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# The Economic Value of Bitcoin: A Portfolio Analysis of Currencies, Gold, Oil and Stocks*

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Economic Gains of US$1 Invested in Portfolios Including and Excluding Bitcoin With 50bp Proportional Transaction Costs with Daily Rebalancing
Equal-Weighted (EW)
Currencies
Gold
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Mixed
Unconstrained Global Minimum-Variance (GMV)
Short-sale Constrained Global Minimum-Variance (CGMV)
Short-sale Constrained Global Minimum-Variance with DCC Covariance Forecasts (CGMV-DCC)
Wealth
Time
2013
2014
2015
2016
2017
Incl. Bitcoin
Excl. Bitcoin

# The Economic Value of Bitcoin: A Portfolio Analysis of Currencies, Gold, Oil and Stocks

**Abstract**

We assess the out-of-sample performance of Bitcoin within portfolios of various asset classes and a well-diversified portfolio under four strategies and estimate the economic gains net of transaction costs. We find statistically significant diversification benefits from the inclusion of Bitcoin which are more pronounced for commodities. Most importantly, the decrease in the overall portfolio risk due to the low correlation of Bitcoin with other assets is not offset by its high volatility. However, the inclusion of Bitcoin pays off little if investors accommodate a battery of economic instruments. Considering non-bubble conditions that are not marked by explosive prices in cryptocurrencies, we document substantially diminished benefits.

Keywords: Bitcoin, Portfolio, Economic value, Non-Bubble, Dynamic Conditional Correlation

# 1. Introduction

Cryptocurrencies offer an increasingly popular decentralized transaction system backed with blockchain technology. The price of Bitcoin, the most widely accepted cryptocurrency, has soared since 2009 climbing from a few cents up to US$14,500 by the end of 2017 increasing its total market value up to 250 billion[1] and counting from 2.9 up to 5.8 million active users (wallets) (Hileman and Rauchs, 2017). Not surprisingly, the rapid growth of cryptocurrencies along with the wide mainstream media coverage have attracted interest from market participants and warnings from policy makers[2]. However, the literature on cryptocurrencies' properties and the potential benefits for investors is still in its infancy. Recent developments such as the launch of the first Bitcoin futures[3] and the higher participation of institutional investors also call for more extensive research in the field.

Our study fills the knowledge gap by undertaking a comprehensive investigation of Bitcoin as a mainstream financial asset in risky portfolios of various assets employing traditional performance metrics to estimate the value added under a battery of trading strategies. Previous empirical analyses (Wu et al., 2014; Brière et al., 2015) come with several limitations as potential benefits of Bitcoin in multi-asset portfolios are examined in-sample without employing statistical tests or assessing the economic value. Moreover, dynamic linkages among assets and transaction costs are ignored. Finally, the findings

[1] As of December 2017. The market capitalization of Bitcoin is taken from Coin Dance (https://coin.dance/stats/), while the Bitcoin Price Index is taken from CoinDesk (https://www.coindesk.com/price/).

[2] Financial Action Task Force (2014) connects virtual currencies to increasing risk for money laundering and potential financing of terrorism (http://www.fatf-gafi.org/media/fatf/documents/reports/Virtual-currency-key-definitions-and-potential-aml-cft-risks.pdf). Similar concerns are also communicated in the Opinion report of European Banking Authority (2014) (https://www.eba.europa.eu/documents/10180/657547/EBA-Op-2014-08+Opinion+on+Virtual+Currencies.pdf).

[3] CME Group launched on December 18, 2017 the CME Bitcoin Reference Rate (BRR) along with the CME Bitcoin Spot Price Index.

are based on limited samples characterized by a long-lasting bubble that questions the economic benefits of Bitcoin (Cheung et al., 2015).

Our contribution is manifold. First, using performance metrics such as the Sharpe ratio, portfolio risk and certainty equivalent, we compare out-of-sample benchmark portfolios with respective portfolios that invest in Bitcoin employing the robust inference tests of Ledoit and Wolf (2008, 2011). In so doing, we answer whether Bitcoin offers statistically higher benefits in an out-of-sample context. Second, we examine the sensitivity of our findings under four portfolio strategies. In addition to the popular equal-weighted portfolio strategy, we consider global minimum-variance portfolio strategies with and without short-sale constraints to evaluate Bitcoin from the scope of a risk-averse investor that is reluctant to the high volatility but desires the low correlation with other assets. To account for volatility clustering and time-varying linkages between Bitcoin and various assets, we forecast one-period ahead conditional correlation based on a multivariate GARCH model extending the findings of Corbet et al. (2018).

Third, we contribute to the literature by estimating the economic gains of these strategies net of transactions costs with frequent daily and weekly rebalancing. Accounting for transaction costs offers useful insights since it has been shown that they can significantly shrink portfolio gains (Balduzzi and Lynch, 1999; DeMiguel et al., 2007). There are also practical implications since cryptocurrencies are particularly appealing to myopic investors who expect profits only from the difference in price in a bullish market with no other financial flows or interest rates from holding them. Fourth, in addition to a well-diversified portfolio that accommodates various asset classes, we extend our findings to study the impact of Bitcoin on less diversified portfolios of traditional and alternative assets. In so doing, we demonstrate the economic significance of Bitcoin as an investment vehicle and answer whether there is merit in introducing cryptocurrencies within portfolios of exchange rates, gold, oil and stocks.

Fifth, we extend the literature using an updated sample spanning from 2011 to 2017. Previous empirical analyses are based on small samples covering the period from 2010 up to 2013 (Wu et al., 2014; Brière et al., 2015). As pointed by Cheung et al. (2015) this period is characterized by a long-lasting bubble that questions the positive impact of Bitcoin. In our sample, the sustainability of Bitcoin has been tested under bullish markets and bubbles, crashes, negative bubbles with long-lasting pessimism and various exogenous market shocks such as the Mt. Gox hack, the closure of Silk Road platform, cyber attacks and the regulations imposed to Chinese banks against Bitcoin investment (Trautman, 2014; Böhme et al., 2015; Cheah and Fry, 2015; Fry and Cheah, 2016; Blau, 2017). This period also corresponds with a great turn of market participants, both retail and institutional investors in cryptocurrencies.[4] In particular, the inflow of short-term investors and noise traders is magnified by the availability of Bitcoin in fractions (up to 8 decimal places) without requiring capital in large bundles as happens with other economic instruments. The growing interest of market participants is also evident from recent advances in derivative markets with the launch of the first Bitcoin future contract which justifies further the context of this analysis and exhibits the managerial implications for funds, investors, institutions, governments and regulatory authorities.

Finally, we examine the diversification opportunities of Bitcoin in a sub-period that is not marked by dramatic price rises. To this end, we employ the multiple bubble test of Phillips et al. (2015) to investigate whether diversification benefits reported in the literature persist when returns are not governed by speculative trading and irrational

[4]Following the Crypto Survey Results of Triad Securities Corp. and DataTrek Research, LLC from November 6, 2017 to November 13, 2017 (http://www.triadsecurities.com/survey/crypto_results/).

behaviour or market inefficiencies such as illiquidity.

In contrast to previous studies, we demonstrate that the value added and the potential for diversification are less prominent in well-diversified portfolios consisting of currency, gold, oil, stock, real estate and bond indices. When it comes to individual asset classes, portfolios with commodities such as gold and oil are most benefited with statistically higher risk-adjusted returns and lower volatility. Bitcoin is also found to be an attractive investment accompanied with economic gains net of transaction costs when investors are reluctant to high risk exposure investing, therefore, in the minimum-variance portfolio. We show that the low correlation of Bitcoin with other asset classes leads to significant portfolio risk reduction, notably that Bitcoin is the most volatile asset in our sample. In a mean-variance strategy that considers dynamic linkages between assets, we provide evidence of a higher contribution of Bitcoin but this requires more frequent rebalancing that soars transaction costs. The study of a sub-period that does not include a long-term bubble reveals that the potential of Bitcoin does not completely vanish in all but the multi-asset portfolio.

Our study adds to the growing literature that studies the cruptocurrency market. In addition to papers that analyze the opportunities and regulations of virtual currencies (e.g., Stokes, 2012; Böhme et al., 2015; Raymaekers, 2015; Vandezande, 2017; Pieters and Vivanco, 2017), there is a stream in the literature that investigates cryptocurrencies as investment vehicles from various perspectives. Significant effort is gathered on answering whether Bitcoin acts entirely as an alternative currency or it maintains similar properties to commodities or speculative assets (e.g., Yermack, 2013; Glaser et al., 2014; Dyhrberg, 2016; Bouri et al., 2017; Blau, 2017; Baur et al., 2018). Other studies measure returns and volatility (e.g., Balcilar et al., 2017; Katsiampa, 2017; Peng et al., 2018), interdependencies (e.g., Ciaian et al., 2018; Corbet et al., 2018; Symitsi and Chalvatzis, 2018), and market inefficiencies of cryptocurrencies (e.g., Urquhart, 2016; Nadarajah and Chu, 2017). We also contribute to the literature that examines diversification benefits in alternative investments such as commodities or futures (e.g., Cheung and Miu, 2010; Baur and Lucey, 2010; Ciner et al., 2013; Bessler and Wolff, 2015; Gao and Nardari, 2018), the effect of transaction costs (e.g., Balduzzi and Lynch, 1999; DeMiguel et al., 2007), and portfolio allocation using time variant covariance forecasts (e.g., Gao and Nardari, 2018).

