Impact onto an ice floe

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ABSTRACT

The unsteady problem of a rigid body impact onto a floating plate is studied. Both the plate and the water are at rest before impact. The plate motion is caused by the impact force transmitted to the plate through an elastic layer with viscous damping on the top of the plate. The hydrodynamic force is calculated by using the second-order model of plate impact by Iafrati and Korobkin (2011). The present study is concerned with the deceleration experienced by a rigid body during its collision with a floating object. The problem is studied also by a fully-nonlinear computational-fluid-dynamics method. The elastic layer is treated with a moving body-fitted grid, the impacting body with an immersed boundary method, and a discrete-element method is used for the contact-force model. The presence of the elastic layer between the impacting bodies may lead to multiple bouncing of them, if the bodies are relatively light, before their interaction is settled and they continue to penetrate together into the water. The present study is motivated by ship slamming in icy waters, and by the effect of ice conditions on conventional free-fall lifeboats.

Keywords: Impact loads; Ice floe; Hertz model; Elastic foundation; Water entry.

1. INTRODUCTION

The reduction in the extent of summer sea ice is the most high-profile indicator of Arctic climate change according to a recent Lloyd's report (2012). The trend towards more ice-free areas of the Arctic Ocean, and a longer ice-free period, is expected to continue. The opening of the Arctic will reduce shipping costs and extend exploration and drilling seasons for offshore oil and gas. On the other hand, the reduction in sea ice increases the distance over which waves gather strength, their 'fetch', making waves stronger and move frequent. The younger and thinner Arctic ice is more prone to break-up by the water waves and ships (Lloyd's report, 2012). We may conclude that the climate changes reduce the extend of the ice cover in Arctic but increase both the amount of broken floating ice and water wave amplitude. Navigation in Arctic Ocean without icebreakers is becoming possible, but ice floes on the ship route in combination with large waves and large amplitude motions of the ship, may cause problems with ice loads in addition to the slamming loads. The Lloyd's report (2012) points that "comprehensive and rigorous risk management is essential for companies seeking to invest in the Arctic. Those companies that can manage their own risks, using technologies and services most adapted to Arctic conditions, are most likely to be commercially successful". In the present study, we are concerned with the effect of a single ice floe on slamming loads for sea-going ships and conventional free-fall lifeboats, see Lubbard *et al.*

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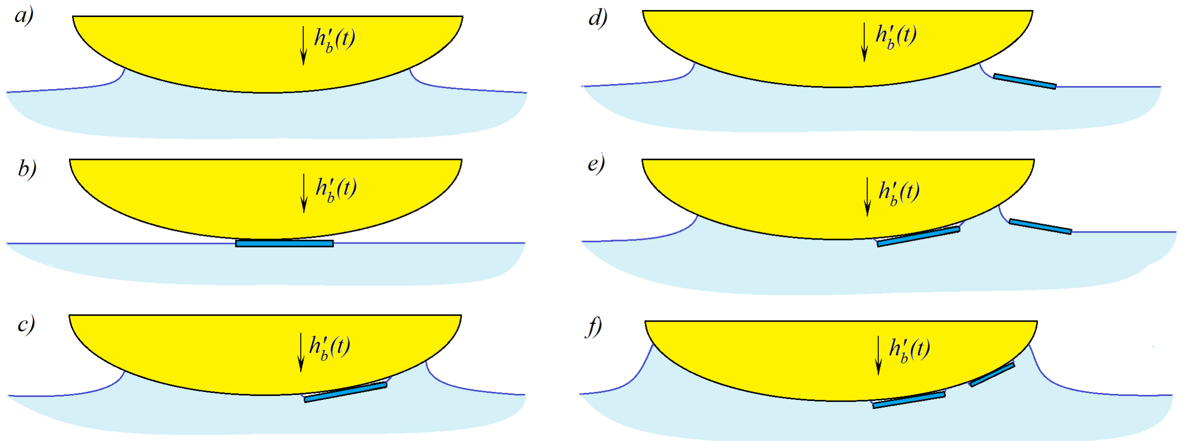

Figure 1: Two-dimensional slamming for open water *(a)*, with ice floe *(b)*, which is attaced to the body surface later on *(c)*, with an ice floe near the impact region *(d)*, several floes under and near the body surface *(e)* and floes on the body surface *(f)*.

(2011) and Re *et al.* (2003).

1.1 Slamming loads with floating ice

The problem under consideration is different from the well studied problem of ship interaction with floating ice without waves and large-amplitude ship motions (Lubbad at al., 2011). We shall estimate the loads caused by a floating ice plate on a rigid body falling from above onto the water surface. The ice plate is relatively short and modelled as rigid (Korobkin, 2000). The problem is sketched in Figure 1 for two-dimensional slamming, where $h'_b(t)$ is the vertical velocity of a ship section. We assume that slamming loads for open water (Figure 1a) are well predicted. If there is an ice floe in the place of impact (Figure 1b), then two stages of the impact can be distinguished. During the first stage, the ship section does not touch the water surface. It accelerates the ice floe up to the velocity of the section. The acceleration of the ice plate is smooth if the impact between the ice plate and the ship section is elastic. In such a case, the ice floe may bounce from the impacting body after a short period of their interaction. Later they collide again but with smaller relative speeds. These short periods of impacts and bouncing back may repeat several times before the ice/body interaction settles down and the body continues to enter the water with the ice plate attached to the body surface. During the second phase the body penetrates the water with the attached ice plate which can move along the body surface changing slightly the impact loads, if the ice plate is short (Figure 1c). However, the impact loads in the presence of ice can be significantly different from the slamming loads in open water for ice plates of moderate size. In this study, we do not account for elastic deflection of the ice floe and its possible breaking caused by the impact.

The situation depicted in Figure 1d is more complicated. Here the body initially enters open water and an ice floe is floating nearby the impact place. Later on the surface of the body approaches the ice floe and comes in contact with it. It is important to note that the floe starts to move before its contact with the entering body, and it is moving towards the body (Figure 1d). Then the speed of the collision between the body and the ice floe is larger that the speed of the body. In addition, the angle between the surface of the body and the ice plate at the impact instant can be such that the plate may cut the body surface in a knife-like manner (Khabakhpasheva *et al.*, 2018). This may happen in the presence of another ice floe attached already to the body surface (Figure 1e). The cases (c) and (d) can be combined leading to multiple ice plates in the impact region (Figure 1f).

