Online Appendix for "Agreements with Reciprocity: Co-financing and MOUs"

Dooseok Jang[*], Amrish Patel[†] and Martin Dufwenberg[‡]

March 20, 2018

Abstract

This online appendix accompanies "Agreements with Reciprocity: Co-financing and MOUs". It contains several additional results mentioned in the paper and their proofs.

In this online appendix we present four further results and proofs not included in the paper "Agreements with Reciprocity: Co-financing and MOUs" (see paper for full details of the notation and model). Observation 2 demonstrates that an analog of Theorem 1(a) cannot be established for high reciprocity sensitivity ($Y > Y^*$). Observation 3 shows that for low reciprocity sensitivity, even if full investment is unattainable, reciprocity can lead to higher investment than with only material preferences. Observations 4 and 5 consider the extent to which Theorem 1 generalises to 3- and 2-player games respectively.

Observation 2 (High reciprocity): For all $n \geq 4$, γ, β and $Y \geq Y^*$ there exists a full investment SRE in $\widehat{\Gamma}_A$. The SRE is described by n signing, then i does not invest iff there are $\lceil \frac{\gamma}{\beta} \rceil - q$ signatories where $q > 0$ and odd.

[*]Korea Advanced Institute of Science and Technology; jangds@kaist.ac.kr
[†]University of East Anglia; amrish.patel@uea.ac.uk
[‡]University of Arizona, University of Gothenburg, CESifo; martind@eller.arizona.edu

Proof: We demonstrate that a particular profile implying full investment, s^*, is a SRE of $\widehat{\Gamma}_A$ for all $Y \geq Y^*$. Consider s^* such that each $i \in N$ signs, then does not invest if a^1 is such that $m\left(a^1\right) = \left\lceil \frac{\gamma}{\beta} \right\rceil - q$ where $q > 0$ and odd, and does invest otherwise. Reason as follows to confirm that for all $Y \geq Y^*$, players have no incentive to deviate at any history.

Consider $h = a^1$ such that $m\left(a^1\right) \in \left(\left\lceil \frac{\gamma}{\beta} \right\rceil, n\right]$. Signatory i has neither a material nor a reciprocity incentive to deviate to not investing at h. Non-signatory i faces identical incentives to the full investment profile in $\widehat{\Gamma}_P$, thus i does not deviate to not investing at h if $Y \geq Y^*$.

Now consider $h = a^1$ such that $m\left(a^1\right) = \left\lceil \frac{\gamma}{\beta} \right\rceil$. Signatory i has neither a material nor a reciprocity incentive to deviate to not investing at h. Non-signatory i has a material incentive to deviate to not investing at h and a reciprocity incentive to not do so. Reason as follows to identify Y such that i does not deviate. Non-signatory i's increase in reciprocity payoff from playing s_i^* rather than $s_i'\left(h, s_i^*\right)$ is expression (6) in Jang et al. (2018). This is no less than the reduction in i 's material payoff from playing s_i^* rather than $s_i'\left(h, s_i^*\right)$ if $Y \geq \overline{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right)$, where

$$\overline{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right) := \frac{2\left(\gamma - \beta\right)}{\beta\left(\left(n\beta - \gamma - \beta\right) m\left(a^1\right) + \left(n-1\right)\beta\right)}.$$

Note that $\overline{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right) \geq Y^*$ iff $\frac{\gamma}{\beta} + 1 \geq n$, however by assumption $\left\lceil \frac{\gamma}{\beta} \right\rceil < n$, therefore $\overline{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right) < Y^*$. Thus $Y \geq Y^*$ is sufficient to prevent non-signatory i deviating at h.

Now consider $h = a^1$ such that $m\left(a^1\right) = \left\lceil \frac{\gamma}{\beta} \right\rceil - q$ where $q > 0$ and even, so all players invest. Non-signatory i, faces identical incentives as a non-signatory at a history with $\left\lceil \frac{\gamma}{\beta} \right\rceil$ signatories, thus i does not deviate to not investing if $Y \geq \overline{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right)$, which holds for $Y \geq Y^*$.