The rest of the study is organized as follows: Section 2 describes the data used in our empirical application, Section 3 presents the methodology and Section 4 discusses the empirical findings. The final Section includes the main conclusions from our analysis.

## 2. Data

We use daily prices for Bitcoin taken from Coindesk for the period spanning from September 20, 2011 to July 14, 2017. The Coindesk Bitcoin Price Index (BPI) averages the Bitcoin prices from four major exchanges namely Bitstamp, Coinbase, itBit and Bitfinex.[5] We collect prices for various risky assets and form four asset class portfolios, namely, exchange rates, gold, oil, and a diversified pool of stocks. We select more than one assets in each asset class portfolio, even in cases they are highly correlated, to reduce the

[5]Employing a price index that blends trading prices of the top Bitcoin exchanges is a standard practice in literature (e.g., see Dyhrberg, 2016; Katsiampa, 2017; Baur et al., 2018). In our robustness checks, we replicate the analysis to account for the risk across exchange platforms using *Thomson Reuters Datastream* Bitcoin prices from Bitstamp Exchange, one of the longest standing Bitcoin marketplaces established in 2011. The correlation of Bitstamp and Coindesk Bitcoin prices is above 99 percent, but the descriptive statistics indicate that this is more volatile with higher extremes. The rationale behind this robustness test is based on the large number of cryptocurrency exchange platforms with differences in depth and market capitalization that raise issues for temporary inefficiencies in prices. Despite the findings of Gandal and Halaburda (2014) who fail to report strong cross-exchange arbitrage opportunities, as pointed by Böhme et al. (2015), risks in cryptocurrency markets are not only related to *shallow markets problem*, but also to the *counterpart risk* (e.g., temporary cease of operations or even closures of exchanges due to security breach, hacks, or denial-of service attacks). We find that an analysis based on Bitstamp Bitcoin prices yields similar conclusions. All the robustness results are available upon request.

idiosyncratic risk from particular components.

In particular, for the currency portfolio we use exchange rates in US Dollar for Australian Dollar (AUD), Euro (EUR), British Pound (GBP), New Zealand Dollar (NZD), Canadian Dollar (CAD), Swiss Franc (CHF), and Japanese Yen (JPY). For gold, we employ the NYSE Arca Gold BUGS Index (HUI), the PHLX Gold/Silver Sector Index (XAU), the Market Vectors Gold Miners ETF (GDX) and the SPDR Gold Shares (GLD). We gather data for three assets related to oil, the NYMEX Light Crude Oil (Pit), the Market Vectors Oil Services ETF (OIH) and the United States Oil Fund (USO). To invest in a portfolio of stocks, we gather data for the Dow Jones Industrial Average (INDU) and the SPDR S&P 500 Growth ETF (SPY). We also maintain a well-diversified portfolio that is composed of assets from all categories including a real estate and a bond index (mixed portfolio). This multi-asset portfolio involves investment in US Dollar Index (DXY), HUI, Pit, INDU, PHLX Housing Sector Index (HGX) and 30-year Treasury Bond Index (TYX).

We investigate the value added from Bitcoin in each of the risky asset classes due to their importance as investment options. In addition to stocks, we study currencies because of the monetary role and properties of Bitcoin. Gold is also an important asset class that serves both as an industrial raw material (commodity) and a store of value during financial distress, currency devaluation and inflation (Capie et al., 2005; Baur and Lucey, 2010). Oil is a significant commodity in industrial production that is connected in the literature with stock price movements and exchange rates (Reboredo, 2012; Ciner, 2013). These assets altogether have various properties in terms of the distribution of their returns offering the chance to study the economic value of Bitcoin under different circumstances.

**Table 1 Description of Variables**

| Variable | Description | Mean (%) | StDev (%) | Min (%) | Max (%) | Skew | Kurt |
|---|---|---|---|---|---|---|---|
| **AUD** | AUDUSD | -0.0160 | 0.6708 | -4.0200 | 3.3300 | -0.0865 | 2.7072 |
| **EUR** | EURUSD | -0.0103 | 0.5615 | -2.2300 | 2.6400 | 0.1013 | 1.8473 |
| **GBP** | GBPUSD | -0.0113 | 0.5642 | -7.9500 | 2.8100 | -1.7572 | 27.4827 |
| **NZD** | NZDUSD | -0.0050 | 0.7349 | -4.3200 | 3.5100 | -0.0808 | 2.5159 |
| **CAD** | CADUSD | -0.0148 | 0.5108 | -2.9100 | 2.9100 | 0.0732 | 2.7139 |
| **CHF** | CHFUSD | -0.0035 | 0.6660 | -2.4900 | 12.1000 | 4.5993 | 77.2008 |
| **JPY** | JPYUSD | -0.0236 | 0.6205 | -3.6200 | 3.4900 | 0.0137 | 3.9200 |
| **DXY** | US Dollar Index | 0.0149 | 0.4496 | -2.3700 | 2.0500 | -0.0342 | 1.9753 |
| **GDX** | Market Vectors Gold Miners ETF | -0.0392 | 2.5828 | -10.7700 | 11.2400 | 0.0547 | 1.2331 |
| **GLD** | SPDR Gold Shares | -0.0216 | 1.0276 | -8.7800 | 4.9100 | -0.5606 | 6.1752 |
| **HUI** | NYSE Arca Gold BUGS Index | -0.0440 | 2.6674 | -12.0500 | 11.5700 | 0.1036 | 1.2966 |
| **XAU** | PHLX Gold/Silver Sector Index | -0.0343 | 2.5184 | -10.2800 | 10.4300 | 0.0589 | 1.1984 |
| **OIH** | Market Vectors Oil Services ETF | -0.0188 | 2.1169 | -10.2300 | 12.3200 | 0.3641 | 3.5503 |
| **Pit** | NYMEX Light Crude Oil | -0.0166 | 1.7987 | -8.9100 | 10.6200 | 0.1062 | 2.0620 |
| **USO** | United States Oil Fund | -0.0634 | 1.9393 | -8.3200 | 9.2400 | 0.1519 | 2.3723 |
| **INDU** | Dow Jones Industrial Average | 0.0453 | 0.7961 | -3.5700 | 4.2400 | -0.1268 | 2.6773 |
| **SPY** | SPDR S&P 500 Growth ETF | 0.0535 | 0.8357 | -4.0000 | 4.3100 | -0.2690 | 2.5030 |
| **HGX** | PHLX Housing Sector Index | 0.0890 | 1.4176 | -6.3300 | 6.0200 | -0.1104 | 1.8802 |
| **TYX** | 30-year Treasury Bond Index | 0.0097 | 1.6145 | -8.3251 | 9.7486 | 0.2721 | 2.1672 |
| **BPI** | Bitcoin Price Index (Coindesk) | 0.5392 | 5.5104 | -35.8400 | 64.8200 | 0.9767 | 19.2864 |

This table describes the risky assets that are considered in the empirical analysis along with basic statistics of their daily returns multiplied by 100, namely mean, standard deviation, median, skewness and kurtosis. All assets prices are nominated in US Dollars. The sample covers the period 20/09/2011-14/07/2017.

Our final dataset consists of 1,519 price observations for all assets. Table 1 displays the mean, standard deviation, median, skewness and kurtosis of daily returns multiplied by 100. Exchange rates have negative average returns that describe the stronger US Dollar the period we test. This is also reflected to DXY which invests in US Dollar against a basket of foreign currencies. Only the portfolios of stocks, real estate and bonds have positive mean returns. With an upward trend in stock market, gold as a store of value generated negative returns on average (see Baur and McDermott, 2010, for a description of the linkages between gold and stock market). The decreasing prices in oil could also fuel positive returns in stock markets (e.g., see Kilian and Park, 2009; Ciner et al., 2013). The average annualized return of the Bitcoin Price Index is 136% but it is accompanied with very high risk. The annualized standard deviation is 87% which is more than double the risk of the gold index HUI, the next most volatile risky asset in our dataset. The marginal distribution of BPI returns is heavy-tailed exhibiting extreme observations with