During the second phase (Figures 1c and 1f), the presence of the ice plates attached to the

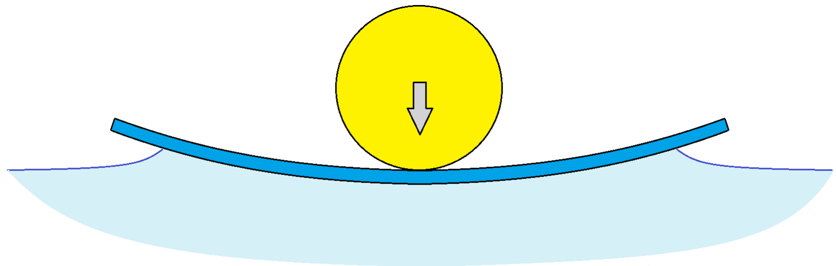

Figure 2: Impact on a floating elastic plate of large length.

wetted surface of the body and moving along this surface can be included in the solution of the water impact problem (Figure 1a) by modifying the shape of the entering body, see Section 4. For short ice plates their effect on the impact force during the second phase can be negligible in the leading order. Then the main changes to the impact loads occur during the first phase (Figure 1b), when the ice floe is accelerated by the impacting body. In general, the body surface can be partly wetted before the floe speed adjusts the speed of the body. For the ice floes of large horizontal extent (Figure 2) elastic deflections of the ice plate are important but the body does not touch water during the impact.

The first phase of impact onto a short floating plate can be divided into the initial stage, when the plate displacements are small compared with the horizontal dimensions of the plate, and the main stage, when the plate displacements are comparable with the plate dimensions but smaller than the dimension of the impacting body. During the main stage the body surface is wetted outside the region shadowed by the plate but the plate motions are still different from the motions of the body. The main stage can be described by the Wagner model of water impact for bodies with small deadrise angles, see Wagner (1932). The Wagner model should be modified to account for the ice plate moving near the rigid surface of the body. During the initial stage, the hydrodynamic loads act only on the floating plate, and the body deceleration is determined by the model of the interaction between the body and the ice floe.

1.2 Impact on a floating body by Joukowski

A floating plate is accelerated by impact on it in short time. The speed of the plate and the speed of the body after this short period of interaction can be obtained by using the concept of the force impulse and a collision model. This problem was solved by Joukowski (1884) for the vertical impact of two spheres, one of which is half-submerged in a spherical vessel, see Figure 3. The notation in the figure is that from the paper by Joukowski. The flow caused by the impact is potential. Let the speed of the floating sphere with mass m' and radius r_1 be u' after the impact, and the speed of the impacting sphere with mass m be v before the impact and u after the impact. The hydrodynamic pressure in the vessel is given by the unsteady Bernoulli equation which can be linearized for the impact stage of short duration. The velocity potential φ of the flow after the impact is obtained in the spherical coordinates within the linearized hydrodynamic impact model. The integrals of the hydrodynamic pressure, $p = -\rho\varphi_t$, over the wetted part of the floating sphere and in time for the duration of the impact provide the impulse of hydrodynamic force as $m_a u'$, where $m_a = \frac{1}{3}\pi\rho r_1^3(r_2^3 + 2r_1^3)/(r_2^3 - r_1^3)$ is the added mass of the floating sphere and ρ is the liquid density.

Combining the equations of the sphere motions and eliminating the interaction force between the spheres during the impact, Joukowski obtained that the change of the total momentum of both spheres is equal to the impulse of the hydrodynamic force,

$$m(v - u) - m'u' = m_a u'.$$

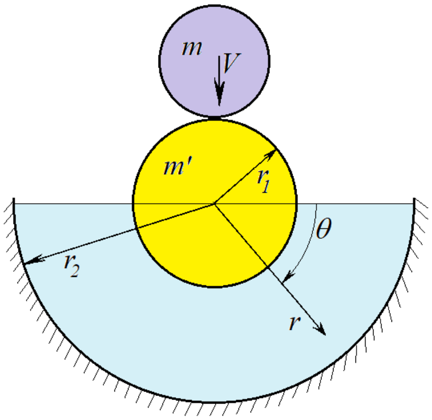

Figure 3: Impact of two spheres studied by Joukowski (1884).

There are two unknowns in this equation, u and u'. Only the mass, but not the size, of the impacting sphere matters.

The second equation describes the physics of the impact. If the impact is inelastic, then the bodies move at the same speed after their collision, $u = u'$, and the kinetic energy of the system is not conserved. The energy can be lost due to heating of the bodies during the impact, the body deformation, or their crushing. Then

$$v/u = 1 + (m' + m_a)/m. \tag{1}$$

Therefore, the speed of the impacting body is reduced by an amount related to the ratio of the mass of the floating sphere together with its added mass, $m' + m_a$, and the mass of the impacting body, m. If the impact is elastic, then the kinetic energy of the system is conserved,

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \frac{1}{2}m'u'^2 + \frac{1}{2}m_a u'^2,$$

where the last term on the right is the kinetic energy of the flow generated by the impulsive motion of the floating sphere, see Joukowski (1884). The conservation of the energy and the conservation of the total momentum provide $v + u = u'$, which is the second required equation. The result for elastic impact is

$$u = v\frac{m - (m' + m_a)}{m + (m' + m_a)}, \qquad u' = 2v\frac{m}{m + (m' + m_a)}. \tag{2}$$

For water of infinite depth, $r_2 \to \infty$, the added mass of the half-submerged sphere is $m_a = \frac{1}{3}\pi\rho r_1^2$. The obtained results (1), (2) and the described models are valid for any shapes of floating and impacting bodies. We may conclude that the ratio of the body speeds before and after impact on a floating body depends on the ratio of the body mass to the mass of the floating object including its added mass. Note that the mass of an ice floe of small thickness is much smaller than its added mass, with the ratio between them being of order of the ratio of the plate thickness to its horizontal dimension. A massive body, such as a ship, experiences a small change of its vertical velocity due to presence of ice floes in the impact region. The velocity reduction for a light body can be significant. Inelastic impact with an ice floe, described by the equation (1), corresponds to high impact velocities, when the ice is crushed in the impact place. Then the kinetic energy of the impacting body is spent on the acceleration of the floating object and its crushing. For low-speed impacts, the ice floe is accelerated without crushing and equations (2) could be used. For a massive impacting body, $(m' + m_a)/m \ll 1$, the equation of elastic impact (2) predicts that the speed of the impacting body does not change significantly but the speed of the floating body after the impact is twice the speed of the impacting body. This means that the floating body moves at a higher speed than the impacting body, where $u' = u + v$ as it follows from the energy conservation law. The floating body may separate from the impacting body at the end of the first impact but then it slows down by the hydrodynamic force and the next collision between the bodies occurs.

