dynamics as well.

One question must be raised here that what is the actual relationship between crude oil and renewable energies (including nuclear energy). Are they actually substitutes for crude oil in the U.S. energy market? We would like to clarify the question according to the available information on the U.S. EIA official website. *Figure 4.1* in the appendix depicts the U.S. primary energy consumption by source and sector in 2017. Crude oil, as one kind of liquid raw material produced from fossil fuels, is refined into petroleum products. Petroleum is the largest U.S. energy source, and the top two are the transportation energy consumption and industrial sector energy consumption, respectively taking 72% and 23% shares of the total in 2017. Whereas, 100% nuclear energy and more than half renewable energy (57% shares of total) have been used to create electricity. *Figure 4.2* demonstrates wind energy consumption and solar energy consumption have respectively increased rapidly from 1990 to 2014. And the U.S. renewable energy consumption experienced the largest percentage growth in renewable consumption from 2001 to 2014. According to *Figure 4.3* and *Figure 4.4*, wind energy takes a much bigger percentage in the U.S. energy consumption than solar energy in 2017, while solar energy is projected to expand much by 2050. Generally speaking, renewable energies are substitutes for crude oil according to energy sectoral consumption in *Figure 4.1*. To be accurately speaking, wind energy, solar energy, and nuclear energy are substitutes for crude oil since electric cars are becoming more popular. Furtherly, nuclear energy can be substituted for natural gas, as well as natural gas is an alternative to crude oil. It is another channel to explain the interaction between crude oil and other energies. Moreover, crude oil and renewable energies can be influent by systematic risk in the energy sector. Therefore, we are interested in the relationships between crude oil, nuclear energy, and renewable energies.

Moreover, the connection between oil and alternative energy resources exists not only in the physical market but also in the financial market. It is widely acknowledged that rising oil prices have a positive effect on the stock prices of alternative energy companies. Henriques and Sadorsky (2008) find oil prices can be Granger causing the stock prices of alternative energy companies. Researches on volatility co-movements and spillovers have provided evidence on how volatility interactions have been working between different energy derivatives. Sadorsky (2012) proposes volatility spillovers study between oil prices and the stock prices of clean energy and technology companies. Chang, McAleer, and Wang (2018a) exam and confirm significant spillover effects in the natural gas spot, futures, and Exchange-Traded Fund (ETF) markets for both USA and UK. And it defines the volatility spillover is the delayed effect of a return shock in one asset to another asset. Reboredo (2015) concludes the oil dynamics significantly affect the risk of renewable energy stock prices. Chang, McAleer, and Wang (2018b) find significant and positive latent volatility Granger causality relationship between crude oil, solar, wind, and nuclear ETFs. Bondia, Ghosh, and Kanjilal (2016) indicate the cointegration between alternative energy company stock prices and oil prices have existed. Reboredo, Rivera-Castro, and Ugolini (2017) find nonlinear causality at higher frequencies and linear causality at lower frequencies between oil prices and renewable energy stock prices. Luqman, Ahmad, and Bakhsh (2019) provide evidence that oil prices have a neutral impact on renewable and nuclear energy consumption in Pakistan economy. Reboredo (2015) documents oil prices contribute to the downside and upside risks of renewable energy companies were around 30%.

The price of crude oil is always considered to have an impact on stock returns. But there are not consistent results about the relationship between crude oil price uncertainty and stock returns. There is no finding about the relationship between oil price changes and stock returns (Chen, Roll, and Ross, 1986)(Huang, Masulis, and Stoll, 1996)(Wei, 2003). Kling (1985), Sadorsky (1999), Park and Ratti (2008), and Jones and Kaul (1996) provide evidence that upward shocks in oil price have a significant effect on the stock returns. Kilian and Park (2009) argue that demand-driven shock in oil prices have more importance for understanding the changes in the stock market. And Alsalman (2016) provides evidence that oil price increases and decreases have symmetric effects on the U.S. aggregate stock return, but the symmetric effects do not exist across all the sectoral stock returns.

One reasonable explanation for the relationship between oil and the stock market is rising oil prices dampen the cash flow and reduce stock prices in sequence as the producing cost increase. So the stock market also reacts to oil price changes. Another one is based on the theory of stock valuation, which explains stock price is the discounted value of expected future cash flows. Wang and Liu (2016) conclude the determinants economics activities ((Hamilton, 1983);(Hamilton, 1996);(Hamilton, 2003b);(Kilian, 2009b)), inflation rates ((LeBlanc and Chinn, 2004);(Chen, 2009)), and exchange rates ((Amano and Van Norden, 1998);(Wang and Wu, 2012);(Akram, 2004)) of expected future cash flows can be affected by oil prices. Oil price increases can result in stock price decreases. Therefore, the oil price fluctuation is expected to be a factor driver in stock market returns. We apply the multivariate GARCH-in-mean system to find the relationship between the uncertainty of oil ETF price and the mean level of the S&P 500 ETF, and vice versa.

S&P 500 (SPX) index is composed of 500 largest U.S. publicly traded companies, basing on the market-capitalization-weighted principal. S&P 500 ETF (SPY) is a tractor of the S&P 500 index. The oil ETF (USO) is a futures-based commodity ETF, which is trying to track the oil commodity price. Renewable energy benchmark ETF (PBW) is trying to track the performance of the stock market index (ECO) within renewable energy and energy conservation companies. The solar energy ETF (TAN) is also an index fund, which holds a concentrated portfolio on solar energy companies. The wind energy ETF (FAN) focuses on index tracking in the wind energy industry. The nuclear energy ETF (NLR) is a market-cap-weighted index fund of nuclear energy companies.

Having briefly discussed the possible channels of interaction between crude oil, renewable energy, and the stock market, we proceed to the discussion on volatility spillovers and dynamic correlations of the previous research in this field.

Volatility transmission mechanism between crude oil and alternative energies have been rarely researched. Lin and Li (2015) find volatility spillovers from the oil market to natural gas market. Reboredo (2015) concludes the oil dynamics significantly affect the risk of renewable energy stock prices. Sadorsky (2012) proposes volatility spillovers study between oil price and stock prices of clean energy and technology companies. Efimova and Serletis (2014) find volatility spillovers return from oil to gas and electricity markets via using GARCH-in-mean model. Wen et al. (2014) also document significant volatility spillovers between Chinese renewable energy and fossil fuel companies. Based on the previous theories and market information of the interaction between crude oil and clean energy resources, we have a strong interest to study and sufficient reasons to believe there are volatility spillovers between crude oil ETF and alternative energy ETFs.

Our study also involves S&P 500 ETF in analysis, because there are substantial and empirical evidence on the impact of crude oil price uncertainty on the stock market in an intensive way. Luo and Qin (2017) find crude oil volatility index (OVX) shocks have a significant and negative effect on the Chinese stock returns, and the effect is more significant after the recent financial crisis. While they find positive price shock effects from oil prices to the Chinese stock market. Sadorsky (1999) also argues that oil price volatility impacts stock returns. Then Oberndorfer (2009) also provides evidence oil price volatility affects the European stock market. Regarding as negative volatility spillovers between oil and stock, it is identified by (Malik and Ewing, 2009)(Chiou and Lee, 2009)(Arouri and Nguyen, 2010). Awartani and Maghyereh (2013) conclude dynamic spillover transmissions have intensified following the financial turmoil in 2008. Oppositely Zhu, Li, and Li (2014) find weak dynamic dependence between crude oil prices and stock market returns, exceptionally rises substantially in the post-crisis. There is a trend to research on the dynamic correlation between crude oil and the stock market. Hassan, Hoque, and Gasbarro (2019) point out the dynamic correlation between the Islamic stock and crude oil increases during the global financial crisis in China and India stock market. Filis, Degiannakis, and Floros (2011) add detailed analysis by finding the contemporaneous correlation does not differ for oil-importing and oil-exporting economies, and lagged oil prices only exhibit a positive correlation with the stock market in the 2008 global financial crisis.

We are interested in four aspects. Firstly, we identify how the volatilities mutually work between diverse energy ETFs (including conventional energy crude oil and non-conventional renewable energies). We not only allow the volatility spillover to the volatility level but also volatility spillover to the mean level. The multivariate GARCH-in-mean model is applied to figure out the impact of crude oil ETF uncertainty on the alternative energy ETFs, and vise versa.

Secondly, as the crude oil price uncertainty on the stock market is substantially addressed in previous literature. We are interested in the volatility dynamics between the crude oil ETF and the S&P 500 ETF. We suppose that the crude oil ETF uncertainty has significant effects on the S&P 500 ETF return in the U.S. ETF market. SPDR S&P 500 ETF trust fund (SPY) is applied here to measure the ETF financial market performance. Further, we wonder if the volatility of renewable energy ETFs have the contemporaneous effect on S&P ETF return as well. Comparing with oil ETF, we expect the same directional but weaker effect from alternative energy ETFs to S&P 500 ETF. The reason for our expected results is the renewable energy ETFs are also in the energy ETF section, but they have smaller market capitalization by contrast with oil ETF.

Volatility links vary depends on different economic conditions. During the recession, volatility spillovers and correlation generally show a high level. Sadorsky (2012) find the evidence on this viewpoint that the dynamic conditional correlation reaches its highest value since the 2008 recession because of the effects of the economic downturn. Thirdly, we figure out that the volatility interactions do have different performances before, during and after a financial crisis.

Moreover, Wang and Liu (2016) find the correlation between oil and the stock market is always positive and stronger for the oil-exporting countries than the oil-importing countries. The view is proved by Jung and Park (2011) and Wang, Wu, and Yang (2013) that higher global economic activities drive higher oil prices. And at the same time, it has more impacts on the oil-exporting economies. The US historically has been a heavy importer of crude oil, but since 2018 December it exports more oil than it ships in (Reuters, December 2018). The U.S. shale revolution has been helping boost overall U.S. oil production. Therefore based on the previous literature and the U.S. shifting from oil importer to exporter, we suppose the correlation between oil and stock markets are stronger and keep positive nowadays than ten years before.

We mainly contribute in two aspects. The first one is to provide evidence for volatility spillovers between crude oil ETF, renewable energy ETFs and the S&P 500 ETF in the U.S. ETF markets. Volatility, as a proxy of risk, is significantly meaningful and crucial not only for researchers but also for the practitioners, such as policymakers, portfolio managers, investors, consumers, and producers. However, volatility modelling rarely has been focused on the energy ETF markets. We focus on the volatility spillovers and furtherly risk management by constructing hedging strategy and portfolio weights in crude oil, renewable energy ETFs and S&P 500 ETF. And we identify the oil ETF uncertainty has a negative and significant effect on the S&P 500 ETF return. The volatility of energy ETFs is driven by two factors. One is relative to the energy market and the other is the stock market. The dynamic correlation results tell us that renewable energy ETF is in tandem with the S&P 500 ETF. It is valuable to investigate in ETF volatilities because it provides risk management implication for practice.

The second contribution is applying the GARCH-in-mean model to analyze the volatility spillover effect of energy ETFs on the mean level of the S&P 500 ETF, and vice versa. The previous energy market research mainly focuses on the volatility spillover in the variance level. They rarely investigate the volatility effect on asset returns. And BEKK GARCH-in-mean is generally applied. Our advantage is empirically applying DCC and CCC VARMA-GARCH-in-mean model to explore the volatility spillover and conditional dynamic correlation together. The most interesting finding is negative volatility spillovers in long persistence between renewable energy ETF and the S&P 500 ETF, which breaks the initial Bollerslev's positive striction to all the elements in the covariance matrix. The conditions are checked to base onConrad and Karanasos (2010)'s study. There is a failure to met all the conditions.

The chapter is organized as follows: *Section* 2 discusses the dataset and variables applied in the empirical analysis. *Section* 3 presents the methodology in multivariate GARCH-in-mean specifications. *Section* 4 illustrates the empirical results. Finally, some conclusions and recommendations are given in *section* 5.

4.2 Data Analysis

4.2.1 ETF Variables Introduction

This chapter applies daily data containing six ETFs to exam the volatility spillover effects, and the data ranges are demonstrated in *Table 4.4* in the appendix. The choice of these ETFs depends on its existence and dominance in the ETF market.

We choose the United States Oil ETF (USO) fund as oil ETF, which is constructed by near-month NYMEX futures contracts on WTI crude oil. The near-month contract is the nearest contract to maturity. The oil ETF is a futures-based commodity ETF, which is trying to track the oil commodity price. As oil is difficult to be stored or would incur high marginal storage cost, oil ETF use oil future contracts to gain exposure to oil commodity (Guedj, Li, and McCann, 2011). NYMEX and the Intercontinental Exchange (ICE) are the main oil futures exchanges. The crude oil future contracts standardized for 1000 barrels of oil on one contract. The New York Mercantile Exchange (NYMEX) West Texas Intermediate (WTI) future price comes from the nearest maturity future contract. The construction method, rolling contract in every month, make it particularly sensitive to short-term changes in the spot price, but still may dramatically deviate from the spot price. The method also results in heavy roll costs because of contango and backwardation. In other words, the spot price is normally higher than the forward price. Then there will be a price difference when future contract roll over. The plot of crude oil ETF (USO) price, oil future price, and the oil spot price is exhibited in *Figure 4.6* and *Figure 4.7*. Therefore, it is efficient to use a near-month contract to explore oil commodity price. We can figure out that the price spread between the oil near-month future price and the oil spot price is very small. The ETF price approaches the crude oil commodity price downward.

SPDR S&P 500 ETF trust fund (SPY) tracks the massively popular U.S. index S&P 500. S&P 500 is weighting constructed by 500 largest market capitalization companies listed on the U.S. stock market. S&P 500 ETF is the best-recognized and oldest ETF, and hold top ranking and greatest trading volume. Therefore, it is used as a proxy of the U.S. market and treated as a performance measurement of the U.S. stock market.

The Invesco WilderHill Clean Energy ETF (PBW) is an index fund, which is trying to track the performance of the stock market index (ECO) of renewable energy and energy conservation companies. The index includes the stocks and sectors are based on their significance for clean energy, technological influence, and relevance to preventing pollution in the first place. The energy industry area of renewable energy companies includes wind, solar, biofuels and geothermal. Especially, it is structured in an equal-weighting way instead of the cap-weighted benchmark.

The Invesco Solar ETF (TAN) is also an index fund, which holds a concentrated portfolio on solar energy companies. It adjusts company weight based on revenues from the solar-related activity. The First Trust Global Wind Energy ETF (FAN) focuses on index tracking in the wind energy industry. The weighting strategy is a float-adjusted market cap with strict limits on individual holdings. The VanEck Vectors Uranium+Nuclear Energy ETF (NLR) is a market-cap-weighted index fund of nuclear energy companies.

4.2.2 Preliminary Analyses

Daily prices of the ETFs are applied in this study, and the sample period varies in the different applied dataset, covers from 10th April 2006 to 2nd January 2019 reported in *Table 4.4*. Because the earliest data available for ETFs is different. All of this dataset is available from Datastream. The ETFs are measured in U.S. dollar, and they are listed active in the U.S. stock market. The availability of data dictates this data set choice.

Raw data plots and time series graphs of the returns in *Figure 4.8*, *Figure 4.9*, *Figure 4.10*, and *Figure 4.11* in the appendix behind provide stylized facts, that each group of time series data plot shows a consistent trend for all three sequences. For each data series, continuously compounded daily return at time t is calculated by price series as below formula shows:

$$y_t = ln\left(\frac{p_t}{p_{t-1}}\right) \times 100 \tag{4.1}$$

Where $ln\left(\frac{p_t}{p_{t-1}}\right)$ is the natural logarithm of the price at date t dividing by the price at date $t-1$. Base on the statistics results for each daily return in *Table 4.5*, the mean and median are close to zero, and Student's t-statistic provides no evidence that the mean is from zero. The skewness and excess kurtosis statistics include a test of the null hypotheses that each is zero. If the skewness and excess kurtosis of the population values are zero if the statistic series is the *i.i.d*. Normal distribution. Jarque-Bera is a test for normality based upon the skewness and kurtosis measures combined. And we can see it significantly rejects the assumption of the Normal distribution for each series. According to the kurtosis statistics, each series shows a higher level than the Normal distribution. It is the reason we choose the multivariate Student's t-distribution for modelling multivariate GARCH-in-mean specification.

From *Figure 4.12* to *Figure 4.19*, we can see volatility clustering (heteroskedasticity) has occurred in all ETF return graphs. Moreover, pronounced volatility clustering between 2008 and 2009, and between 2011 and 2012, have occurred contemporaneously in all three six graphs. It motivates us to apply the multivariate GARCH model. There is a strong positive unconditional correlation between S&P 500 ETF and each renewable energy ETF. While the unconditional correlation between oil ETF and each other ETF is roughly half of the unconditional correlation between S&P 500 ETF and each renewable energy ETF, but still positive. The unconditional correlation between squared returns of each pair estimated series follow the similar pattern as return correlation.

The multivariate Q statistic (Hosking, 1981) test result is reported in *Table 4.12*. The null hypothesis of this test is that there is no serial correlation within each ETF return and also between them. In other words, all autocorrelations and lagged cross-correlations are zero. According to the MQ statistic results, we have enough evidence to reject the null hypothesis and we should include VAR process in the mean equation. Combining the information criterion (AIC/BIC/HQ/Chi-Squared Test) reported by VAR lag selection tables (*Table 4.8*, *Table 4.9*, *Table 4.10*, and *Table 4.11*), we select the lag order of 1. Preliminary regression analysis showed there is no significant difference between a VAR with one lag and a VAR with three lags. We increase the VAR lag number when the estimation result does not converge. For example, normally the lag order of 1 is selected. However, if we got a non-convergence in the estimation result we have to move to order 2 for modelling the returns.

4.3 Methodology

4.3.1 Conditional Mean Definition

In the light of multivariate volatility spillovers analyses addressed by GARCH-in-mean models, previous works identify how the volatility spillovers the mean level. Engle, Lilien, and Robins (1987) extend the original ARCH framework to allow the mean of a sequence to depend on its own conditional variance. Bollerslev, Chou, and Kroner (1992) apply the GARCH-in-mean model to improve asset pricing theory, by allowing time-varying conditional variances of asset returns and a time-varying risk premium. The structural vector autoregression (VAR) model accommodated GARCH-in-mean errors are broadly applied ((Elder, 1995);(Elder, 2004);(Elder and Serletis, 2010b);(Alsalman, 2016)). They just focus on the effect of oil uncertainty on the output mean level by allowing zero restriction, not vice versa. Ratti and Hasan (2013) measure the effect of the return and volatility of oil price on the return and volatility in the Australian sectoral stock. Some other works just focus on how its own conditional variance affect on asset return (Efimova and Serletis, 2014). We improved GARCH-in-mean models mentioned above, by allowing the variances of all returns in each mean equation ((Alsalman, 2016);(Rahman and Serletis, 2012);(Shields et al., 2005);(Grier and Perry, 2000);(Rahman and Serletis, 2011)). However, the difference is we apply the vector ARMA-GARCH constant conditional correlation (VARMA-GARCH-in-mean CCC) model and VARMA-GARCH-in-mean dynamic conditional correlations (DCC) model for the variance modelling part. The VARMA-GARCH model is proposed by Ling and McAleer (2003). The detail is showed in the next part.

There are two components defined in multivariate GARCH specifications. One is used to model the returns, while another one aims at the variances and covariances modelling. We introduce three multivariate GARCH specifications (BEKK-GARCH model, CCC-VARMA-GARCH model, and DCC-VARMA-GARCH model) to measure the conditional variance series of renewable energy ETF, the S&P 500 ETF, and crude oil ETF. Using GARCH-in-mean terms explore if and how volatility spillovers influent to the mean level of each return.

In the specification of the GARCH-in-mean model, all three variances are assumed effect in each mean equation. Rahman and Serletis (2012) use the GARCH-in-mean specification to study oil uncertainty and the Canadian economy. The difference is we adopted conditional variances instead of conditional volatility.

$$\boldsymbol{y}_t = \boldsymbol{c} + \boldsymbol{\Gamma}^1 \boldsymbol{y}_{t-1} + \boldsymbol{\Gamma}^2 \boldsymbol{y}_{t-2} + \boldsymbol{\Gamma}^3 \boldsymbol{y}_{t-3} + \boldsymbol{\Psi}\boldsymbol{\Sigma}_{ii,t} + \boldsymbol{u}_t \tag{4.2}$$

$$\boldsymbol{u}_t = \boldsymbol{\sigma}_t \boldsymbol{v}_t, \quad \boldsymbol{v}_t \sim N(\boldsymbol{0}, \boldsymbol{I}_N) \tag{4.3}$$

$$\boldsymbol{u}_t | \boldsymbol{F}_{t-1} \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}_{u,t}) \tag{4.4}$$

where, $\boldsymbol{F}_{t-1}$ denotes the available information set in period $t-1$, and $\mathbf{0}$ is a null vector. $\boldsymbol{I_N}$ is the identity matrix of order N. Moreover,

$$\boldsymbol{y}_t = \begin{bmatrix} y_{re,t} \\ y_{stock,t} \\ y_{oil,t} \end{bmatrix}, \quad \boldsymbol{\Sigma}_{ii,t} = \begin{bmatrix} \sigma^2_{re,t} \\ ,\sigma^2_{stock,t} \\ \sigma^2_{oil,t} \end{bmatrix}, \quad \boldsymbol{u}_t = \begin{bmatrix} u_{re,t} \\ u_{stock,t} \\ u_{oil,t} \end{bmatrix} \tag{4.5}$$

$$\boldsymbol{\Sigma}_{u,t} = \begin{bmatrix} \sigma^2_{re-re,t} & \sigma^2_{re-stock,t} & \sigma^2_{re-oil,t} \\ \sigma^2_{stock-re,t} & \sigma^2_{stock-stock,t} & \sigma^2_{stock-oil,t} \\ \sigma^2_{oil-re,t} & \sigma^2_{oil-stock,t} & \sigma^2_{oil-oil,t} \end{bmatrix} \tag{4.6}$$

$$\boldsymbol{c} = \begin{bmatrix} c_{re} \\ c_{stock} \\ c_{oil} \end{bmatrix}, \boldsymbol{\Gamma^i_1} = \begin{bmatrix} \gamma^i_{11} & \gamma^i_{12} & \gamma^i_{13} \\ \gamma^i_{21} & \gamma^i_{22} & \gamma^i_{23} \\ \gamma^i_{31} & \gamma^i_{32} & \gamma^i_{33} \end{bmatrix}, \boldsymbol{\Psi} = \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ \psi_{21} & \psi_{22} & \psi_{23} \\ \psi_{31} & \psi_{32} & \psi_{33} \end{bmatrix} \tag{4.7}$$

Where the vector $\boldsymbol{y_t}$ includes three series of returns of renewable energy ETF (wind ETF/solar ETF/nuclear ETF/renewable energy benchmark ETF), S&P 500 ETF, and oil ETF. $\boldsymbol{c}$ is the constant vector. $\boldsymbol{\Gamma}^i, i = 1,2,3$ is the parameter matrix in last i period of $\boldsymbol{y}_{t-i}$. $\boldsymbol{\Psi}$ catches the information of how conditional variances influent the mean level of each return. $\boldsymbol{\Sigma}_{ii,t}$ is stacking all conditional variances in a vector.

Historical stock returns generally have serial correlations. And the risk premium term introduces serial correlations in the return series $\boldsymbol{y}$. The volatility processes in the variances equation are the reason for serial correlations introduced. The model characterizes the evolution of the mean and the variance of a time series simultaneously. Thus,

$$\boldsymbol{y}_t | \boldsymbol{F}_{t-1} \sim N((\boldsymbol{c} + \boldsymbol{\Gamma}_i \boldsymbol{y}_{t-i} + \boldsymbol{\Psi\Sigma_{ii,t}}), \boldsymbol{\Sigma}_{u,t}) \tag{4.8}$$

$\boldsymbol{F}$ is the information set.

The parameter ψ in the mean equation is risk premium parameter. Positive ψ indicates that the return is positively related to its volatility and vice versa. We apply conditional time-varying variances σ^2 as a regressor in the mean equation, to capture the risk-return relationship. σ^2 is the risk premium term. A risk premium of an asset is a form of compensation for investors who tolerate the extra risk. In applications, there are several specifications of risk premium are used, such as standard deviation σ and taking the natural logarithm of variances $ln(\sigma^2)$ (Engle, Lilien, and Robins (1987), Elyasiani and Mansur (1998), Ryan and Worthington (2004)). The GARCH-in-mean model allows a time-varying risk premium.

Given the evidence of the Jarque-Bera normality test, we characterize the joint data generating process underlying ETF returns as a trivariate GARCH-in-mean model within the errors from the multivariate Student's t-distribution. The Student's t-distribution is fatter-tailed distribution than the Normal distribution. We allow for a fat-tailed distribution for the financial dataset we applied in this chapter, then the multivariate Student's t-distribution is selected. Under the Student's t-distribution assumption, the standardized errors $\boldsymbol{v}_t$ follow a multivariate Student's t-distribution with ν degree of freedom.

4.3.2 Conditional Variances Definition

Turning on the definition of the conditional variances, we use three specifications: the BEKK-GARCH model, the CCC-VARMA-GARCH model, and the DCC-VARMA-GARCH model. Engle and Kroner (1995b) introduce the BEKK model, which using the quadratic form for the conditional covariance matrix to impose its positivity. In fact, an early version of this chapter was written by Baba, Engle, Kraft, and Kroner in 1991, which led to the acronym BEKK. In the BEKK model below, $\boldsymbol{C}^*$, $\boldsymbol{A}_i^*$ and $\boldsymbol{B}_j^*$ are $N * N$ matrices, while constant $\boldsymbol{C}^*$ is represented by a lower triangular matrix. The resulting product matrix will be the same in either upper triangular matrix or lower triangular matrix definition for the constant $\boldsymbol{C}^*$. The ARCH and GARCH terms are formed by a sandwich product with an $N \times N$ matrix of coefficients around a symmetric matrix $\boldsymbol{u}_{t-i}\boldsymbol{u}_{t-i}^T$. In keeping with the literature, we define the premultiplying matrix is the transposed one:

$$\boldsymbol{\Sigma}_{u,t} = \boldsymbol{C}^*\boldsymbol{C}^{*T} + \sum_{i=1}^{i} \boldsymbol{A}_i^{*T}\boldsymbol{u}_{t-i}\boldsymbol{u}_{t-i}^T\boldsymbol{A}_i^* + \sum_{j=1}^{j} \boldsymbol{B}_j^{*T}\boldsymbol{\Sigma}_{u,t-j}\boldsymbol{B}_j^* \tag{4.9}$$

When we try to interpret the result of the BEKK-GARCH model, we should be careful. The parameter of the BEKK model does not represent directly the impact of the different lagged terms of the elements of the covariance matrix ($\mathbf{\Sigma}_{u,t}$). Because of the standard use of the transpose of $\boldsymbol{A}_i^*$ and $\boldsymbol{B}_j^*$ as the pre-multiplying matrix, the coefficients have the opposite interpretation as a_{ij} is the effect of residual i on variable j, rather than j on i in most other forms of GARCH models. The number of parameters in the BEKK(1,1,1) model is $N(5N+1)/2$. The difficulty when estimating a BEKK model is the high number of unknown parameters. Therefore it is not surprising that the BEKK model is rarely used when the number of the sequence is larger than 3 or 4 (Bauwens, Laurent, and Rombouts, 2006a). And the BEKK model can have a poorly behaved likelihood function, which also makes the estimation difficult. The reason for this difficulty is changing the signs of all elements of $\boldsymbol{C}^*$, $\boldsymbol{A}_i^*$ or $\boldsymbol{B}_j^*$ will have no effect on the value of the likelihood function (Enders, 2008).

Ling and McAleer (2003) propose variances of the multivariate GARCH model follow a VARMA process. Conditional variances include not just own lagged squared residuals and own variances, but all the other returns' lagged squared residuals and variances in each variance equation (not to be confused with VARMA model for the mean).

$$\sigma^2_{ii,t} = c_{ii} + \sum_{j=1}^{3} a_{ij} u^2_{j,t-1} + \sum_{j=1}^{3} b_{ij} \sigma^2_{jj,t-1} \tag{4.10}$$

Where, a_{ij} is the parameter for lagged squared residuals for variable j with effect on variance i, which also called ARCH effect term. While b_{ij} is the GARCH effect term, which is the parameter for the lagged variance of variable j with effect on variance i. We adopt the VARMA-GARCH specification as the first model in CCC and DCC system for calculating the conditional variances.