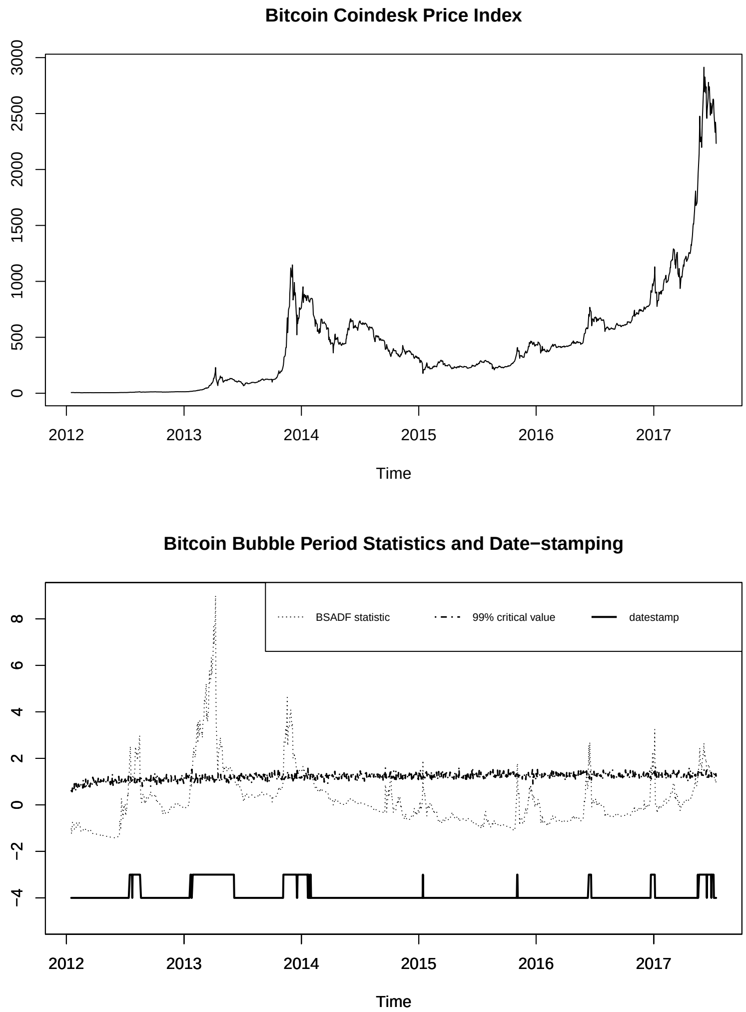


**Figure 1 Bitcoin Price Index and Bubble Date-Stamping** The top figure shows the Coindesk Bitcoin Prices for the period from 20/09/2011 to 14/07/2017. The bottom figure presents the backward sup ADF statistics (dotted line) along with 99% critical values (dot-dash line) and a dummy variable that stamps the date a bubble is captured (solid line).

large positive skewness and kurtosis. This is in line with the cryptocurrencies' rally noted during the period under examination with prices ranging from 2.05 up to US$2,914.08. ,

Bitcoin has experienced plenty of bubbles and crashes since its foundation. Similar to Cheung et al. (2015) and Baur et al. (2018), we employ the methodology of Phillips et al. (2015) that allows the identification of multiple bubbles in a sample period. In particular, we apply the backward sup ADF test using 99% critical values obtained from Monte Carlo simulation.[6] As shown in Figure 1, there are short-lived bubbles from 1 to 5 days and longer period bubbles such as the July-August 2012, January-June 2013, November 2013-



[6]Matlab code for this test is taken from the website of Shuping Shi (https://sites.google.com/site/shupingshi/home/research).

January 2014 and June-July 2017. Major events in cryptocurrency markets coincide with the burst and collapse of these bubbles such as the Mt. Gox failure to cover the increasing demand for Bitcoin after Cyprus's bailout in April 2013, major cryptocurrency exchanges attacks and Mt. Gox collapse in February 2014, and increasing interest of mainstream investors in mid 2017.

The hedging properties and correlation across the asset classes are extensively studied in the literature (Ciner et al., 2013; Caporale et al., 2014). The unconditional correlation of BPI with the other risky assets is remarkably low and insignificant suggesting that the inclusion of BPI could increase the diversification benefits of a portfolio. This is very important given the stronger positive associations between the risky assets in currencies, gold, oil and stock portfolios with average correlations 0.4503, 0.9429, 0.8159, and 0.9257, respectively. Adding Bitcoin within these portfolios, the average correlations reduce to 0.3488, 0.7907, 0.5187 and 0.4982, respectively. The multi-asset portfolio includes significant negative correlations of the risky assets with the US Dollar Index as well as insignificant correlation between the bond index and the gold, oil, stock and real estate indices.[7] The average correlations in the mixed asset portfolio is 0.0812 and reduces only to 0.0628 with the inclusion of Bitcoin.

We examine the volatility dynamics of Bitcoin with every asset in our sample accounting for volatility clustering and time-varying dependencies. The dynamic conditional correlations estimated by a multivariate GARCH model in Figure 2 document that volatility spills over from Bitcoin to several assets in periods that coincide with major events in cryptocurrency markets. For instance Bitcoin prices are sensitive to the closure of Silk Road and Silk Road 2.0 in October 2013 and November 2014, respectively, the collapse of

[7] The strong negative correlation of gold and US Dollar Index is indicative of the safe heaven properties of gold.

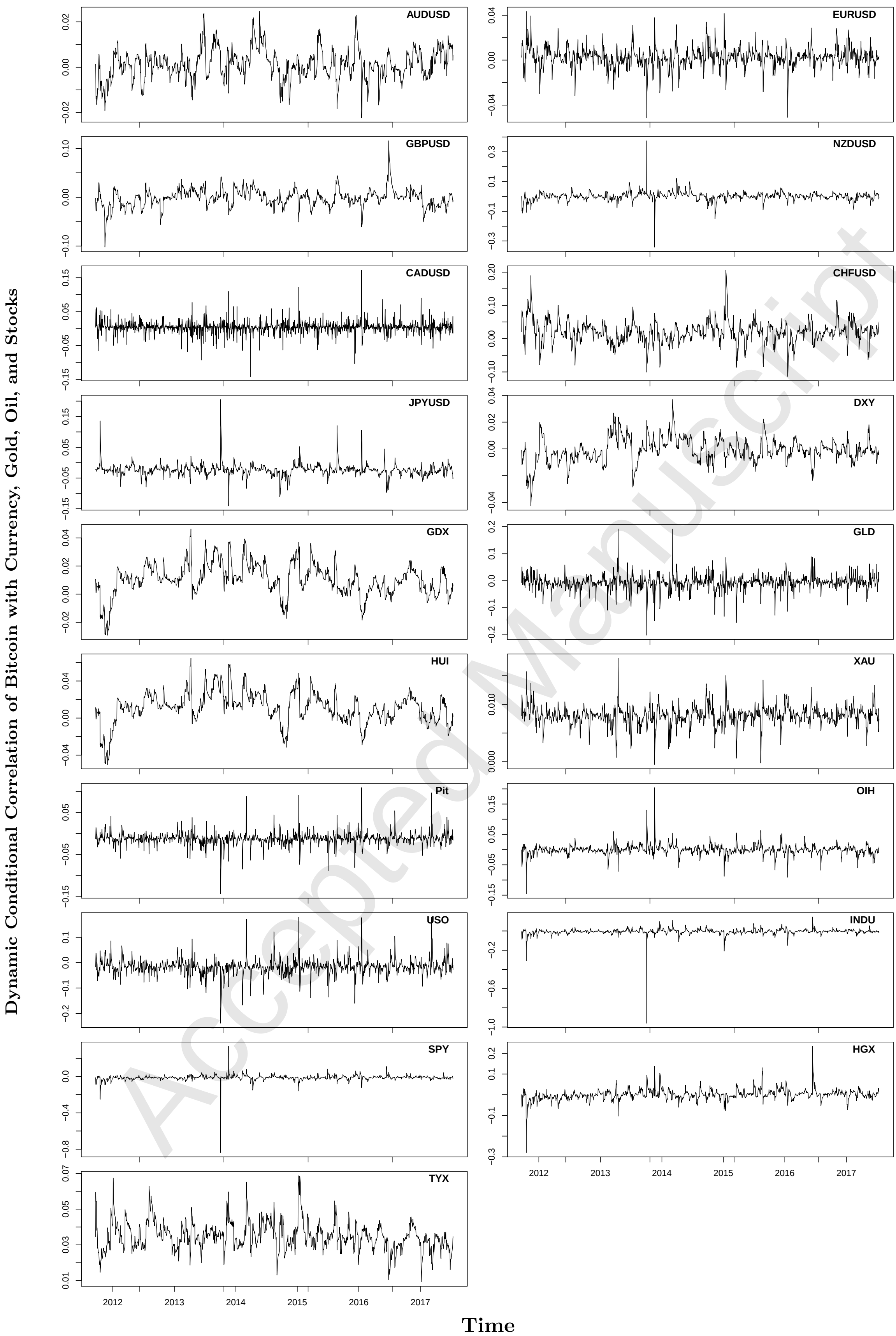


**Figure 2 Dynamic Conditional Correlation of Bitcoin with Currency, Gold, Oil, Stocks, Real Estate and Bonds** The figures display the dynamic conditional correlations of Bitcoin with each of the assets under examination in the period spanning from 20/09/2011 to 14/07/2017.