Signatory i has a material incentive to deviate to not investing. Using expression (7) in Jang et al. (2018), signatory i's change in reciprocity payoff from playing s_i^* rather than $s_i'\left(h, s_i^*\right)$ is strictly positive iff $\frac{Y\Omega}{2} > 0$ where

$$\Omega := \left(m\left(a^1\right) - 1\right)\left(\beta - \frac{\gamma}{m\left(a^1\right)}\right)\Delta\lambda_S + \left(n - m\left(a^1\right)\right)\beta\Delta\lambda_N,$$

$\Delta\lambda_S := \left(n+1\right)\beta - \frac{m\left(a^1\right)+1}{m\left(a^1\right)}\gamma$ and $\Delta\lambda_N := \left(n-1\right)\beta - \frac{m\left(a^1\right)}{m\left(a^1\right)+1}\gamma$. Note that $\Delta\lambda_S < \Delta\lambda_N$. To determine the sign of Ω reason as follows. Clearly $m\left(a^1\right) - 1 > 0$, $\beta - \frac{\gamma}{m\left(a^1\right)} < 0$ and $n - m\left(a^1\right) > 0$, thus we need only sign $\Delta\lambda_S$

and $\Delta\lambda_N$. Given $n > \lceil\frac{\gamma}{\beta}\rceil$, it follows that $(n-1)\beta > \gamma$, which implies that $\Delta\lambda_N > 0$. Consider how $\Delta\lambda_S$ influences the sign of Ω. If $\Delta\lambda_S \leq 0$, then $\Omega > 0$. If $\Delta\lambda_S > 0$, since $\Delta\lambda_S < \Delta\lambda_N$ we can write

$$\Omega > \left(\left(m\left(a^1\right)-1\right)\left(\beta - \frac{\gamma}{m\left(a^1\right)}\right) + \left(n - m\left(a^1\right)\right)\beta\right)\Delta\lambda_S > 0.$$

where the final inequality follows from $\left(m\left(a^1\right)-1\right)(\beta - \frac{\gamma}{m(a^1)}) + \left(n - m\left(a^1\right)\right)\beta > \Delta\lambda_N > 0$. Therefore $\Omega > 0$ and signatory i's reciprocity payoff is strictly higher playing s_i^* instead of $s_i'\left(h, s_i^*\right)$. This increase in reciprocity payoff is no less than i's reduction in material payoff from playing s_i^* rather than $s_i'\left(h, s_i^*\right)$ if $Y \geq \widehat{Y}\left(n,\beta,\gamma,m\left(a^1\right)\right)$ where

$$\widehat{Y}\left(n,\beta,\gamma,m\left(a^1\right)\right) := \frac{2\left(\gamma/m\left(a^1\right)-\beta\right)}{\Omega}.$$

Now argue that $\widehat{Y}\left(n,\beta,\gamma,m\left(a^1\right)\right) < Y^*$. To do so, take a function, $\tilde{Y}\left(n,\beta,\gamma,m\left(a^1\right)\right)$, such that $\tilde{Y}\left(n,\beta,\gamma,m\left(a^1\right)\right) > \widehat{Y}\left(n,\beta,\gamma,m\left(a^1\right)\right)$. To identify an appropriate function, reason as follows. For a given $\Delta\lambda_N$, Ω is decreasing in $\Delta\lambda_S$, and $\Delta\lambda_S$ is bounded by $\Delta\lambda_N$, thus let $\Delta\lambda_S = \Delta\lambda_N$ to minimise Ω. Furthermore, note that Ω is increasing in $\Delta\lambda_N$, and that $\Delta\lambda_N$ is strictly greater than β. To see this, note that $\beta n - \gamma > \beta$ since $n > \frac{\gamma}{\beta} + 1$ by assumption. Also note that $\frac{\gamma}{m(a^1)+1} - \beta > 0$ for all $m\left(a^1\right) \in \{1, \ldots, \lceil\frac{\gamma}{\beta}\rceil - 2\}$. Putting this together gives $\Delta\lambda_N > \beta$. Overall then, substitute $\Delta\lambda_S = \Delta\lambda_N = \beta$ into $\widehat{Y}\left(n,\beta,\gamma,m\left(a^1\right)\right)$ to give