The CCC and DCC models can be viewed as nonlinear combinations of univariate GARCH models. We define the individual conditional variances follow a VARMA process mentioned above. Then we model the conditional correlation matrix by imposing its positive definiteness at any time t. The CCC model and the DCC model have the advantage that it can be applied to very large numbers of variables compared with the BEKK model.

Bollerslev et al. (1990) propose a constant conditional correlation GARCH (CCC-GARCH) model. And Engle (2002) introduces dynamic conditional correlations GARCH (DCC-GARCH) model, in which conditional correlations are time dependent.

The CCC model is defined in matrix express as:

$$\boldsymbol{\Sigma}_{u,t} = \boldsymbol{D}_t \boldsymbol{R} \boldsymbol{D}_t, \quad \sigma^2_{ij,t} = (\rho_{ij}\sigma_{ii,t}\sigma_{jj,t}) \tag{4.11}$$

$$\boldsymbol{D}_t = diag(\sigma_{11,t} \cdots \sigma_{NN,t}), \quad \boldsymbol{R} = (\rho_{ij}) \tag{4.12}$$

The CCC model contains $2N^2 + 2N$ parameters. $\boldsymbol{\Sigma}_{u,t}$ is positive definite if and only if all the N conditional variances are positive and $\boldsymbol{R}$ is a symmetric positive definite matrix with $\rho_{ii} = 1, \forall i$. The restriction of constant conditional correlations in the CCC model probably seems unrealistic in many empirical applications. The DCC-GARCH model is introduced by setting the conditional correlations time dependent.

The Engle (2002) *DCC* is defined as:

$$\boldsymbol{\Sigma}_{u,t} = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t \tag{4.13}$$

$$\boldsymbol{R}_t = diag(q_{11,t}^{-1/2} \cdots q_{NN,t}^{-1/2}) \boldsymbol{Q}_t diag(q_{11,t}^{-1/2} \cdots q_{NN,t}^{-1/2}), \tag{4.14}$$

$$\boldsymbol{Q}_t = (q_{ij,t}) = (1 - \alpha - \beta)\overline{\boldsymbol{Q}} + \alpha v_{t-1} v_{t-1}^T + \beta \boldsymbol{Q}_{t-1} \tag{4.15}$$

$\boldsymbol{Q}_t$ is an $N * N$ symmetric positive definite matrix. $\overline{\boldsymbol{Q}}$ is the $N * N$ unconditional covariances matrix of $\boldsymbol{v}_t$. α and β are nonnegative scalar parameter satisfying $\alpha + \beta < 1$.

The correlation estimator is,

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}} \tag{4.16}$$

In the CCC model, $\sigma^2_{ij,t} = \rho_{ij}(\sigma_{ii,t}\sigma_{jj,t})$. Whereas in the DCC model, covariance $\sigma^2_{ij,t}$ is generated using variances from the GARCH model and correlation ρ_{ij} computed by q_{ij}. Actually, it shows $\sigma^2_{ij,t} = \frac{q_{ij,t}\sigma_{ii,t}\sigma_{jj,t}}{\sqrt{q_{ii,t}q_{jj,t}}}$. Restricted correlation models (CCC and DCC) allows two-step procedures to estimate. GARCH parameters are estimated at first and then correlation matrix $\boldsymbol{R}_t$ is computed. To be more explicit, The first one is employing the quasi-maximum likelihood estimation for conditional variances modelling. The variances are computed using separate equations for each variable. Then the conditional correlation matrix $\boldsymbol{R}_t$ is estimated by using the previous results (conditional variances). Finally, the joint covariance matrix can be estimated.

We use four types of information criteria to select a best-fitting model. The information criteria provide model assessment introducing penalty terms for the number of parameters in the empirical estimated model. The penalty terms in the BIC is bigger than the AIC. The Schwarz Bayesian Criterion is also called the Bayesian Information Criterion.

$$AIC\ (Akaike\ Information\ Criterion) = -2logL + k \times 2 \tag{4.17}$$

$$BIC\ (Schwarz\ Bayesian\ Criterion) = -2logL + k \times log\boldsymbol{T} \tag{4.18}$$

$$HQ\ (Hannan - Quinn) = -2logL + k \times 2log(log\boldsymbol{T}) \tag{4.19}$$

$$FPE\ (log)\ (Final\ Prediction\ Error) = -2logL + \boldsymbol{T}log(\frac{\boldsymbol{T}+k}{\boldsymbol{T}-k}) \tag{4.20}$$

Where K is the number of estimated parameters (or regressors) and $\boldsymbol{T}$ is the number of observations. The best model holds the minimizes the chosen criterion.

4.3.3 Estimation Method

Given that the errors $\boldsymbol{v_t}$ are multivariate normal distributed. In this case, the joint distribution has density function below:

$$f\left(\boldsymbol{v}_t\right) = \prod_{t=1}^{\boldsymbol{T}} \frac{1}{(2\pi)^{n/2}} \exp\left\{-\frac{1}{2}\boldsymbol{v}_t^T\boldsymbol{v}_t\right\} \tag{4.21}$$

Where $\boldsymbol{T}$ is the number of observations. Here $E(\boldsymbol{v}_t) = \mathbf{0}$ and $E(\boldsymbol{v}_t\boldsymbol{v}_t^T) = \boldsymbol{I}$.

Then the error vector $\boldsymbol{u_t} = \boldsymbol{\sigma_t v_t}$ are conditionally multivariate-normally distributed as well. The joint density is the product of all the conditional densities, so the log-likelihood function of the joint distribution is the sum of the log-likelihood functions of the conditional distributions. Therefore we estimate the parameters in BEKK-GARCH specification with the errors from the multivariate normal distribution using the log-likelihood function below equation 23 for the heteroskedastic system equation 22:

$$\begin{aligned} \boldsymbol{y}_t =& E(\boldsymbol{y}_t|\boldsymbol{F}_{t-1}) + \boldsymbol{u}_t \\ Var(\boldsymbol{u}_t|\boldsymbol{F}_{t-1}) =& \boldsymbol{\Sigma_{u,t}} \end{aligned} \tag{4.22}$$

$$\begin{aligned} L(\theta) &= \sum_{t=1}^{\boldsymbol{T}} L_t(\theta) \\ \ln(L(\boldsymbol{\theta}))^{BEKK,N} &= -\frac{1}{2}\sum_{t=1}^{\boldsymbol{T}}(n\ln(2\pi) + \ln|\boldsymbol{\Sigma_{u,t}}| + \boldsymbol{u_t^T\Sigma_{u,t}^{-1}u_t}) \end{aligned} \tag{4.23}$$

Where n is the number of variables, three in our case and θ denotes the unknown parameters to be computed in $\boldsymbol{u,t}$ and $\boldsymbol{\Sigma_{u,t}}$.

Respectively the log-likelihood function of CCC-GARCH model and DCC-GARCH model with the errors from the multivariate normal distribution are shown below, by substituting $\boldsymbol{\Sigma}_{u,t} = \boldsymbol{D}_t\boldsymbol{R}\boldsymbol{D}_t$, and $\boldsymbol{\Sigma}_{u,t} = \boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t$:

$$\ln(L(\boldsymbol{\theta}))^{CCC,N} = -\frac{1}{2}\sum_{t=1}^{\boldsymbol{T}}\left(n\ln(2\pi) + 2\ln\left(|\boldsymbol{D}_t|\right) + \ln\left(|\boldsymbol{R}|\right) + \boldsymbol{u_t}^T\boldsymbol{D}_t^{-1}\boldsymbol{R}^{-1}\boldsymbol{D}_t^{-1}\boldsymbol{u_t}\right) \tag{4.24}$$

$$
\begin{aligned}
\ln(L(\boldsymbol{\theta}))^{DCC,N} &= -\frac{1}{2}\sum_{t=1}^{\boldsymbol{T}}\left(n\ln(2\pi)+\ln\left(|\boldsymbol{\Sigma_{u,t}}|\right)+\boldsymbol{u_t}^T\boldsymbol{\Sigma_{u,t}^{-1}}\boldsymbol{u_t}\right)\\
&= -\frac{1}{2}\sum_{t=1}^{\boldsymbol{T}}\left(n\ln(2\pi)+\ln\left(|\boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t)\right)+\boldsymbol{u_t}_t^T\boldsymbol{D}_t^{-1}\boldsymbol{R}_t^{-1}\boldsymbol{D}_t^{-1}\boldsymbol{u_t}\right)\\
&= -\frac{1}{2}\sum_{t=1}^{\boldsymbol{T}}\left(n\ln(2\pi)+2\ln\left(|\boldsymbol{D}_t|\right)+\ln\left(|\boldsymbol{R}_t|\right)+\boldsymbol{u_t}^T\boldsymbol{D}_t^{-1}\boldsymbol{R}_t^{-1}\boldsymbol{D}_t^{-1}\boldsymbol{u_t}\right)
\end{aligned}
\tag{4.25}
$$

When the errors $\boldsymbol{v_t}$ are multivariate student's t-distributed with the degree of freedom ν. In this case, the joint distribution has density function below:

$$
f\left(\boldsymbol{z}_t|\nu\right)=\prod_{t=1}^{\boldsymbol{T}}\frac{\Gamma\left(\frac{\nu+n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\left[\pi(\nu-2)\right]^{n/2}}\left[1+\frac{\boldsymbol{v}_t^T\boldsymbol{v}_t}{\nu-2}\right]^{-\frac{n+\nu}{2}} \tag{4.26}
$$

The likelihood function of $\boldsymbol{u_t}=\boldsymbol{\sigma_t}\boldsymbol{v_t}$ is shown:

$$
L(\boldsymbol{\theta})=\prod_{t=1}^{\boldsymbol{T}}\frac{\Gamma\left(\frac{\nu+n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\left[\pi(\nu-2)\right]^{n/2}|\boldsymbol{\Sigma_{u,t}}|^{1/2}}\left[1+\frac{\boldsymbol{u}_t^T\boldsymbol{\Sigma_{u,t}}^{-1}\boldsymbol{u}_t}{\nu-2}\right]^{-\frac{n+\nu}{2}} \tag{4.27}
$$

Therefore, the log-likelihood function of BEKK-GARCH model, CCC-GARCH model, and DCC-GARCH model with the errors from the multivariate Student's t-distribution are shown below,

$$
\ln(L(\boldsymbol{\theta}))^{BEKK,T}=\prod_{t=1}^{\boldsymbol{T}}\frac{\Gamma\left(\frac{\nu+n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\left[\pi(\nu-2)\right]^{n/2}|\boldsymbol{\Sigma_{u,t}}|^{1/2}}\left[1+\frac{\boldsymbol{u}_t^T\boldsymbol{\Sigma_{u,t}}^{-1}\boldsymbol{u}_t}{\nu-2}\right]^{-\frac{n+\nu}{2}} \tag{4.28}
$$

$$
\begin{aligned}
\ln(L(\boldsymbol{\theta}))^{CCC,T} = &\sum_{t=1}^{\boldsymbol{T}}\left(\ln\left[\Gamma\left(\frac{\nu+n}{2}\right)\right]-\ln\left[\Gamma\left(\frac{\nu}{2}\right)\right]-\frac{n}{2}\ln[\pi(\nu-2)]-\frac{1}{2}\ln\left[|\boldsymbol{D}_t\boldsymbol{R}\boldsymbol{D}_t|\right]\right.\\
&-\frac{\nu+n}{2}\ln\left[1+\frac{\boldsymbol{u}_t^T\boldsymbol{D}_t^{-1}\boldsymbol{R}^{-1}\boldsymbol{D}_t^{-1}\boldsymbol{u}_t}{\nu-2}\right])
\end{aligned}
\tag{4.29}
$$

$$\ln(L(\boldsymbol{\theta}))^{DCC,T} = \sum_{t=1}^{T} \Bigg(\ln \left[\Gamma \left(\frac{\nu + n}{2} \right) \right] - \ln \left[\Gamma \left(\frac{\nu}{2} \right) \right] - \frac{n}{2} \ln[\pi(\nu - 2)] - \frac{1}{2} \ln \left[|\boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t| \right] - \frac{\nu + n}{2} \ln \left[1 + \frac{\boldsymbol{u}_t^T \boldsymbol{D}_t^{-1} \boldsymbol{R}_t^{-1} \boldsymbol{D}_t^{-1} \boldsymbol{u}_t}{\nu - 2} \right] \Bigg) \tag{4.30}$$

The quasi-maximum likelihood method is applied in Estima RATS to estimate multivariate GARCH systems. The estimation for the CCC-GARCH and the DCC-GARCH is adopted a two-stage approach. First, the parameters in the univariate GARCH model is estimated for each data series by replacing $\boldsymbol{R_t}$ with the identity matrix $\boldsymbol{I_n}$ in the likelihood function. In the second stage, the likelihood function above is applied to compute the rest parameters (ρij in CCC, α and β in DCC). When the errors are assumed to be Student's t-distributed, the first stage is the same as a normal distribution, which means assuming univariate GARCH model following a normal distribution. Then in the second stage, ν, α, and β are estimated.

Ling and McAleer (2003) investigate the asymptotic normality of the quasi-maximum-likelihood estimator for the Vector ARMA-GARCH model (VARMA-GARCH). In this study, they make the assumption that all the parameters on ARCH effect (a_{ij}) and GARCH effect (b_{ij}) are nonnegative. However, it concludes that the assumption may be too strong in practice. The estimation results will show some minor modifications (Ahn and Reinsel, 1988). The negative volatility spillovers in the unrestricted extended constant conditional correlation GARCH (UECCC-GARCH) model is discussed by Conrad and Karanasos (2010). It relaxes the assumption of nonnegative volatility feedback, which is firstly imposed by Bollerslev (1986) to restrict parameters in the univariate GARCH process. It is a sufficient condition on a positive definition of the conditional variance-covariance matrix but not necessary (Nelson and Cao, 1992)(Tsai and Chan, 2008). Conrad and Karanasos (2010) also suppose and suggest to derive a further asymptotic theory for the VARMA-GARCH model by relaxing nonnegative volatility spillovers assumption.

In practice, the negative volatility spillovers get high-end adjustments using NLPAR in the Estima RATS. NLPAR allows changing the parameter settings (the estimation method used, the initial guess values, and the number of iterations) during the iterative estimation process. It makes the estimation process avoiding "dead-end" parameter paths, otherwise a fairly large negative covariance (b_{ij}) this period might push the relative variance zero or negative in the next period.

4.4 Empirical Results and Discussions

The multivariate GARCH-in-mean model specifications are estimated by quasi-maximum likelihood methods (QMLE) using the BFGS algorithm. The BEKK model is the most computationally intensive of the models we applied.

Based on the estimation results of multivariate GARCH-in-mean models (BEKK-GARCH, CCC-VARMA-GARCH, DCC-VARMA-GARCH), fitting data of four types of renewable energy ETFs in different model specifications are compared and contrasted. And we find some significant volatility spillover effect, by using renewable energy ETFs, S&P 500 ETF, and crude oil ETF. The details are furtherly discussed in an explicit way below. In order to measure the efficiency of the parameterized model in the model performances of fitness, we compare four types of information criteria reported in *Table 4.13* (AIC, BIC, HQ and FPE). All the criteria show that the DCC model is the best model, and they rank the BEKK model as the second best.

According to the information criteria values, the model is fitting substantially better by applying the multivariate Student's t-distribution to model the errors comparing with the Normally distributed errors. The shape parameter ν for the degree of the Student's t-distribution is listed in the estimation output. Moreover, the empirical probability of a residual being in the left 0.05 tail is computed basing on saved residuals and variance-covariance matrices. We can conclude that the S&P 500 ETF has the fattest tail, and any one of the renewable ETF has a fatter tail than oil ETF shown by *Table 4.41*.

4.4.1 Volatility Spillovers in Mean Level

Turning first to the GARCH-in-mean terms estimation results reported in *Table 4.14*, *Table 4.15*, and *Table 4.16*, we have some results discussed below. In this model, we allow each variance of three returns to enter each mean level. The point estimates for the free elements in Ψ from the mean equation in multivariate GARCH-in-mean system individually represent the effect of the conditional variances of the renewable energy ETF, the S&P 500 ETF and the oil ETF on each return of their own. The elements in matrix Ψ capture the effects of uncertainty on ETFs.

In *Table 4.16*, the coefficient on the conditional variance of the S&P 500 ETF in its own return equation equals 0.0742 with a p-value of 0.0400, is given by ψ_{22} in DCC model with the multivariate Student's t-distribution when we apply wind ETF return as the renewable energy ETF for estimation. It indicates that the S&P 500 ETF volatility has a significant and positive effect on its own return. We can find the matching volatility spillover effect when we change the wind renewable energy ETF to solar renewable energy ETF, but not in renewable energy nuclear ETF neither renewable energy benchmark ETF. These findings are consistent with the theoretical model made in the literature on the Capital Asset Pricing Model (CAPM), which is introduced by (Sharpe, 1964)(Lintner, 1975). It describes the relationship between risk and return. The positive relationship indicates investors, who are assumed to be risk averse, require a high return as compensation for bearing extra risk. We also find the oil ETF volatility has a significant negative effect on the S&P 500 ETF return, as it is shown by in bold (ψ_{23}) in *Table 4.14*, *Table 4.15*, and *Table 4.16*. We get consistent results when different renewable energy ETFs is applied except the renewable energy benchmark ETF. It is consistent with some previous literature that oil uncertainty has a significant and negative effect on stock performance. However, the uncertainty effect of renewable energy ETFs does not show out the results we expected. It implicates the renewable energy benchmark ETF has dampened even eliminated the effect of oil uncertainty on ETF stock market. It is plausible the renewable energy benchmark ETF (PBW) changed the dominant role of oil ETF (USO) in energy ETF sector because PBW tracks an index (ECO) of highly diverse in the range of underlying assets about wind, solar, biofuels, and geothermal energy-related companies.

More precisely, the GARCH-in-mean effects are shown in the third mean equation (oil ETF) when we apply the nuclear ETF data. The coefficient 0.0788 with a p-value of 0.0220 (ψ_{31}) in *Table 4.15* nuclear column, indicates the conditional volatility of the nuclear ETF has a significant positive effect on the oil ETF. We get similar results as Alsalman (2016) proposed the stock return of coal industry level response positively to uncertainty in oil prices. Oppositely, the S&P 500 ETF uncertainty negatively spillovers to the return of the oil ETF, as it is -0.1308 with a p-value of 0.0217 shown by ψ_{32} in *Table*

4.15. It indicates the S&P 500 ETF uncertainty has a significant and negative effect on the oil ETF return. By contrast, nuclear ETF uncertainty has a significant and positive effect on the oil ETF return.

Turning to the VAR process in the returns, the most important finding is The previous period returns of S&P 500 ETF and oil ETF consistently have negative effects on its own current returns, which means they both are in the mean reverse process. One of the strongest effects in the renewable energy mean equation is that period lag of S&P 500 ETF return negatively affects current period wind ETF return and solar ETF, while it has positive effects on current period nuclear ETF and renewable energy ETF (γ_{12}^{i}). And the previous period return of oil negatively affect the current period S&P 500 ETF return, demonstrated by (γ_{23}^{i}) in *Table 4.32* and *Table 4.36*.

4.4.2 Volatility Spillovers between Conditional Variances

Based on the estimation results of the conditional variance of three multivariate GARCH specifications, different models are compared and constructed. And the volatility spillovers between renewable energy ETFs, S&P 500 ETF, and crude oil ETF are discussed further in an explicit way below.

Starting with the own conditional GARCH effect terms b_{ii}, it measures the long-term persistence and the own conditional ARCH effect terms a_{ii} measure the short-term persistence. They are clearly important in explaining conditional volatility. The b_{ii} terms and the a_{ii} terms are statistically significant at the 1% level in each of multivariate GARCH models. The coefficient b_{11} refers to the GARCH term in the renewable energy ETFs equation, while the coefficient b_{22} refers to the GARCH term in the S&P 500 ETF equation and the coefficient b_{33} refers to the GARCH term in the oil ETF equation. The renewable energy ETFs and the oil ETF has shown a larger amount of long-term persistence than the S&P 500 ETF. For each variance, the a_{ii} is much smaller than the b_{ii}, which indicates the own volatility long-run (GARCH) persistence is larger than the short-run (ARCH) persistence. We also conclude that the “own" effect on variance is always dominant (the diagonal elements are larger than the off-diagonals).

Comparing volatility spillovers in BEKK, CCC and DCC models, we find the strongest evidence is found from the estimates of the BEKK model. The restricted correlation models (CCC and DCC) shows less evidence of it. We start the volatility spillover analysis in BEKK models. For the short-term persistence when applying wind ETF, there is evidence of volatility spillovers from S&P 500 ETF to wind ETF (a_{21}) and from S&P 500 ETF to oil ETF (a_{23}). There is also evidence of long-term persistence volatility spillovers from wind ETF to S&P 500 ETF (b_{12}) and from S&P 500 ETF to wind ETF (b_{21}). Solar ETF has shown the consistent volatility spillovers in the BEKK model fitting in as the solar ETF shows. For nuclear ETF, there is evidence of short-term persistence volatility spillovers from S&P 500 ETF to nuclear ETF (a_{21}). There is also evidence of long-term persistence volatility spillovers from nuclear ETF (b_{12}) to S&P 500 ETF, from nuclear ETF to oil ETF (b_{13}), from S&P 500 ETF to nuclear ETF (b_{21}), and from S&P 500 to ETF oil ETF (b_{23}). For renewable energy benchmark ETF, there is evidence of short-term persistence volatility spillovers from S&P 500 ETF to renewable energy benchmark ETF (a_{21}) and from S&P 500 ETF to oil ETF (a_{23}). There is also evidence of long-term persistence volatility spillovers from S&P 500 ETF to renewable energy benchmark ETF (b_{21}) and from S&P 500 ETF to oil ETF (b_{23}).

Looking across the full suite of models (BEKK, CCC, and DCC with the multivariate Normal distribution or the multivariate Student's t-distribution), there is some consistent evidence for volatility spillovers. When wind ETF is applied, there is the consistent evidence that the short-term persistence volatility spillover effect from S&P 500 ETF to renewable energy ETF (a_{21} in BEKK, a_{12} in CCC and a_{12} in DCC) in six models. The long-term volatility persistence and the short-term volatility persistence both spillover from S&P 500 ETF to solar ETF (a_{21} and b_{21} in BEKK model, a_{12} and b_{12} in CCC/DCC model) is shown in six models. The nuclear ETF and the renewable ETF are shown the consistent results as the results in solar ETF. Previous literature rarely finds consistent estimation results in applying different multivariate GARCH specifications, as their results are somewhat mixed across different models. Then we conclude that S&P 500 ETF volatility has a significant effect on the volatility of renewable energy ETFs, which shows the positive short-term persistence volatility spillover and the negative long-term persistence volatility spillover.

In the CCC model, all correlations between renewable energy ETFs, the S&P 500 ETF, and oil ETF are positive and strong significant at the 1% level. The correlation between renewable energy ETFs and the S&P 500 ETF (ρ_{21}) is the highest. Except for the renewable energy benchmark ETF, the correlation between the S&P 500 ETF and oil ETF (ρ_{32}) ranks second high, but slightly higher the correlation between renewable energy and oil ETF (ρ_{31}). Outstandingly, the oil ETF has a higher correlation with renewable energy benchmark ETF(ρ_{31}), rather than with S&P 500 ETF(ρ_{32}). We conclude that renewable energy ETF has moved much closer to S&P 500 ETF than with oil ETF. Moreover, S&P 500 ETF has moved much closer to renewable energy ETF than with oil ETF. Each pair of returns moves in lockstep as all the correlations are positive.

Based on the time-varying conditional correlation estimated by the DCC model, there is volatility clustering pattern for each series. And the dynamic conditional correlation really changes a lot compared with the constant correlation in the CCC model. Each pair of the correlations has dramatically decreased most to negative before the financial crises respectively in September 2008 and 2011. In the post-crisis periods, there is a coincidence that the correlation has risen getting the highest level historically. We conclude the correlation increases after the financial crisis (individually in 2008 and in 2011). The most important finding is that the dynamic correlation between oil ETF and S&P 500 ETF are always positive since U.S. net imports of crude oil and petroleum products gradually decrease commencing 2005 (shown in *Figure 4.20*, *Figure 4.21*, *Figure 4.22*, and *Figure 4.23*). It is consistent evidence in Wang and Liu (2016)'s study.

4.4.3 Negative Volatility Spillovers

In *Section 4.4.2*, the volatility spillover estimates reported in *Tables 4.17-4.40* in *Appendix* were interpretated. The estimates of the a_{ij} parameters were interpreted in terms of short-term volatility spillovers, and the b_{ij} parameters in terms of long-term volatility spillovers.

Most of these spillover estimates are positive, indicating positive volatility spillovers. However, some are negative. Negative volatility spillover is an intriguing estimation result in above-presenting outcomes. To be explicit, the negative long-term persistence volatility spillover (b_{12}) is strongly significant, consistent, and robust from stock to renewable energy, whatever which kind of renewable energy ETF (wind ETF/ solar ETF/ nuclear ETF/ renewable energy benchmark ETF) is applied in estimation. For example, consider the estimate of b_{12} in *Table 4.39* of Appendix (b_{12} = -0.1635; p-value=0.0000).

This estimate represents a strongly significant negative long-term volatility spillover from the renewable energy benchmark ETF to the S&P 500 ETF.

Negative volatility spillovers have been found in other contexts. For example, Conrad and Karanasos (2010) find evidence that variability in industrial production (real variability) affects variation in consumer prices (nominal uncertainty) negatively, and this result supports theories of optimal monetary policy proposed by Fuhrer (1997) and others.

Negative volatility spillovers also raise interesting theoretical issues. In early versions of multivariate GARCH models, it was assumed that all volatility spillovers need to be positive in order to guarantee that the conditional variance is positive (e.g., see (Jeantheau, 1998)). The assumption of nonnegative volatility feedback, which is firstly imposed by Bollerslev (1986) to restrict parameters in the univariate GARCH process. It is a sufficient condition for positive-definiteness of the conditional variance-covariance matrix but not necessary (Nelson and Cao, 1992)(Tsai and Chan, 2008). Ling and McAleer (2003) investigate the asymptotic normality of the quasi-maximum-likelihood estimator for the Vector ARMA-GARCH model (VARMA-GARCH). In this study, they followed Bollerslev (1986)'s step to assume that all the parameters on ARCH effect (a_{ij}) and GARCH effect (b_{ij}) are nonnegative. He and Teräsvirta (2004) examed the fourth-moment structure of an extended CCC GARCH model, which model is first defined by Jeantheau (1998). This study also mentioned that this model specification is also seen in Ling and McAleer (2003)'s paper. Therefore, we conclude that Jeantheau (1998), Ling and McAleer (2003), He and Teräsvirta (2004) adopted the same model specification having rich autocorrelation structure, but they research different question on econometric theories.

However, the assumption of the specification seems too strong in practice. That is the reason we got negative volatility spillover estimation results by applying Ling and McAleer (2003)'s methodology. The lucky thing is the negative volatility spillovers in the unrestricted extended constant conditional correlation GARCH (UECCC-GARCH) model is discussed by Conrad and Karanasos (2010). More recent work has established that this is a sufficient but not a necessary condition. The necessary condition is specified in Proposition 1 of Conrad and Karanasos (2010), and this condition allows negative volatility spillovers. In this study, Conrad and Karanasos (2010) also suppose and suggest to derive a further asymptotic theory for the VARMA-GARCH model by relaxing nonnegative volatility spillovers assumption.

Conrad and Karanasos (2010) provided the conditions for positive conditional variance when the negative volatility spillovers are allowed. We conclude the conditions below referring to Conrad and Karanasos (2010)'s study. Conrad and Karanasos (2010) intensively discuss the conditions for the most often applied specification-the bivariate model of order (1, 1).