Mt. Gox in February 2014, the hack of Bitfinex exchange in August 2016. High reaction of Bitcoin prices is also observed with exogenous events such as the financial crisis of Cyprus and the British EU referendum in 2016.

# 3. Methodology

In this section, we describe the portfolio strategies and performance measures we use to compare the benchmark portfolios of currencies, gold, oil, stocks, and mixed against respective portfolios including Bitcoin. Our purpose is to understand whether there are diversification opportunities and sizeable gains by investing in cryptocurrencies.

## 3.1. Portfolio Strategies

### 3.1.1. Equal-Weighted Portfolio

We consider the equal-weighted portfolio, an easy to implement and widely applied strategy. DeMiguel et al. (2007) show that the naive 1/N strategy that does not involve any optimization often outperforms optimal strategies. Inaccurate return and risk forecasting models along with increased sample error are commonly used to interpret this finding.

### 3.1.2. Global Minimum-Variance Portfolio

The global minimum-variance portfolio solves the following optimization problem:

$$min\, w_t'\Sigma_t w_t \quad \text{s.t.} \quad w_t'I = 1, \tag{1}$$

where $w_t$ is an $N \times 1$ vector of portfolio weights; $\Sigma_t$ is the $N \times N$ covariance matrix; and $I$ is an $N \times 1$ vector of ones. The constraint ensures that the sum of portfolio weights is equal to one allowing negative weights. The optimal weights of the GMV portfolio are, then, given by:

$$w_t = \frac{\Sigma_t^{-1} I}{I' \Sigma_t^{-1} I} \tag{2}$$

The solution for the optimal portfolio weights depends only on the sample variance-covariance matrix resulting in two benefits from the consideration of the minimum-variance portfolio. First, ignoring the expected returns which are based on sample means the estimation error is reduced (DeMiguel et al., 2007). Second, we focus solely on the variance of Bitcoin and its covariance with the other assets in the portfolio. This is extremely important following the recent upward trends in Bitcoin, which would favour its weights.

#### 3.1.3. Constrained Global Minimum-Variance Portfolio

Similar to DeMiguel et al. (2013), we consider the constrained global minimum-variance portfolio (CGMV) which imposes short-selling constraints by setting:

$$w_t \geq 0 \tag{3}$$

This is in line with a stream in the literature that suggests that the out-of-sample performance of the minimum-variance portfolio can be improved imposing constraints on the weights (Jagannathan and Ma, 2003; Ledoit and Wolf, 2004).

#### 3.1.4. Constrained Global Minimum-Variance Portfolio with Dynamic Conditional Correlation Forecasts

To accommodate time-varying interdependencies between assets and mitigate concerns for inaccurate estimation of covariance matrix, we also employ the constrained global minimum-variance portfolio strategy with covariance forecasts estimated by the Dynamic Conditional Correlation model (Engle, 2002) from the multivariate GARCH family

(CGMV-DCC). The one-period ahead covariance matrix, $\hat{\Sigma}_{t+1|t}$ is decomposed as:

$$\hat{\Sigma}_{t+1|t} = \hat{D}_{t+1|t}\hat{R}_{t+1|t}\hat{D}_{t+1|t} \tag{4}$$

$$\hat{R}_{t+1|t} = \hat{V}_{t+1|t}^{-1}\hat{Q}_{t+1|t}\hat{V}_{t+1|t}^{-1} \tag{5}$$

$$\hat{Q}_{t+1|t} = (1-\alpha-\beta)\bar{Q} + \alpha z_t z_t' + \beta Q_t, \tag{6}$$

where $\hat{D}_{t+1|t} = diag\{\sqrt{\hat{\sigma}_{11,t+1|t}}, \sqrt{\hat{\sigma}_{22,t+1|t}}, ..., \sqrt{\hat{\sigma}_{NN,t+1|t}}\}$ is a diagonal matrix with the square root of conditional variances ($\hat{\sigma}_{ii,t+1|t}$) of the $N$ assets on the main diagonal modelled through univariate GJR-GARCH(1,1) processes (Glosten et al., 1993). $\hat{R}_{t+1|t}$ is the $N \times N$ unconditional correlation matrix of the $z_{it}$ standardized residuals given by $z_{it} = e_{it}/\sqrt{\sigma_{ii,t}}$, where $e_{it}$ are the innovations from a random walk model for returns. Positive conditional variances and the positive definite matrix $\hat{R}_{t+1|t}$ ensure the positive definiteness of $\hat{\Sigma}_{t+1|t}$. $\hat{V}_{t+1|t} = diag\{\sqrt{\hat{q}_{11,t+1|t}}, \sqrt{\hat{q}_{22,t+1|t}}, ..., \sqrt{\hat{q}_{NN,t+1|t}}\}$ is a diagonal matrix with the square root of quasi-correlations ($\hat{q}_{ij,t+1|t}$). The quasi-correlations $\hat{q}_{ij,t+1|t}$ of $\hat{Q}_{t+1|t}$, re-scaled within [-1,1], are used to calculate conditional correlations as $\hat{\sigma}_{ij,t+1|t} = \hat{q}_{ij,t+1|t}/\sqrt{\hat{q}_{ii,t+1|t}\hat{q}_{jj,t+1|t}}$. $\bar{Q}$ is the unconditional covariance matrix of the $z's$. DCC forecasts are generated through a two-step process: (i) we predict each element on the main diagonal of $D_t$ with univariate GJR-GARCH(1,1) models, (ii) we use residuals of returns from a random walk model standardized by their conditional standard deviations and feed them into a multivariate GARCH model to obtain the conditional correlation matrix $\hat{R}_{t+1|t}$. Employing DCC forecasts in portfolio covariance forecasting is also applied in Gao and Nardari (2018).

### 3.2. Portfolio Performance Measures

We assess the out-of-sample performance of the portfolios based on five criteria: (i) out-of-sample portfolio variance; (ii) out-of-sample Sharpe ratio; (iii) certainty equivalent return of an investor with quadratic utility and risk aversion parameter $\gamma = 1$; (iv) portfolio turnover; and (v) return loss.

We consider short-term investing horizons with daily (1-day) and weekly (5-day) rebalancing to increase the statistical power of our analysis. To compute the portfolio weights, we use a rolling window method with the in-sample estimation period for daily (weekly) data being $h = 250$ days ($h = 60$ weeks). The covariance matrix is estimated based on 250-day rolling samples that discard the oldest and include the newest observation in each step (historical covariance). One period ahead DCC forecasts are produced using the parameters of the model with the most recent 250 observations, then move forward and repeat the process.[8] Thus, the out-of-sample observations are 1,268 and 243 with daily and weekly rebalancing, respectively. For the $1/N$ strategy the weights remain the same throughout the period. For each portfolio in the minimum-variance strategies we compute the optimal portfolio weights using (2) and, then, we estimate the out-of-sample daily portfolio return at time $t+1$ as $r_{t+1}^{portfolio} = w_{i,t}r_{i,t+1}$, where $r_{i,t+1}$ are the returns for each asset $i$ in the portfolio at time $t+1$.

Based on the computed time series of out-of-sample portfolio returns ($r_{t+1}^{portfolio}$) we estimate the variance ($\hat{VR}^{portfolio}$), Sharpe ratio ($\hat{SR}^{portfolio}$), and certainty equivalent

[8]The analysis is replicated with 500 rolling sample observations.

return ($\hat{CE}^{portfolio}$) as follows:

$$\hat{VR}^{portfolio} = 1/(T-h-1-k)\sum_{t=h}^{T-1}\left(r_{t+1}^{portfolio} - \mu^{portfolio}\right)^2 \tag{7}$$

$$\hat{SR}^{portfolio} = \hat{\mu}^{portfolio}/\hat{\sigma}^{portfolio} \tag{8}$$

$$\hat{CE}^{portfolio} = \hat{\mu}^{portfolio} - (\gamma/2)\hat{VR}^{portfolio}, \tag{9}$$

where $\hat{\mu}^{portfolio}$ is the average out-of-sample portfolio return calculated as $\hat{\mu}^{portfolio} = 1/(T-h-1)\sum_{t=h}^{T-1} r_{t+1}^{portfolio}$; $\hat{\sigma}^{portfolio}$ is the portfolio volatility gauged as the square root of variance; $k$ is the rebalancing period (either 1-day or 5-day); and $N$ is the number of assets in each portfolio. The Sharpe ratio measures the risk-adjusted returns while portfolio variance is a measure of risk. According to the expected utility theory, risk aversion is the rational attitude towards risk. Under this assumption, the certainty equivalent of returns is the certain return that an investor would accept rather than investing to the uncertain returns of a portfolio with risky assets. Based on the increasing and concave utility function that summarizes the preferences of risk-averse investors, an approximation of the $\hat{CE}^{portfolio}$ is given by (9), where $\gamma$ corresponds to the risk tolerance. It follows that the risk premium for a risk-averse investor should be positive.