$$\tilde{Y}\left(n,\beta,\gamma,m\left(a^1\right)\right) := \frac{2(\frac{\gamma}{m(a^1)} - \beta)}{\beta((n-1)\beta - \frac{m(a^1)-1}{m(a^1)}\gamma)}.$$

Suppose that $\tilde{Y}\left(n,\beta,\gamma,m\left(a^1\right)\right) > Y^*$. This requires

$$(\gamma - \beta)\left((n-1)\beta - \frac{m(a^1)-1}{m(a^1)}\gamma\right) < \left(\frac{\gamma}{m(a^1)} - \beta\right)\beta(n-1).$$

Note that the LHS is increasing in $m\left(a^1\right)$ and that the RHS is decreasing in $m\left(a^1\right)$. Substituting $m\left(a^1\right) = 1$ gives $(\gamma-\beta)(n-1)\beta < (\gamma-\beta)(n-1)\beta$, a contradiction. Therefore $\tilde{Y}\left(n,\beta,\gamma,m\left(a^1\right)\right) \leq Y^*$. Overall, $Y^* \geq \tilde{Y}\left(n,\beta,\gamma,m\left(a^1\right)\right) > \widehat{Y}\left(n,\beta,\gamma,m\left(a^1\right)\right)$, thus $Y \geq Y^*$ is sufficient to prevent signatory i deviating from s_i^* to $s_i'\left(h, s_i^*\right)$ at h.

Now consider $h = a^1$ such that $m(a^1) = \lceil \frac{\gamma}{\beta} \rceil - q$ where $q > 1$ and odd, so zero players invest. Non-signatory i has neither material nor reciprocity incentives to deviate to investing at h. Signatory i has no material incentive to deviate to investing at h. Using expression (7) in Jang et al. (2018), the change in signatory i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is $Y\Omega/2$, which is strictly positive as already established, thus i does not deviate at h.

Now consider $h = a^1$ such that $m(a^1) = \lceil \frac{\gamma}{\beta} \rceil - 1$, so zero invest. Non-signatory i has neither a material nor a reciprocity incentive to deviate to investing. Signatory i has no material incentive to deviate to investing. Using expression (7) in Jang et al. (2018), the change in signatory i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is $\frac{Y}{2}\left(\Omega + (n - m(a^1))\,\beta\,(\beta - \gamma/(m(a^1)+1))\right)$, which is strictly positive as we know $\Omega > 0$ and that $\frac{\gamma}{\beta} < m(a^1) + 1$, implies $\frac{\beta - \gamma}{m(a^1)+1} > 0$, thus i does not deviate at h.

Now consider $h = a^1$ such that $m(a^1) = 0$, so zero invest for $\lceil \frac{\gamma}{\beta} \rceil$ odd and n invest otherwise. For $\lceil \frac{\gamma}{\beta} \rceil$ odd, player i has neither material nor reciprocity incentives to deviate to investing. For $\lceil \frac{\gamma}{\beta} \rceil$ even, player i faces identical incentives as a non-signatory following a history of $\lceil \frac{\gamma}{\beta} \rceil$ signatories, thus does not deviate if $Y \geq \overline{Y}(n, \beta, \gamma, m(a^1))$, which is satisfied for all $Y \geq Y^*$.