1.3 Hydrodynamic forces of water entry

The deceleration of the impacting body, as well as the second and the following collisions, cannot be described by using the force impulse only. The time-dependent hydrodynamic loads for moderate penetration depth are needed together with the buoyancy force. The hydrodynamic forces can be calculated by the following formula,

$$F_h(t) = \rho h_p'' F_w(h_p) + \rho (h_p')^2 F_v(h_p), \tag{3}$$

see Korobkin *et al.* (2014), for the water entry problems and by similar formulae for impact on floating plates, see Iafrati *et al.* (2008) for a two-dimensional plate and Iafrati *et al.* (2011) for a circular plate on the surface of water of infinite depth. Here $h_p(t)$ is the vertical displacement of a floating body. The functions $F_w(h_p)$ and $F_v(h_p)$ can be obtained approximately for small displacements or numerically for moderate displacements of the body. Iafrati *et al.* (2008, 2011) obtained these functions by using the method of matched asymptotic expansions for plates of zero draft. The approximate formulae are

$$\rho F_w(h_p) = m_a, \tag{4}$$

where $m_a = \frac{4}{3}\rho R_p^3$ for a circular disc of radius R_p and $m_a = \frac{\pi}{2}\rho L^2$ for a two-dimensional plate of length $2L$, $F_v(h_p) = R_p^2 f(h_p/R_p)$ for a circular disc and $F_v(h_p) = L f(h_p/L)$ for a two-dimensional plate, where

$$f(x) = a_1 x^{-1/3} + a_2 \log x + a_0. \tag{5}$$

The coefficients in (5) are given in Table 1 of Iafrati *et al.* (2011) for the circular disc and the flat plate. The forces predicted by the equations (3)-(5) were compared with the numerical forces computed by a boundary element method, see Iafrati *et al.* (2008, 2011) for constant speed of impact. It was shown that the formula (3) can be used for penetration depth up to 10% of the plate radius. Similar formulae for hydrodynamic forces and moments acting on floating two-dimensional plates, and circular and elliptic discs moving with more than one degree of freedom can be derived using the method by Iafrati *et al.* (2008, 2011). This analysis has not been done yet. The buoyancy force acting on a flat disc is given by the formula $F_A(t) = S\rho g h_p$, where g is the gravitational acceleration, S is the disc area, $S = \pi R_p^2$ for the circular disc, and $S = 2L$ for the two-dimensional plate. Note that the gravity effect on the flow and the hydrodynamic force (3) is not taken into account.

The magnitudes of the impact forces transmitted to the rigid body during each collision with a floating ice floe, as well as the corresponding body decelerations, cannot be obtained by using the force impulse formulae (1) or (2). A model of interaction between the colliding bodies is needed.

1.4 Interaction models of colliding bodies

For high-speed impact and a relatively small body, the ice can crush in the impact region (Kurdyumov *et al.*, 1976, Jordan *et al.*, 1988). The kinetic energy of the impacting body is used for the crushing until the speeds of the body and the ice floe become equal. It is written by Kurdyumov *et al.* (1976): "Experiments on impact on ice have shown that the insertion of a solid in ice occurs because of local shattering of the ice surface. Plastic strains did not succeed in developing. The elastic strains were also insignificant. The shattered ice from the intermediate layer (between the surface of the body and the main crystalline ice) is displaced to the free surface during the insertion. A certain quantity of water is apparently also present in the shattered material under pressure". The flow in the intermediate layer was modelled as viscoelastic. In the experiments, the impacts were inelastic for impact speeds higher than 1.5 m/s (Kurdyumov *et al.*, 1976). Dependence of the inelastic impact conditions on the thickness of ice and the radius of the solid spheres used in the experiments were not reported. We may assume that impacts on ice are elastic for low impact speeds of light bodies with large radius of curvature in the impact region. Elastic effects are expected to dominate for thin ice and bodies with small deadrise angles.

Forces between two elastic three-dimensional bodies pressed to each other by a given force P were calculated by Hertz (1882). His theory assumes elliptic contact regions which are much smaller than both the dimensions of the bodies and the relative radii of curvature of their surfaces (Johnson, 1987). In the Hertz theory, each body is approximated by an elastic half-space loaded over a small elliptical region of its plane surface. Correspondingly, the contact stresses localised near the contact region are treated separately from the stresses in the main parts of the elastic bodies. The Hertz theory is quasi-static. This implies, in particular, that the speed of the contact area expansion is much smaller than the speeds of elastic waves in the solids. This is not valid for early stage of the contact of bodies with small deadrise angles. For a rigid sphere of radius R pressed by a force P to the elastic lower half-space $z < 0$, the position of the body in the contact region can be approximated by $z = r^2/(2R) - h$, where h is the indentation depth and r is the radial coordinate. The Young's modulus of the elastic half-space is E and the Poisson ratio is ν. Then the Hertz theory provides (Johnson, 1987)

$$h = \left(\frac{3P(1-\nu^2)}{4E} \right)^{2/3} R^{-1/3}.$$

In the present study, this equation is used in the form

$$P = \frac{4}{3} \frac{E\sqrt{R}}{1-\nu^2} h^{3/2}, \tag{6}$$

where $E = 4.2 \cdot 10^9$ N/m^2 is Young's modulus of ice, $\nu = 0.3$, R is the radius of the curvature of a rigid axisymmetric body impacting a floating ice floe and P is the interaction (impact) force between the body and the floe. The indentation depth of the body h is equal to the difference between rigid-body displacements of the impacting body and the ice floe.

The rigid displacements of the body and the floe are described by Newton's second law with account for the hydrodynamic (3) and the buoyancy forces, and the interaction force (6). For a rigid body of double curvature, the interaction force P is still proportional to $h^{3/2}$ but the coefficient of proportionality now depends on both radii of the body curvature. The body shape near the impact place is approximated by an elliptic paraboloid, in the general case. A corresponding formula of the force, similar to (6), for the two-dimensional case does not exist. Such a formula would be helpful for estimation of ice loads during ship slamming in icy water. Calculations of two-dimensional interaction loads should account for the shape and size of the bodies. Such calculations are difficult to perform in most practical problems (Johnson, 1987).