To compare the differences in the Sharpe ratio and certainty equivalent return of the Bitcoin portfolios from the respective benchmark portfolio that excludes the cryptocurrency, we perform the robust statistical tests of Ledoit and Wolf (2008). The test for differences in portfolio variance follows the robust method developed in Ledoit and Wolf (2011). The p-values are inferred via bootstrapping methodology with 5,000 trials and block size 10.[9]

[9] The p-values are not sensitive with various block sizes and trials. The code for bootstrap inference in R is taken from the website of Professor Michael Wolf, University of Zyrich (http://www.econ.uzh.ch/en/people/faculty/wolf/publications.html).

The portfolio turnover measures the average change of the wealth traded in each day and reflects a metric of portfolio stability. This is computed as follows:

$$\hat{\tau}^{portfolio} = 1/(T-h-1)\sum_{t=h}^{T-1}\sum_{i=1}^{N}\Big(|w_{i,t+1} - w_{i,t^+}|\Big), \tag{10}$$

where $w_{i,t^+}$ is the weight invested in asset $i$ before rebalancing at the beginning of $t+1$ and $w_{i,t+1}$ is the desired portfolio weight at $t+1$. $w_{i,t^+}$ differs from $w_{i,t}$ since the prices change from $t$ to $t+1$.

We also report the return loss (gain) from investing to portfolios augmented with Bitcoin against the benchmark portfolios. The return loss measures the extra return required from the Bitcoin portfolio to perform equally well to the benchmark portfolio and is estimated as:

$$Return \quad Loss_s = \hat{\mu}_b \hat{\sigma}_s / \hat{\sigma}_b - \hat{\mu}_s, \tag{11}$$

where $s$ is the strategy that includes Bitcoin, and $b$ represents the strategy of the benchmark portfolios. A positive return loss indicates that in the presence of transaction costs the benchmark portfolio risk-adjusted return outperforms that of the Bitcoin portfolio and vice versa.

### 3.3. Economic Gains

We estimate the economic value of each strategy net of transaction costs. Setting transaction costs equal to 50 basis points ($c = 0.0050$) (similar to Balduzzi and Lynch, 1999; DeMiguel et al., 2007; Kirby and Ostdiek, 2012) and under the assumption that this is constant across various assets, we estimate the evolution of our initial wealth of US\$1 invested in the tested portfolios under the four investing strategies. This is a fair conjecture following the findings of Kim (2017) who estimates that the trading costs of Bitcoin are somewhat less than the trading costs of other financial assets.[10] The wealth $W_{t+1}$ evolves as follows:

$$W_{t+1} = W_t\Big(1 + r_{t+1}^{portfolio}\Big)\Big(1 - c\sum_{i=1}^{N}(|w_{i,t+1} - w_{i,t^+}|)\Big) \tag{12}$$

# 4. Empirical results

## 4.1. Bitcoin and Portfolio Performance

In this section, we present the main findings of our analysis. Table 2 describes the weights of each trading strategy for daily and weekly rebalancing rounded at the fourth decimal point. The composition of the equal-weighted strategy ($EW$) remains constant over time. We present the mean, standard deviation, minimum and maximum weights (in percentage) for the global minimum-variance ($GMV$) and the short-sale constrained global minimum-variance strategies with historical ($CGMV$) and DCC covariance forecasts (*CGMV-DCC*). Bitcoin has higher and more volatile contribution to portfolios of oil and gold. This is explained by the high volatility of these instruments that is accompanied with downward price movements. For the optimal strategies that the decision can be either a negative weight (unconstrained strategy) or a zero investment (constrained strategies), we also present the number of times (in percentage) Bitcoin has a positive weight scaled by the total number of trading periods (see column *Inv* in Table 2).

While there are differences across portfolios which are attributed to the particular characteristics of the included assets during the sample period, there are consistencies in the findings. By and large, the high volatility of Bitcoin is penalized in the minimum-variance strategies allowing small proportions of the cryptocurrency with average weight

[10] See also percentage fee on Bitcoin.com charts (https://charts.bitcoin.com/chart/fee-percentage#0). In our robustness checks we also examine for proportional transaction costs equal to 100 basis points. As expected, higher proportional transaction costs yield a parallel downward shift of the economic gains for all strategies but the effect of Bitcoin on each portfolio is preserved.

**Table 2 Description of Portfolio Weights**

| | EW | GMV | | | | | CGMV | | | | | CGMV-DCC | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Mean | SD | Min | Max | Inv | Mean | SD | Min | Max | Inv | Mean | SD | Min | Max | Inv |
| *Panel A: Daily Rebalancing* | | | | | | | | | | | | | | | | |
| **Currencies** | 12.50 | 1.00 | 0.74 | 0.02 | 3.17 | 100.00 | 1.00 | 0.75 | 0.02 | 3.22 | 100.00 | 1.57 | 1.73 | 0.00 | 10.24 | 99.13 |
| **Gold** | 20.00 | 4.30 | 2.57 | 0.00 | 10.25 | 96.53 | 4.30 | 2.57 | 0.00 | 10.25 | 96.53 | 8.00 | 7.46 | 0.00 | 54.51 | 99.13 |
| **Oil** | 25.00 | 14.35 | 10.86 | 1.74 | 36.70 | 100.00 | 14.38 | 10.82 | 1.74 | 36.70 | 100.00 | 18.93 | 16.29 | 0.17 | 71.78 | 100.00 |
| **Stocks** | 33.33 | 3.43 | 2.57 | 0.16 | 9.67 | 100.00 | 3.81 | 2.76 | 0.12 | 10.40 | 100.00 | 5.45 | 5.05 | 0.00 | 25.68 | 98.50 |
| **Mixed** | 14.28 | 0.50 | 0.55 | 0.00 | 1.83 | 66.06 | 2.39 | 1.83 | 0.04 | 7.53 | 100.00 | 3.34 | 3.28 | 0.00 | 18.95 | 95.82 |
| *Panel B: Weekly Rebalancing* | | | | | | | | | | | | | | | | |
| **Currencies** | 12.50 | 0.93 | 1.25 | 0.00 | 5.53 | 93.83 | 0.90 | 1.23 | 0.00 | 5.52 | 93.83 | 0.75 | 0.50 | 0.00 | 2.70 | 98.35 |
| **Gold** | 20.00 | 4.31 | 3.15 | 0.00 | 10.78 | 75.31 | 4.31 | 3.15 | 0.00 | 10.78 | 75.31 | 7.59 | 5.56 | 0.31 | 35.71 | 100.00 |
| **Oil** | 25.00 | 11.13 | 9.59 | 0.00 | 32.87 | 99.59 | 11.14 | 9.59 | 0.00 | 32.87 | 99.59 | 10.15 | 7.30 | 0.17 | 27.60 | 100.00 |
| **Stocks** | 33.33 | 1.97 | 2.11 | 0.00 | 8.17 | 81.48 | 2.26 | 2.26 | 0.00 | 8.67 | 85.19 | 3.66 | 3.24 | 0.00 | 14.41 | 98.77 |
| **Mixed** | 14.28 | 0.08 | 0.15 | 0.00 | 0.70 | 35.12 | 1.58 | 1.70 | 0.00 | 7.17 | 83.88 | 3.36 | 3.06 | 0.00 | 15.05 | 97.11 |

This table presents the Bitcoin weights (in percentage) of the naive $1/N$ strategy ($EW$) and basic statistics of Bitcoin weights of the global minimum-variance ($GMV$), the short-sale constrained global minimum-variance trading strategies with historical ($CGMV$) and DCC covariance forecasts ($CGMV\text{-}DCC$) namely, the mean, standard deviation, minimum and maximum weights for daily and weekly rebalancing in Panel A and B, respectively. Column *Inv* calculates how many times Bitcoin is included in portfolios scaled by the total number of trading periods.

below 10% in all but one case. Not surprisingly, the high Bitcoin returns during this period make it unsuitable for short-selling (see the unconstrained *GMV* strategy) and along with the low correlation with all assets offer diversification benefits to investors concerned with their portfolio risk. This is also apparent from the limited periods where zero investment in Bitcoin is the most optimal solution. Specifically, in more than 95% of the rebalancing decisions for the mean-variance strategies, Bitcoin is selected and improve the portfolio performance.

In a well-diversified portfolio of assets, though, the benefits of Bitcoin are eliminated for the unconstrained mean-variance portfolio. In almost one third of the portfolio allocation decisions Bitcoin does not contribute successfully either to minimize the risk or maximize the return.[11] However, in the more realistic constrained optimal strategies, *CGMV* and *CGMV-DCC*, Bitcoin has contribution more than 95% when mixed assets are selected. The consideration of time-varying correlation also increases the participation of Bitcoin in optimal portfolios exhibiting that dynamic dependencies across assets are better modelled with a multivariate GARCH model. Daily portfolio rebalancing decisions seem to exploit better the diversification opportunities of Bitcoin than weekly rebalancing with the exception of the *CGMV-DCC* strategy that invests similarly in the cryptocurrency in both horizons.