Finally consider the initial node. Player i has neither material nor reciprocity incentives to deviate. Hence s^* is a SRE. ■

Observation 3 (Low reciprocity): For all $n \geq 4$ and γ, there exists $Y''' \in (0, Y^*)$, $\beta'' \in (\gamma/n, \gamma)$ and $m'(Y) \in \left[\lceil \frac{\gamma}{\beta} \rceil, \lfloor \frac{\sqrt{8n-7}-1}{2} \rfloor\right]$ such that

- **(a)** if $\beta \geq \beta''$ and $Y \leq Y'''$ there exists a SRE where $m'(Y)$ players sign and invest on path. The SRE is described by $m'(Y)$ players signing, then i invests iff i signed and there are at least $m'(Y)$ signatories.
- **(b)** $m'(Y)$ is non-decreasing in Y.

Proof: (a) Consider s^* such that $m(Y)$ sign, then i invests iff i signed and there are at least $m(Y)$ signatories. For β sufficiently high, we first identify non-deviation conditions for signatories in the investment stage, then do the same for non-signatories, and then consider the sign-up stage. Using these conditions, we show that for all $Y \in (0, Y''')$ where $Y''' \in (0, Y^*)$, some s^* is a SRE.

Consider $h = a^1$ such that $m(a^1) > m(Y)$. Signatory i has no material incentive to deviate to not investing. Using expression (7) in Jang et al. (2018), the change in i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is

$$Y\frac{\beta}{2}\left[(\beta - \frac{\gamma}{m(a^1)})(m(a^1) - 1) - \beta(n - m(a^1))\right]. \tag{1}$$

Note that if $m(a^1) = \frac{\gamma}{\beta}$ then (1) < 0, if $m(a^1) = n$ then (1) > 0 and that (1) is strictly increasing in $m(a^1)$. There must then exist some $\widetilde{m} \in (\frac{\gamma}{\beta}, n)$ such that if $m(a^1) = \widetilde{m}$ then (1) $= 0$. For $m(a^1) \geq \widetilde{m}$, signatory i does not deviate at h. For $m(a^1) \in [m(Y) + 1, \widetilde{m})$, signatory i does not deviate to not investing at h if $Y \leq Y_1(m(a^1))$, where

$$Y_1(m(a^1)) \equiv \frac{\beta - \frac{\gamma}{m(a^1)}}{-\frac{\beta}{2}\left[(\beta - \frac{\gamma}{m(a^1)})(m(a^1) - 1) - \beta(n - m(a^1))\right]}.$$

As $Y(m(a^1))$ is strictly increasing in $m(a^1)$, $Y \leq Y_1(m(Y) + 1)$ is a sufficient condition for signatory i to not deviate to not investing at h.

Consider $h = a^1$ such that $m(a^1) = m(Y)$. Signatory i has a material incentive to not deviate to not investing. Using expression (7) in Jang et al. (2018), the change in i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is

$$f(m(a^1)) \equiv \begin{cases} (\beta - \frac{\gamma}{m(a^1)})\frac{m(a^1)\beta - \gamma}{2}(m(a^1) - 1) - \frac{\beta^2}{2}(n - m(a^1)) & \text{if } m(a^1) \geq 3, \\ (\beta - \frac{\gamma}{m(a^1)})(m(a^1)\beta - \frac{\beta}{2} - \gamma)(m(a^1) - 1) - \frac{\beta^2}{2}(n - m(a^1)) & \text{if } m(a^1) = 2. \end{cases}$$

Note that $f(m(a^1))$ is strictly increasing in $m(a^1)$. If $f(m(a^1)) \geq 0$, signatory i does not deviate to not investing at h. If $f(m(a^1)) < 0$, signatory i does not deviate to not investing at h if $Y \leq Y_2(m(a^1))$, where

$$Y_2(m(a^1)) \equiv \frac{\beta - \frac{\gamma}{m(a^1)}}{-f(m(a^1))}.$$

If $m(Y) \geq 3$, then for $m(a^1) \geq m(Y)$, signatory i does not deviate to not investing if $Y \leq min\{Y_1(m(Y) + 1), Y_2(m(Y)\}$. This can be rewritten as $Y \leq Y_1(m(Y))$ since for $m(a^1) \geq 3$, $Y_1(m(a^1)) < Y_2(m(a^1))$ and both are strictly increasing in $m(a^1)$. If $m(Y) = 2$, then for $m(a^1) \geq 2$, signatory

i does not deviate to not investing if $Y \leq Y_2(m(Y))$ (since $Y_2(m(a^1)) < Y_1(m(a^1)) < Y_1(m(a^1)+1)$).