Proposition 1 of Conrad and Karanasos (2010) is as follows:

> PROPOSITION 1: Let Assumptions A1 and A2 be satisfied and $\phi_1 \neq \phi_2$. The following conditions are necessary and sufficient for $\sigma_{it} > 0$, $i = 1, 2$, for all t in the bivariate UECCC-GARCH(1, 1) model:
>
> (a) For the two constants, we require
> $w_1 = (1 - b_{22})c_{11} + b_{12}c_{22} > 0,$
> $w_2 = (1 - b_{11})c_{22} + b_{21}c_{11} > 0.$
> (b) Condition (C1) in Theorem 1 reduces to (C1′)
> $(b_{11} - b_{22})^2 > -4b_{12}b_{21} \quad and \quad \phi_1 > 0.$
> Condition (C2) becomes (C2′)
> $(b_{11} - \phi_2)a_{11} + b_{12}a_{21} > 0,$
> $(b_{11} - \phi_2)a_{12} + b_{12}a_{22} > 0,$
> $b_{21}a_{11} + (b_{22} - \phi_2)a_{21} > 0,$
> $b_{21}a_{12} + (b_{22} - \phi_2)a_{22} > 0.$
> Condition (C3) amounts to (C3′a)
> $a_{11} \geq 0, \quad a_{12} \geq 0,$
> $a_{21} \geq 0, \quad a_{22} \geq 0.$
> and (C3′b)
> $b_{11}a_{11} + b_{12}a_{21} \geq 0, \quad b_{11}a_{12} + b_{12}a_{22} \geq 0,$
> $b_{21}a_{11} + b_{22}a_{21} \geq 0, \quad b_{21}a_{12} + b_{22}a_{22} \geq 0.$

ϕ_1 and ϕ_2 are the inverse roots of $\beta(z)$. And we order them as follows: $| \phi_1 | \geq | \phi_2 |$. c_{ii}, a_{ij}, and b_{ij}, $i = 1, 2$, are the parameters in a bivariate GARCH model.

Correspondingly, we estimate the model with only two variables (renewable energy ETFs and the S&P 500 ETF), which show the negative volatility spillover between. Consistently, the negative volatility spillover has shown up again in the two-variable estimation results as *Table 4.1* presents below.

After estimation, we particular check the conditions of Conrad and Karanasos (2010)'s Proposition 1 are satisfied in my results. Here, we verify this condition for the set of estimates reported in *Table 4.1*.

The condition is checked using *Table 4.2* and *Table 4.3*. All the checking results reject C2′, but the values are slightly under zero (condition requires the value above zero). The checking results are reasonable, and we expected. Based on what we are introducing concerning the econometric theory, the methodology of Ling and McAleer (2003) (generating estimation results) and the conditions from Conrad and Karanasos (2010)'s study are under different assumptions. We arbitrarily check the conditions to confirm the necessary for our further research. It is beneficial to investigate whether the Ling and McAleer (2003)'s theory holds under the necessary and sufficient conditions derived by Conrad and Karanasos (2010).

In *Table 4.2* and *Table 4.3*, we see that in all model estimated. There is a failure to meet all of the conditions. To be specific, particular problems arise to meet C2′. However, the violation appears to be slight. This leads to the question of whether a statistical test of the conditions can be developed. This is an interesting question for further research.

For example, consider the estimate of b_{12} in *Table 4.1* of RE DCC N column ($b_{12} = -0.1817$ with statistically highly significant as p-value < 0.001). It is a piece of strong evidence for negative volatility spillovers from renewable energy benchmark ETF to S&P 500 ETF. The corresponding condition check result is in *Table 4.3* in the second to the last column. We can see that besides two conditions of C2′ are slightly violated the rest conditions met. Other evidence is shown by various renewable energy ETF is substitute applied in *Table 4.1*.

4.4.4 Implications

During the post-financial crisis period, the dynamic correlation between oil ETF and S&P 500 ETF has been getting a very high level. It implicates the oil ETF or oil future is not a safe haven to protect stock investor's profits when they construct stock portfolios. And most GARCH-in-mean estimation results provide evidence that oil ETF uncertainty negatively affects S&P 500 ETF. It will be a double risky portfolio strategy to hold oil ETF and S&P 500 ETF during an economic downturn condition or during world turmoil.

TABLE 4.1: *Multivariate GARCH-in-mean Model Estimation Result of Renewable Energy ETF and S&P 500 ETF*

Coefficient	Wind CCC N	Wind CCC T	Wind DCC N	Wind DCC T	Solar CCC N	Solar CCC T	Solar DCC N	Solar DCC T
c_{11}	0.0665	0.0429	0.0325	0.0097	0.0499	0.0429	0.0451	0.0377
c_{22}	0.0360	0.0234	0.0203	0.0045	0.0240	0.0128	0.0209	0.0099
a_{11}	0.0713	0.0562	0.0548	0.0375	0.0333	0.0353	0.0309	0.0330
a_{12}	0.0448	0.0282	0.0808	0.0559	0.1164	0.1487	0.1675	0.2103
a_{21}	0.0136	0.0067	0.0090	-0.0001	0.0009	0.0015	0.0000	0.0006
a_{22}	0.1034	0.1046	0.1225	0.1205	0.1231	0.1273	0.1339	0.1368
b_{11}	0.8988	0.9255	0.9390	0.9641	0.9661	0.9668	0.9666	0.9665
b_{12}	-0.0536	-0.0360	-0.0855^{*}	-0.0578^{*}	-0.1673^{***}	-0.1844^{***}	-0.1750^{***}	-0.1982^{***}
b_{21}	-0.0098	0.0001	0.0031	0.0161	0.0021	0.0025	0.0017	0.0019
b_{22}	0.8490	0.8563	0.8440	0.8537	0.8334	0.8472	0.8467	0.8595

Coefficient	Nuclear CCC N	Nuclear CCC T	Nuclear DCC N	Nuclear DCC T	RE CCC N	RE CCC T	RE DCC N	RE DCC T
c_{11}	0.0378	0.0220	0.0155	0.0063	0.0724	0.0620	0.0605	0.0415
c_{22}	0.0340	0.0253	0.0191	0.0084	0.0326	0.0213	0.0256	0.0124
a_{11}	0.0263	0.0214	0.0157	0.0059	0.0351	0.0450	0.0329	0.0440
a_{12}	0.0932	0.0592	0.0822	0.0711	0.1294	0.1120	0.1813	0.1600
a_{21}	0.0008	-0.0020	-0.0011	-0.0031	0.0047	0.0061	0.0041	0.0050
a_{22}	0.1145	0.1096	0.1195	0.1063	0.1060	0.1005	0.1187	0.1147
b_{11}	0.9594	0.9675	0.9978	1.0009	0.9556	0.9459	0.9557	0.9479
b_{12}	-0.1086^{***}	-0.0616	-0.1138^{***}	-0.0820^{***}	-0.1782^{***}	-0.1419^{**}	-0.1817^{***}	-0.1418^{***}
b_{21}	-0.0021	0.0047	0.0214	0.0185	-0.0034	-0.0039	-0.0049	-0.0034
b_{22}	0.8531	0.8675	0.8345	0.8714	0.8549	0.8714	0.8704	0.8818

1

*** *indicates p-value < 0.001*
** *indicates p-value < 0.01*
* *indicates p-value < 0.05*

TABLE 4.2: *(Part1) Condition Check for Multivariate GARCH-in-mean Model Estimation Result of Various Renewable Energy ETF and S&P 500 ETF*

Conditions	Wind CCC N	Wind CCC T	Wind DCC N	Wind DCC T	Solar CCC N	Solar CCC T	Solar DCC N	Solar DCC T
(a)								
$w_1 = (1 - b_{22})c_{11} + b_{12}c_{22}$	0.0081	0.0053	0.0033	0.0012	0.0043	0.0042	0.0033	0.0033
$w_2 = (1 - b_{11})c_{22} + b_{21}c_{11}$	0.0030	0.0017	0.0013	0.0003	0.0009	0.0005	0.0008	0.0004
Satisfied $w_1 > 0$ and $w_2 > 0$	yes	yes	yes	yes	yes	yes	yes	yes
(b) (C1′)								
$(b_{11} - b_{22})^2$	0.0025	0.0048	0.0090	0.0122	0.0176	0.0143	0.0144	0.0115
$-4b_{12}b_{21}$	-0.0021	0.0000	0.0011	0.0037	0.0014	0.0019	0.0012	0.0015
ϕ_1	0.9078	0.9255	0.9361	0.9549	0.9634	0.9627	0.9640	0.9628
$(b_{11} - b_{22})^2 > -4b_{12}b_{21}$ and $\phi_1 > 0$	yes	yes	yes	yes	yes	yes	yes	yes
(b) (C2′)								
$(b_{11} - \phi_2)a_{11} + b_{12}a_{21}$	0.0035	0.0036	0.0043	0.0038	0.0042	0.0038	0.0036	0.0033
$(b_{11} - \phi_2)a_{12} + b_{12}a_{22}$	-0.0029	-0.0018	-0.0030	-0.0013	-0.0055	-0.0063	-0.0038	-0.0054
$b_{21}a_{11} + (b_{22} - \phi_2)a_{21}$	-0.0006	0.0000	0.0001	0.0006	0.0001	0.0001	0.0001	0.0001
$b_{21}a_{12} + (b_{22} - \phi_2)a_{22}$	0.0005	0.0000	-0.0001	-0.0002	-0.0001	-0.0001	-0.0001	-0.0001
$(b_{11} - \phi_2)a_{11} + b_{12}a_{21} > 0$	yes	yes	yes	yes	yes	yes	yes	yes
$(b_{11} - \phi_2)a_{12} + b_{12}a_{22} > 0$	no	no	no	no	no	no	no	no
$b_{21}a_{11} + (b_{22} - \phi_2)a_{21} > 0$	no	yes	yes	yes	yes	yes	yes	yes
$b_{21}a_{12} + (b_{22} - \phi_2)a_{22} > 0$	yes	yes	no	no	no	no	no	no
(b) (C3′a)								
a_{11}	0.0713	0.0562	0.0548	0.0375	0.0333	0.0353	0.0309	0.0330
a_{12}	0.0448	0.0282	0.0808	0.0559	0.1164	0.1487	0.1675	0.2103
a_{21}	0.0136	0.0067	0.0090	-0.0001	0.0009	0.0015	0.0000	0.0006
a_{22}	0.1034	0.1046	0.1225	0.1205	0.1231	0.1273	0.1339	0.1368
$a_{11} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
$a_{12} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
$a_{21} \geq 0$	yes	yes	yes	no	yes	yes	yes	yes
$a_{22} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
(b) (C3′b)								
$b_{11}a_{11} + b_{12}a_{21}$	0.0633	0.0517	0.0507	0.0362	0.0320	0.0338	0.0299	0.0318
$b_{11}a_{12} + b_{12}a_{22}$	0.0347	0.0223	0.0654	0.0470	0.0918	0.1202	0.1385	0.1761
$b_{21}a_{11} + b_{22}a_{21}$	0.0109	0.0057	0.0078	0.0005	0.0008	0.0014	0.0001	0.0006
$b_{21}a_{12} + b_{22}a_{22}$	0.0874	0.0895	0.1037	0.1038	0.1029	0.1082	0.1137	0.1180
$b_{11}a_{11} + b_{12}a_{21} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
$b_{11}a_{12} + b_{12}a_{22} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
$b_{21}a_{11} + b_{22}a_{21} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
$b_{21}a_{12} + b_{22}a_{22} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
All conditions met?	no	no	no	no	no	no	no	no

TABLE 4.3: *(Part 2)Condition Check for Multivariate GARCH-in-mean Model Estimation Result of Various Renewable Energy ETF and S&P 500 ETF*

Conditions	Nuclear CCC N	Nuclear CCC T	Nuclear DCC N	Nuclear DCC T	RE CCC N	RE CCC T	RE DCC N	RE DCC T
(a)								
$w_1 = (1-b_{22})c_{11} + b_{12}c_{22}$	0.0019	0.0014	0.0004	0.0001	0.0047	0.0050	0.0032	0.0031
$w_2 = (1-b_{11})c_{22} + b_{21}c_{11}$	0.0013	0.0009	0.0004	0.0001	0.0012	0.0009	0.0008	0.0005
Satisfied $w_1 > 0$ and $w2 > 0$	yes	yes	yes	yes	yes	yes	yes	yes
(b) (C1′)								
$(b_{11}-b_{22})^2$	0.0113	0.0100	0.0267	0.0168	0.0101	0.0056	0.0073	0.0044
$-4b_{12}b_{21}$	-0.0009	0.0012	0.0097	0.0061	-0.0024	-0.0022	-0.0036	-0.0019
ϕ_1	0.9615	0.9645	0.9812	0.9879	0.9613	0.9527	0.9651	0.9545
$(b_{11}-b_{22})^2 > -4b_{12}b_{21}$ and $\phi_1 > 0$	yes	yes	yes	yes	yes	yes	yes	yes
(b) (C2′)								
$(b_{11}-\phi_2)a_{11} + b_{12}a_{21}$	0.0028	0.0022	0.0024	0.0009	0.0029	0.0028	0.0024	0.0025
$(b_{11}-\phi_2)a_{12} + b_{12}a_{22}$	-0.0023	-0.0010	-0.0015	-0.0004	-0.0051	-0.0051	-0.0044	-0.0046
$b_{21}a_{11} + (b_{22}-\phi_2)a_{21}$	-0.0001	0.0001	0.0004	0.0001	-0.0001	-0.0001	-0.0001	-0.0001
$b_{21}a_{12} + (b_{22}-\phi_2)a_{22}$	0.0000	0.0000	-0.0002	-0.0001	0.0002	0.0002	0.0002	0.0002
$(b_{11}-\phi_2)a_{11} + b_{12}a_{21} > 0$	yes	yes	yes	yes	yes	yes	yes	yes
$(b_{11}-\phi_2)a_{12} + b_{12}a_{22} > 0$	no	no	no	no	no	no	no	no
$b_{21}a_{11} + (b_{22}-\phi_2)a_{21} > 0$	no	yes	yes	yes	no	no	no	no
$b_{21}a_{12} + (b_{22}-\phi_2)a_{22} > 0$	yes	yes	no	no	yes	yes	yes	yes
(b) (C3′a)								
a_{11}	0.0263	0.0214	0.0157	0.0059	0.0351	0.0450	0.0329	0.0440
a_{12}	0.0932	0.0592	0.0822	0.0711	0.1294	0.1120	0.1813	0.1600
a_{21}	0.0008	-0.0020	-0.0011	-0.0031	0.0047	0.0061	0.0041	0.0050
a_{22}	0.1145	0.1096	0.1195	0.1063	0.1060	0.1005	0.1187	0.1147
$a_{11} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
$a_{12} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
$a_{21} \geq 0$	yes	no	no	no	yes	yes	yes	yes
$a_{22} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
(b) (C3′b)								
$b_{11}a_{11} + b_{12}a_{21}$	0.0252	0.0209	0.0158	0.0062	0.0327	0.0417	0.0307	0.0410
$b_{11}a_{12} + b_{12}a_{22}$	0.0769	0.0505	0.0684	0.0625	0.1048	0.0917	0.1517	0.1354
$b_{21}a_{11} + b_{22}a_{21}$	0.0006	-0.0017	-0.0005	-0.0026	0.0039	0.0051	0.0034	0.0042
$b_{21}a_{12} + b_{22}a_{22}$	0.0975	0.0954	0.1015	0.0940	0.0902	0.0871	0.1024	0.1006
$b_{11}a_{11} + b_{12}a_{21} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
$b_{11}a_{12} + b_{12}a_{22} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
$b_{21}a_{11} + b_{22}a_{21} \geq 0$	yes	no	no	no	yes	yes	yes	yes
$b_{21}a_{12} + b_{22}a_{22} \geq 0$	yes	yes	yes	yes	yes	yes	yes	yes
All conditions met?	no	no	no	no	no	no	no	no

Therefore, investors should take the sector rotation investment strategy and switch to an investment vehicle negatively or not correlated to the target ETF asset to maintain positions in whether bullish market or bearish of an economic circle.

The oil ETF uncertainty has no significant effect on S&P 500 ETF while renewable energy benchmark ETF applying in estimation. It implicates the renewable energy benchmark ETF has dampened even eliminated the effect of oil uncertainty on ETF stock return. It is plausible the renewable energy benchmark ETF (PBW) changed the dominant role of oil ETF (USO) in energy ETF sector.

The nuclear ETF uncertainty has a significant and positive effect on oil ETF. The connection is originally from the U.S. energy consumption market. *Figure 4.1* and *Figure 4.3* shows the natural gas is a substitute respectively to petroleum and nuclear electric power in the different energy sector in the U.S. market. Therefore, the effect of nuclear energy on the crude oil market can not be negligible. *Figure 4.18* demonstrates the dynamic correlation between nuclear ETF and oil ETF has been upward reaching roughly zero commencing 2014 beginning, which is the significantly booming phrase for the U.S. shale oil revolution. The U.S. is predominantly an oil-importer before the U.S. shale oil revolution, and the worldwide oil supply disruptions influent the U.S. oil market furtherly transmit to the whole energy sector. Nevertheless, the transmission mechanism has diminished, before the U.S. oil price has not been as vulnerable as before. Our finding suggests investors construct oil derivative portfolio with nuclear ETF or nuclear energy-related companies stocks. We confirm that the role of nuclear ETF as safe-haven against crude oil ETF in the U.S. financial market.

The dynamic correlation between renewable energy ETFs and S&P 500 ETF are relatively stable comparing other correlation pairs. Especially the renewable benchmark ETF correlate to S&P 500 ETF in a very high and firm level. Entering one asset in long position can be hedged with a short position in a second asset. Furtherly, investors should adopt a market-timing strategy when they hold the renewable benchmark ETF. The reason is the renewable benchmark ETF perform in tandem with the S&P 500 ETF. The high dependence level to S&P 500 ETF provides a barometer for the renewable benchmark ETF investment.

Finally, we give a suggestion to investors who are interested in energy derivatives. The policy and the physical market situation have driven the financial instruments for energy markets dramatically. And the U.S. dependence level of oil imports has influent the energy-related company stock price. Moreover, the dynamic correlation trends during business circle might provide useful information for adopting efficient investment strategies in future similar scenarios.

4.5 Conclusion

There is a large amount of previous literature studying how the oil price uncertainty affects the financial market but very limit research has focused on the ETF market. And the GARCH-in-mean model is less used to explore the volatility spillover effects. In this chapter, we investigate the volatility spillovers and the dynamic correlation between crude oil ETF, renewable energy ETFs, and S&P 500 ETF via applying three multivariate GARCH specifications respectively with errors from the multivariate Normal distribution and the multivariate Student's t-distribution. We mainly have four findings.

First, the S&P 500 ETF volatility has a significant and positive effect on the return of itself, which is consistent with the theoretical CAPM model. We also find oil ETF volatility has a significant negative effect on the S&P 500 ETF return, which is consistent with previous literature on oil uncertainty has a significant and negative effect on stock performance. Outstandingly, the renewable energy benchmark ETF (PBW) changed the dominant role of oil ETF (USO) in energy ETF sector. The uncertainty of oil ETF has a consistent and negative effect on the S&P 500 ETF return, whereas the effect diminishes when PBW ETF is applied instead of other renewable ETFs. In one word, the PBW ETF has changed the channel of volatility spillovers to return. When nuclear ETF data is applied, the conditional volatility of the nuclear ETF has a significant positive effect on the oil ETF, oppositely the volatility of S&P 500 ETF negatively spillovers to the return of the oil ETF. However, there is no evidence that the volatility spillovers to renewable energy ETF mean level as we expected. And the volatility of renewable energy does not affect the S&P 500 ETF return.

Second, we find consistent evidence of volatility spillovers in the conditional variance level in applying different multivariate GARCH specifications (BEKK, CCC, and DCC). The S&P 500 ETF volatility has a significant effect on the volatility of renewable energy ETFs, which shows the positive short-term persistence volatility spillover and the negative long-term persistence volatility spillover. To be more specific, the increase of squared errors in the last period of the S&P 500 ETF results in a growth of this period conditional variance of renewable energy ETFs. On the other hand, this period conditional variance of renewable energy ETFs will decrease when the last period conditional variance of S&P 500 ETF increases.

Third, we can see that the correlation between renewable energy ETF and S&P 500 ETF is significantly higher than the other two pairs of correlations according to the restricted constant correlation estimation results. About the dynamic correlation results, we conclude the correlation decrease before the financial crisis (in 2008 and in 2011 individually) then dramatically increases after the financial crisis. More importantly, we find the consistent evidence in Wang and Liu (2016)'s study that the dynamic correlation between oil ETF and S&P 500 ETF are always positive commencing 2005 since U.S. net imports of crude oil and petroleum products gradually decrease.

Finally, we find the DCC model is the best model, and the BEKK model is the second best according to the information criteria. The log-likelihood is substantially better when the multivariate Student's t-distributed errors are applied comparing with using the model with the multivariate Normal distributed errors.

Our finding as well as provides several valuable implications for investors. oil ETF or oil future is not a safe haven to construct S&P 500 ETF portfolios especially during a downturn economy. Therefore, investors should take the sector rotation investment strategy. Investors are suggested to construct oil derivative portfolio with nuclear ETF or nuclear energy-related companies stocks. We confirm that the role of nuclear ETF as safe-haven against crude oil ETF in the U.S. financial market. Entering renewable benchmark ETF in long position can be hedged with a short position in S&P 500 ETF and verse visa. Furtherly, investors should adopt a market-timing strategy when they hold the renewable benchmark ETF.

As regarding the future research, risk management can apply in practice by constructing hedging strategy and portfolio weights between crude oil, renewable energy ETFs and S&P 500 ETF. Secondly, we might explore the forecasting abilities in different multivariate GARCH specifications. If there are not consistent results as the in-sample estimation ability contrast results, we have to consider the overfitting problem. The last suggestion is applying the impulse response function to check the price shocks and volatility shocks in ETFs.

4.6 Appendix to Chapter 4

The advantage of renewable energy is being replenished naturally over a relatively short period, such as energy producing by wind, solar, nuclear and so on. While the conventional energy, crude oil resource we focused on, naturally takes millions of years to have been formed. Obviously, the consuming time of crude oil is much quicker than the formation process. Renewable energy resources do not suffer from long-term availability problems, but conventional energy is finite, they will eventually run out in the future. The globally rising demand for energy, especially in the emerging market, like China and India, is definitely a fact we have to face and a problem we have to deal with, but it is not the biggest challenge brought by fossil fuel resource.

It is well known that carbon dioxide is released during the consumption of crude oil production, which has strengthened the greenhouse effect and caused global warming. Oil spills, during transport across the sea, can damage the sea environment and endanger marine ecosystem to the ocean animals as well. Especially it has caused deadly harm when ocean creatures are coated with oil. In addition, separating crude oil into a wide array of petroleum production creates toxins, which is not only directly damage human health, also generate air pollution to threaten climate change.

4.6.1 ETF in Financial Market

A **stock index** or **stock market index** is statistical measurement (typically a weighted average) of the changes in a portfolio (constructed by several stocks). The financial market participants use indices to track the performance of the stock market, as the trades of every single stock is difficult to track. The NASDAQ Composite, Dow Jones Industrial Average, and *S&P* 500 are the three major U.S. stock indices. An index fund, also called index tracker, is used to track the index. Mutual fund and the exchange-traded fund can be used as an index fund. And the major type if ETF is index ETF, which is an attempt to replicate the performance of a specific index.

Except for the index ETF, there are also other types of ETFs, such as stock ETF, bond ETF, commodity ETF, currency ETF and so on. ETF is traded on stock exchanges, which means it traded like a stock. ETF is traded at a different time compared with a mutual fund, and it sometimes weighted by revenue rather than market capitalization. ETFs are structured variously in distinct regions. It contains assets of bonds, stocks, commodities and so on. The ETF shareholders hold the ownership of ETF shares. Ideally, ETF is traded close to its underlying asset value. In this way, ETF closely tracks the return (performance) of the underlying asset (securities).

Underlying assets could be stocks, bonds, commodities, currencies, interest rates, and stock market indices. A derivative is a financial security (contract) which value is based on underlying assets, such as futures contracts, forward contracts, swaps an options.

The primary market refers to where the securities are created. It is the first time for the firm to sell new stocks and bonds to the public. The secondary market is where the trades have activities for selling and buying stocks. The secondary market could be broken down into action market and dealer market. Especially, the “over-the-counter" market refers to the trading did not occur at a physical market. The third market and fourth markets deal with the transaction of significant volumes of shares between broker-dealers and large institutions through over-the-counter electronic networks.

4.6.2 Bi-variate GARCH Models

4.6.2.1 Overview of Multivariate GARCH Model

The GARCH(I,J) specification with N dimensions for a conditional variance-covariance matrix at period t is:

$$u_{t-i}u_{t-i}^T = \begin{bmatrix} u_1 \\ \cdots \\ u_N \end{bmatrix} \begin{bmatrix} u_1 & \cdots & u_N \end{bmatrix} = \begin{bmatrix} u_1 u_2 & \cdots & u_1 u_N \\ \cdots & \cdots & \cdots \\ u_N u_1 & \cdots & u_N u_N \end{bmatrix} \tag{4.31}$$

$$\Sigma_{u,t} = \begin{bmatrix} \sigma_1^2 \\ \cdots \\ \sigma_N^2 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_N^2 \end{bmatrix} = \begin{bmatrix} \sigma_{11}^2 & \cdots & \sigma_{1N}^2 \\ \cdots & \cdots & \cdots \\ \sigma_{N1}^2 & \cdots & \sigma_{NN}^2 \end{bmatrix} \tag{4.32}$$

Consider a bi-variate full-VEC model in GARCH(1,1) process with full parameters :

$$\begin{bmatrix} \sigma_{11,t}^2 \\ \sigma_{12,t}^2 \\ \sigma_{22,t}^2 \end{bmatrix} = \begin{bmatrix} c_{10} \\ c_{20} \\ c_{30} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 \\ u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} \sigma_{11,t-1}^2 \\ \sigma_{12,t-1}^2 \\ \sigma_{22,t-1}^2 \end{bmatrix} \tag{4.33}$$

Indivadual variances and covariance expression:

$$\sigma_{11,t}^2 = c_{10} + a_{11}u_{1,t-1}^2 + a_{12}u_{1,t-1}u_{2,t-1} + a_{13}u_{2,t-1}^2 + b_{11}\sigma_{11,t-1}^2 + b_{12}\sigma_{12,t-1}^2 + b_{13}\sigma_{22,t-1}^2 \tag{4.34}$$

$$\sigma_{12,t}^2 = c_{20} + a_{21}u_{1,t-1}^2 + a_{22}u_{1,t-1}u_{2,t-1} + a_{23}u_{2,t-1}^2 + b_{21}\sigma_{11,t-1}^2 + b_{22}\sigma_{12,t-1}^2 + b_{23}\sigma_{22,t-1}^2 \tag{4.35}$$

$$\sigma_{22,t}^2 = c_{30} + a_{31}u_{1,t-1}^2 + a_{32}u_{1,t-1}u_{2,t-1} + a_{33}u_{2,t-1}^2 + b_{31}\sigma_{11,t-1}^2 + b_{32}\sigma_{12,t-1}^2 + b_{33}\sigma_{22,t-1}^2 \tag{4.36}$$

Consider a DVEC model in a bi-variate example:

$$\begin{bmatrix} \sigma_{11,t}^2 \\ \sigma_{12,t}^2 \\ \sigma_{22,t}^2 \end{bmatrix} = \begin{bmatrix} c_{10} \\ c_{20} \\ c_{30} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 \\ u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} \sigma_{11,t-1}^2 \\ \sigma_{12,t-1}^2 \\ \sigma_{22,t-1}^2 \end{bmatrix} \tag{4.37}$$

Individual variances and covariance expression:

$$\sigma_{11,t}^2 = c_{10} + a_{11}u_{1,t-1}^2 + b_{11}\sigma_{11,t-1}^2 \tag{4.38}$$

$$\sigma^2_{12,t} = c_{20} + a_{22}u_{1,t-1}u_{2,t-1} + b_{22}\sigma^2_{12,t-1} \tag{4.39}$$

$$\sigma^2_{22,t} = c_{30} + a_{33}u^2_{2,t-1} + b_{33}\sigma^2_{22,t-1} \tag{4.40}$$

Considering a two-variable case for BEKK model with lag 1 for parameter matrics:

$$\Sigma_{u,t} = \begin{bmatrix} \sigma^2_{11,t} & \sigma^2_{12,t} \\ \sigma^2_{12,t} & \sigma^2_{22,t} \end{bmatrix} \tag{4.41}$$

$$C^* = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} \tag{4.42}$$

$$A^* = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{12} & \alpha_{22} \end{bmatrix} \tag{4.43}$$

$$B^* = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{12} & \beta_{22} \end{bmatrix} \tag{4.44}$$

Consider a bi-variate BEKK GARCH(1,1) under matrix multiplications:

$$\begin{bmatrix} \sigma^2_{11,t} & \sigma^2_{12,t} \\ \sigma^2_{12,t} & \sigma^2_{22.t} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}^T + \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} u^2_{1,t-1} & u_{1,t-1}u_{2,t-1} \\ u_{1,t-1}u_{2,t-1} & u^2_{2,t-1} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}^T \tag{4.45}$$

$$+ \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} \sigma^2_{11,t-1} & \sigma^2_{12,t-1} \\ \sigma^2_{12,t-1} & \sigma^2_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}^T \tag{4.46}$$