While the contribution of Bitcoin to the various portfolios (positive weight) indicates possible diversification benefits, we examine whether they are statistically significant. Table 3 presents performance metrics of portfolios that include Bitcoin against benchmark portfolios for the four examined portfolio strategies with daily rebalancing. In particular, in panel A, B, and C we report the out-of-sample Sharpe ratio, annualized variance,

[11] This number is increased due to the rounded weights up to 4 decimal places. Thus, extremely small weights are not considered.

**Table 3 Portfolio Performance with Daily Rebalancing**

| Portfolio | EW | | Diff | GMV | | Diff | CGMV | | Diff | CGMV-DCC | | Diff |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Excl BPI | Incl BPI | | Excl BPI | Incl BPI | | Excl BPI | Incl BPI | | Excl BPI | Incl BPI | |
| *Panel A: Sharpe Ratio* | | | | | | | | | | | | |
| **Currencies** | −0.0295 | 0.0784 | −0.1078*** | −0.0645 | −0.0362 | −0.0283*** | −0.0504 | −0.0364 | −0.0140*** | −0.0410 | −0.0206 | −0.0204*** |
| **Gold** | −0.0121 | 0.0441 | −0.0562*** | −0.0214 | 0.0059 | −0.0273 | −0.0223 | 0.0059 | −0.0282*** | −0.0269 | 0.0004 | −0.0273** |
| **Oil** | −0.0253 | 0.0562 | −0.0815*** | −0.0336 | 0.0261 | −0.0598*** | −0.0253 | 0.0262 | −0.0514*** | −0.0276 | 0.0207 | −0.0483*** |
| **Stocks** | 0.0625 | 0.1181 | −0.0556 | 0.0582 | 0.0838 | −0.0256*** | 0.0583 | 0.0918 | −0.0336*** | 0.0564 | 0.0833 | −0.0269** |
| **Mixed** | 0.0139 | 0.0873 | −0.0734*** | 0.0405 | 0.0501 | −0.0096*** | 0.0414 | 0.0570 | −0.0157 | 0.0838 | 0.3861 | −0.3024* |
| *Panel B: Portfolio Variance* | | | | | | | | | | | | |
| **Currencies** | 1.1317 | 3.7308 | −2.5991*** | 1.0366 | 0.9467 | 0.0899 | 0.9548 | 0.9524 | 0.0024 | 0.9116 | 0.9115 | 0.0001 |
| **Gold** | 30.5274 | 27.8620 | 2.6654 | 4.2704 | 6.2034 | −1.9329*** | 6.2386 | 6.2034 | 0.0352 | 6.4088 | 6.3023 | 0.1065 |
| **Oil** | 20.8263 | 23.2141 | −2.3878 | 18.0947 | 15.3804 | 2.7142*** | 18.078 | 15.3623 | 2.7156*** | 18.0779 | 14.1626 | 3.9153*** |
| **Stocks** | 3.5278 | 22.0675 | −18.5397*** | 3.4761 | 3.3926 | 0.0835 | 3.4670 | 3.7996 | −0.3326*** | 3.5732 | 3.8073 | −0.2340 |
| **Mixed** | 4.0394 | 6.8894 | −2.8500*** | 0.7428 | 0.7409 | 0.0019 | 0.7391 | 2.6427 | −1.9037*** | 3.5984 | 7.2050 | −3.6066** |
| *Panel C: Certainty Equivalent* | | | | | | | | | | | | |
| **Currencies** | −0.5972 | −1.7140 | 1.1169*** | −0.5840 | −0.5086 | −0.0754*** | −0.5267 | −0.5118 | −0.0149*** | −0.4949 | −0.4754 | −0.0195*** |
| **Gold** | −15.3305 | −13.6984 | −1.6321*** | −2.1794 | −3.0870 | 0.9076 | −3.1749 | −3.0870 | −0.0880*** | −3.2724 | −3.1501 | −0.1223** |
| **Oil** | −10.5286 | −11.3361 | 0.8075*** | −9.1904 | −7.5877 | −1.6027*** | −9.1464 | −7.5786 | −1.5678*** | −9.1562 | −7.0035 | −2.1527*** |
| **Stocks** | −1.6465 | −10.4787 | 8.8323*** | −1.6296 | −1.5419 | −0.0876*** | −1.6250 | −1.7208 | 0.0958*** | −1.6800 | −1.7411 | 0.0611** |
| **Mixed** | −1.9917 | −3.2164 | 1.2247*** | −0.3365 | −0.3273 | −0.0092*** | −0.3340 | −1.2286 | 0.8947 | −1.6403 | −2.5660 | 0.9256** |

This table presents the out-of-sample Sharpe ratio, variance and certainty equivalent at a daily rebalancing frequency for equal-weighted ($EW$), global minimum-variance ($GMV$) and short-sale constrained global minimum-variance strategies with historical ($CGMV$) and DCC covariance forecasts ($CGMV$-$DCC$). Columns (4), (7) and (10) perform the robust statistical inference method of Ledoit and Wolf (2008, 2011) to test the null hypothesis of no differences in performance of the Bitcoin portfolios from the benchmark portfolios. *** $p < 0.01$ ** $p < 0.05$ * $p < 0.10$ denote the level of significance.

**Table 4 Portfolio Turnover and Return Loss**

| Portfolio | EW | | | GMV | | | CGMV | | | CGMV-DCC | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Turnover | | Return | Turnover | | Return | Turnover | | Return | Turnover | | Return |
| | Excl BPI | Incl BPI | Loss | Excl BPI | Incl BPI | Loss | Excl BPI | Incl BPI | Loss | Excl BPI | Incl BPI | Loss |
| *Panel A: Daily Rebalancing* | | | | | | | | | | | | |
| **Currencies** | 0.0030 | 0.0082 | −0.2082 | 0.0364 | 0.0179 | −0.0276 | 0.0175 | 0.0167 | −0.0137 | 0.1869 | 0.1713 | −0.0195 |
| **Gold** | 0.0059 | 0.0144 | −0.2964 | 0.0564 | 0.0029 | −0.0679 | 0.0000 | 0.0029 | −0.0702 | 0.0056 | 0.0308 | −0.0685 |
| **Oil** | 0.0054 | 0.0152 | −0.3928 | 0.0347 | 0.0242 | −0.2344 | 0.0197 | 0.0154 | −0.2016 | 0.2590 | 0.1361 | −0.1816 |
| **Stocks** | 0.0011 | 0.0145 | −0.2613 | 0.0208 | 0.0177 | −0.0472 | 0.0162 | 0.0026 | −0.0654 | 0.2668 | 0.0247 | −0.0525 |
| **Mixed** | 0.0103 | 0.0141 | −0.1917 | 0.0102 | 0.0085 | −0.0083 | 0.0082 | 0.0103 | −0.0255 | 0.0428 | 0.0426 | −0.8117 |
| *Panel B: Weekly Rebalancing* | | | | | | | | | | | | |
| **Currencies** | 0.0071 | 0.0227 | −0.2734 | 0.1540 | 0.0808 | −0.0335 | 0.0775 | 0.0755 | −0.0140 | 0.3050 | 0.2683 | −0.0253 |
| **Gold** | 0.0126 | 0.0373 | −0.3732 | 0.2009 | 0.0088 | −0.0621 | 0.0000 | 0.0088 | −0.0746 | 0.0165 | 0.0688 | −0.1032 |
| **Oil** | 0.0121 | 0.0393 | −0.4952 | 0.2188 | 0.1026 | −0.2788 | 0.0940 | 0.0544 | −0.1888 | 0.3347 | 0.2337 | −0.2327 |
| **Stocks** | 0.0026 | 0.0397 | −0.2404 | 0.0898 | 0.0635 | −0.0221 | 0.0624 | 0.0066 | −0.0365 | 0.4023 | 0.0376 | −0.0701 |
| **Mixed** | 0.0227 | 0.03513 | −0.2314 | 0.0437 | 0.0277 | 0.0019 | 0.0274 | 0.0365 | 0.0304 | 0.1323 | 0.1962 | −0.0468 |

This table presents the average turnover for daily and weekly rebalancing in Panel A and B, respectively, which is indicative of the transaction costs incurred from the equal-weighted (*EW*), global minimum-variance (*GMV*) and short-sale constrained global minimum-variance strategies with historical (*CGMV*) and DCC covariance forecasts (*CGMV-DCC*). The return loss measures the additional risk-adjusted return required by the Bitcoin strategy with respect to the benchmark strategy.

and annualized certainty equivalent, respectively. Columns (4), (7), (10) and (13) test the null hypotheses that the particular metric for portfolios including Bitcoin is not statistically different from that of the benchmark portfolio. In all cases the Sharpe ratios of portfolios with Bitcoin are higher than those of the benchmark portfolios with 17 out of 20 differences being statistically significant. On average, the Sharpe ratios of equal-weighted portfolios that include Bitcoin are positive. In the optimal mean-variance strategies positive Sharpe ratios are observed for all portfolios including Bitcoin except for the currencies. The slightly better performance of the constrained optimal portfolio with historical covariance forecasts compared to the *GMV* and the *CGMV-DCC* is in line with the findings of Jagannathan and Ma (2003) who suggest that constraints offer an advantage to the portfolio metrics when the sampling error is large.