Consider $h = a^1$ such that $m(a^1) = m(Y) - 1 \geq 2$. Signatory i has a material incentive to deviate to investing. Using expression (7) in Jang et al. (2018), the change in i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is

$$Y\frac{\beta}{2}\left[(n-m(a^1))((m(a^1)+1)\beta-\gamma)-(m(a^1)-1)(\frac{\gamma}{m(a^1)}-\beta)\right]. \qquad (2)$$

Note that (2) > 0. Thus signatory i does not deviate to not investing at h if $Y \geq Y_3(m(a^1))$, where

$$Y_3(m(a^1)) \equiv \frac{\beta-\frac{\gamma}{m(a^1)}}{\frac{\beta}{2}\left[(n-m(a^1))((m(a^1)+1)\beta-\gamma)-(m(a^1)-1)(\frac{\gamma}{m(a^1)}-\beta)\right]}.$$

Consider $h = a^1$ such that $m(a^1) \in [2, m(Y)-1)$. Signatory i has a material incentive to deviate to investing. Using expression (7) in Jang et al. (2018), the change in i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is

$$Y\frac{\beta}{2}\left[(n-m(a^1))\beta-(m(a^1)-1)(\frac{\gamma}{m(a^1)}-\beta)\right]. \qquad (3)$$

Note that (3) > 0. Thus signatory i does not deviate to not investing at h if $Y \geq Y_4(m(a^1))$, where

$$Y_4(m(a^1)) \equiv \frac{\beta-\gamma/m(a^1)}{\frac{\beta}{2}\left[(n-m(a^1))\beta-(m(a^1)-1)(\frac{\gamma}{m(a^1)}-\beta)\right]}.$$

Since $Y_4(.)$ is increasing in $m(a^1)$, for all $m(a^1) \in [2, m(Y)-1)$, signatory i does not deviate if $Y \geq Y_4(m(Y)-2)$.

Consider $h = a^1$ such that $m(a^1) = 1$. Signatory i has no material incentive to deviate to investing. Using expression (7) in Jang et al. (2018), the change in i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is $Y\frac{\beta^2}{2}(n-1) > 0$, thus i has no reciprocity incentive to deviate either. Finally, Consider $h = a^1$ such that $m(a^1\ ) \in \{0, 1\}$, i has neither material nor reciprocity incentive to deviate to investing.

Now consider non-signatories. Clearly non-signatory i has no material incentive to invest. For all $h = a^1$ such that $m(a^1) \notin \{m(Y)-1, m(Y)\}$, non-signatory i perceives others as no more kind than in the full investment profile in $\widehat{\Gamma}_P$, thus does not deviate to investing for $Y < Y^*$. For $h = a^1$ such that $m(a^1) = m(Y) - 1$, all other players are unkind to i so he has no reciprocity incentive to deviate to investing. For $h = a^1$ such that $m(a^1) = m(Y)$, using expression (6) in Jang et al. (2018), non-signatory i does not deviate to investing if

$$\beta - \gamma + Y\frac{\beta^2}{2}\left[(m(a^1))^2 + m(a^1) + 1 - n\right] < 0.$$

If [.] is non-positive then the inequality holds. If [.] is positive, then i does not deviate if $Y \leq Y_5(m(a^1))$, where

$$Y_5(m(a^1)) \equiv \frac{2(\beta-\gamma)}{\beta^2\left[(m(a^1))^2 + m(a^1) + 1 - n\right]}.$$

Note that if

$$m(a^1) \leq \frac{\sqrt{8n-7}-1}{2}, \tag{4}$$

then $Y_5(m(a^1)) > Y^*$. In this case it must be that $Y < Y_5(m(a^1))$ since $Y < Y^*$. If $m(a^1) > (\sqrt{8n-7}-1)/2$, then i does not deviate if $Y \leq Y_5(m(a^1))$.