For a floating elastic plate, which models an ice floe, and an impacting body of large size, as a ship section, the contact region size can be comparable to the plate thickness and even to the plate horizontal dimensions. The ice floe is pressed by the rigid body from above and by the hydrodynamic force from below. Separating the rigid displacement of the plate and its elastic compression, we arrive at the approximation similar to the strip theory, where the ice plate is compressed in the one-dimensional manner with the vertical stress σ being proportional to the relative compression of the plate, $\sigma = Eh/H_s$ with E being Young's modulus of the ice, and h the local indentation depth, and H_s is the elastic sheet thickness. This formula is valid inside the contact region, where the horizontal stresses are small compared with the vertical one. Near the periphery of the contact region, where the stress field is essentially three-dimensional, this approximation is not valid. However, the stresses near the contact region periphery are small and give negligibly small contribution to the total interaction force. The global deflection of the floating plate may significantly modify the local indentation h and, as a result, the total

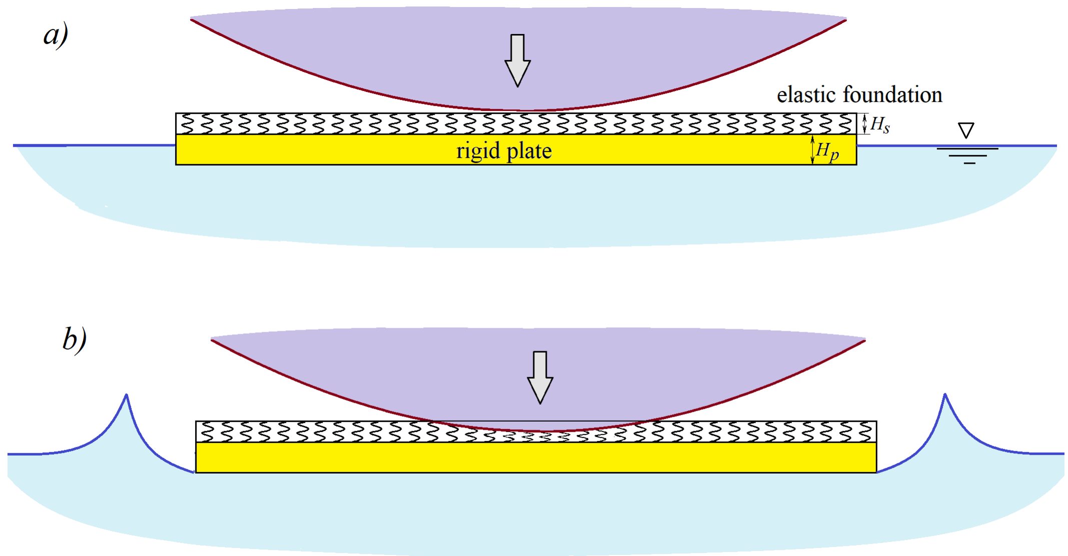

Figure 4: Impact on a floating plate with an elastic layer on the top of it before the interaction *(a)* and during the interaction *(b)*.

interaction force. In order to simplify the complex problem of impact onto a floating elastic plate, we model the ice floe as a short rigid plate of constant thickness H_p, see Figure 4a, with an elastic thin layer of thickness H_s on it. Therefore, the plate behaves as rigid in computations of the hydrodynamic forces, and as elastic in calculations of the impact forces, see Figure 4b. The ice floe is modelled as a Winkler elastic foundation rather than an elastic half-space as in the Hertz theory. The elastic foundation of thickness H_s rests on a rigid movable base and is compressed by a rigid body (Winkler, 1867). A Winkler foundation can be viewed as a set of linear springs with rigidity E/H_s, or k_s in general. The idea of the Winkler foundation is simple and flexible. It makes it possible to account for the bending of the underlying base and complex motions of both the plate and the impacting body with 6DoF in general. Fryba (1995) wrote "The Winkler foundation was many times cursed and refused, but the scientists return to this simple law again and again. I thing that it shares the same lot as the other simple theories like Bernoulli-Euler beam, Palmgren-Miner rules etc.: their simplicity wins against more precise formulations".

The interaction force for a Winkler foundation model and a two-dimensional parabolic shape, $z = x^2/(2R) - h(t)$, is obtained as

$$P = \frac{4}{3}k_s\sqrt{2R}h^{3/2}. \tag{7}$$

This formula is similar to (6), which is for the axisymmetric case. For a rigid sphere of radius R penetrating into the elastic layer with rigidity k_s the interaction force reads

$$P = \pi k_s R h^2 \left(1 - \frac{1}{3}\frac{h}{R}\right), \tag{8}$$

where the term with h/R can be neglected for small indentations. It is seen that both the Hertz theory and the model of the Winkler foundation yield non-linear interaction forces even for small indentation depth.

1.5 Physical effects on hydroelastic interaction of bodies

The dissipation of energy during collision of bodies can be taken into account, for example, by using the Kelvin model, $\sigma = Eh/H_s + \eta h'(t)/H_s$, which provides the interaction force as $(1 + \tau\, d/dt)P(t)$, where $P(t)$ is given by (7) in two-dimensional and by (8) in three-dimensional cases. Here τ is the retardation time. As τ increases, a larger amount of the kinetic energy is dissipated during a collision.

In the limiting case of a body with flat bottom impacting a floating plate with flat upper surface, we arrive at the experimental (Ermanyuk *et al.*, 2005) and theoretical (Korobkin *et al.*, 2004) studies of two circular plates, one of which is floating on a thin layer of liquid. It was shown both theoretically and experimentally that the presence of compressed air between the two colliding plates is the governing factor in this problem. However, the air layer cannot be modelled as elastic layer. The damping effect of the air is due to its escape at a finite speed from the gap between the plates. This effect can be used in the problem of lifeboat landing in icy waters by making the bottom of the boat nearly flat.

Impact onto an elastic beam was studied by Timoshenko *et al.* (1955, §66). For a beam of negligible mass, the beam response is approximately quasi-static. It was assumed that the kinetic energy of the impacting body transferred completely to the potential energy of the deformed beam. The impact was inelastic. It was shown that in these conditions the dynamic deflection of the beam is more than twice larger than the static deflection for the same weight of the impacting body. The elastic impact of a sphere onto a beam with non-negligible mass is a more complicated process because the beam oscillations should be taken into account. The impact occurred at the middle of the beam. The load magnitude was calculated by (5), where $h(t)$ is the difference between the displacement of the body and the displacement of the beam at the impact point. The impact force $P(t)$ and the beam deflection at the impact point were evaluated numerically. It was shown that the body bounces several times from the beam. The impact force during the second impact was higher than that during the first one. The beam deflection increased monotonically at least to the end of the second impact.