Surprisingly, despite the extreme volatility of Bitcoin during the examined period, less significant differences are reported for the variance. This means that when Sharpe ratios increase because of the Bitcoin inclusion, risk does not always increase significantly. Bitcoin increases the portfolio variance significantly in 7 out of 20 cases, while the portfolio variance is statistically reduced at the 1 percent level of significance for the mean-variance strategies of the oil portfolio. Since the portfolio risk is decomposed into variance and covariance matrices and Bitcoin has the highest variance, we deduce that the low correlation of Bitcoin with the assets contributes to this outcome. As expected, the variance of the *EW* strategy is higher than the respective variance in the *GMV*, *CGMV* and *CGMV-DCC* strategies. The relatively lower portion of Bitcoin in the optimal portfolios can explain the less notable differences in risk. These findings are extremely important to an investor that is not tolerant towards risk.

Another notable result of our empirical application is linked to the significant contribution of Bitcoin in increasing the certainty equivalent returns in 11 out of 20 cases with higher impact on the optimal strategies where Bitcoin is maintained at a lower level. In the equal-weighted portfolio in all but the gold portfolio, a higher risk premium is required for risk-averse investors to accept the risk that comes with Bitcoin. We also demonstrate that Bitcoin decreases significantly the required certain return in the currency, stock, oil and mixed portfolios for the $GMV$, and in currencies, gold and oil portfolios for the $CGMV$ and $CGMV$-$DCC$ strategies. In all cases the negative certainty equivalent indicates that a higher risk premium should compensate investors for the risk.

As the rebalancing frequency decreases from daily to weekly, the Sharpe ratios increase and the risk decreases in magnitude.[12] Performing the same comparisons between the Bitcoin portfolios and the respective benchmark portfolios, we find that the significant differences in risk-adjusted returns are reduced (12 out of 20 cases) while the portfolio variance does not increase significantly in most of cases. There is also evidence that the inclusion of Bitcoin dwarfs the required risk premium for risk-averse investors since we find 8 cases where the certainty equivalent of the Bitcoin portfolios significantly outperforms those of the benchmark portfolios.

Table 4 shows the portfolio turnover and return loss for daily and weekly rebalancing in Panel A and B, respectively. The equal-weighted strategy, that maintains the largest amounts of Bitcoin and soars the Sharpe ratios significantly, is accompanied by a decrease in portfolio stability (higher turnover) compared to the respective portfolios that exclude Bitcoin. However, Bitcoin inclusion in the optimal strategies is more successful in decreasing portfolio turnover. As expected, the equal-weighted strategy has lower turnover in magnitude than the optimal strategies, and the constrained strategy has lower turnover than the unconstrained ($CGMV$) (DeMiguel et al., 2007). However,

[12] To conserve space, portfolio performance results for weekly rebalancing are not tabulated.

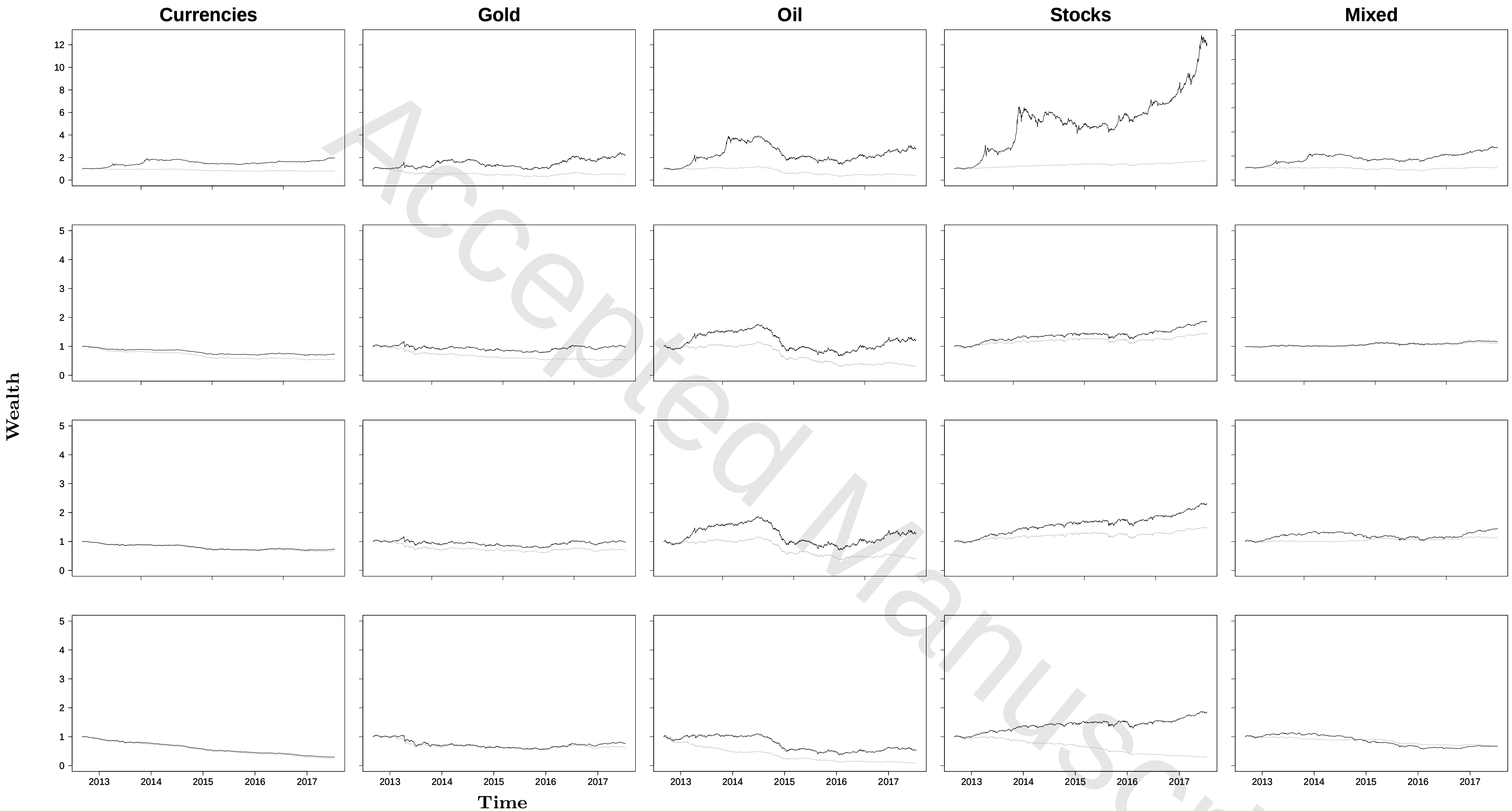


**Figure 3 Economics Gains with Daily Rebalancing** The figures display the economic gains net of transaction costs for the portfolios of currencies, gold, oil, stock, and mixed assets excluding (grey line) and including the Bitcoin (black line). The top, upper middle, lower middle and bottom rows of graphs show how the US$1 initial wealth of an investor changes following the $1/N$ strategy, the minimum-variance strategy and the short-sale constrained minimum-variance strategy with historical and DCC covariance forecasts, respectively.

the turnover is rocketed when we use covariance forecasts from a dynamic conditional correlation model (*CGMV-DCC*). This means that accounting for time-varying relationships between assets requires more frequent rebalancing that can trade off the gains of this strategy. The results remain qualitatively unchanged with weekly rebalancing but with manifold turnover in magnitude. Nevertheless, the return loss is negative indicating that even after accounting for proportional transaction costs the Bitcoin portfolios compensate investors with higher risk-adjusted returns. In weekly rebalancing, the mixed asset portfolios seem to be less benefited from Bitcoin after accounting for the significant rebalancing costs.

Figure 3 plots the change in the initial US$1 wealth of an investor with daily rebalancing decisions for all the strategies accounting for the required transaction costs to update the portfolio positions. The top, upper-middle, lower-middle and bottom panels show the evolution of wealth for equal-weighted, minimum-variance, constrained minimum-variance portfolios with historical covariance forecasts and constrained minimum-variance with DCC covariance forecasts, respectively, excluding Bitcoin (grey line) and including Bitcoin (black line). To facilitate comparability across graphs, all the axes at the mean-variance strategies have the same scaling. The equal-weighted portfolio shows that Bitcoin's inclusion within portfolios of various assets offers significant economic gains that do not vanish after the consideration of transaction costs. The volatility of wealth, though, increases significantly, indicating that the gains are driven by the extremely high performance in cryptocurrency markets the period we consider. The currency portfolio involved with Bitcoin offers a more stable evolution of wealth over time followed by the gold and mixed portfolios.