Now consider the sign-up stage. Signatory i has identical incentives to a signatory's incentives at $h = a^1$ such that $m(a^1) = m(Y)$. Non-signatory i has identical incentives to a non-signatory's incentives at $h = a^1$ such that $m(a^1) = m(Y)$. Thus if players have no incentive to deviate in the investment stage, they also have no incentive to deviate in the sign-up stage.

In sum, if Y is sufficiently small such that $m(Y) \leq (\sqrt{8n-7}-1)/2$, then there exists a SRE where $m(Y) = 2$ if $Y \in I_2 \equiv \{Y : Y < Y_2(m(Y))\}$; $m(Y) = 3$ if $Y \in I_3 \equiv \{Y : Y \in [Y_3(m(Y)-1), Y_1(m(Y))]\}$; $m(Y) \in [4, \widetilde{m})$ if $Y \in I_{[4,\widetilde{m})} \equiv \{Y : Y \in [Y_4(m(Y)-1), Y_1(m(Y))]\}$ and $m(Y) \geq \widetilde{m}$ if $Y \in I_{\geq \widetilde{m}} \equiv \{Y : Y \geq Y_4(m(Y)-1)\}$. To show that there exists a SRE for all $Y \in (0, Y''')$, verify that $I_2 \cup I_3 \cup I_{[4,\widetilde{m})} \cup I_{\geq \widetilde{m}}$ covers $\mathbb{R}^+$ as follows. First, $I_2 \cap I_3 \neq \varnothing$ as $Y_3(m(Y)-1) < Y_2(m(Y)-1) < Y_2(m(Y))$. Second, $I_3 \neq \varnothing$ since $Y_3(m(Y)-1) < Y_1(m(Y)-1) < Y_1(m(Y))$. Third, since $Y_4(m(Y)) < Y_1(m(Y))$ and both increase in $m(Y)$, it holds that $Y_4(m(Y)-1) < Y_4(m(Y)) < Y_1(m(Y)) < Y_1(m(Y)+1)$. Therefore the intersections for the intervals for all $m(Y) \geq 3$ are also non-empty.

(b) See final paragraph of part (a) of this proof. ■

Observation 4 (3-players): For $n = 3$, all β and γ, there exists $Y'' \in (0, Y^*)$ such that,

(a) if $Y \in [Y'', Y^*)$ there exists a full investment SRE in $\widehat{\Gamma}_A$ with 3 signatories. The SRE is described by 3 signing, then i does not invest iff there is only 1 signatory.

(b) if $Y \in [Y'', Y^*)$ there exists a full investment SRE in $\widehat{\Gamma}_A$ with 0 signatories. The SRE is described by 0 signing, then i does not invest iff there is only 1 signatory.

Proof: **(a)** Let $n = 3$. Consider s^* such that each $i \in N$ signs, then does not invest if a^1 is such that $m\left(a^1\right) = 1$ and does invest otherwise. Reason as follows to verify the profile is SRE for an interval of Y less than Y^*. Consider $h = a^1$ such that $m\left(a^1\right) = 3$, so all invest. Signatory i has neither material nor reciprocity incentive to deviate to not investing. Now consider $h = a^1$ such that $m(a^1) = 2$, so all invest. Signatory i has neither a material nor a reciprocity incentive to deviate to not investing at h. Non-signatory i has a material incentive to deviate to not investing at h and a reciprocity incentive to not do so. Reason as follows to identify Y such that i does not deviate. Using expression (6) in Jang et al. (2018), non-signatory i's increase in reciprocity payoff from playing s_i^* rather than $s_i'\left(h, s_i^*\right)$ is no less than his reduction in material payoff if $Y \geq Y''(\beta, \gamma)$, where