Consider a individual variance expression:

$$\sigma^2_{11,t} = (c^2_{11} + c^2_{12}) + (\alpha^2_{11}u^2_{1,t-1} + 2\alpha_{11}\alpha_{21}u_{1,t-1}u_{2,t-1} + \alpha^2_{21}u^2_{2,t-1}) \tag{4.47}$$

$$+ (b^2_{11}\sigma^2_{11,t-1} + 2b_{11}\beta_{21}\sigma^2_{12,t-1} + b^2_{21}\sigma^2_{22,t-1}) \tag{4.48}$$

In the case of N dimensional CCC GARCH model, the conditional covariances matrix can be written as:

$$\Sigma_t = \begin{pmatrix} \sigma_{1,t} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{N,t} \end{pmatrix} \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & 1 & \cdots & \vdots \\ \vdots & \vdots & \cdots & \rho_{N-1N} \\ \rho_{N1} & \cdots & \rho_{NN-1} & 1 \end{pmatrix} \begin{pmatrix} \sigma_{1,t} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{N,t} \end{pmatrix} \tag{4.49}$$

When $N = 2$,

$$\Sigma_t = \begin{pmatrix} \sigma_{1,t} & 0 \\ 0 & \sigma_{2,t} \end{pmatrix} \begin{pmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{pmatrix} \begin{pmatrix} \sigma_{1,t} & 0 \\ 0 & \sigma_{2,t} \end{pmatrix} = \begin{pmatrix} \sigma^2_{1,t} & \rho_{12}\sigma_{1,t}\sigma_{2,t} \\ \rho_{12}\sigma_{1,t}\sigma_{2,t} & \sigma^2_{2,t} \end{pmatrix} \tag{4.50}$$

Consider a expression of bi-variate DCC_E,

$$\rho_{12,t} = \frac{(1-\alpha-\beta)\bar{q}_{12} + \alpha v_{1,t-1}v_{2,t-1} + \beta q_{12,t-1}}{\sqrt{((1-\alpha-\beta)\bar{q}_{11} + \alpha v^2_{1,t-1} + \beta q_{11,t-1})((1-\alpha-\beta)\bar{q}_{22} + \alpha v^2_{2,t-1} + \beta q_{22,t-1})}} \tag{4.51}$$

Then consider a expression of bi-variate DCC_T,

$$\rho_{12,t} = (1-\theta_1-\theta_2)\rho_{12} + \theta_2\rho_{12,t-1} + \theta_1 \frac{\sum_{m=1}^{M} v_{1,t-m}v_{2,t-m}}{\sqrt{(\sum_{m=1}^{M} v^2_{1,t-m})(\sum_{h=1}^{M} v^2_{2,t-m})}} \tag{4.52}$$