In the optimal strategies that account for the sensitivity of investors towards risk by keeping Bitcoin in small quantities, the significant differences in portfolio risk-adjusted returns are shrunk in the presence of proportional transaction costs. This is more obvious in the *CGMV-DCC* portfolios. Bitcoin leads to marginal wealth increase in cases of currencies, gold, and mixed portfolios. However, in the more realistic cases of a short-sale constrained minimum-variance portfolios, there are periods in a well-diversified portfolio where the inclusion of Bitcoin deteriorates investors' wealth.[13]

Overall, the oil portfolio profits most from the investment in Bitcoin with statistically higher risk-adjusted returns, significant decrease in volatility and risk premium and higher portfolio stability for the optimal strategies. The gold and currency portfolios present significantly higher Sharpe ratios which are accompanied with insignificant increase in portfolio risk and significant decrease in the required risk premium for the constrained minimum-variance strategies. The stock portfolio presents diversification benefits for the unconstrained minimum-variance portfolios but the benefits are eliminated for the more realistic constrained optimal strategies. When investors maintain well-diversified portfolios the benefits are eliminated considerably.

## 4.2. Bitcoin and Portfolio Performance in Non-Bubble Period

We examine whether the benefits are preserved in a sub-period that is not marked by explosive increases in Bitcoin prices. To this end, we follow the methodology of Phillips et al. (2015) and find that the period between 01/02/2014 and 01/05/2017 is not described by long-lasting increases in prices (see Figure 2). The minimum and maximum Bitcoin prices range from US$177.28 to US$1,329.19. The average daily returns are reduced to (0.14%), but the volatility remains high (4%). The minimum and maximum returns are -21.9% and 21.84%, respectively. Skewness and kurtosis are significantly reduced to 0.08

[13] To conserve space, we do not report the economic gains with weekly rebalancing, as they do not change significantly

**Table 5 Portfolio Performance with Daily Rebalancing Excluding Bubble Periods**

| Portfolio | EW | | Diff | GMV | | Diff | CGMV | | Diff | CGMV-DCC | | Diff |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Excl BPI | Incl BPI | | Excl BPI | Incl BPI | | Excl BPI | Incl BPI | | Excl BPI | Incl BPI | |
| *Panel A: Sharpe Ratio* | | | | | | | | | | | | |
| **Currencies** | −0.0385 | 0.0038 | −0.0423 | −0.0580 | −0.0336 | −0.0244** | −0.0444 | −0.0349 | −0.0096* | −0.0407 | −0.0227 | −0.0180** |
| **Gold** | 0.0087 | 0.0218 | −0.0131 | −0.0021 | 0.0170 | −0.0191 | 0.0041 | 0.0170 | −0.0129 | 0.0041 | 0.0313 | −0.0272** |
| **Oil** | −0.0345 | −0.0100 | −0.0245 | −0.0458 | −0.0033 | −0.0425** | −0.0348 | −0.0033 | −0.0314* | −0.0365 | 0.0059 | −0.0425** |
| **Stocks** | 0.0509 | 0.0514 | −0.0006 | 0.0488 | 0.0623 | −0.0135* | 0.0488 | 0.0674 | −0.0186 | 0.0427 | 0.0634 | −0.0207 |
| **Mixed** | 0.0132 | 0.0319 | −0.0187 | 0.0571 | 0.0623 | −0.0052 | 0.0568 | 0.0260 | 0.0308 | 0.0323 | 0.0226 | 0.0096 |
| *Panel B: Portfolio Variance* | | | | | | | | | | | | |
| **Currencies** | 1.2633 | 2.5429 | −1.2796*** | 1.2105 | 1.0575 | 0.153 | 1.0697 | 1.0697 | 0.0000 | 1.0387 | 1.0314 | 0.0073 |
| **Gold** | 32.4488 | 25.5089 | 6.9398*** | 3.0454 | 4.9996 | −1.9541*** | 5.2066 | 4.9996 | 0.2070** | 5.2066 | 4.9073 | 0.2992** |
| **Oil** | 26.7649 | 21.2018 | 5.5631*** | 22.6341 | 18.2451 | 4.3891*** | 22.6502 | 18.2405 | 4.4097*** | 22.5737 | 16.9440 | 5.6297*** |
| **Stocks** | 3.9722 | 12.8899 | −8.9176*** | 3.9270 | 3.7854 | 0.1416 | 3.9134 | 4.1737 | −0.2603* | 4.0219 | 4.1817 | −0.1598 |
| **Mixed** | 4.4524 | 5.3925 | −0.9401*** | 0.8674 | 0.8648 | 0.0026 | 0.8655 | 2.9133 | −2.0478*** | 0.8411 | 2.8756 | −2.0345*** |
| *Panel C: Certainty Equivalent* | | | | | | | | | | | | |
| **Currencies** | −0.6750 | −1.2655 | 0.5905 | −0.6691 | −0.5633 | −0.1057* | −0.5808 | −0.5671 | −0.0137* | −0.5608 | −0.5387 | −0.0221** |
| **Gold** | −16.1748 | −12.6445 | −3.5303 | −1.5264 | −2.4618 | 0.9354 | −2.5940 | −2.4618 | −0.1322 | −2.5940 | −2.3844 | −0.2096** |
| **Oil** | −13.5610 | −10.6470 | −2.9140 | −11.5351 | −9.1368 | −2.3983*** | −11.4906 | −9.1344 | −2.3561** | −11.4605 | −8.4475 | −3.0129** |
| **Stocks** | −1.8847 | −6.2602 | 4.3755 | −1.8669 | −1.7715 | −0.0954* | −1.8601 | −1.9491 | 0.0890 | −1.9254 | −1.9613 | 0.0359 |
| **Mixed** | −2.1983 | −2.6222 | 0.4238 | −0.3806 | −0.3745 | −0.0061 | −0.3799 | −1.4122 | 1.0323 | −0.3910 | −1.3994 | 1.0084 |

This table presents the out-of-sample Sharpe ratio, variance and certainty equivalent at a daily rebalancing frequency for equal-weighted (*EW*), global minimum-variance (*GMV*) and short-sale constrained global minimum-variance strategies with historical (*CGMV*) and DCC covariance forecasts (*CGMV-DCC*) for the sub-period from 01/02/2014 to 01/05/2017 that does not accommodate explosive price increases according to bubble test of (Phillips et al., 2015). Columns (4), (7) and (10) perform the robust statistical inference method of Ledoit and Wolf (2008, 2011) to test the null hypothesis of no differences in performance of the Bitcoin portfolios from the benchmark portfolios. *** $p < 0.01$ ** $p < 0.05$ * $p < 0.10$ denote the level of significance.

and 6.31, respectively.

As expected, the high Sharpe ratio previously found in the equal-weighted strategy are totally driven by the high performance of Bitcoin. However, the effect of Bitcoin on Sharpe ratios in mean-variance strategies remains consistently positive, especially in non-diversified portfolios. The findings for the overall portfolio risk are still inconclusive. In particular, portfolios of commodities such as gold and oil most profit from Bitcoin, since it leads to higher risk-adjusted returns and lower volatility. Decreases in portfolio variance arise solely from the low correlation of Bitcoin with the other assets since its volatility is the largest. The role of Bitcoin on decreasing the required certainty equivalent returns is maintained in the minimum-variance strategies. Yet, these benefits disappear when investors keep multi-asset portfolios from various classes. Such findings, presented in Table 5, oppose previous studies that find diversification opportunities in Bitcoin within multi-asset portfolios.

## 5. Conclusion

This paper presents an extensive analysis of the statistical performance of benchmark portfolios of currencies, gold, oil and stocks as well as a multi-asset portfolio of currencies, gold, oil, stock, real estate and bond with respective portfolios that invest additionally in Bitcoin under four trading strategies. We also estimate the economic gains added from Bitcoin in a sample period that includes both bullish and bearish cryptocurrency market conditions.

We document significant diversification benefits for equal-weighted and optimal minimum-variance portfolios with daily and weekly rebalancing, performing the robust statistical inference tests of Ledoit and Wolf (2008, 2011). In most cases, Bitcoin portfolios come with high Sharpe ratios which do not necessarily involve a statistically significant increase in variances or required risk premia. Such performance is maintained even in cases where the participation of Bitcoin is very small, such as in minimum-variance portfolios, indicating that risk-averse investors can profit as well. We also demonstrate that the proportion of Bitcoin within each portfolio increases under a minimum-variance strategy that accounts for time-variant correlations between assets predicted from a multivariate GARCH model.

Another important finding is that the economic gains are not reduced after the consideration of transaction costs. However, the high economic value added in equal-weighted portfolios should be taken with precaution as in all but one cases leads to more risky portfolios and more volatile evolution of wealth. This is also apparent from the insignificant differences in risk-adjusted returns when we consider a non-bubble sub-period with less extreme market conditions in cryptocurrencies applying the Phillips et al. (2015) test. During this period, the benefits from minimum-variance strategies are also reduced, yet they do not completely vanish, particularly when investors keep less diversified portfolios of commodities. However, the advantages for a well-diversified portfolio are substantially eliminated since we report less statistically significant differences in Sharpe ratios, portfolio variances and uncertainty equivalents.

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