Our present problem differs from that studied by Timoshenko *et al.* (1955) in several aspects: (*i*) our plate is floating on the water surface, (*ii*) our plate is short with free-free edges, (*iii*) the impacting body is large in our problem with the contact area being comparable with the size of the plate. The force restoring the position of the plate after each impact is due to the hydrodynamic force and the buoyancy force in our problem. The fluid-inertia force given by the first term in (3) does not contribute to the restoring force. This part of the hydrodynamic force leads to the added mass of the plate, see (4), increasing the effective mass of the plate, see also equation (1) and (2) for the role played by the added mass of the floating plate. However, in our problem of impact onto a short floating plate, we could also expect multiple impacts as in the problem of a rigid body impact onto an elastic beam.

1.6 The aims of the present study

Through the analysis of the present problem, we should answer several questions. Does the maximum deceleration of the vertically impacting body always occur during the first impact with a floating body within the Hertz theory and the model of elastic layer? What is the effect of the impact force damping on the body deceleration? How does the energy dissipation in the collision model of section 1.5 relate to the ice crushing model by Kurdyumov and Kheisin (1976)? How does the horizontal velocity of the body change its deceleration and its trajectory after the body impacts onto a floating ice plate? What is the motion of the plate and its effect on the hydrodynamic forces acting on the body penetrating deep into the water with the attached plate? To answer the question about oblique impact of a body onto an ice floe, the study of the relevant problem for shallow water without ice, see Khabakhpasheva *et al.* (2013) can be useful.

In the next section, central impacts onto a two-dimensional and a circular ice floe are studied using the Winkler foundation model (7), (8) and the Hertz model (6) for the impact force with viscous damping.

2. CENTRAL IMPACT

Problems of impact are multi-scale problems. The collision time is usually much smaller than the characteristic time of the body motions. The collision force depends on the indentation depth $h(t)$. It is reasonable to separate short-time equations of the collisions, which are the equations for $h(t)$, and the equations of the body motions. During the early stage of the impact onto a floating short plate, time intervals between subsequent collisions are also short. All equations of the body motions should be numerically integrated with a time step being small enough to describe the fine details of the collision processes.

The vertical motion of an impacting body is described by the equation

$$m_b \frac{\mathrm{d}^2 h_b}{\mathrm{d}t^2} = m_b g - F_{imp}(t), \tag{9}$$

where m_b is the mass of the body, $F_{imp}(t)$ is the impact force, and $h_b(t)$ is the displacement of the body downwards starting from the time, $t = 0$, when the body touches the upper surface of the floating plate at a single point. The impact force accounts for dissipation of energy during collisions, $F_{imp}(t) = (1 + \tau\, \mathrm{d}/\mathrm{d}t)P(t)$, where the retardation time τ is a characteristic of the colliding materials and $P(t)$ is given by (7), (8) or (6) depending on the collision model and the dimension of the problem. The impacting body is assumed symmetric with respect to the vertical axis and impacts a floating plate at its centre. The impact force has only the vertical component. Initially,

$$h_b = 0, \qquad h_b'(0) = V_0, \tag{10}$$

where V_0 is the velocity of the impact.

The motion of the floating plate is governed by the impact force and the hydrodynamic force (3)-(5) for small displacements of the plate $h_p(t)$:

$$(m_p + m_a)\frac{\mathrm{d}^2 h_p}{\mathrm{d}t^2} = F_{imp}(t) - \rho \left(\frac{\mathrm{d}h_p}{\mathrm{d}t}\right)^2 F_v(h_p) - F_A(h_p). \tag{11}$$

Here m_p is the mass of the plate, $h_p(t)$ is positive downwards, $F_v(h_p)$ is given by (5) for both axisymmetric and two-dimensional cases, $F_A(h_p)$ is the buoyancy force, $F_A(h_p) = \pi R_p^2 \rho g h_p$ for a circular plate of radius R_p and $F_A(h_p) = 2L\rho g h_p$ for a two-dimensional floating plate of length $2L$. Equation (11) is solved with the following initial conditions

$$h_p = 0, \qquad h_p'(0) = 0. \tag{12}$$

The indentation depth $h(t)$ is defined by

$$h(t) = h_b(t) - h_p(t). \tag{13}$$

Combining equation (9) and (11), we find the equation for the indentation depth and, therefore, for the impact force,

$$\frac{\mathrm{d}^2 h}{\mathrm{d}t^2} + D_n \left(1 + \tau \frac{\mathrm{d}}{\mathrm{d}t}\right) h^{\alpha_n} = g + \frac{\rho F_v(h_p)\, h_{p,t}^2 + F_A(h_p)}{m_p + m_a}, \tag{14}$$

where $n = 1$ correspond to the Hertz model of elastic impact, $n = 2$ and $n = 3$ correspond to the elastic foundation model for two-dimensional and axisymmetric cases respectively,

$$D_n = \left(\frac{1}{m_b} + \frac{1}{m_p + m_a}\right) K_n,$$

$$K_1 = \frac{4}{3}\frac{E\sqrt{R}}{1 - \nu^2}, \qquad K_2 = \frac{4}{3} k_s \sqrt{2R}, \qquad K_3 = \pi k_s R,$$

$$\alpha_1 = \frac{3}{2}, \qquad \alpha_2 = \frac{3}{2}, \qquad \alpha_3 = 2,$$

where R is the radius of the body curvature at the impact point, k_s is the rigidity of the elastic foundation. In the plate model with a rigid base, we have $k_s = E/H_s$ or $k_s = E/H_p$ if the whole plate is made of ice. Note that the right-hand side of (14) depends on the elements of the hydrodynamic force acting on the floating plate. This right-hand side is zero in the model of impact by Joukowski (1884). Each term on the right-hand side of (14) increases the indentation depth $h(t)$ and, therefore, increases the impact force and the body deceleration, see equation (9). The initial conditions for equation (14) are

$$h(0) = 0, \qquad h'(0) = V_0. \tag{15}$$

To estimate the maximum of the impact force during the first collision and the duration of this collision, we neglect the right-hand side in (14) and set the retardation time to zero. The resulting equation can be integrated once using the conditions (15),

$$\left(\frac{\mathrm{d}h}{\mathrm{d}t}\right)^2 + 2D_n\frac{h^{\alpha_n+1}}{\alpha_n+1} \approx V_0^2. \tag{16}$$