4.6.3 Figures

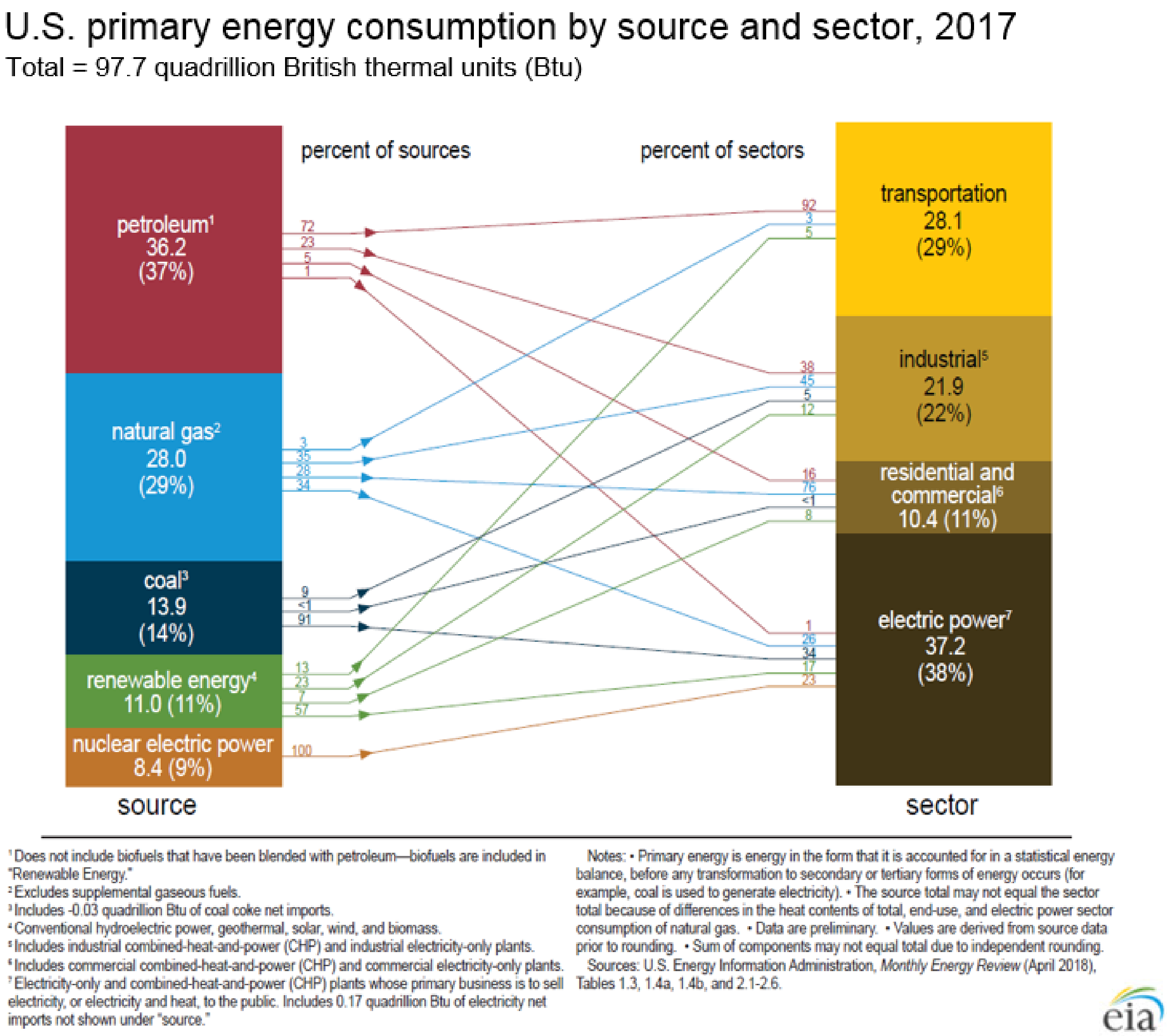

FIGURE 4.1: U.S. Primary Energy Consumption by Source and Sector in 2017

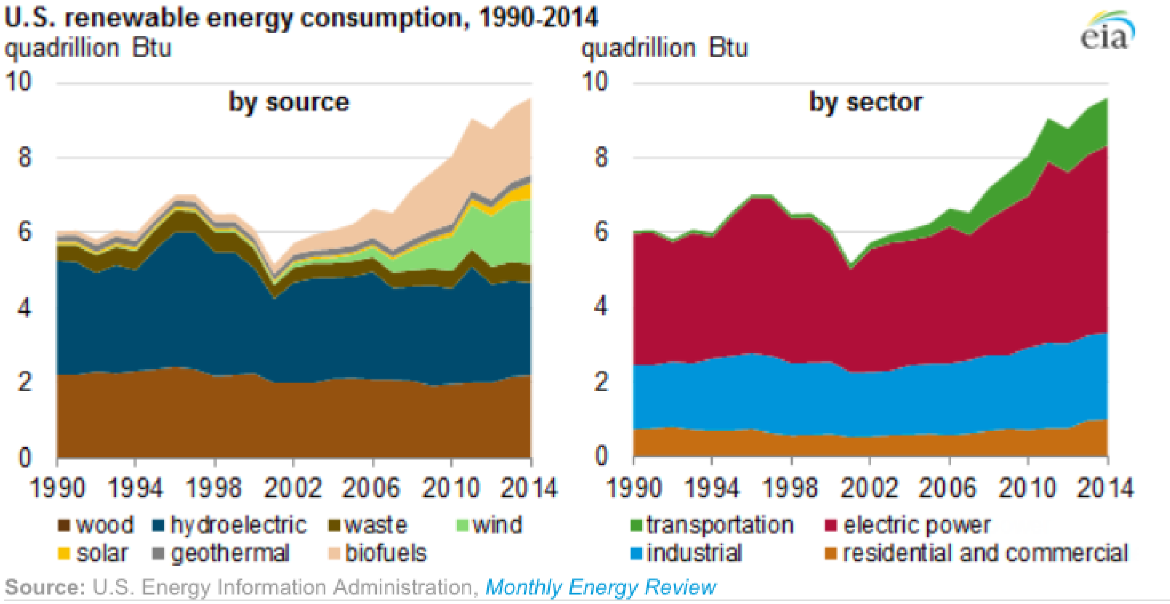

FIGURE 4.2: U.S. Renewable Energy Consumption 1990-2014

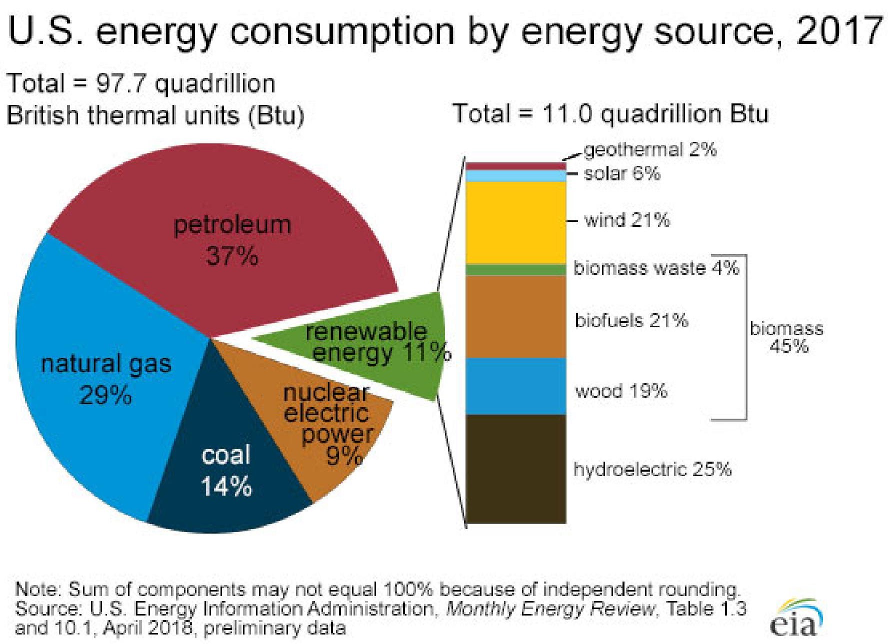

FIGURE 4.3: U.S. Energy Consumption by Energy Source in 2017

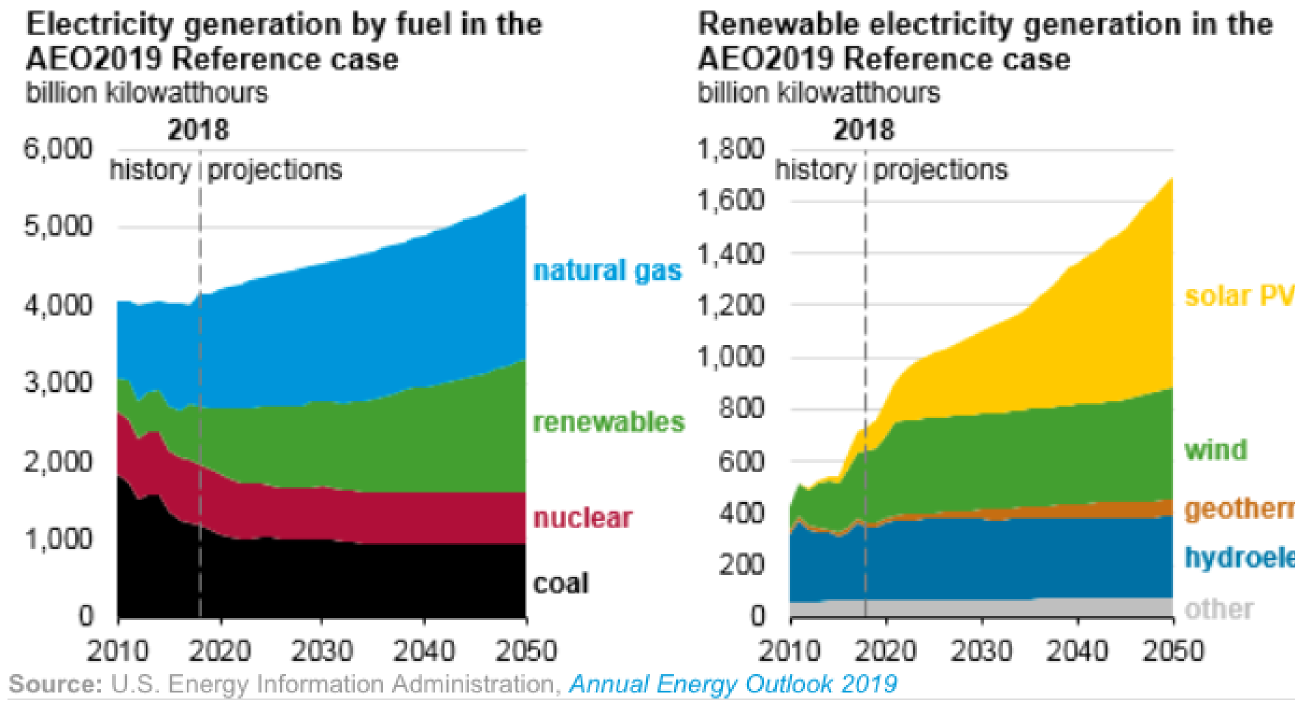

FIGURE 4.4: Electricity Energy Generation History and Projections

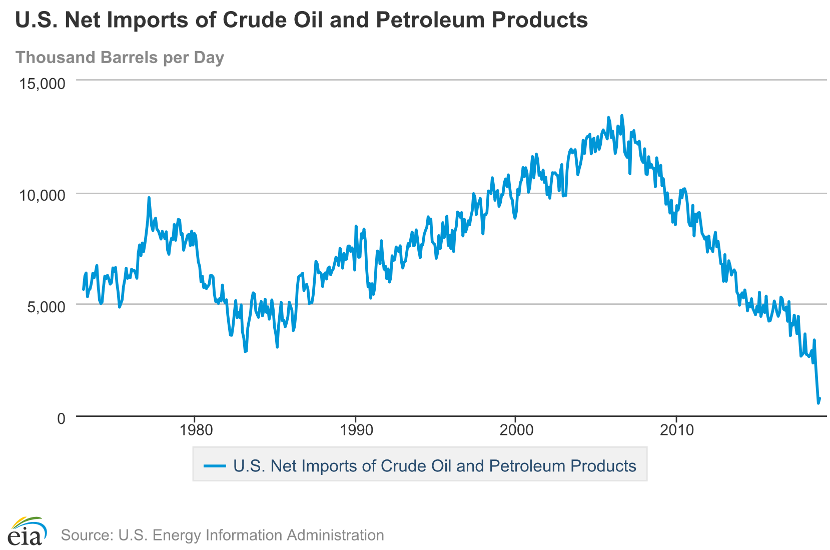

FIGURE 4.5: U.S. Net Imports of Crude oil and Petroleum Products

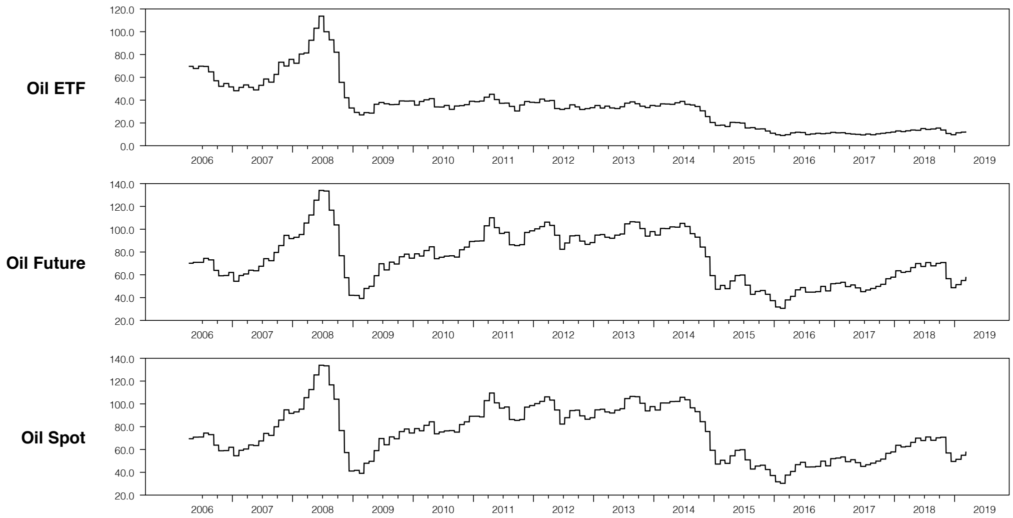

FIGURE 4.6: WTI ETF Price, WTI Future Price, and WTI Spot Price

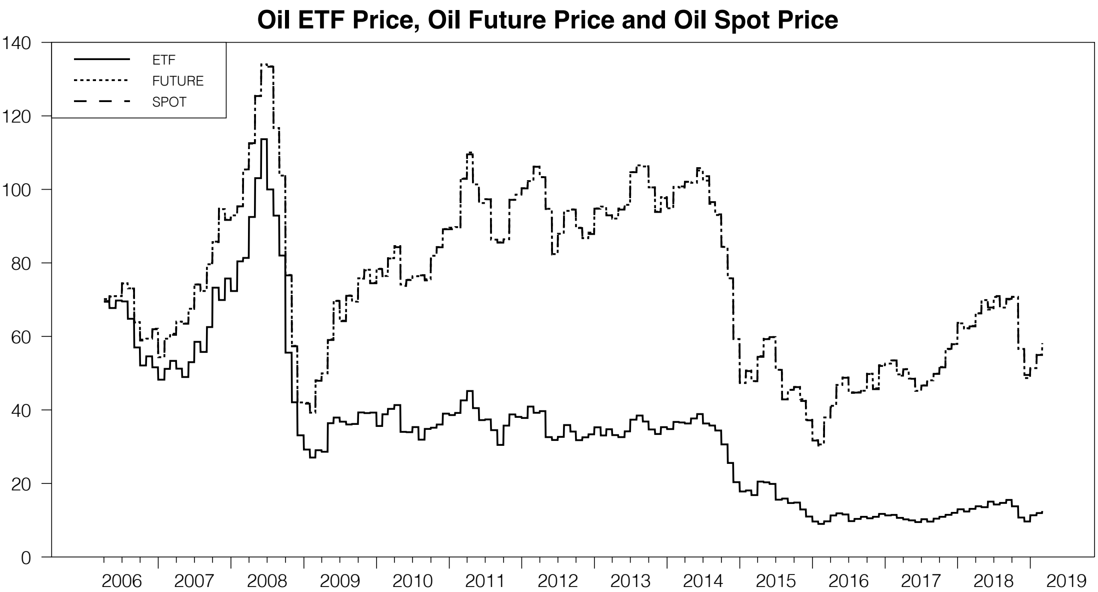

FIGURE 4.7: WTI ETF Price, WTI Future Price, and WTI Spot Price

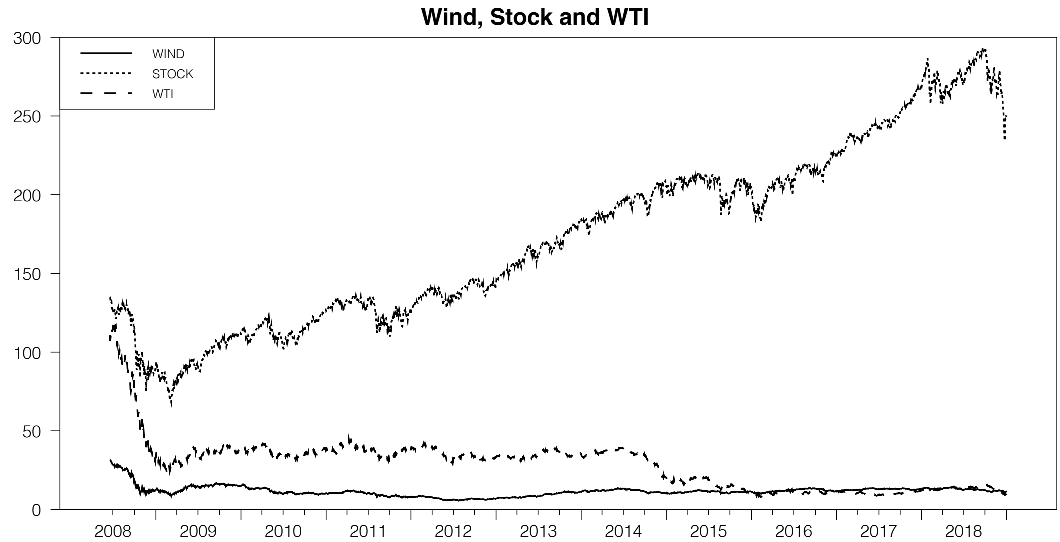

FIGURE 4.8: Wind, Stock and Oil ETF PRICE

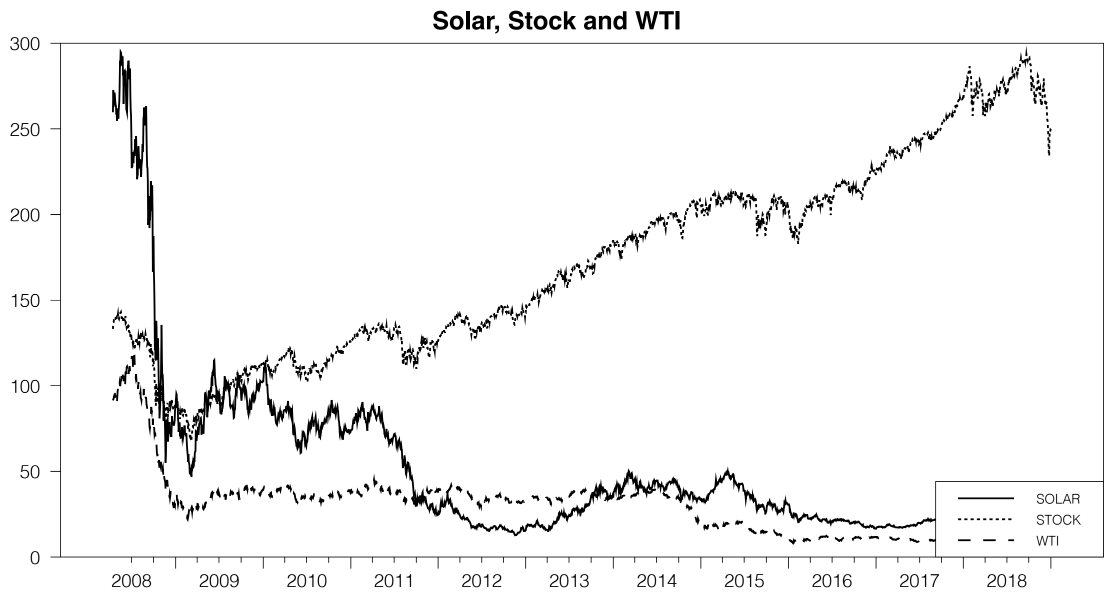

FIGURE 4.9: Solar, Stock and Oil ETF PRICE

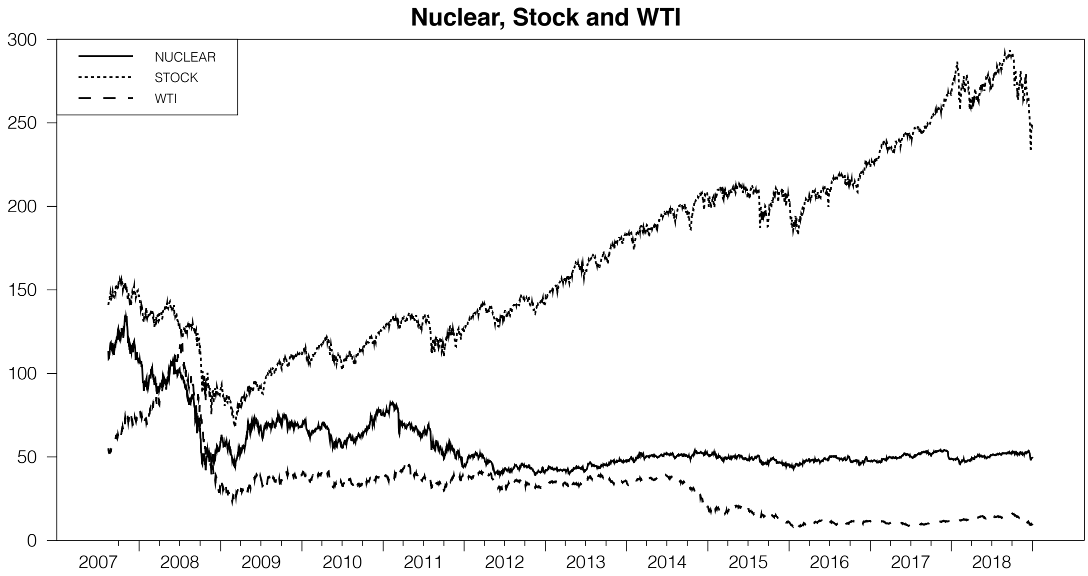

FIGURE 4.10: Nuclear, Stock and Oil ETF PRICE

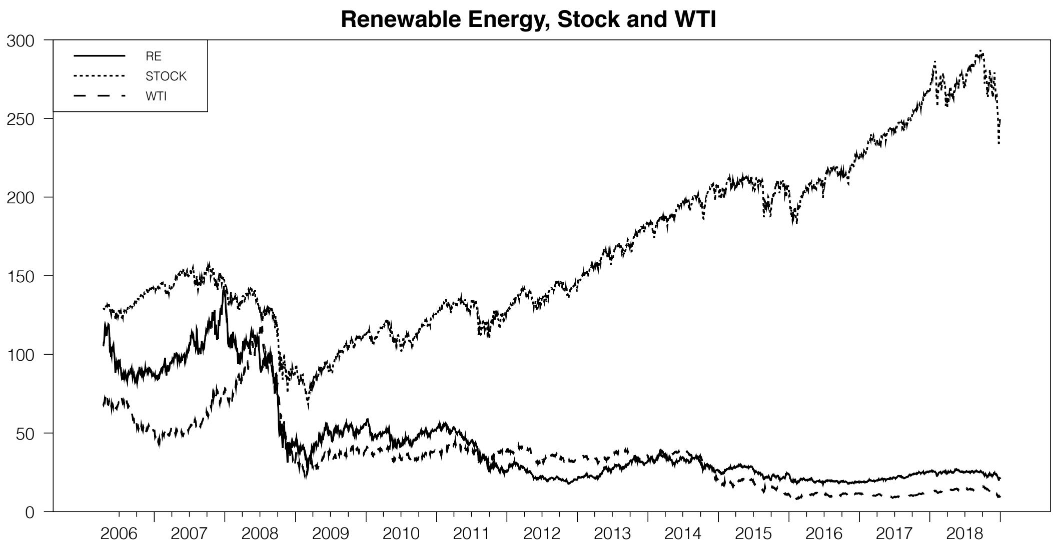

FIGURE 4.11: Renewable Energy, Stock and Oil ETF PRICE

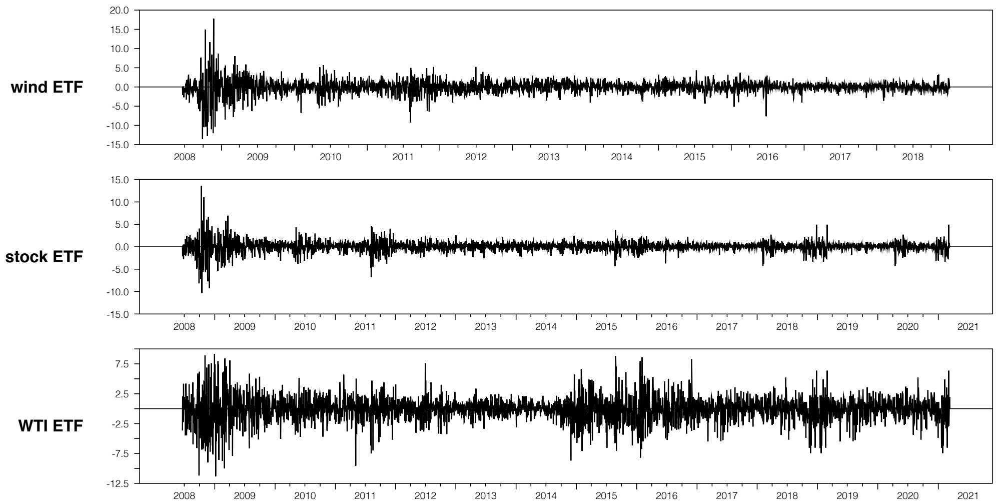

FIGURE 4.12: Returns including Wind ETF

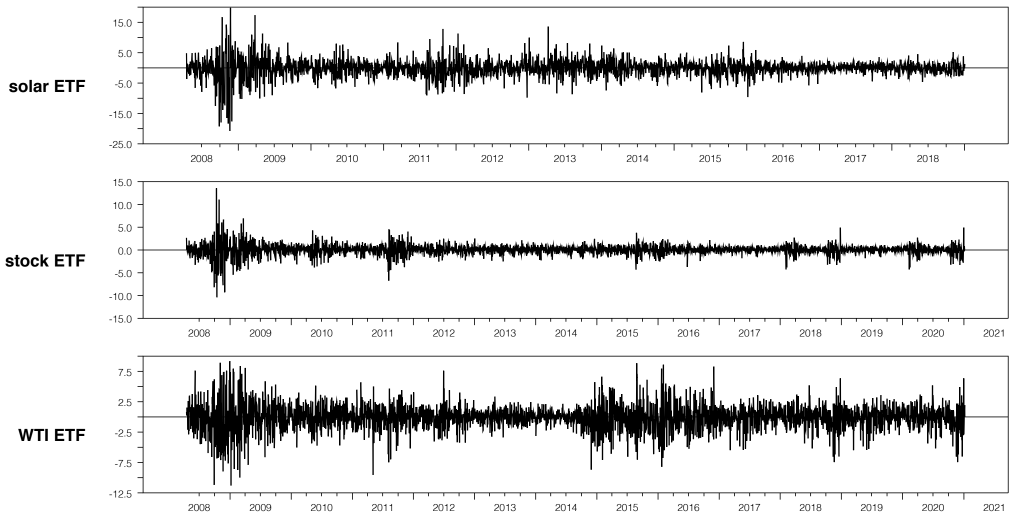

FIGURE 4.13: Returns including Solar ETF

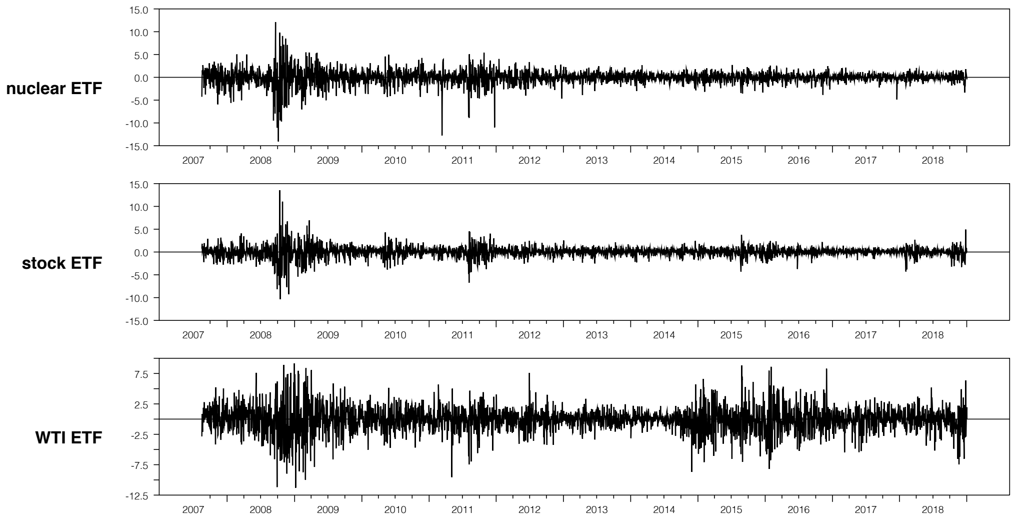

FIGURE 4.14: Returns including Nuclear ETF

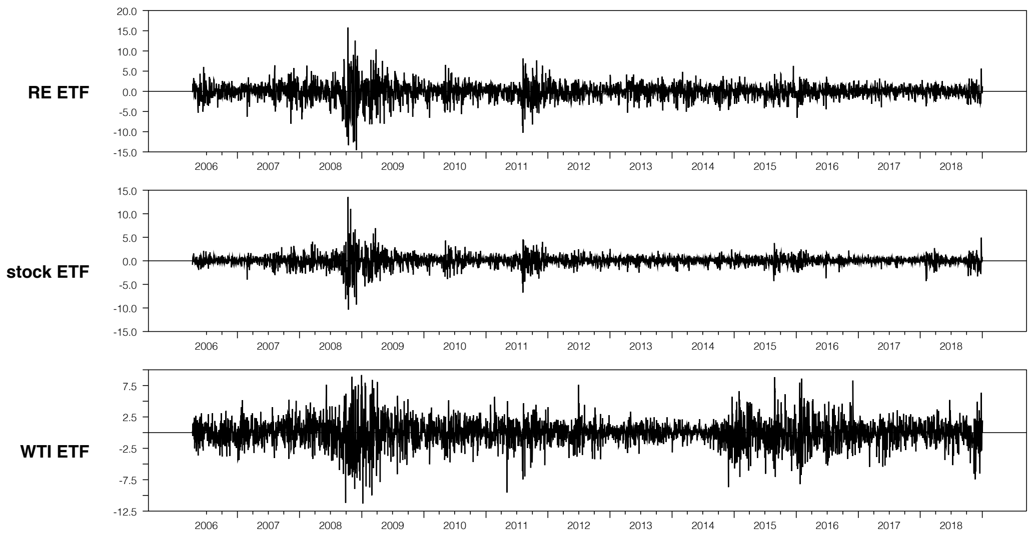

FIGURE 4.15: Returns including RE Benchmark ETF

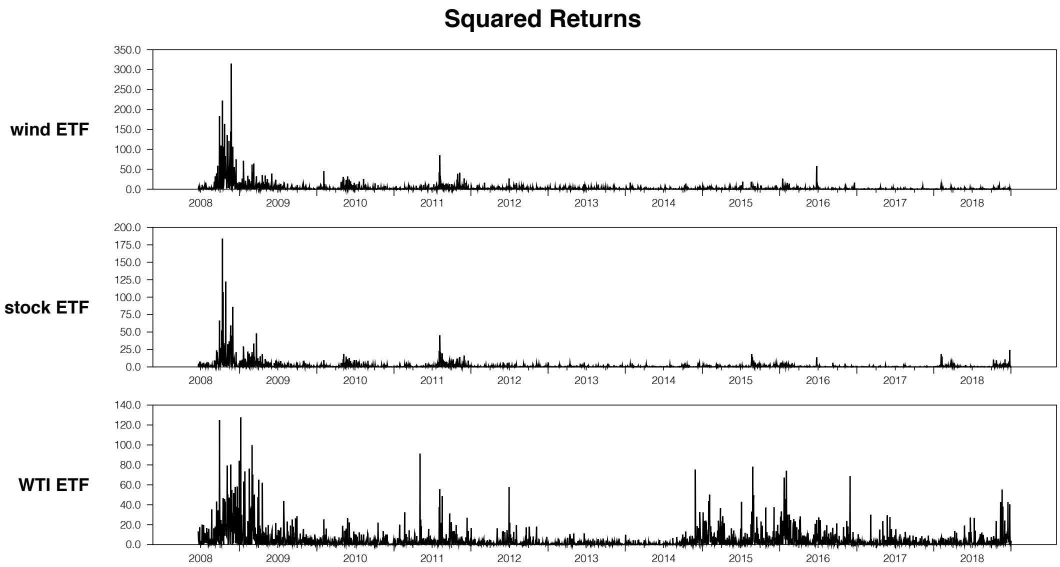

FIGURE 4.16: Returns including Wind ETF

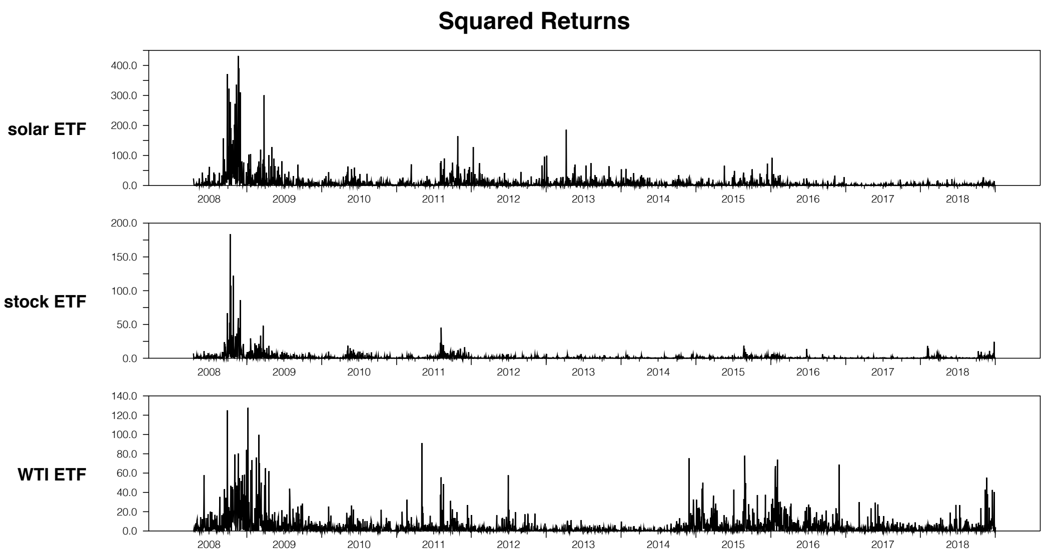

FIGURE 4.17: Returns including Solar ETF

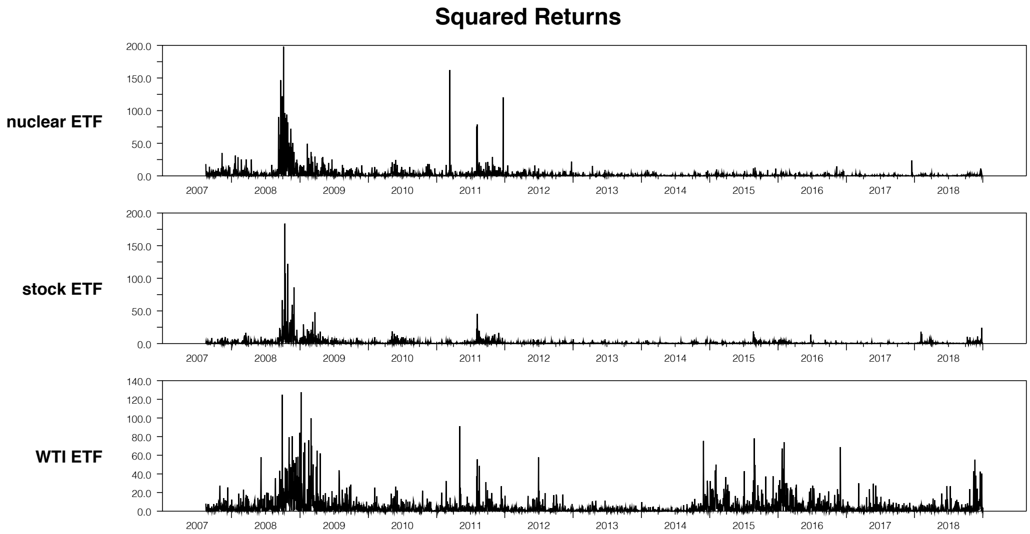

FIGURE 4.18: Returns including Nuclear ETF

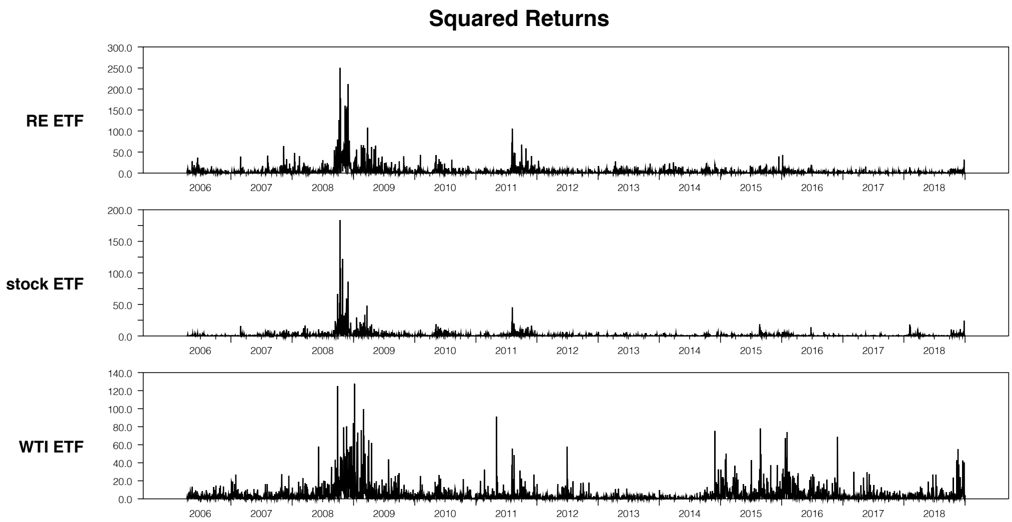

FIGURE 4.19: Returns including RE Benchmark ETF

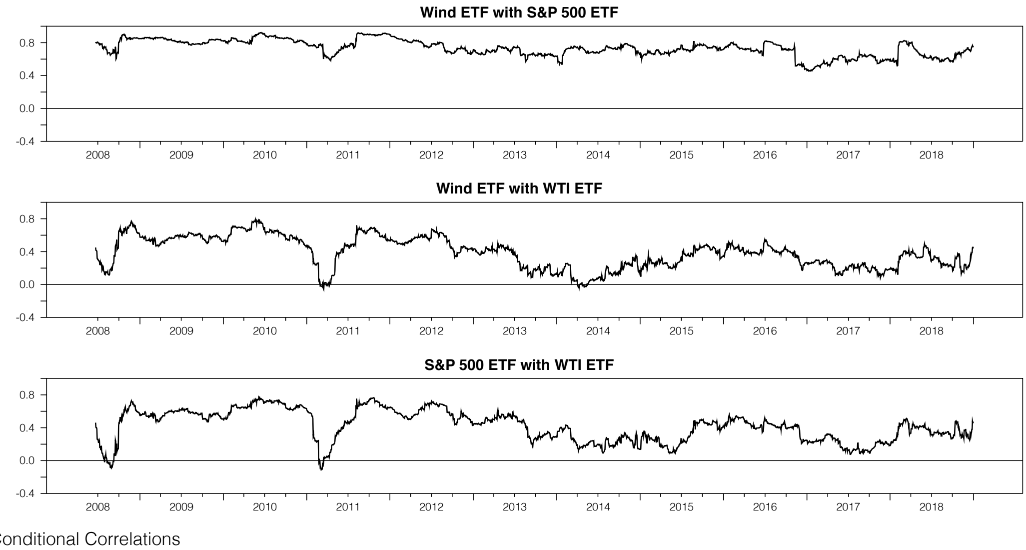

FIGURE 4.20: Dynamic Conditional Correlation-Wind ETF

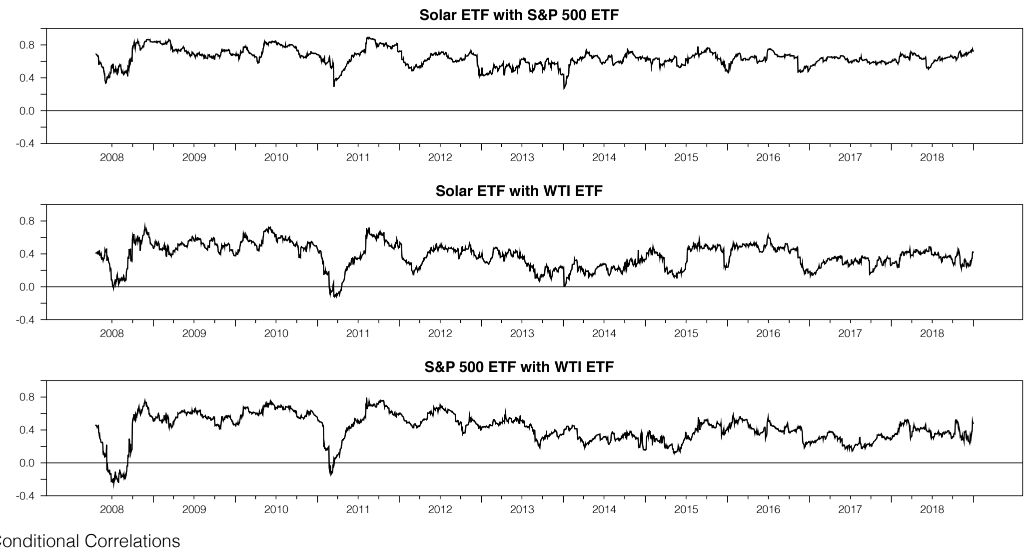

FIGURE 4.21: Dynamic Conditional Correlation-Solar ETF

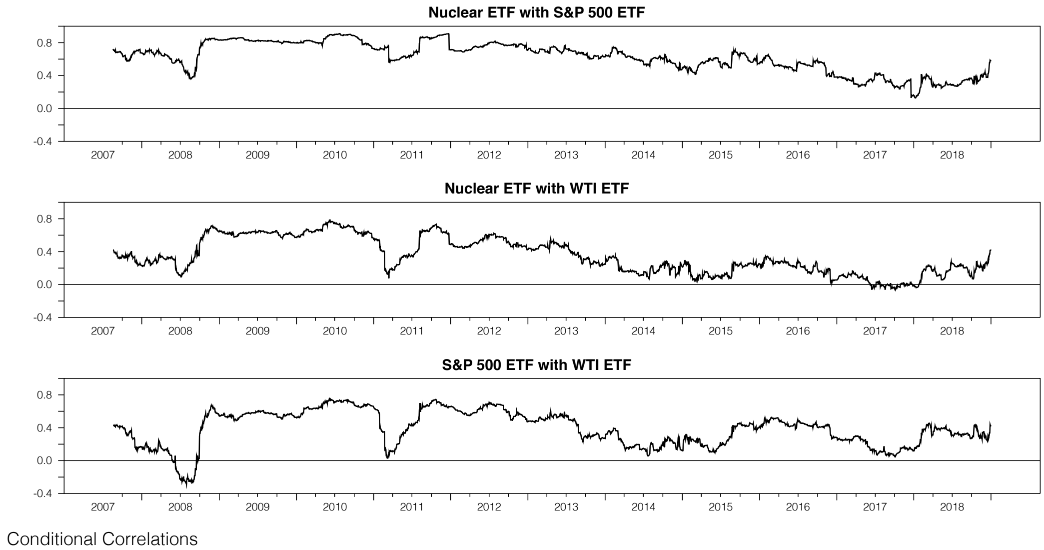

FIGURE 4.22: Dynamic Conditional Correlation-Nuclear ETF

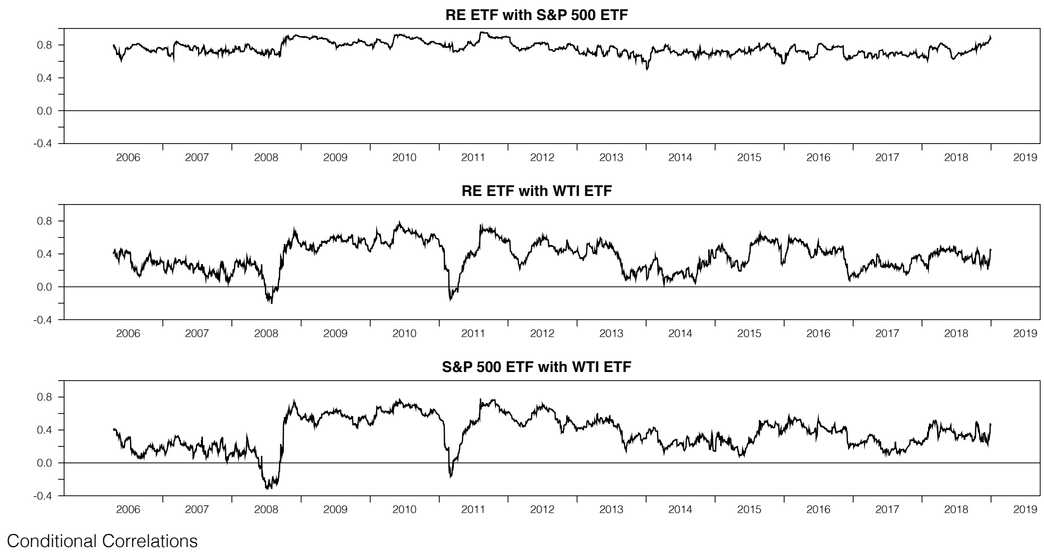

FIGURE 4.23: Dynamic Conditional Correlation-RE ETF

4.6.4 Tables

TABLE 4.4: *ETF Data Information*

Symbol	ETF Name	ETF Categories	Asset Class	Timespan	Observations	Frequency	Market	Currency
USO	United States Oil Fund	Crude Oil	Equity	10/04/2006	02/01/2019	Daily	United States	Dollar
SPY	SPDR S&P 500	S&P 500	Equity	10/04/2006	02/01/2019	Daily	United States	Dollar
FAN	First Trust Global Wind Energy	Wind	Equity	18/06/2008	02/01/2019	Daily	United States	Dollar
TAN	Invesco Solar	Solar	Equity	15/04/2008	02/01/2019	Daily	United States	Dollar
NLR	Vanv Uranium + Nuclear Energy	Nuclear	Equity	15/08/2007	02/01/2019	Daily	United States	Dollar
PBW	Wilderhill Clean Energy	Renewable Energy Benchmark	Equity	10/04/2006	02/01/2019	Daily	United States	Dollar

TABLE 4.5: *Statistics for daily returns*

	WTI	SP 500	Wind	Solar	Nuclear	Benchmark
Mean	-0.0581	0.0197	-0.0367	-0.0939	-0.0280	-0.0476
Median	0.0000	0.0284	0.0000	0.0000	0.0000	0.0000
Maximum	9.1691	13.5577	17.7453	19.7607	12.1101	15.8199
Minimum	-11.2995	-10.3637	-13.5412	-20.7752	-14.0703	-14.5550
Std. dev.	2.0999	1.1972	1.7879	2.8792	1.5715	2.0703
Skewness	-0.1774	-0.1306	-0.3307	-0.3481	-0.8846	-0.3871
Siginif Level(Sk=0)	0.0000	0.0021	0.0000	0.0000	0.0000	0.0000
(Excess)Kurtosis	2.5368	14.7867	12.7429	7.1314	11.0825	5.8442
Siginif Level(Ku=0)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Student's t(Mean=0)	-1.5965	0.9515	-1.0773	-1.7249	-0.9723	-1.3268
Siginif Level(Mean=0)	0.1104	0.3413	0.2814	0.0846	0.3309	0.1846
Jarque-Bera	908.2228	30273.9330	18656.4622	5981.4199	15586.5782	4810.5665
Siginif Level(JB=0)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	3322	3322	2750	2796	2970	3322

TABLE 4.6: *Correlations between daily returns*

	WTI	SP 500	Wind	Solar	Nuclear	Benchmark
WTI	1	0.4117	0.4496	0.4020	0.4222	0.4050
SP 500	0.4117	1	0.7930	0.6815	0.7193	0.7945
Wind	0.4496	0.7930	1	\	\	\
Solar	0.4020	0.6815	\	1	\	\
Nuclear	0.4222	0.7193	\	\	1	\
Benchmark	0.4050	0.7945	\	\	\	1

TABLE 4.7: *Correlations between squared daily returns*

	WTI	SP 500	Wind	Solar	Nuclear	Benchmark
WTI	1	0.3327	0.4080	0.4479	0.4033	0.3665
SP 500	0.3327	1	0.7263	0.6288	0.6218	0.7754
Wind	0.4080	0.7263	1	\	\	\
Solar	0.4479	0.6288	\	1	\	\
Nuclear	0.4033	0.6218	\	\	1	\
Benchmark	0.3665	0.7754	\	\	\	1

TABLE 4.8: *VAR Lag Selection-including Wind ETF Return*

Lags	AIC	SBC/BIC	HQ	Chi-Squared Test
0	10.4008764	10.4073426*	10.4032122	0.0000000
1	10.3868743	10.4127295	10.3962076*	56.4709497
2	10.3822864	10.4275162	10.3986029	30.6682492
3	10.3794263	10.4440162	10.4027116	25.9652536
4	10.3774958	10.4614314	10.4077354	23.4531602
5	10.3765471*	10.4798138	10.4137263	20.7981961*

TABLE 4.9: *VAR Lag Selection-including Solar ETF Return*

Lags	AIC	SBC/BIC	HQ	Chi-Squared Test
0	11.7324308	11.7388083	11.7347325	0.0000000
1	11.7098732	11.7353739*	11.7190709	80.9927335
2	11.7005276*	11.7451376	11.7166074*	44.1569583
3	11.7018771	11.7655825	11.7248250	14.3457819
4	11.7014686	11.7842553	11.7312706	19.2914077
5	11.7017889	11.8036428	11.7384309	17.2968909*

TABLE 4.10: *VAR Lag Selection-Nuclear ETF Return*

Lags	AIC	SBC/BIC	HQ	Chi-Squared Test
0	10.4061485	10.4122129*	10.4083306	0.0000000
1	10.3947310	10.4189807	10.4034512	51.8852416
2	10.3857119*	10.4281345	10.4009578*	44.8107102
3	10.3887269	10.4493101	10.4104862	9.1662183
4	10.3889774	10.4677087	10.4172376	17.3997923
5	10.3890273	10.4858943	10.4237761	18.0313472*

TABLE 4.11: *VAR Lag Selection-RE Benchmark ETF*

Lags	AIC	SBC/BIC	HQ	Chi-Squared Test
0	10.6122576	10.6177801	10.6142334	0.000000
1	10.5832823	10.6053656*	10.5911791	114.140042
2	10.5738702	10.6125046	10.5876781*	49.281534
3	10.5722372*	10.6274126	10.5919462	23.511217
4	10.5747981	10.6465048	10.6003984	9.632681
5	10.5750976	10.6633256	10.6065793	17.166800*

TABLE 4.12: *Multivariate Q Test*

	Wind ETF Return	Solar ETF Return	Nuclear ETF Return	RE Benchmark ETF Return
Test Run Over	2006:04:11 to 2016:10:24	2008:04:16 to 2019:01:02	2007:08:16 to 2019:01:02	2006:04:11 to 2019:01:02
Lags Tested 1, Degrees of Freedom 9				
Q Statistic	29.95587	79.46677	51.87136	112.7274
Signif Level	0.00045	0.00000	0.00000	0.0000
Lags Tested 5, Degrees of Freedom 45				
	Degrees of Freedom 45	Degrees of Freedom 45	Degrees of Freedom 45	Degrees of Freedom 45
Q Statistic	89.33166	173.8159	140.5165	209.8704
Signif Level	0.00009	0.0000	0.0000	0.0000

TABLE 4.13: *Information Criteria*

		Normal Distribution				Student-t Distribution			
		AIC	BIC	HQ	FPE	AIC	BIC	HQ	FPE
BEKK	Wind	9.211	9.308	9.246	9.211	9.062	9.161	9.098	9.062
	Solar	10.573	10.668	10.607	10.573	10.389	10.487	10.425	10.389
	Nuclear	9.297	9.387	9.329	9.297	9.016	9.109	9.049	9.016
	RE	9.603	9.685	9.632	9.603	9.468	9.553	9.498	9.468
CCC	Wind	9.278	9.394	9.320	9.278	9.115	9.214	9.151	9.115
	Solar	10.640	10.755	10.682	10.640	10.435	10.532	10.470	10.435
	Nuclear	9.396	9.486	9.428	9.396	9.130	9.259	9.176	9.130
	RE	9.670	9.753	9.700	9.670	9.528	9.613	9.558	9.528
DCC	Wind	9.196	9.219	9.230	9.196	9.053	9.150	9.089	9.054
	Solar	10.564	10.677	10.605	10.564	10.376	10.490	10.417	10.376
	Nuclear	9.291	9.380	9.323	9.291	8.995	9.086	9.028	8.995
	RE	9.579	9.660	9.608	9.579	9.450	9.533	9.480	9.450