The maximum of $h(t)$ is achieved at t_*, when $h'(t_*) = 0$. Equation (16) at $t = t_*$ provides,

$$h_{max} = \left[\frac{\alpha_n + 1}{2D_n}\,V_0^2\right]^{\frac{1}{\alpha_n+1}}, \tag{17}$$

and the maximum of the impact force,

$$F_{imp}^{(max)} = D_n h_{max}^{\alpha_n}.$$

For $n = 1$ and $n = 2$ with $\alpha_{1,2} = 3/2$ we obtain

$$F_{imp}^{(max)} = \left(\frac{5}{4}\right)^{2/5} D_n^{3/5} V_0^{4/5},$$

and for $n = 3$ with $\alpha_3 = 2$,

$$F_{imp}^{(max)} = \left(\frac{3}{2}\right)^{1/3} D_n^{2/3} V_0^{2/3}.$$

Equation (16) in the nondimensional variables $h = h_{max}u(\tau)$, $t = T\tau$ takes the form

$$u_\tau^2 = 1 - u^{\alpha_n+1} \qquad u(0) = 0,$$

where $T = h_{max}/V_0$. Note that $T \sim V_0^{-1/5}$ for $n = 1, 2$ and $T \sim V_0^{-1/3}$ for $n = 3$. The function $u(\tau)$ is given by

$$\int_0^{u(\tau)} \frac{du_0}{\sqrt{1 - u_0^{\alpha_n+1}}} = \tau.$$

The time τ_* from the start of the impact to the maximum of the indentation depth, when $u(\tau_*) = 1$, is

$$\tau_* = \sqrt{\pi}\beta\frac{\Gamma(\beta)}{\Gamma(\beta + 1/2)}, \qquad \beta = \frac{1}{\alpha_n + 1},$$

where $\Gamma(\beta)$ is the Gamma function. Calculations yield $\tau_* \approx 1.47165$ for $n = 1$ and $n = 2$, and $\tau_* \approx 1.4023$ for $n = 3$.

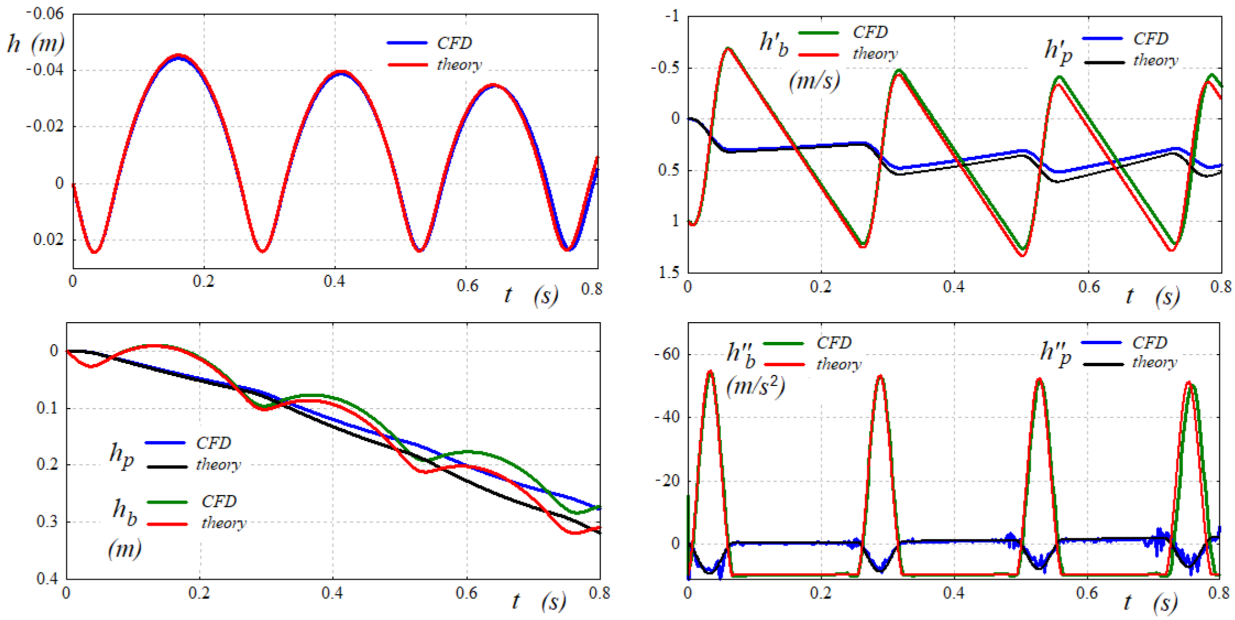

Figure 5: Comparison of the theoretical and numerical results for the plate indentation h, displacements h_p and h_b, speeds h'_p and h'_b and the accelerations h''_p and h''_b of the plate and the body, respectively, as functions of time t.

3. NUMERICAL RESULTS

The numerical results are obtained by solving the incompressible Navier-Stokes Equations using the finite-volume method. The numerical solver is developed based on the open-source CFD (Computational-Fluid-Dynamics) library, OpenFOAM. The OpenFOAM solver interDyMFoam is extended to use a direct-forcing immersed-boundary method to solve for the flow around moving bodies through an unstructured discretization of the flow domain. More details of the solver can be found in Ye at al. (2018).

For this study, the impacting body is represented by an immersed-boundary and the floating plate is modeled by the body-fitted grid. The volume-of-fluid method is used to capture the air-water interface. When contact between the two bodies occur, the collision force is modeled using equation (7). The collision force and the force due to the fluid flow are supplied to a rigid-body motion solver (Piro, 2013). The motion of the bodies are integrated using a second-order symplectic scheme.

The two-dimensional numerical results are compared with the theoretical predictions of displacements, speeds and accelerations of both the impacting body and the floating plate by equations (11) and (14). These equations are integrated numerically by the Euler method. The results of the comparisons are shown in Figure 5 for the following conditions of the impact: the half length of the floating plate $L = 2$ m, the plate thickness $H_p = 0.2$ m, the mass of the plate $m_p = 400$ kg, the rigidity of the elastic foundation $k_s = 8 \times 10^5$ kg/(m^{2}s^2), the retardation time $\tau = 0.00025$ s, the mass of the impacting body $m_b = 1000$ kg, the initial impact velocity $V_0 = 1$ m/s, the water density 1000 kg/m^3. The impacting body is parabolic with the radius of curvature $R = 5$ m.