TABLE 4.14: *GARCH-in-mean Coefficients Ψ Estimation Result (Part 1)*

		Wind	Solar	Nuclear	RE
BEKK Normal Distribution	ψ_{11}	-0.0110	-0.0019	0.0094	-0.0217
		(0.6677)	(0.8386)	(0.6918)	(0.3753)
	ψ_{12}	0.0449	0.0548	0.0548	0.0228
		(0.3332)	(0.2485)	(0.4648)	(0.0804)
	ψ_{13}	-0.0025	-0.0126	-0.0051	-0.0057
		(0.7647)	(0.3339)	(0.5243)	(0.6189)
	ψ_{21}	-0.0181	-0.0001	-0.0037	0.0036
		(0.3880)	(0.9808)	(0.8183)	(0.7892)
	ψ_{22}	0.0683	0.0376	0.0322	0.0284
		(0.0659)	(0.1424)	(0.2376)	(0.4120)
	ψ_{23}	-0.0089	-0.0086	-0.0078	-0.0083
		(0.0974)	(0.0976)	(0.2261)	(0.1752)
	ψ_{31}	-0.0251	0.0027	0.0566	0.0244
		(0.4684)	(0.7537)	(0.0848)	(0.2984)
	ψ_{32}	0.0250	-0.0291	**-0.0872**	-0.0660
		(0.6360)	(0.3523)	**(0.0372)**	(0.1775)
	ψ_{33}	0.0073	0.0098	0.0020	0.0073
		(0.6187)	(0.3920)	(0.8986)	(0.6276)
BEKK Student's t-distribution	ψ_{11}	-0.0067	-0.0038	-0.0092	-0.0198
		(0.7952)	(0.7020)	(0.6561)	(0.4254)
	ψ_{12}	0.0328	0.0299	0.0492	0.0833
		(0.4127)	(0.5008)	(0.1269)	(0.1328)
	ψ_{13}	0.0009	-0.0030	-0.0056	-0.0078
		(0.9256)	(0.6574)	(0.1955)	(0.3922)
	ψ_{21}	-0.0162	0.0014	0.0003	0.0022
		(0.3878)	(0.7638)	(0.9863)	(0.8667)
	ψ_{22}	**0.0669**	0.0266	0.0374	0.0267
		(0.0462)	(0.2494)	(0.1234)	(0.4547)
	ψ_{23}	-0.0094	**-0.0069**	**-0.0102**	-0.0085
		(0.1172)	**(0.0289)**	**(0.0074)**	(0.0985)
	ψ_{31}	-0.0087	0.0003	**0.0747**	0.0143
		(0.8044)	(0.9568)	**(0.0242)**	(0.5510)
	ψ_{32}	-0.0084	-0.0311	**-0.0940**	-0.0491
		(0.8660)	(0.2491)	**(0.0331)**	(0.3731)
	ψ_{33}	0.0053	0.0099	-0.0046	0.0052
		(0.7351)	(0.2625)	(0.4291)	(0.5426)

TABLE 4.15: *GARCH-in-mean Coefficients Ψ Estimation Result (Part 2)*

		Wind	Solar	Nuclear	RE
CCC Normal Distribution	ψ_{11}	-0.0121	-0.0125	-0.0250	-0.0197
		(0.7513)	(0.2920)	(0.4565)	(0.5569)
	ψ_{12}	0.0461	0.0872	0.0667	0.0924
		(0.4962)	(0.1411)	(0.2009)	(0.2393)
	ψ_{13}	0.0043	-0.0078	-0.0075	-0.0065
		(0.6862)	(0.6038)	(0.3848)	(0.6027)
	ψ_{21}	-0.0255	-0.0033	-0.0103	0.0124
		(0.3365)	(0.4864)	(0.5998)	(0.4952)
	ψ_{22}	0.0925	**0.0572**	0.0616	0.0122
		(0.0762)	**(0.0286)**	(0.0863)	(0.7908)
	ψ_{23}	-0.0064	-0.0089	-0.0108	-0.0067
		(0.3626)	(0.1761)	(0.1263)	(0.3493)
	ψ_{31}	-0.0333	0.0046	0.0720	0.0349
		(0.4910)	(0.5288)	(0.0840)	(0.1052)
	ψ_{32}	0.0159	-0.0500	**-0.1443**	**-0.1024**
		(0.8557)	(0.1811)	**(0.0364)**	**(0.0343)**
	ψ_{33}	0.0224	0.0178	0.0178	0.0156
		(0.1937)	(0.2148)	(0.2854)	(0.3020)
CCC Student's t-distribution	ψ_{11}	-0.0138	-0.0111	-0.0054	-0.0050
		(0.6211)	(0.4021)	(0.8480)	(0.8802)
	ψ_{12}	0.0428	0.0649	0.0451	0.0587
		(0.3862)	(0.2742)	(0.2961)	(0.4452)
	ψ_{13}	0.0013	-0.0038	-0.0053	-0.0063
		(0.8418)	(0.7958)	(0.4372)	(0.5521)
	ψ_{21}	-0.0225	-0.0010	-0.0043	0.0162
		(0.2811)	(0.8401)	(0.8159)	(0.3481)
	ψ_{22}	**0.0893**	0.05125	0.0525	0.0006
		(0.0177)	(0.0701)	(0.0664)	(0.9887)
	ψ_{23}	**-0.0091**	-0.0082	**-0.0115**	-0.0059
		(0.0179)	(0.1723)	**(0.0424)**	(0.3300)
	ψ_{31}	-0.0257	0.0009	**0.0788**	0.0291
		(0.5218)	(0.8988)	**(0.0220)**	(0.3495)
	ψ_{32}	0.0022	-0.0362	**-0.1308**	-0.0837
		(0.9739)	(0.3091)	**(0.0217)**	(0.2464)
	ψ_{33}	0.0147	0.0136	0.0070	0.0099
		(0.2649)	(0.3130)	(0.5670)	(0.5188)

TABLE 4.16: *GARCH-in-mean Coefficients Ψ Estimation Result (Part 3)*

		Wind	Solar	Nuclear	RE
DCC Normal Distribution	ψ_{11}	-0.0008	-0.0081	-0.0134	-0.0194
		(0.9816)	(0.4424)	(0.6548)	(0.5579)
	ψ_{12}	0.0268	0.0756	0.0568	0.0926
		(0.6649)	(0.1696)	(0.2122)	(0.2425)
	ψ_{13}	0.0028	-0.0091	-0.0057	-0.0061
		(0.7717)	(0.5543)	(0.4867)	(0.6323)
	ψ_{21}	-0.0139	-0.0022	-0.0114	0.0067
		(0.5646)	(0.6352)	(0.5793)	(0.6984)
	ψ_{22}	0.0673	0.0451	0.0518	0.0242
		(0.1535)	(0.0989)	(0.1647)	(0.5970)
	ψ_{23}	-0.0089	-0.0091	-0.0107	-0.0085
		(0.1714)	(0.1714)	(0.1121)	(0.2345)
	ψ_{31}	-0.0256	0.0043	0.0413	0.0342
		(0.3748)	(0.6030)	(0.3009)	(0.2622)
	ψ_{32}	0.0224	-0.0398	-0.0797	-0.0959
		(0.6724)	(0.3259)	(0.1802)	(0.1849)
	ψ_{33}	0.0144	0.0134	0.0078	0.0130
		(0.3511)	(0.3281)	(0.6220)	(0.4188)
DCC Student's t-distribution	ψ_{11}	-0.0082	-0.0072	-0.0159	-0.0130
		(0.7523)	(0.1165)	(0.6091)	(0.6670)
	ψ_{12}	0.0361	0.0530	0.0574	0.0691
		(0.4386)	(0.0705)	(0.1358)	(0.3489)
	ψ_{13}	0.0016	-0.0041	-0.0048	-0.0056
		(0.8559)	(0.5416)	(0.4765)	(0.6162)
	ψ_{21}	-0.0172	-0.0001	0.0044	0.0074
		(0.3608)	(0.9598)	(0.8485)	(0.6422)
	ψ_{22}	**0.0742**	**0.0410**	0.0459	0.0165
		(0.0400)	**(0.0078)**	(0.1614)	(0.6997)
	ψ_{23}	-0.0109	**-0.0089**	**-0.0130**	-0.0078
		(0.0594)	**(0.0017)**	**(0.0214)**	(0.2092)
	ψ_{31}	-0.0114	0.0024	**0.0688**	0.0228
		(0.7470)	(0.7401)	**(0.0410)**	(0.4176)
	ψ_{32}	-0.0044	-0.0357	-0.0826	-0.0713
		(0.9386)	(0.2146)	(0.0694)	(0.2981)
	ψ_{33}	0.0094	0.0113	-0.0030	0.0089
		(0.5308)	(0.2000)	(0.8292)	(0.5444)

TABLE 4.17: *BEKK GARCH-in-mean Parameter Estimates with Normal Distribution [Wind Energy]*

Wind Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Wind (1) γ_{11}^1	0.0032	0.8663	c_{11}	0.1046	0.0042
Stock (1) γ_{12}^1	-0.0462	0.0762	c_{21}	0.0964	0.0004
WTI (1) γ_{13}^1	0.0005	0.9673	c_{22}	0.1169	0.0000
Constant c_1	0.0328	0.4049	c_{31}	-0.0065	0.8907
$\sigma_{11}^2\psi_{11}$	-0.0111	0.6677	c_{32}	0.0885	0.0299
$\sigma_{22}^2\psi_{12}$	0.0450	0.3332	c_{33}	0.0813	0.0868
$\sigma_{33}^2\psi_{13}$	-0.0025	0.7647	a_{11}	0.2149	0.0000
Stock Mean			a_{12}	0.0382	0.0455
Wind (1) γ_{21}^1	0.0154	0.1706	a_{13}	-0.0027	0.9146
Stock (1) γ_{22}^1	-0.0573	0.0019	a_{21}	0.1013	0.0065
WTI (1) γ_{23}^1	-0.0038	0.6501	a_{22}	0.3181	0.0000
Constant c_2	0.0911	0.0002	a_{23}	0.1119	0.0141
$\sigma_{11}^2\psi_{21}$	-0.0182	0.3880	a_{31}	0.0037	0.7419
$\sigma_{22}^2\psi_{22}$	0.0684	0.0660	a_{32}	-0.0016	0.8682
$\sigma_{33}^2\psi_{23}$	-0.0090	0.0974	a_{33}	0.1895	0.0000
Oil Mean			b_{11}	0.9904	0.0000
Wind (1) γ_{31}^1	-0.0461	0.0942	b_{12}	0.0148	0.0478
Stock (1) γ_{32}^1	0.0666	0.1123	b_{13}	0.0131	0.1654
WTI (1) γ_{33}^1	-0.0453	0.0240	b_{21}	-0.0602	0.0002
Constant c_3	0.0353	0.4591	b_{22}	0.9061	0.0000
$\sigma_{11}^2\psi_{31}$	-0.0251	0.4685	b_{23}	-0.0503	0.0208
$\sigma_{22}^2\psi_{32}$	0.0250	0.6361	b_{31}	0.0027	0.3117
$\sigma_{33}^2\psi_{33}$	0.0073	0.6188	b_{32}	0.0020	0.4067
			b_{33}	0.9818	0.0000

TABLE 4.18: *BEKK GARCH-in-mean Parameter Estimates with t-distribution [Wind Energy]*

Wind Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Wind (1) γ_{11}^1	0.0191	0.4270	c_{11}	0.0278	0.3680
Stock (1)γ_{12}^1	-0.0616	0.0727	c_{21}	0.0934	0.2402
WTI (1) γ_{13}^1	0.0001	0.9957	c_{22}	0.0880	0.2682
Constant c_1	0.0324	0.3733	c_{31}	-0.0417	0.6924
$\sigma_{11}^2\psi_{11}$	-0.0067	0.7952	c_{32}	0.1110	0.0257
$\sigma_{22}^2\psi_{12}$	0.0328	0.4128	c_{33}	-0.0001	0.9999
$\sigma_{33}^2\psi_{13}$	0.0009	0.9257	a_{11}	0.1630	0.0000
Stock Mean			a_{12}	0.0081	0.6780
Wind (1) γ_{21}^1	0.0290	0.0544	a_{13}	-0.0215	0.3897
Stock (1) γ_{22}^1	-0.0798	0.0011	a_{21}	0.1220	0.0000
WTI (1) γ_{23}^1	-0.0038	0.6208	a_{22}	0.3278	0.0000
Constant c_2	0.1097	0.0000	a_{23}	0.0878	0.0557
$\sigma_{11}^2\psi_{21}$	-0.0162	0.3879	a_{31}	0.0009	0.9298
$\sigma_{22}^2\psi_{22}$	0.0669	0.0462	a_{32}	-0.0060	0.5480
$\sigma_{33}^2\psi_{23}$	-0.0094	0.1172	a_{33}	0.1902	0.0000
Oil Mean			b_{11}	0.9992	0.0000
Wind (1) γ_{31}^1	-0.0353	0.2585	b_{12}	0.0189	0.0001
Stock (1) γ_{32}^1	0.0002	0.9961	b_{13}	0.0105	0.1289
WTI (1) γ_{33}^1	-0.0307	0.1142	b_{21}	-0.0538	0.0000
Constant c_3	0.0545	0.2348	b_{22}	0.9143	0.0000
$\sigma_{11}^2\psi_{31}$	-0.0087	0.8045	b_{23}	-0.0284	0.1185
$\sigma_{22}^2\psi_{32}$	-0.0084	0.8661	b_{31}	0.0014	0.5338
$\sigma_{33}^2\psi_{33}$	0.0053	0.7351	b_{32}	0.0020	0.4041
			b_{33}	0.9812	0.0000
			Shape ν (t degrees)	6.7653	0.0000

TABLE 4.19: *BEKK GARCH-in-mean Parameter Estimates with Normal Distribution [Solar Energy]*

Solar Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Solar (1) γ_{11}^1	0.0740	0.0000	c_{11}	0.1673	0.0000
Stock (1) γ_{12}^1	-0.0482	0.1254	c_{21}	0.0993	0.0000
WTI (1) γ_{13}^1	0.0018	0.9244	c_{22}	0.1154	0.0000
Constant c_1	0.0308	0.6306	c_{31}	0.0455	0.0121
$\sigma_{11}^2\psi_{11}$	-0.0019	0.8387	c_{32}	0.0528	0.0517
$\sigma_{22}^2\psi_{12}$	0.0548	0.2485	c_{33}	0.1111	0.0000
$\sigma_{33}^2\psi_{13}$	-0.0126	0.3339	a_{11}	0.1730	0.0000
Stock Mean			a_{12}	0.0070	0.3671
Solar (1) γ_{21}^1	0.0000	0.9960	a_{13}	0.0113	0.2526
Stock (1) γ_{22}^1	-0.0448	0.0065	a_{21}	0.1848	0.0000
WTI (1) γ_{23}^1	-0.0048	0.5031	a_{22}	0.3538	0.0000
Constant c_2	0.0815	0.0025	a_{23}	0.1044	0.0011
$\sigma_{11}^2\psi_{21}$	-0.0001	0.9808	a_{31}	0.0004	0.9752
$\sigma_{22}^2\psi_{22}$	0.0376	0.1424	a_{32}	0.0048	0.5734
$\sigma_{33}^2\psi_{23}$	-0.0086	0.0976	a_{33}	0.1987	0.0000
Oil Mean			b_{11}	0.9856	0.0000
Solar (1) γ_{31}^1	-0.0270	0.0165	b_{12}	0.0033	0.0813
Stock (1) γ_{32}^1	0.0548	0.0183	b_{13}	0.0009	0.7179
WTI (1) γ_{33}^1	-0.0445	0.0121	b_{21}	-0.0544	0.0001
Constant c_3	-0.0055	0.9151	b_{22}	0.9161	0.0000
$\sigma_{11}^2\psi_{31}$	0.0027	0.7538	b_{23}	-0.0416	0.0007
$\sigma_{22}^2\psi_{32}$	-0.0291	0.3524	b_{31}	0.0016	0.6221
$\sigma_{33}^2\psi_{33}$	0.0098	0.3927	b_{32}	0.0006	0.7738
			b_{33}	0.9798	0.0000

TABLE 4.20: *BEKK GARCH-in-mean Parameter Estimates with t-distribution [Solar Energy] (Table 17)*

Solar Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Solar (1) γ_{11}^1	0.0457	0.0167	c_{11}	0.1535	0.0000
Stock (1) γ_{12}^1	-0.0120	0.8118	c_{21}	0.0936	0.0000
WTI (1) γ_{13}^1	-0.0098	0.5874	c_{22}	0.0913	0.0000
Constant c_1	0.0441	0.3473	c_{31}	0.0244	0.5457
$\sigma_{11}^2\psi_{11}$	-0.0038	0.7021	c_{32}	0.0349	0.3168
$\sigma_{22}^2\psi_{12}$	0.0299	0.5008	c_{33}	-0.1180	0.0000
$\sigma_{33}^2\psi_{13}$	-0.0030	0.6575	a_{11}	0.1495	0.0000
Stock Mean			a_{12}	-0.0022	0.8197
Solar (1) γ_{21}^1	-0.0027	0.6989	a_{13}	-0.0048	0.6717
Stock (1) γ_{22}^1	-0.0362	0.0666	a_{21}	0.2262	0.0000
WTI (1) γ_{23}^1	-0.0045	0.5266	a_{22}	0.3542	0.0000
Constant c_2	0.0891	0.0000	a_{23}	0.0920	0.0154
$\sigma_{11}^2\psi_{21}$	0.0014	0.7639	a_{31}	0.0026	0.8775
$\sigma_{22}^2\psi_{22}$	0.0266	0.2494	a_{32}	0.0029	0.7731
$\sigma_{33}^2\psi_{23}$	-0.0069	0.0290	a_{33}	0.2078	0.0000
Oil Mean			b_{11}	0.9912	0.0000
Solar (1) γ_{31}^1	-0.0242	0.0776	b_{12}	0.0044	0.0449
Stock (1) γ_{32}^1	0.0078	0.8468	b_{13}	0.0026	0.2827
WTI (1) γ_{33}^1	-0.0312	0.0688	b_{21}	-0.0675	0.0000
Constant c_3	0.0307	0.4091	b_{22}	0.9249	0.0000
$\sigma_{11}^2\psi_{31}$	0.0003	0.9569	b_{23}	-0.0267	0.0450
$\sigma_{22}^2\psi_{32}$	-0.0311	0.2491	b_{31}	0.0018	0.6193
$\sigma_{33}^2\psi_{33}$	0.0099	0.2625	b_{32}	0.0010	0.6716
			b_{33}	0.9787	0.0000
			Shape ν (t degrees)	6.0516	0.0000

TABLE 4.21: *BEKK GARCH-in-mean Parameter Estimates with Normal Distribution [Nuclear Energy] (Table 18)*

Nuclear Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Nuclear (1) γ^1_{11}	-0.0211	0.3379	c_{11}	0.0361	0.0829
Stock (1) γ^1_{12}	0.0572	0.0391	c_{21}	-0.0087	0.8783
WTI (1) γ^1_{13}	-0.0138	0.1968	c_{22}	0.1390	0.0000
Constant c_1	0.0041	0.8974	c_{31}	-0.1057	0.0000
$\sigma^2_{11}\psi_{11}$	0.0094	0.6918	c_{32}	0.0257	0.6081
$\sigma^2_{22}\psi_{12}$	0.0228	0.4648	c_{33}	0.0000	1.0000
$\sigma^2_{33}\psi_{13}$	-0.0051	0.5243	a_{11}	0.1389	0.0000
Stock Mean			a_{12}	-0.0183	0.1620
Nuclear (1) γ^1_{21}	-0.0073	0.6314	a_{13}	-0.0020	0.9152
Stock (1) γ^1_{22}	-0.0305	0.1822	a_{21}	0.1118	0.0000
WTI (1) γ^1_{23}	-0.0138	0.0812	a_{22}	0.3497	0.0000
Constant c_2	0.0811	0.0009	a_{23}	0.0718	0.0255
$\sigma^2_{11}\psi_{21}$	-0.0037	0.8183	a_{31}	-0.0025	0.7576
$\sigma^2_{22}\psi_{22}$	0.0322	0.2376	a_{32}	-0.0031	0.7269
$\sigma^2_{33}\psi_{23}$	-0.0078	0.2261	a_{33}	0.1873	0.0000
Oil Mean			b_{11}	0.9938	0.0000
Nuclear (1) γ^1_{31}	0.0113	0.7099	b_{12}	0.0174	0.0000
Stock (1) γ^1_{32}	0.0151	0.7169	b_{13}	0.0119	0.0218
WTI (1) γ^1_{33}	-0.0551	0.0039	b_{21}	-0.0308	0.0000
Constant c_3	0.0023	0.9614	b_{22}	0.9181	0.0000
$\sigma^2_{11}\psi_{31}$	0.0566	0.0848	b_{23}	-0.0357	0.0019
$\sigma^2_{22}\psi_{32}$	-0.0872	0.0372	b_{31}	0.0015	0.3851
$\sigma^2_{33}\psi_{33}$	0.0020	0.8986	b_{32}	0.0035	0.1293
			b_{33}	0.9823	0.0000

TABLE 4.22: *BEKK GARCH-in-mean Parameter Estimates with t-distribution [Nuclear Energy] (Table 19)*

Nuclear Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Nuclear (1) γ_{11}^1	-0.0332	0.0595	c_{11}	0.0421	0.0201
Stock (1) γ_{12}^1	0.0514	0.0257	c_{21}	-0.0530	0.1749
WTI (1) γ_{13}^1	-0.0003	0.9657	c_{22}	0.0938	0.0002
Constant c_1	0.0384	0.1125	c_{31}	-0.1164	0.0000
$\sigma_{11}^2\psi_{11}$	-0.0092	0.6562	c_{32}	-0.0246	0.6357
$\sigma_{22}^2\psi_{12}$	0.0492	0.1269	c_{33}	0.0000	1.0000
$\sigma_{33}^2\psi_{13}$	-0.0056	0.1955	a_{11}	0.1406	0.0000
Stock Mean			a_{12}	-0.0184	0.1704
Nuclear (1) γ_{21}^1	-0.0017	0.8959	a_{13}	-0.0147	0.5439
Stock (1) γ_{22}^1	-0.0331	0.0978	a_{21}	0.0634	0.0013
WTI (1) γ_{23}^1	-0.0101	0.0974	a_{22}	0.2915	0.0000
Constant c_2	0.0946	0.0000	a_{23}	0.0685	0.0772
$\sigma_{11}^2\psi_{21}$	0.0003	0.9863	a_{31}	-0.0006	0.9355
$\sigma_{22}^2\psi_{22}$	0.0374	0.1234	a_{32}	-0.0049	0.5570
$\sigma_{33}^2\psi_{23}$	-0.0102	0.0074	a_{33}	0.1835	0.0000
Oil Mean			b_{11}	0.9916	0.0000
Nuclear (1)γ_{31}^1	-0.0042	0.8647	b_{12}	0.0160	0.0001
Stock (1)γ_{32}^1	-0.0071	0.8435	b_{13}	0.0135	0.0412
WTI (1) γ_{33}^1	-0.0512	0.0021	b_{21}	-0.0163	0.0027
Constant c_3	0.0244	0.5431	b_{22}	0.9427	0.0000
$\sigma_{11}^2\psi_{31}$	0.0747	0.0242	b_{23}	-0.0279	0.0222
$\sigma_{22}^2\psi_{32}$	-0.0940	0.0331	b_{31}	0.0007	0.6376
$\sigma_{33}^2\psi_{33}$	-0.0046	0.4292	b_{32}	0.0017	0.3608
			b_{33}	0.9829	0.0000
			Shape ν (t degrees)	5.7590	0.0000

TABLE 4.23: *BEKK GARCH-in-mean Parameter Estimates with Normal Distribution [RE Energy] (Table 20)*

RE Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
RE (1) γ_{11}^1	0.0170	0.3942	c_{11}	0.1760	0.0000
Stock (1) γ_{12}^1	0.0849	0.0085	c_{21}	0.1077	0.0000
WTI (1) γ_{13}^1	-0.0119	0.4092	c_{22}	0.0977	0.0000
Constant c_1	0.0303	0.5118	c_{31}	0.0452	0.0576
$\sigma_{11}^2\psi_{11}$	-0.0217	0.3754	c_{32}	0.0575	0.0609
$\sigma_{22}^2\psi_{12}$	0.0934	0.0804	c_{33}	0.1124	0.0000
$\sigma_{33}^2\psi_{13}$	-0.0057	0.6189	a_{11}	0.1734	0.0000
Stock Mean			a_{12}	-0.0008	0.9475
RE (1) γ_{21}^1	-0.0185	0.0568	a_{13}	-0.0006	0.9746
Stock (1) γ_{22}^1	-0.0111	0.5595	a_{21}	0.1552	0.0002
WTI (1) γ_{23}^1	-0.0104	0.1628	a_{22}	0.3435	0.0000
Constant c_2	0.0695	0.0072	a_{23}	0.1202	0.0026
$\sigma_{11}^2\psi_{21}$	0.0036	0.7893	a_{31}	0.0034	0.7922
$\sigma_{22}^2\psi_{22}$	0.0284	0.4120	a_{32}	0.0013	0.8780
$\sigma_{33}^2\psi_{23}$	-0.0084	0.1753	a_{33}	0.1937	0.0000
Oil Mean			b_{11}	0.9860	0.0000
RE (1) γ_{31}^1	-0.0218	0.2910	b_{12}	0.0077	0.0639
Stock (1) γ_{32}^1	0.0575	0.1458	b_{13}	0.0078	0.1444
WTI (1) γ_{33}^1	-0.0539	0.0026	b_{21}	-0.0494	0.0010
Constant c_3	-0.0324	0.5445	b_{22}	0.9202	0.0000
$\sigma_{11}^2\psi_{31}$	0.0244	0.2984	b_{23}	-0.0498	0.0009
$\sigma_{22}^2\psi_{32}$	-0.0660	0.1775	b_{31}	0.0004	0.8992
$\sigma_{33}^2\psi_{33}$	0.0073	0.6276	b_{32}	0.0004	0.8462
			b_{33}	0.9799	0.0000

TABLE 4.24: BEKK GARCH-in-mean Parameter Estimates with t-distribution [RE Energy]

RE Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
RE (1) γ^1_{11}	0.0134	0.3764	c_{11}	0.1576	0.0000
Stock (1) γ^1_{12}	0.0873	0.0017	c_{21}	0.0823	0.0000
WTI (1) γ^1_{13}	-0.0108	0.4046	c_{22}	0.0746	0.0000
Constant c_1	0.0760	0.1029	c_{31}	0.0096	0.7815
$\sigma^2_{11}\psi_{11}$	-0.0198	0.4255	c_{32}	0.0701	0.0203
$\sigma^2_{22}\psi_{12}$	0.0833	0.1328	c_{33}	0.0941	0.0042
$\sigma^2_{33}\psi_{13}$	-0.0078	0.3922	a_{11}	0.1636	0.0000
Stock Mean			a_{12}	-0.0026	0.8451
RE (1) γ^1_{21}	-0.0099	0.1976	a_{13}	-0.0177	0.3341
Stock (1) γ^1_{22}	-0.0186	0.2706	a_{21}	0.1765	0.0000
WTI (1) γ^1_{23}	-0.0113	0.0906	a_{22}	0.3197	0.0000
Constant c_2	0.0914	0.0008	a_{23}	0.1439	0.0001
$\sigma^2_{11}\psi_{21}$	0.0022	0.8666	a_{31}	-0.0004	0.9740
$\sigma^2_{22}\psi_{22}$	0.0267	0.4547	a_{32}	0.0040	0.6290
$\sigma^2_{33}\psi_{23}$	-0.0085	0.0985	a_{33}	0.1949	0.0000
Oil Mean			b_{11}	0.9873	0.0000
RE (1) γ^1_{31}	-0.0134	0.4991	b_{12}	0.0044	0.2672
Stock (1) γ^1_{32}	0.0218	0.5478	b_{13}	0.0104	0.0263
WTI (1) γ^1_{33}	-0.0499	0.0034	b_{21}	-0.0508	0.0001
Constant c_3	0.0144	0.7903	b_{22}	0.9370	0.0000
$\sigma^2_{11}\psi_{31}$	0.0143	0.5510	b_{23}	-0.0506	0.0000
$\sigma^2_{22}\psi_{32}$	-0.0491	0.3731	b_{31}	0.0012	0.6716
$\sigma^2_{33}\psi_{33}$	0.0052	0.5426	b_{32}	0.0000	0.9959
			b_{33}	0.9805	0.0000
			Shape ν (t degrees)	7.2554	0.0000

TABLE 4.25: *CCC GARCH-in-mean Parameter Estimates with Normal Distribution [Wind Energy]*

Wind Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Wind (1) γ_{11}^1	-0.0020	0.9381	c_{11}	0.0684	0.0000
Wind (2) γ_{11}^2	0.0144	0.4745	c_{22}	0.0361	0.0000
Stock (1) γ_{12}^1	-0.0262	0.5112	c_{33}	0.0447	0.0000
Stock (2) γ_{12}^2	-0.0226	0.4141	a_{11}	0.0668	0.0000
WTI (1) γ_{13}^1	0.0061	0.6106	a_{12}	0.0400	0.1057
WTI (2) γ_{13}^2	0.0130	0.3272	a_{13}	0.0003	0.9121
Constant c_1	0.0133	0.7534	a_{21}	0.0122	0.0130
$\sigma_{11}^2\psi_{11}$	-0.0121	0.7513	a_{22}	0.0923	0.0000
$\sigma_{22}^2\psi_{12}$	0.0461	0.4962	a_{23}	0.0037	0.0163
$\sigma_{33}^2\psi_{13}$	0.0043	0.6862	a_{31}	-0.0044	0.7147
Stock Mean			a_{32}	0.0351	0.1641
Wind (1) γ_{21}^1	0.0066	0.6989	a_{33}	0.0458	0.0000
Wind (2) γ_{21}^2	0.0120	0.3194	b_{11}	0.9003	0.0000
Stock (1) γ_{22}^1	-0.0503	0.0670	b_{12}	-0.0574	0.1828
Stock (2) γ_{22}^2	-0.0404	0.0328	b_{13}	0.0020	0.5672
WTI (1) γ_{23}^1	-0.0010	0.9017	b_{21}	-0.0106	0.2000
WTI (2) γ_{23}^2	0.0126	0.1588	b_{22}	0.8514	0.0000
Constant c_2	0.0809	0.0031	b_{23}	-0.0011	0.5370
$\sigma_{11}^2\psi_{21}$	-0.0255	0.3365	b_{31}	-0.0114	0.5575
$\sigma_{22}^2\psi_{22}$	0.0925	0.0762	b_{32}	-0.0256	0.5523
$\sigma_{33}^2\psi_{23}$	-0.0064	0.3626	b_{33}	0.9488	0.0000
Oil Mean			R(2,1) ρ_{21}	0.7246	0.0000
Wind (1) γ_{31}^1	-0.0443	0.1587	R(3,1) ρ_{31}	0.3862	0.0000
Wind (2) γ_{31}^2	0.0048	0.8414	R(3,2) ρ_{32}	0.4155	0.0000
Stock (1) γ_{32}^1	0.0406	0.3498			
Stock (2) γ_{32}^2	-0.0116	0.7329			
WTI (1) γ_{33}^1	-0.0334	0.0920			
WTI (2) γ_{33}^2	0.0109	0.6061			
Constant c_3	-0.0041	0.9389			
$\sigma_{11}^2\psi_{31}$	-0.0333	0.4910			
$\sigma_{22}^2\psi_{32}$	0.0157	0.8557			
$\sigma_{33}^2\psi_{33}$	0.0224	0.1937			

TABLE 4.26: *CCC GARCH-in-mean Parameter Estimates with t-distribution [Wind Energy]*

Wind Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Wind (1) γ^1_{11}	0.0140	0.4695	c_{11}	0.0486	0.0001
Stock (1) γ^1_{12}	-0.0461	0.0953	c_{22}	0.0257	0.0000
WTI (1) γ^1_{13}	0.0024	0.8278	c_{33}	0.0378	0.0003
Constant c_1	0.0463	0.1324	a_{11}	0.0571	0.0001
$\sigma^2_{11}\psi_{11}$	-0.0139	0.6212	a_{12}	0.0211	0.3577
$\sigma^2_{22}\psi_{12}$	0.0429	0.3862	a_{13}	0.0004	0.9002
$\sigma^2_{33}\psi_{13}$	0.0013	0.8418	a_{21}	0.0082	0.1346
Stock Mean			a_{22}	0.0921	0.0000
Wind (1) γ^1_{21}	0.0196	0.1154	a_{23}	0.0034	0.0395
Stock (1) γ^1_{22}	-0.0711	0.0002	a_{31}	0.0092	0.4750
WTI (1) γ^1_{23}	-0.0026	0.7271	a_{32}	0.0122	0.6528
Constant c_2	0.1049	0.0000	a_{33}	0.0486	0.0000
$\sigma^2_{11}\psi_{21}$	-0.0225	0.2812	b_{11}	0.9216	0.0000
$\sigma^2_{22}\psi_{22}$	0.0893	0.0178	b_{12}	-0.0359	0.3219
$\sigma^2_{33}\psi_{23}$	-0.0091	0.0179	b_{13}	0.0005	0.8653
Oil Mean			b_{21}	-0.0040	0.6411
Wind (1) γ^1_{31}	-0.0327	0.2663	b_{22}	0.8572	0.0000
Stock (1) γ^1_{32}	-0.0206	0.6341	b_{23}	-0.0008	0.6032
WTI (1) γ^1_{33}	-0.0184	0.2963	b_{31}	-0.0244	0.2079
Constant c_3	0.0431	0.3657	b_{32}	-0.0029	0.9479
$\sigma^2_{11}\psi_{31}$	-0.0257	0.5218	b_{33}	0.9495	0.0000
$\sigma^2_{22}\psi_{32}$	0.0022	0.9740	R(2,1) ρ_{21}	0.7119	0.0000
$\sigma^2_{33}\psi_{33}$	0.0147	0.2650	R(3,1) ρ_{31}	0.3717	0.0000
			R(3,2) ρ_{32}	0.4163	0.0000
			Shape ν (t degrees)	6.5691	0.0000

TABLE 4.27: *CCC GARCH-in-mean Parameter Estimates with Normal Distribution [Solar Energy]*

Solar Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Solar (1) γ_{11}^1	0.0882	0.0000	c_{11}	0.0461	0.0001
Solar (2) γ_{11}^2	0.0447	0.0073	c_{22}	0.0197	0.0000
Stock (1) γ_{12}^1	-0.0512	0.2795	c_{33}	0.0540	0.0000
Stock (2) γ_{12}^2	-0.0927	0.0236	a_{11}	0.0380	0.0000
WTI (1) γ_{13}^1	0.0017	0.9317	a_{12}	0.1026	0.0001
WTI (2) γ_{13}^2	0.0168	0.3568	a_{13}	-0.0091	0.0102
Constant c_1	0.0507	0.3745	a_{21}	0.0012	0.1395
$\sigma_{11}^2\psi_{11}$	-0.0125	0.2920	a_{22}	0.1135	0.0000
$\sigma_{22}^2\psi_{12}$	0.0872	0.1411	a_{23}	0.0030	0.0460
$\sigma_{33}^2\psi_{13}$	-0.0078	0.6038	a_{31}	-0.0004	0.8260
Stock Mean			a_{32}	0.0215	0.0889
Solar (1) γ_{21}^1	0.0031	0.6531	a_{33}	0.0498	0.0000
Solar (2) γ_{21}^2	0.0195	0.0002	b_{11}	0.9625	0.0000
Stock (1) γ_{22}^1	-0.0548	0.0093	b_{12}	-0.1762	0.0000
Stock (2) γ_{22}^2	-0.0630	0.0006	b_{13}	0.0131	0.0044
WTI (1) γ_{23}^1	-0.0028	0.7335	b_{21}	0.0022	0.0894
WTI (2) γ_{23}^2	0.0141	0.0652	b_{22}	0.8282	0.0000
Constant c_2	0.0872	0.0014	b_{23}	0.0006	0.7395
$\sigma_{11}^2\psi_{21}$	-0.0033	0.4864	b_{31}	-0.0033	0.1583
$\sigma_{22}^2\psi_{22}$	0.0572	0.0286	b_{32}	-0.0198	0.3695
$\sigma_{33}^2\psi_{23}$	-0.0089	0.1761	b_{33}	0.9424	0.0000
Oil Mean			R(2,1) ρ_{21}	0.6096	0.0000
Solar (1) γ_{31}^1	-0.0192	0.1322	R(3,1) ρ_{31}	0.3659	0.0000
Solar (2) γ_{31}^2	0.0159	0.2060	R(3,2) ρ_{32}	0.3993	0.0000
Stock (1) γ_{32}^1	0.0199	0.5606			
Stock (2) γ_{32}^2	-0.0353	0.3293			
WTI (1) γ_{33}^1	-0.0297	0.1207			
WTI (2) γ_{33}^2	0.0112	0.4912			
Constant c_3	-0.0296	0.5853			
$\sigma_{11}^2\psi_{31}$	0.0046	0.5288			
$\sigma_{22}^2\psi_{32}$	-0.0500	0.1811			
$\sigma_{33}^2\psi_{33}$	0.0178	0.2148			

TABLE 4.28: *CCC GARCH-in-mean Parameter Estimates with t-distribution [Solar Energy]*

Solar Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Solar (1) γ_{11}^1	0.0489	0.0036	c_{11}	0.0399	0.0016
Stock (1) γ_{12}^1	0.0084	0.8288	c_{22}	0.0074	0.1390
WTI (1) γ_{13}^1	-0.0154	0.3722	c_{33}	0.0370	0.0020
Constant c_1	0.0571	0.3794	a_{11}	0.0346	0.0000
$\sigma_{11}^2\psi_{11}$	-0.0111	0.4021	a_{12}	0.1180	0.0016
$\sigma_{22}^2\psi_{12}$	0.0649	0.2743	a_{13}	0.0025	0.6942
$\sigma_{33}^2\psi_{13}$	-0.0038	0.7959	a_{21}	0.0016	0.1381
Stock Mean			a_{22}	0.1118	0.0000
Solar (1) γ_{21}^1	-0.0029	0.6506	a_{23}	0.0037	0.0707
Stock (1) γ_{22}^1	-0.0351	0.0599	a_{31}	0.0008	0.7409
WTI (1) γ_{23}^1	-0.0070	0.3267	a_{32}	0.0250	0.2612
Constant c_2	0.0848	0.0017	a_{33}	0.0566	0.0000
$\sigma_{11}^2\psi_{21}$	-0.0010	0.8401	b_{11}	0.9681	0.0000
$\sigma_{22}^2\psi_{22}$	0.0513	0.0702	b_{12}	-0.1894	0.0005
$\sigma_{33}^2\psi_{23}$	-0.0082	0.1724	b_{13}	0.0014	0.8404
Oil Mean			b_{21}	0.0030	0.0683
Solar (1) γ_{31}^1	-0.0215	0.0960	b_{22}	0.8373	0.0000
Stock (1) γ_{32}^1	-0.0083	0.8137	b_{23}	0.0007	0.7370
WTI (1) γ_{33}^1	-0.0234	0.1911	b_{31}	-0.0021	0.5221
Constant c_3	0.0236	0.6327	b_{32}	-0.0278	0.4243
$\sigma_{11}^2\psi_{31}$	0.0009	0.8989	b_{33}	0.9419	0.0000
$\sigma_{22}^2\psi_{32}$	-0.0362	0.3091	R(2,1) ρ_{21}	0.6167	0.0000
$\sigma_{33}^2\psi_{33}$	0.0136	0.3130	R(3,1) ρ_{31}	0.3777	0.0000
			R(3,2) ρ_{32}	0.4113	0.0000
			Shape ν (t degrees)	5.8869	0.0000

TABLE 4.29: *CCC GARCH-in-mean Parameter Estimates with Normal Distribution [Nuclear Energy]*

Nuclear Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Nuclear (1) γ_{11}^1	-0.0006	0.9784	c_{11}	0.0392	0.0000
Stock (1) γ_{12}^1	0.0318	0.3125	c_{22}	0.0314	0.0000
WTI (1) γ_{13}^1	-0.0053	0.6356	c_{33}	0.0387	0.0001
Constant c_1	0.0415	0.2246	a_{11}	0.0372	0.0081
$\sigma_{11}^2\psi_{11}$	-0.0249	0.4565	a_{12}	0.0883	0.0000
$\sigma_{22}^2\psi_{12}$	0.0668	0.2009	a_{13}	0.0003	0.8838
$\sigma_{33}^2\psi_{13}$	-0.0075	0.3848	a_{21}	0.0023	0.4284
Stock Mean			a_{22}	0.1011	0.0000
Nuclear (1) γ_{21}^1	-0.0072	0.6531	a_{23}	0.0042	0.0081
Stock (1) γ_{22}^1	-0.0336	0.1701	a_{31}	-0.0068	0.1241
WTI (1) γ_{23}^1	-0.0125	0.1580	a_{32}	0.0411	0.0425
Constant c_2	0.0835	0.0018	a_{33}	0.0534	0.0000
$\sigma_{11}^2\psi_{21}$	-0.0103	0.5999	b_{11}	0.9408	0.0000
$\sigma_{22}^2\psi_{22}$	0.0616	0.0864	b_{12}	-0.1088	0.0001
$\sigma_{33}^2\psi_{23}$	-0.0108	0.1263	b_{13}	0.0036	0.1399
Oil Mean			b_{21}	-0.0047	0.3196
Nuclear (1) γ_{31}^1	0.0242	0.4542	b_{22}	0.8584	0.0000
Stock (1) γ_{32}^1	-0.0180	0.6926	b_{23}	-0.0013	0.4926
WTI (1) γ_{33}^1	-0.0423	0.0400	b_{31}	0.0097	0.2758
Constant c_3	-0.0045	0.9306	b_{32}	-0.0652	0.0284
$\sigma_{11}^2\psi_{31}$	0.0720	0.0840	b_{33}	0.9428	0.0000
$\sigma_{22}^2\psi_{32}$	-0.1443	0.0364	R(2,1) ρ_{21}	0.6177	0.0000
$\sigma_{33}^2\psi_{33}$	0.0178	0.2854	R(3,1) ρ_{31}	0.3458	0.0000
			R(3,2) ρ_{32}	0.3827	0.0000

TABLE 4.30: *CCC GARCH-in-mean Parameter Estimates with t-distribution [Nuclear Energy]*

Nuclear Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Nuclear (1) γ_{11}^1	-0.0334	0.0152	c_{11}	0.0250	0.0001
Nuclear (2) γ_{11}^2	-0.0036	0.8235	c_{22}	0.0235	0.0000
Nuclear (3) γ_{11}^3	0.0076	0.5696	c_{33}	0.0290	0.0035
Stock (1) γ_{12}^1	0.0551	0.0032	a_{11}	0.0371	0.0054
Stock (2) γ_{12}^2	0.0006	0.9791	a_{12}	0.0502	0.0019
Stock (3) γ_{12}^3	-0.0021	0.9073	a_{13}	0.0011	0.5853
WTI (1) γ_{13}^1	0.0031	0.6801	a_{21}	0.0002	0.9559
WTI (2) γ_{13}^2	0.0237	0.0013	a_{22}	0.0975	0.0000
WTI (3) γ_{13}^3	-0.0117	0.1549	a_{23}	0.0050	0.0093
Constant c_1	0.0465	0.0730	a_{31}	-0.0056	0.3635
$\sigma_{11}^2\psi_{11}$	-0.0055	0.8480	a_{32}	0.0367	0.1128
$\sigma_{22}^2\psi_{12}$	0.0451	0.2962	a_{33}	0.0560	0.0000
$\sigma_{33}^2\psi_{13}$	-0.0053	0.4372	b_{11}	0.9403	0.0000
Stock Mean			b_{12}	-0.0556	0.0364
Nuclear (1) γ_{21}^1	-0.0033	0.7332	b_{13}	0.0021	0.3461
Nuclear (2) γ_{21}^2	0.0150	0.2023	b_{21}	0.0008	0.8778
Nuclear (3) γ_{21}^3	-0.0050	0.6166	b_{22}	0.8636	0.0000
Stock (1) γ_{22}^1	-0.0339	0.0152	b_{23}	-0.0012	0.5074
Stock (2) γ_{22}^2	-0.0366	0.0334	b_{31}	0.0037	0.7615
Stock (3) γ_{22}^3	-0.0134	0.3450	b_{32}	-0.0438	0.2288
WTI (1) γ_{23}^1	-0.0072	0.2256	b_{33}	0.9450	0.0000
WTI (2) γ_{23}^2	0.0143	0.0115	R(2,1) ρ_{21}	0.6301	0.0000
WTI (3) γ_{23}^3	-0.0197	0.0028	R(3,1) ρ_{31}	0.3521	0.0000
Constant c_2	0.1021	0.0000	R(3,2) ρ_{32}	0.3866	0.0000
$\sigma_{11}^2\psi_{21}$	-0.0043	0.8160	Shape ν (t degrees)	5.5580	0.0000
$\sigma_{22}^2\psi_{22}$	0.0525	0.0664			
$\sigma_{33}^2\psi_{23}$	-0.0115	0.0424			
Oil Mean					
Nuclear (1) γ_{31}^1	0.0004	0.9864			
Nuclear (2) γ_{31}^2	0.0435	0.0894			
Nuclear (3) γ_{31}^3	-0.0366	0.1027			
Stock (1) γ_{32}^1	-0.0187	0.4865			
Stock (2) γ_{32}^2	-0.0661	0.0717			
Stock (3) γ_{32}^3	0.0034	0.9103			
WTI (1) γ_{33}^1	-0.0410	0.0083			
WTI (2) γ_{33}^2	0.0069	0.6489			
WTI (3) γ_{33}^3	0.0066	0.6784			
Constant c_3	0.0368	0.3490			
$\sigma_{11}^2\psi_{31}$	0.0789	0.0220			
$\sigma_{22}^2\psi_{32}$	-0.1309	0.0217			
$\sigma_{33}^2\psi_{33}$	0.0070	0.5670			

TABLE 4.31: *CCC GARCH-in-mean Parameter Estimates with Normal Distribution [RE Energy]*

RE Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
RE (1) γ_{11}^1	0.0092	0.7156	c_{11}	0.0637	0.0000
Stock (1) γ_{12}^1	0.1072	0.0228	c_{22}	0.0265	0.0000
WTI (1) γ_{13}^1	-0.0076	0.6178	c_{33}	0.0491	0.0001
Constant c_1	0.0349	0.5755	a_{11}	0.0354	0.0000
$\sigma_{11}^2\psi_{11}$	-0.0198	0.5570	a_{12}	0.1194	0.0000
$\sigma_{22}^2\psi_{12}$	0.0924	0.2393	a_{13}	-0.0004	0.8665
$\sigma_{33}^2\psi_{13}$	-0.0065	0.6027	a_{21}	0.0048	0.0401
Stock Mean			a_{22}	0.1032	0.0000
RE (1) γ_{21}^1	-0.0198	0.1066	a_{23}	0.0010	0.3627
Stock (1) γ_{22}^1	-0.0078	0.7556	a_{31}	-0.0120	0.0204
WTI (1) γ_{23}^1	-0.0134	0.0905	a_{32}	0.0514	0.0154
Constant c_2	0.0517	0.1379	a_{33}	0.0496	0.0000
$\sigma_{11}^2\psi_{21}$	0.0124	0.4953	b_{11}	0.9608	0.0000
$\sigma_{22}^2\psi_{22}$	0.0122	0.7908	b_{12}	-0.1913	0.0000
$\sigma_{33}^2\psi_{23}$	-0.0067	0.3493	b_{13}	0.0034	0.3286
Oil Mean			b_{21}	-0.0003	0.9500
RE (1) γ_{31}^1	-0.0204	0.3929	b_{22}	0.8419	0.0000
Stock (1) γ_{32}^1	0.0403	0.3963	b_{23}	0.0016	0.3387
WTI (1) γ_{33}^1	-0.0374	0.0528	b_{31}	0.0038	0.6446
Constant c_3	-0.0487	0.3383	b_{32}	-0.0392	0.2804
$\sigma_{11}^2\psi_{31}$	0.0349	0.1052	b_{33}	0.9418	0.0000
$\sigma_{22}^2\psi_{32}$	-0.1024	0.0343	R(2,1) ρ_{21}	0.7463	0.0000
$\sigma_{33}^2\psi_{33}$	0.0156	0.3020	R(3,1) ρ_{31}	0.3644	0.0000
			R(3,2) ρ_{32}	0.3469	0.0000

TABLE 4.32: *CCC GARCH-in-mean Parameter Estimates with t-distribution [RE Energy]*

RE Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
RE (1) γ_{11}^1	0.0079	0.7343	c_{11}	0.0603	0.0001
Stock (1) γ_{12}^1	0.1000	0.0249	c_{22}	0.0194	0.0021
WTI (1) γ_{13}^1	-0.0089	0.5054	c_{33}	0.0339	0.0242
Constant c_1	0.0566	0.3304	a_{11}	0.0419	0.0001
$\sigma_{11}^2\psi_{11}$	-0.0050	0.8802	a_{12}	0.1015	0.0022
$\sigma_{22}^2\psi_{12}$	0.0587	0.4453	a_{13}	0.0038	0.3145
$\sigma_{33}^2\psi_{13}$	-0.0063	0.5521	a_{21}	0.0059	0.0342
Stock Mean			a_{22}	0.0952	0.0000
RE (1) γ_{21}^1	-0.0101	0.3577	a_{23}	0.0025	0.1259
Stock (1) γ_{22}^1	-0.0201	0.3694	a_{31}	-0.0079	0.2507
WTI (1) γ_{23}^1	-0.0141	0.0366	a_{32}	0.0454	0.0768
Constant c_2	0.0631	0.0486	a_{33}	0.0557	0.0000
$\sigma_{11}^2\psi_{21}$	0.0162	0.3481	b_{11}	0.9510	0.0000
$\sigma_{22}^2\psi_{22}$	0.0006	0.9887	b_{12}	-0.1500	0.0009
$\sigma_{33}^2\psi_{23}$	-0.0059	0.3300	b_{13}	-0.0021	0.6092
Oil Mean			b_{21}	-0.0023	0.6302
RE (1) γ_{31}^1	-0.0110	0.6161	b_{22}	0.8638	0.0000
Stock (1) γ_{32}^1	0.0173	0.6997	b_{23}	-0.0008	0.6541
WTI (1) γ_{33}^1	-0.0403	0.0231	b_{31}	0.0041	0.7054
Constant c_3	-0.0034	0.9596	b_{32}	-0.0390	0.3376
$\sigma_{11}^2\psi_{31}$	0.0291	0.3496	b_{33}	0.9398	0.0000
$\sigma_{22}^2\psi_{32}$	-0.0838	0.2465	R(2,1) ρ_{21}	0.7379	0.0000
$\sigma_{33}^2\psi_{33}$	0.0099	0.5188	R(3,1) ρ_{31}	0.3668	0.0000
			R(3,2) ρ_{32}	0.3490	0.0000
			Shape ν (t degrees)	7.0138	0.0000

TABLE 4.33: *DCC GARCH-in-mean Parameter Estimates with Normal Distribution [Wind Energy]*

Wind Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Wind (1) γ_{11}^1	0.0085	0.7164	c_{11}	0.0283	0.0003
Stock (1) γ_{12}^1	-0.0449	0.2034	c_{22}	0.0162	0.0004
WTI (1) γ_{13}^1	0.0023	0.8598	c_{33}	0.0259	0.0001
Constant c_1	0.0056	0.8780	a_{11}	0.0492	0.0002
$\sigma_{11}^2\psi_{11}$	-0.0008	0.9817	a_{12}	0.0807	0.0006
$\sigma_{22}^2\psi_{12}$	0.0268	0.6649	a_{13}	0.0020	0.3556
$\sigma_{33}^2\psi_{13}$	0.0029	0.7717	a_{21}	0.0094	0.0505
Stock Mean			a_{22}	0.1126	0.0000
Wind (1) γ_{21}^1	0.0163	0.2682	a_{23}	0.0041	0.0048
Stock (1) γ_{22}^1	-0.0574	0.0188	a_{31}	0.0048	0.6317
WTI (1) γ_{23}^1	-0.0046	0.5904	a_{32}	0.0352	0.1467
Constant c_2	0.0810	0.0005	a_{33}	0.0443	0.0000
$\sigma_{11}^2\psi_{21}$	-0.0140	0.5647	b_{11}	0.9492	0.0000
$\sigma_{22}^2\psi_{22}$	0.0673	0.1535	b_{12}	-0.0955	0.0053
$\sigma_{33}^2\psi_{23}$	-0.0090	0.1714	b_{13}	-0.0007	0.7720
Oil Mean			b_{21}	0.0057	0.4996
Wind (1) γ_{31}^1	-0.0347	0.2321	b_{22}	0.8450	0.0000
Stock (1) γ_{32}^1	0.0429	0.3558	b_{23}	-0.0021	0.2034
WTI (1) γ_{33}^1	-0.0440	0.0247	b_{31}	-0.0112	0.4608
Constant c_3	0.0208	0.6519	b_{32}	-0.0220	0.5466
$\sigma_{11}^2\psi_{31}$	-0.0257	0.3748	b_{33}	0.9520	0.0000
$\sigma_{22}^2\psi_{32}$	0.0225	0.6725	α	0.0316	0.0000
$\sigma_{33}^2\psi_{33}$	0.0144	0.3512	β	0.9630	0.0000

TABLE 4.34: *DCC GARCH-in-mean Parameter Estimates with t-distribution [Wind Energy]*

Wind Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Wind (1) γ_{11}^1	0.0284	0.1286	c_{11}	0.0124	0.0163
Stock (1) γ_{12}^1	-0.0636	0.0330	c_{22}	0.0040	0.3815
WTI (1) γ_{13}^1	0.0000	0.9983	c_{33}	0.0145	0.0469
Constant c_1	0.0279	0.4035	a_{11}	0.0363	0.0006
$\sigma_{11}^2\psi_{11}$	-0.0083	0.7523	a_{12}	0.0520	0.0201
$\sigma_{22}^2\psi_{12}$	0.0361	0.4387	a_{13}	0.0014	0.5145
$\sigma_{33}^2\psi_{13}$	0.0016	0.8560	a_{21}	0.0008	0.8649
Stock Mean			a_{22}	0.1112	0.0000
Wind (1) γ_{21}^1	0.0284	0.0131	a_{23}	0.0030	0.0629
Stock (1)γ_{22}^1	-0.0764	0.0001	a_{31}	0.0064	0.5511
WTI (1) γ_{23}^1	-0.0052	0.4724	a_{32}	0.0239	0.3914
Constant c_2	0.1082	0.0000	a_{33}	0.0471	0.0000
$\sigma_{11}^2\psi_{21}$	-0.0173	0.3608	b_{11}	0.9667	0.0000
$\sigma_{22}^2\psi_{22}$	0.0742	0.0401	b_{12}	-0.0600	0.0408
$\sigma_{33}^2\psi_{23}$	-0.0109	0.0595	b_{13}	-0.0015	0.4561
Oil Mean			b_{21}	0.0168	0.0383
Wind (1) γ_{31}^1	-0.0228	0.4078	b_{22}	0.8529	0.0000
Stock (1) γ_{32}^1	-0.0152	0.7078	b_{23}	-0.0015	0.3511
WTI (1) γ_{33}^1	-0.0289	0.0722	b_{31}	-0.0052	0.7312
Constant c_3	0.0485	0.2655	b_{32}	-0.0191	0.6240
$\sigma_{11}^2\psi_{31}$	-0.0115	0.7471	b_{33}	0.9516	0.0000
$\sigma_{22}^2\psi_{32}$	-0.0044	0.9386	α	0.0220	0.0000
$\sigma_{33}^2\psi_{33}$	0.0095	0.5308	β	0.9761	0.0000
			Shape ν (t degrees)	6.9901	0.0000

TABLE 4.35: DCC GARCH-in-mean Parameter Estimates with Normal Distribution [Solar Energy]