Agreement between the theoretical and numerical results is very good, especially during the initial stage of the interaction when the plate penetration is smaller than 0.1 m. Several consequent impacts are observed with similar maximum accelerations of the impacting body and similar durations, which implies that the selected damping is negligible. Between the impacts the only force acting on the body is the gravity force, and the plate moves under the actions of the hydrodynamic and buoyancy forces. For greater retardation time every next impact is less violent. Finally the body and the plate move together without separating each other. If the rigidity of the elastic layer, k_s, increases, the time periods of interaction become shorter and deceleration of the body increases significantly. The body deceleration also increases if the mass of the impacting body decreases. It should be noticed, that in the condition of Figure 5, the maximum of the body deceleration is about $5.5g$.

4. WATER ENTRY WITH ATTACHED ICE FLOE

In this section, we assume that the two-dimensional parabolic contour $y = x^2/(2R) - h_b(t)$ of mass m_b per unit length impacts the floating rigid plate, $-L < x < L$, $y = 0$, at its centre, and then penetrates the water, $y < 0$, together with the plate. The body and the plate interact through an elastic layer with rigidity k_s. The radius R of the body is much larger than the plate length. The early stage, when the body surface is not in contact with water is short. We assume that the body surface is wetted in $L < |x| < c(t)$, where $c(0) = L$ and $c'(t) > 0$, see Figure 6. The problem is solved within the Wagner theory, where the body together with the plate is described by the equation $y = y_b(x,t)$, $y_b(x,t) = -h_p(t)$, where $|x| < L$, and $y_b(x,t) = x^2/(2R) - h_b(t)$, where $|x| > L$. Note that the shape of the fictitious body, $y = y_b(x,t)$, is not continuous at $x = \pm L$. Here $h_p(t)$ is the displacement of the plate, and $h(t) = h_b(t) - h_p(t)$ is the indentation depth of the elastic layer which is on the top of the plate.

The Wagner condition that the elevations of the water surface at the contact points $x = \pm c(t)$ are equal to the vertical coordinates of the entering body, $y_b[c(t), t]$, at these points, leads to the equation for the function $c(t)$ (Korobkin, 1996),

$$\int_0^{\pi/2} y_b[c(t)\sin\theta, t]\, d\theta = 0.$$

Evaluating the integral for our compound body, we find

$$h_b(t) = h(t)B_1(\sigma) + B_2(\sigma), \qquad \sigma = c(t)/L, \tag{18}$$

$$B_1(\sigma) = \frac{2}{\pi}\arcsin\frac{1}{\sigma}, \qquad B_2(\sigma) = \frac{L^2}{4R}\left(\sigma^2 + \frac{2}{\pi}\sqrt{\sigma^2-1} - \frac{2}{\pi}\sigma^2\arcsin\frac{1}{\sigma}\right).$$

The hydrodynamic force acting on the plate is obtained as

$$F_p(t) = \rho L^2 \frac{\mathrm{d}}{\mathrm{d}t}\Big[h_b'(t)G_b(\sigma) - h'(t)G_s(\sigma)\Big], \tag{19}$$

and the force acting on the wetted part of the main body as

$$F_b(t) = \rho L^2 \frac{\mathrm{d}}{\mathrm{d}t}\left[h_b'(t)\Big\{\frac{\pi}{2}\sigma^2 - G_b(\sigma)\Big\} + h'(t)\Big\{G_s(\sigma) - G_b(\sigma)\Big\}\right], \tag{20}$$

where

$$G_b(\sigma) = \sigma^2\arcsin\frac{1}{\sigma} + \sqrt{\sigma^2-1}, \qquad G_s(\sigma) = 2\log\sigma + 2\sqrt{\sigma^2-1}\arcsin\frac{1}{\sigma} + \sigma^2\arcsin^2\frac{1}{\sigma}.$$

The motions of the body and the plate are governed by the equations

$$m_b\frac{\mathrm{d}^2 h_b}{\mathrm{d}t^2} = -F_{imp}(t) - F_b(t), \tag{21}$$

$$m_p\frac{\mathrm{d}^2 h_p}{\mathrm{d}t^2} = F_{imp}(t) - F_p(t), \tag{22}$$

where $F_{imp}(t) = \frac{4}{3}k_s\sqrt{2R}h^{3/2}$, see equation (7), $F_{imp}(t) > 0$, where $h > 0$, and $F_{imp}(t) = 0$, where $h \le 0$, the gravity forces are not included in the Wagner model of water impact. The initial conditions for the differential equations (21) and (22) are

$$h_b(0) = 0, \qquad h_b'(0) = V_0, \qquad h_p(0) = 0, \qquad h_p'(0) = 0. \tag{23}$$

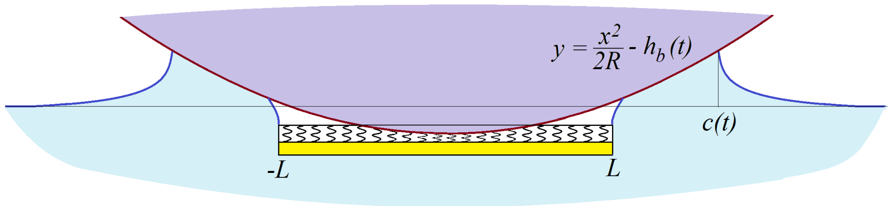

Figure 6: Impact of a large body onto a floating plate with pile-up effect.

The sum of equations (21) and (22) does not contain $F_{imp}(t)$ and can be integrated in time using conditions (23) and equations (19) and (20):

$$h_b'\Big[m_b + \rho L^2\big(\frac{\pi}{2}\sigma^2 - G_b\big)\Big] + h'\rho L^2\big[G_s - G_b\big] + m_p h_p' + \rho L^2 h_b' G_b - \rho L^2 h' G_s = V_0 m_b. \tag{24}$$

Here $\sigma = \sigma(t)$, $\sigma(0) = 1$ and $G_b(1) = \pi/2$. Using $h_p(t) = h_b(t) - h(t)$, equation (24) can be written in terms of h_b', h' and σ,

$$h_b' B_3(\sigma) - h' B_4(\sigma) = V_0 m_b, \tag{25}$$

$$B_3(\sigma) = m_b + m_p + \frac{\pi}{2}\rho L^2\sigma^2, \qquad B_4(\sigma) = \rho L^2 G_b(\sigma) + m_p.$$

Equations (19) and (22) yield

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big[m_p(h_b' - h') + h_b'\rho L^2 G_b - h'\rho L^2 G_s\Big] = F_{imp}(t). \tag{26}$$

It is convenient to introduce new function $q(t)$ by

$$q = h_b' B_4(\sigma) + h' B_5(\sigma), \qquad B_5(\sigma) = -m_p - \rho L^2 G_s(\sigma). \tag{27}$$