Solar Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Solar (1) γ_{11}^1	0.0768	0.0000	c_{11}	0.0335	0.0029
Solar (2) γ_{11}^2	0.0337	0.0354	c_{22}	0.0134	0.0017
Stock (1) γ_{12}^1	-0.0403	0.4152	c_{33}	0.0402	0.0001
Stock (2) γ_{12}^2	-0.0792	0.0493	a_{11}	0.0368	0.0000
WTI (1) γ_{13}^1	0.0002	0.9904	a_{12}	0.1435	0.0006
WTI (2) γ_{13}^2	0.0103	0.5880	a_{13}	-0.0112	0.0129
Constant c_1	0.0393	0.5432	a_{21}	0.0005	0.5474
$\sigma_{11}^2\psi_{11}$	-0.0082	0.4425	a_{22}	0.1250	0.0000
$\sigma_{22}^2\psi_{12}$	0.0757	0.1696	a_{23}	0.0016	0.3400
$\sigma_{33}^2\psi_{13}$	-0.0091	0.5543	a_{31}	-0.0005	0.5189
Stock Mean			a_{32}	0.0371	0.0420
Solar (1) γ_{21}^1	-0.0008	0.9079	a_{33}	0.0477	0.0000
Solar (2) γ_{21}^2	0.0169	0.0071	b_{11}	0.9612	0.0000
Stock (1) γ_{22}^1	-0.0403	0.0540	b_{12}	-0.1588	0.0018
Stock (2) γ_{22}^2	-0.0545	0.0022	b_{13}	0.0146	0.0085
WTI (1) γ_{23}^1	-0.0064	0.4321	b_{21}	0.0019	0.1542
WTI (2) γ_{23}^2	0.0112	0.1545	b_{22}	0.8466	0.0000
Constant c_2	0.0889	0.0010	b_{23}	0.0014	0.5185
$\sigma_{11}^2\psi_{21}$	-0.0023	0.6353	b_{31}	-0.0027	0.0903
$\sigma_{22}^2\psi_{22}$	0.0452	0.0989	b_{32}	-0.0206	0.3930
$\sigma_{33}^2\psi_{23}$	-0.0091	0.1714	b_{33}	0.9456	0.0000
Oil Mean			α	0.0327	0.0000
Solar (1) γ_{31}^1	-0.0247	0.0827	β	0.9575	0.0000
Solar (2) γ_{31}^2	0.0057	0.6814			
Stock (1) γ_{32}^1	0.0377	0.3533			
Stock (2)γ_{32}^2	-0.0221	0.5574			
WTI (1) γ_{33}^1	-0.0380	0.0420			
WTI (2) γ_{33}^2	0.0047	0.8017			
Constant c_3	-0.0210	0.6868			
$\sigma_{11}^2\psi_{31}$	0.0043	0.6030			
$\sigma_{22}^2\psi_{32}$	-0.0399	0.3260			
$\sigma_{33}^2\psi_{33}$	0.0134	0.3282			

TABLE 4.36: *DCC GARCH-in-mean Parameter Estimates with t-distribution [Solar Energy]*

Solar Mean	Coefficient	p-value	Variance Equation	Coefficient	p-value
Solar (1) γ^1_{11}	0.0487	0.0001	c_{11}	0.0299	0.0160
Solar (2)γ^2_{11}	0.0284	0.0147	c_{22}	0.0041	0.3741
Stock (1) γ^1_{12}	-0.0022	0.9357	c_{33}	0.0264	0.0127
Stock (2) γ^2_{12}	-0.0679	0.0191	a_{11}	0.0333	0.0000
WTI (1) γ^1_{13}	-0.0107	0.4598	a_{12}	0.1569	0.0002
WTI (2) γ^2_{13}	0.0019	0.8873	a_{13}	-0.0003	0.9625
Constant c_1	0.0498	0.1173	a_{21}	0.0008	0.4387
$\sigma^2_{11}\psi_{11}$	-0.0073	0.1166	a_{22}	0.1207	0.0000
$\sigma^2_{22}\psi_{12}$	0.0530	0.0705	a_{23}	0.0021	0.2495
$\sigma^2_{33}\psi_{13}$	-0.0041	0.5416	a_{31}	0.0005	0.8282
Stock Mean			a_{32}	0.0395	0.0714
Solar (1) γ^1_{21}	-0.0027	0.5995	a_{33}	0.0535	0.0000
Solar (2) γ^2_{21}	0.0112	0.0198	b_{11}	0.9659	0.0000
Stock (1) γ^1_{22}	-0.0333	0.0063	b_{12}	-0.1663	0.0017
Stock (2) γ^2_{22}	-0.0427	0.0003	b_{13}	0.0024	0.7057
WTI (1) γ^1_{23}	-0.0055	0.3783	b_{21}	0.0024	0.1060
WTI (2) γ^2_{23}	0.0117	0.0392	b_{22}	0.8589	0.0000
Constant c_2	0.0916	0.0000	b_{23}	0.0007	0.7039
$\sigma^2_{11}\psi_{21}$	-0.0001	0.9599	b_{31}	-0.0018	0.5552
$\sigma^2_{22}\psi_{22}$	0.0410	0.0079	b_{32}	-0.0249	0.3773
$\sigma^2_{33}\psi_{23}$	-0.0090	0.0018	b_{33}	0.9445	0.0000
Oil Mean			α	0.0292	0.0000
Solar (1) γ^1_{31}	-0.0233	0.0534	β	0.9623	0.0000
Solar (2) γ^2_{31}	0.0046	0.6691	Shape ν (t degrees)	6.1483	0.0000
Stock (1) γ^1_{32}	0.0026	0.9326			
Stock (2)γ^2_{32}	-0.0360	0.2003			
WTI (1) γ^1_{33}	-0.0289	0.0417			
WTI (2) γ^2_{33}	0.0102	0.4513			
Constant c_3	0.0220	0.6531			
$\sigma^2_{11}\psi_{31}$	0.0024	0.7402			
$\sigma^2_{22}\psi_{32}$	-0.0357	0.2147			
$\sigma^2_{33}\psi_{33}$	0.0113	0.2000			

TABLE 4.37: *DCC GARCH-in-mean Parameter Estimates with Normal Distribution [Nuclear Energy]*

Nuclear Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Nuclear (1) γ_{11}^{1}	-0.0114	0.6143	c_{11}	0.0128	0.0001
Stock (1) γ_{12}^{1}	0.0397	0.1840	c_{22}	0.0150	0.0001
WTI (1) γ_{13}^{1}	-0.0118	0.2591	c_{33}	0.0238	0.0021
Constant c_1	0.0167	0.5968	a_{11}	0.0155	0.0000
$\sigma_{11}^{2}\psi_{11}$	-0.0135	0.6548	a_{12}	0.0809	0.0000
$\sigma_{22}^{2}\psi_{12}$	0.0568	0.2122	a_{13}	0.0013	0.3345
$\sigma_{33}^{2}\psi_{13}$	-0.0057	0.4867	a_{21}	-0.0006	0.8311
Stock Mean			a_{22}	0.1086	0.0000
Nuclear (1) γ_{21}^{1}	-0.0129	0.4025	a_{23}	0.0033	0.0110
Stock (1) γ_{22}^{1}	-0.0173	0.4463	a_{31}	-0.0039	0.4218
WTI (1) γ_{23}^{1}	-0.0179	0.0308	a_{32}	0.0352	0.0740
Constant c_2	0.0861	0.0005	a_{33}	0.0498	0.0000
$\sigma_{11}^{2}\psi_{21}$	-0.0114	0.5793	b_{11}	0.9982	0.0000
$\sigma_{22}^{2}\psi_{22}$	0.0518	0.1648	b_{12}	-0.1160	0.0000
$\sigma_{33}^{2}\psi_{23}$	-0.0107	0.1121	b_{13}	0.0005	0.7885
Oil Mean			b_{21}	0.0204	0.0012
Nuclear (1) γ_{31}^{1}	-0.0006	0.9854	b_{22}	0.8442	0.0000
Stock (1) γ_{32}^{1}	0.0186	0.6630	b_{23}	-0.0014	0.3660
WTI (1) γ_{33}^{1}	-0.0545	0.0051	b_{31}	0.0150	0.0856
Constant c_3	0.0083	0.8650	b_{32}	-0.0479	0.1020
$\sigma_{11}^{2}\psi_{31}$	0.0414	0.3010	b_{33}	0.9450	0.0000
$\sigma_{22}^{2}\psi_{32}$	-0.0797	0.1803	α	0.0119	0.0000
$\sigma_{33}^{2}\psi_{33}$	0.0078	0.6221	β	0.9872	0.0000

TABLE 4.38: *DCC GARCH-in-mean Parameter Estimates with t-distribution [Nuclear Energy]*

Nuclear Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
Nuclear (1) γ_{11}^{1}	-0.0356	0.0470	c_{11}	0.0053	0.0416
Stock (1) γ_{12}^{1}	0.0509	0.0185	c_{22}	0.0072	0.0411
WTI (1) γ_{13}^{1}	-0.0040	0.6452	c_{33}	0.0163	0.0480
Constant c_1	0.0439	0.1012	a_{11}	0.0046	0.1439
$\sigma_{11}^{2}\psi_{11}$	-0.0159	0.6092	a_{12}	0.0699	0.0000
$\sigma_{22}^{2}\psi_{12}$	0.0575	0.1358	a_{13}	0.0027	0.0638
$\sigma_{33}^{2}\psi_{13}$	-0.0049	0.4765	a_{21}	-0.0017	0.5224
Stock Mean			a_{22}	0.1009	0.0000
Nuclear (1) γ_{21}^{1}	-0.0054	0.6683	a_{23}	0.0032	0.0391
Stock (1) γ_{22}^{1}	-0.0292	0.0908	a_{31}	-0.0071	0.1708
WTI (1) γ_{23}^{1}	-0.0122	0.0794	a_{32}	0.0356	0.1018
Constant c_2	0.0915	0.0000	a_{33}	0.0531	0.0000
$\sigma_{11}^{2}\psi_{21}$	0.0044	0.8486	b_{11}	1.0072	0.0000
$\sigma_{22}^{2}\psi_{22}$	0.0460	0.1615	b_{12}	-0.0951	0.0000
$\sigma_{33}^{2}\psi_{23}$	-0.0129	0.0215	b_{13}	-0.0012	0.4584
Oil Mean			b_{21}	0.0224	0.0008
Nuclear (1) γ_{31}^{1}	-0.0119	0.6167	b_{22}	0.8599	0.0000
Stock (1) γ_{32}^{1}	-0.0039	0.9110	b_{23}	-0.0013	0.4225
WTI (1) γ_{33}^{1}	-0.0494	0.0057	b_{31}	0.0144	0.2200
Constant c_3	0.0239	0.5828	b_{32}	-0.0334	0.3122
$\sigma_{11}^{2}\psi_{31}$	0.0688	0.0410	b_{33}	0.9454	0.0000
$\sigma_{22}^{2}\psi_{32}$	-0.0827	0.0694	α	0.0164	0.0000
$\sigma_{33}^{2}\psi_{33}$	-0.0030	0.8292	β	0.9830	0.0000
			Shape ν (t degrees)	5.9585	0.0000

TABLE 4.39: *DCC GARCH-in-mean Parameter Estimates with Normal Distribution [RE Energy]*

RE Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
RE (1) γ_{11}^1	0.0080	0.7457	c_{11}	0.0361	0.0002
Stock (1) γ_{12}^1	0.1013	0.0301	c_{22}	0.0128	0.0119
WTI (1) γ_{13}^1	-0.0128	0.3876	c_{33}	0.0370	0.0007
Constant c_1	0.0275	0.6446	a_{11}	0.0327	0.0000
$\sigma_{11}^2\psi_{11}$	-0.0194	0.5580	a_{12}	0.1413	0.0000
$\sigma_{22}^2\psi_{12}$	0.0927	0.2425	a_{13}	-0.0032	0.2326
$\sigma_{33}^2\psi_{13}$	-0.0061	0.6324	a_{21}	0.0037	0.1103
Stock Mean			a_{22}	0.1093	0.0000
RE (1) γ_{21}^1	-0.0208	0.0821	a_{23}	-0.0001	0.9436
Stock (1) γ_{22}^1	-0.0043	0.8645	a_{31}	-0.0124	0.0111
WTI (1) γ_{23}^1	-0.0137	0.0761	a_{32}	0.0550	0.0120
Constant c_2	0.0644	0.0510	a_{33}	0.0473	0.0000
$\sigma_{11}^2\psi_{21}$	0.0068	0.6984	b_{11}	0.9648	0.0000
$\sigma_{22}^2\psi_{22}$	0.0243	0.5971	b_{12}	-0.1635	0.0000
$\sigma_{33}^2\psi_{23}$	-0.0085	0.2345	b_{13}	0.0058	0.0998
Oil Mean			b_{21}	0.0009	0.8312
RE (1) γ_{31}^1	-0.0244	0.2869	b_{22}	0.8626	0.0000
Stock (1) γ_{32}^1	0.0485	0.2918	b_{23}	0.0027	0.1576
WTI (1) γ_{33}^1	-0.0509	0.0056	b_{31}	0.0032	0.6761
Constant c_3	-0.0471	0.4715	b_{32}	-0.0253	0.4407
$\sigma_{11}^2\psi_{31}$	0.0342	0.2622	b_{33}	0.9454	0.0000
$\sigma_{22}^2\psi_{32}$	-0.0960	0.1849	α	0.0291	0.0000
$\sigma_{33}^2\psi_{33}$	0.0130	0.4188	β	0.9638	0.0000

TABLE 4.40: DCC GARCH-in-mean Parameter Estimates with t-distribution [RE Energy]

RE Mean	Coefficient	P-value	Variance Equation	Coefficient	P-value
RE (1) γ_{11}^{1}	0.0079	0.7364	c_{11}	0.0346	0.0072
Stock (1) γ_{12}^{1}	0.0968	0.0255	c_{22}	0.0081	0.1046
WTI (1) γ_{13}^{1}	-0.0128	0.3662	c_{33}	0.0260	0.0442
Constant c_1	0.0587	0.2765	a_{11}	0.0407	0.0005
$\sigma_{11}^2 \psi_{11}$	-0.0130	0.6671	a_{12}	0.1142	0.0014
$\sigma_{22}^2 \psi_{12}$	0.0691	0.3490	a_{13}	0.0006	0.8555
$\sigma_{33}^2 \psi_{13}$	-0.0056	0.6162	a_{21}	0.0040	0.1346
Stock Mean			a_{22}	0.0996	0.0000
RE (1) γ_{21}^{1}	-0.0108	0.3199	a_{23}	0.0013	0.3466
Stock (1) γ_{22}^{1}	-0.0155	0.5048	a_{31}	-0.0079	0.2399
WTI (1) γ_{23}^{1}	-0.0143	0.0463	a_{32}	0.0485	0.0590
Constant c_2	0.0802	0.0069	a_{33}	0.0541	0.0000
$\sigma_{11}^2 \psi_{21}$	0.0075	0.6422	b_{11}	0.9531	0.0000
$\sigma_{22}^2 \psi_{22}$	0.0165	0.6997	b_{12}	-0.1077	0.0178
$\sigma_{33}^2 \psi_{23}$	-0.0078	0.2093	b_{13}	-0.0007	0.8552
Oil Mean			b_{21}	-0.0006	0.8991
RE (1) γ_{31}^{1}	-0.0150	0.4845	b_{22}	0.8871	0.0000
Stock (1) γ_{32}^{1}	0.0229	0.5996	b_{23}	-0.0004	0.7777
WTI (1) γ_{33}^{1}	-0.0483	0.0034	b_{31}	0.0024	0.8161
Constant c_3	-0.0001	0.9985	b_{32}	-0.0219	0.5445
$\sigma_{11}^2 \psi_{31}$	0.0229	0.4177	b_{33}	0.9417	0.0000
$\sigma_{22}^2 \psi_{32}$	-0.0713	0.2982	α	0.0265	0.0000
$\sigma_{33}^2 \psi_{33}$	0.0090	0.5444	β	0.9685	0.0000
			Shape ν (t degrees)	7.5681	0.0000

TABLE 4.41: *Probability of a residual being below 0.05 level-left 0.05 tail*

	Wind			Solar			Nuclear			RE		
	Wind	Stock	WTI	Solar	Stock	WTI	Nuclear	Stock	WTI	RE	Stock	WTI
BEKK	0.0648	0.0688	0.0597	0.0601	0.0655	0.0555	0.0603	0.0674	0.0543	0.0638	0.0684	0.0542
CCC	0.0706	0.0720	0.0622	0.0626	0.0694	0.0590	0.0674	0.0708	0.0563	0.0684	0.0708	0.0599
DCC	0.0593	0.0648	0.0593	0.0616	0.0655	0.0583	0.0626	0.0653	0.0553	0.0635	0.0647	0.0578

TABLE 4.42: *Variance Equation Estimation Results of Bivariate GARCH Model on S&P 500 ETF and various Renewable Energy ETFs (for Negative Volatility Spillovers) (Part 1)*

		GARCH-CCC Normal Distribution		GARCH-CCC Student's t-Distribution		GARCH-DCC Normal Distribution		GARCH-DCC Student's t-Distribution	
		Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value
Wind	c_{11}	0.0665	0.0000	0.0429	0.0002	0.0325	0.0001	0.0097	0.0392
	c_{22}	0.0360	0.0000	0.0234	0.0000	0.0203	0.0000	0.0045	0.2289
	a_{11}	0.0713	0.0000	0.0562	0.0002	0.0548	0.0003	0.0375	0.0009
	a_{12}	0.0448	0.0614	0.0282	0.2700	0.0808	0.0014	0.0559	0.0153
	a_{21}	0.0136	0.0070	0.0067	0.2450	0.0090	0.0975	-0.0001	0.9893
	a_{22}	0.1034	0.0000	0.1046	0.0000	0.1225	0.0000	0.1205	0.0000
	b_{11}	0.8988	0.0000	0.9255	0.0000	0.9390	0.0000	0.9641	0.0000
	b_{12}	-0.0536	0.1595	-0.0360	0.3310	-0.0855	0.0158	-0.0578	0.0391
	b_{21}	-0.0098	0.2178	0.0001	0.9926	0.0031	0.7201	0.0161	0.0398
	b_{22}	0.8490	0.0000	0.8563	0.0000	0.8440	0.0000	0.8537	0.0000
Solar	c_{11}	0.0499	0.0000	0.0429	0.0002	0.0451	0.0000	0.0377	0.0012
	c_{22}	0.0240	0.0000	0.0128	0.0028	0.0209	0.0000	0.0099	0.0100
	a_{11}	0.0333	0.0000	0.0353	0.0000	0.0309	0.0000	0.0330	0.0001
	a_{12}	0.1164	0.0032	0.1487	0.0009	0.1675	0.0003	0.2103	0.0003
	a_{21}	0.0009	0.3058	0.0015	0.2285	0.0000	0.9820	0.0006	0.6007
	a_{22}	0.1231	0.0000	0.1273	0.0000	0.1339	0.0000	0.1368	0.0000
	b_{11}	0.9661	0.0000	0.9668	0.0000	0.9666	0.0000	0.9665	0.0000
	b_{12}	-0.1673	0.0009	-0.1844	0.0004	-0.1750	0.0010	-0.1982	0.0008
	b_{21}	0.0021	0.1173	0.0025	0.1536	0.0017	0.1951	0.0019	0.2392
	b_{22}	0.8334	0.0000	0.8472	0.0000	0.8467	0.0000	0.8595	0.0000

TABLE 4.43: *Variance Equation Estimation Results of Bivariate GARCH Model on S&P 500 ETF and various Renewable Energy ETFs (for Negative Volatility Spillovers) (Part 2)*

		GARCH-CCC Normal Distribution		GARCH-CCC Student's t-Distribution		GARCH-DCC Normal Distribution		GARCH-DCC Student's t-Distribution	
		Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value
Nuclear	c_{11}	0.0378	0.0000	0.0220	0.0257	0.0155	0.0000	0.0063	0.0056
	c_{22}	0.0340	0.0000	0.0253	0.0000	0.0191	0.0000	0.0084	0.0073
	a_{11}	0.0263	0.1877	0.0214	0.6182	0.0157	0.0000	0.0059	0.0836
	a_{12}	0.0932	0.0000	0.0592	0.0706	0.0822	0.0000	0.0711	0.0000
	a_{21}	0.0008	0.7526	-0.0020	0.5728	-0.0011	0.6856	-0.0031	0.1264
	a_{22}	0.1145	0.0000	0.1096	0.0000	0.1195	0.0000	0.1063	0.0000
	b_{11}	0.9594	0.0000	0.9675	0.0000	0.9978	0.0000	1.0009	0.0000
	b_{12}	-0.1086	0.0002	-0.0616	0.3474	-0.1138	0.0000	-0.0820	0.0000
	b_{21}	-0.0021	0.6283	0.0047	0.6316	0.0214	0.0008	0.0185	0.0035
	b_{22}	0.8531	0.0000	0.8675	0.0000	0.8345	0.0000	0.8714	0.0000
RE Benchmark	c_{11}	0.0724	0.0000	0.0620	0.0000	0.0605	0.0000	0.0415	0.0009
	c_{22}	0.0326	0.0000	0.0213	0.0001	0.0256	0.0000	0.0124	0.0053
	a_{11}	0.0351	0.0000	0.0450	0.0001	0.0329	0.0000	0.0440	0.0001
	a_{12}	0.1294	0.0000	0.1120	0.0019	0.1813	0.0000	0.1600	0.0000
	a_{21}	0.0047	0.0551	0.0061	0.0343	0.0041	0.0879	0.0050	0.0756
	a_{22}	0.1060	0.0000	0.1005	0.0000	0.1187	0.0000	0.1147	0.0000
	b_{11}	0.9556	0.0000	0.9459	0.0000	0.9557	0.0000	0.9479	0.0000
	b_{12}	-0.1782	0.0000	-0.1419	0.0013	-0.1817	0.0000	-0.1418	0.0008
	b_{21}	-0.0034	0.4421	-0.0039	0.4068	-0.0049	0.2093	-0.0034	0.4078
	b_{22}	0.8549	0.0000	0.8714	0.0000	0.8704	0.0000	0.8818	0.0000

Chapter 5

Conclusion

5.1 Main Findings and Contributions

There are a number of conclusions draw from the second chapter. Based on the crude oil market, the volatility responses of Brent and WTI to the market stimulation are distinct. Specifically, WTI is generally more sensitive than Brent to the market changes, and its fluctuation also keeps a little longer than Brent. Based on the estimation results of the GJR-GARCH model, the leverage effect is strongly significant in both crude oil futures markets (Brent and WTI). Notably, negative shocks will increase more volatility in WTI futures price than Brent counterpart. In a nutshell, the leverage effect in the WTI futures market is stronger than the Brent market. We can also say that the investors holding WTI are more sensitive to the negative news (negative shocks in the market) than the Brent futures holders. In terms of the volatility forecast, I mainly conclude that GARCH-type models perform better than the stochastic volatility models either in Brent or WTI crude oil futures market. OVX index can provide the optimal forecast in the volatility of Brent futures. By contrast, for WTI, the ARCH model exhibits the most accurate forecast ability. Outstandingly, over-fitting is important. Thus, Researchers should pay attention to this. For example, GJR-GARCH model and stochastic volatility models perform poorly in out-of-sample forecasting as suffering from over-fitting verified by this chapter.

The main finding of the third chapter is threefold. A breakpoint in 2014 is identified by the structural break threshold vector autoregressive (SBT-VAR) model. After successfully the structural form (SBT-SVAR) extended, I further construct the impulse response functions and find the evidence that the impulse response functions of oil prices to a unity structural geopolitical risk shock are smoother after the breakpoint in 2014 compared with those before the break. The most important find is the covariance response between geopolitical risk and oil price is reduced by $5 \sim 20 \times 10^{-3}$ with the extra unit shale production shock. It provides evidence that shale oil production has suppressed the impact of geopolitical risk on the U.S. crude oil price volatility.

There are a large number of interesting findings in the fourth chapter. I state them in an explicite way as following.

The Capital Asset Pricing Model (CAPM) introduced by (Sharpe, 1964)(Lintner, 1975) describes the relationship between risk and return. The positive relationship indicates investors, who are assumed to be risk averse, require a high return as compensation for bearing extra risk. First, the S&P 500 ETF volatility has a significant and positive effect on the return of itself, which is consistent with the theoretical CAPM model. We also find oil ETF volatility has a significant negative effect on the S&P 500 ETF return, which is consistent with previous literature on oil uncertainty has a significant and negative effect on stock performance. Outstandingly, the renewable energy benchmark ETF (PBW) changed the dominant role of oil ETF (USO) in energy ETF sector. The uncertainty of oil ETF has a consistent and negative effect on the S&P 500 ETF return, whereas the effect diminishes when PBW ETF is applied instead of other renewable ETFs. In one word, the PBW ETF has changed the channel of volatility spillovers to return. When nuclear ETF data is used, the conditional volatility of the nuclear ETF has a significant positive effect on the oil ETF, oppositely the volatility of S&P 500 ETF negatively spillovers to the return of the oil ETF. However, there is no evidence that the volatility spillovers to renewable energy ETF mean level as we expected. And the volatility of renewable energy does not affect the S&P 500 ETF return.

Second, we find consistent evidence of volatility spillovers in the conditional variance level in applying different multivariate GARCH specifications (BEKK, CCC, and DCC). The S&P 500 ETF volatility has a significant effect on the volatility of renewable energy ETFs, which shows the positive short-term persistence volatility spillover and the negative long-term persistence volatility spillover. To be more specific, the increase of squared errors in the last period of the S&P 500 ETF results in a growth of this period conditional variance of renewable energy ETFs. On the other hand, this period conditional variance of renewable energy ETFs will decrease when the last period conditional variance of S&P 500 ETF increases.

Based on this part, the strongly significantly negative volatility spillovers from renewable energy benchmark ETF to S&P 500 ETF is noticeable. And the conditions from Conrad and Karanasos (2010)'s study is checked using bivariate GARCH estimation results. The reason is the conditions (Conrad and Karanasos (2010)) we checked is different from the model (Ling and McAleer (2003)) we applied. There is a failure to meet all of the conditions, so we suggest a statistical test of the conditions can be developed for further research.

Third, we can see that the correlation between renewable energy ETF and S&P 500 ETF is significantly higher than the other two pairs of correlations according to the restricted constant correlation estimation results. About the dynamic correlation results, we conclude the correlation decrease before the financial crisis (in 2008 and 2011 individually) then dramatically increases after the financial crisis. More importantly, we find the consistent evidence in Wang and Liu (2016)'s study that the dynamic correlation between oil ETF and S&P 500 ETF are always positive commencing 2005 since U.S. net imports of crude oil and petroleum products gradually decrease.

Finally, we find the DCC model is the best, and the BEKK model is the second best according to the information criteria. The log-likelihood is substantially better when the multivariate Student's t-distributed errors are applied to compare with using the model with the multivariate Normal distributed errors.

Our finding as well as provides several valuable implications for investors. oil ETF or oil futures is not a safe haven to construct S&P 500 ETF portfolios, especially during a downturn economy. Therefore, investors should take the sector rotation investment strategy. Investors are suggested to construct oil derivative portfolio with nuclear ETF or nuclear energy-related companies stocks. We confirm that the role of nuclear ETF as safe-haven against crude oil ETF in the U.S. financial market. Entering renewable benchmark ETF in long position can be hedged with a short position in S&P 500 ETF and verse visa. Furtherly, investors should adopt a market-timing strategy when they hold the renewable benchmark ETF.

5.2 Recommendations for Future Research

Based on what I discussed in *chapter 2*, there are several aspects to extend in future research. First, GARCH-type models with Student's t-distribution is expected to provide efficient predictive ability. So it is valuable to investigate further. Second, it would be worthwhile to compare multivariate GARCH models in fitting multiple energy prices. Third, it would be interesting to add some macroeconomic variables in multivariate modelling. I can further identify the interaction between the crude oil market and the macroeconomics. Finally, it would improve the forecasting evaluation by using high-frequency intraday data as proxy volatility. And I advise that arbitrarily choosing a volatility model to forecast is not wise. The findings presented in this chapter provide evidence on how to select a volatility forecasting model for financial practitioners, energy economists, and policymakers. However, the data sample length and the choice for loss functions and proxy variance make evaluation vary as regard as the forecasting performance of the different models.

In terms of the further research about *chapter 3*, firstly the impact of shale oil production on the U.S. crude oil market is valuable to study by adding other crude oil market variables, such as crude oil inventory in the U.S. market. Also, it is interesting to discuss in further if the U.S. becomes more dominant in the world crude oil market by shale oil production boosting. Finally, it probably results in different impose response if other structure models identify method is applied. I encourage researchers to use a different approach to redo the same analysis. It will be interesting to compare the difference and valuable to make some conclusion.

As regarding the future research for *chapter 4*, risk management can apply in practice by constructing hedging strategy and portfolio weights between crude oil, renewable energy ETFs, and S&P 500 ETF. Secondly, we might explore the forecasting abilities in different multivariate GARCH specifications. If there are not consistent results as the in-sample estimation ability contrast results, we have to consider the overfitting problem. The third suggestion is applying the impulse response function to check the price shocks and volatility shocks in ETFs. Finally, we strongly suggest a statistical test of the conditions can be developed for further research, as there is a failure to meet all of the conditions.

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