Then the equation (26) takes the form

$$\frac{\mathrm{d}q}{\mathrm{d}t} = F_{imp}(h), \qquad q(0) = V_0\big(m_p + \frac{\pi}{2}\rho L^2\big). \tag{28}$$

Equations (28), (27), (25) and (18) are to determine the unknown functions $q(t)$, $h_b(t)$, $h(t)$ and $\sigma(t)$. Equations (25) and (27) can be resolved with respect to h_b' and h',

$$\frac{\mathrm{d}h_b}{\mathrm{d}t} = U_b(\sigma, q), \qquad \frac{\mathrm{d}h}{\mathrm{d}t} = U_s(\sigma, q), \tag{29}$$

$$U_b = \frac{m_b V_0 B_5(\sigma) - q B_4(\sigma)}{B_3(\sigma)B_5(\sigma) + B_4^2(\sigma)}, \qquad U_s = \frac{-m_b V_0 B_4(\sigma) + q B_3(\sigma)}{B_3(\sigma)B_5(\sigma) + B_4^2(\sigma)}.$$

Equation (18) is differentiated in time and resolved with respect to $\mathrm{d}\sigma/\mathrm{d}t$ using (29),

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = Q(\sigma, q, h), \qquad \sigma(0) = 1, \tag{30}$$

$$Q(\sigma, q, h) = \frac{\pi\sigma\sqrt{\sigma^2 - 1}}{2}\,\frac{U_b(\sigma, q) - B_1(\sigma)U_s(\sigma, q)}{B_6(\sigma) - h}, \; B_6(\sigma) = \frac{L^2}{2R}\Big[\big(\frac{\pi}{2} - \arcsin\frac{1}{\sigma}\big)\sigma^2\sqrt{\sigma^2 - 1} + \sigma^2\Big].$$

The body deceleration, $h_b''(t)$, is calculated by numerical differentiation of $U_b(\sigma, t)$ in time, see (29).

If the impact is inelastic, then $h \equiv 0$ and the equations (18) and (25) provide

$$h_b = B_2(\sigma), \qquad h_b' = V_0 m_b / B_3(\sigma), \qquad \sigma = c(t)/L, \tag{31}$$

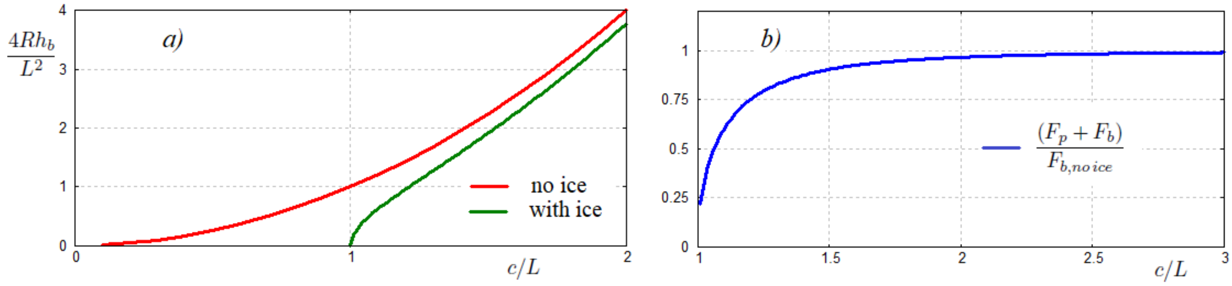

Figure 7: Non-dimensional penetration depth of the body *(a)* and the non-dimensional hydrodynamic force *(b)* as functions of the scaled length, c/l, of the wetted part of the body surface.

which correspond to equation (1) at $t = 0$. In particular, the body velocity instantly drops from V_0 to $h_b'(0)$ given by (31) at the impact instant. Equations (31) can be reduced to a single equation for $\sigma(t)$ by differentiating the first equation and then using the second one. However, it is simpler to use (30), where $h = 0$ and $U_s = 0$ now. We find

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{V_0 R}{L^2} \frac{m_b}{m_b + m_a + \pi\rho L^2\sigma^2/2} Q_i(\sigma), \qquad Q_i(\sigma) = \frac{\pi\sqrt{\sigma^2-1}/\sigma}{(\frac{\pi}{2} - \arcsin\frac{1}{\sigma})\sqrt{\sigma^2-1}+1}. \tag{32}$$

This equation can be simplified, if we introduce a new unknown function $w(t)$ by $c(t) = \sqrt{L^2+w^2}$. Then equation (32) yields

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{V_0 R m_b}{m_b + m_a + \frac{\pi}{2}\rho L^2 + \frac{\pi}{2}\rho w^2} \frac{\pi}{w\arctan(w/L)+L}, \qquad w(0) = 0. \tag{33}$$

For $c(t) \gg L$, we have $w \sim c$, $w/L \gg 1$ and equation (33) is approximated by

$$\frac{\mathrm{d}c}{\mathrm{d}t} \approx V_0 R \frac{m_b}{m_b + m_a + \frac{\pi}{2}\rho w^2} \frac{2}{c} = \frac{2V_0 R h_b'(t)}{c}. \tag{34}$$

This is the equation for the motion of the contact points in the problem of entry without a floating plate. The solution of (34) is

$$c(t) \approx 2\sqrt{Rh_b(t)}, \tag{35}$$

see Korobkin (1996). The solution (35), scaled with L, is compared to the exact relation (31) in Figure 7*a* as a function of the non-dimensional penetration depth $4Rh_b/L^2$. It is seen that the effect of the floating plate can be neglected for $c(t) > 3L$, which is for $h_b/L > 9L/(4R)$. The total hydrodynamic force, $F_p(t) + F_b(t)$, acting on the body scaled with the force calculated without the ice plate, $2\pi\rho Rh_b'$, is shown in Figure 7*b*. The impulsive force at the impact instant is not shown here. After a short initial stage, the total force $F_p + F_b$ is approximately zero and then quickly approaches the force calculated without the ice plate.

5. CONCLUSION

Impact onto a single ice floe of small dimensions floating on the water surface has been studied by theoretical and computational means. The interaction between the impacting body and the ice floe during the early stage consists of several impacts and bounces of the bodies. The body deceleration can be very high during this stage. During the following main stage of the interaction, the body penetrates the water together with the plate sliding along the body surface and slightly changing the hydrodynamic loads. It was shown that the presence of the attached ice floe is negligible when the size of the body wetted area is three times larger than the size of the floe.

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