$$Y''(\beta, \gamma) := \frac{\gamma - \beta}{\beta(3\beta - \gamma)}.$$

Note that $Y''(\beta, \gamma) \geq Y^*$ iff $\frac{\gamma}{\beta} + 1 > 3$, however given $\lceil \frac{\gamma}{\beta} \rceil < n = 3$, then $Y''(\beta, \gamma) < Y^*$. Thus i does not deviate at h if $Y \in (Y'', Y^*)$. Now consider $h = a^1$ such that $m(a^1) = 1$, so zero invest. Player i has neither a material nor a reciprocity incentive to deviate to investing. Then consider $h = a^1$ such that $m\left(a^1\right) = 0$, so all players invest. Non-signatory i, faces identical incentives as a non-signatory at a history with 2 signatories, i does not deviate at h if $Y \in (Y'', Y^*)$. Finally, at the initial node, i has neither reciprocity nor material incentives to deviate.

(b) Let $n = 3$. Consider s^* such that each $i \in N$ does not sign, then does not invest if a^1 is such that $m\left(a^1\right) = 1$ and does invest otherwise. Reason as follows to verify the profile is SRE for an interval of Y less than Y^*. Stage

2 behaviour is optimal (part (a) of this proof). At the initial node, i has neither reciprocity nor material incentives to deviate. ■

For 2-player games, drop the assumptions that $\frac{\gamma}{\beta}$ is non-integer and $n > \lceil \frac{\gamma}{\beta} \rceil$ as they would contradict that $n\beta > \gamma > \beta > 0$.

Observation 5 (2-players): For $n = 2$, all γ and $Y \in (0, Y^*)$,

(a) if $\beta \geq \frac{2}{3}\gamma$, then there exists a full investment SRE in $\widehat{\Gamma}_A$ with 2 signatories. The SRE is described by 2 signing, then i invests iff there are 2 signatories.

(b) for all β, there does not exist a full investment SRE in $\widehat{\Gamma}_A$ with 0 signatories.

Proof: **(a)** Let $n = 2$. Consider s^* such that each $i \in N$ signs, then invests iff a^1 is such that $m(a^1) = 1$. Note that i has no material incentive to deviate at any history. Furthermore if $\beta \geq \frac{2}{3}\gamma$, i has no reciprocity incentive to deviate either.

(b) Let $n = 2$. Consider any s^* such that each $i \in N$ does not sign, then invests on path. Consider $h = a^1$ such that $m(a^1) = 0$, so all invest. Non-signatory i has a material incentive to deviate to not investing. Non-signatory i's increase in reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is expression (6) in Jang et al. (2018). We now demonstrate that this increase is reciprocity payoff is not sufficient to prevent deviation for $Y < Y^*$. Consider the following 4 exhaustive cases.

Case (i): s^* is such that j does not invest if only i signs and i does not invest if only j signs. Note that expression (6) in Jang et al. (2018) is no less than the reduction in i's material payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ iff $Y > \frac{2(\gamma-\beta)}{\beta(2\beta-\gamma)}$. However the RHS is less than Y^* iff $\beta > \gamma$, which is false.

Case (ii): s^* is such that j does not invest if only i signs and i does invest if only j signs. Note that expression (6) in Jang et al. (2018) is no less than the reduction in i's material payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ iff $Y \geq Y^*$, which is false.

Case (iii): s^* is such that j does invest if only i signs and i does not invest if only j signs. Note that expression (6) in Jang et al. (2018) is no less than the reduction in i's material payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ iff $Y > \frac{2(\gamma-\beta)}{\beta(2\beta-\gamma)}$. However the RHS is less than Y^* iff $\beta > \gamma$, which is false.

Case (iv): s^* is such that j does invest if only i signs, i does invest if only j signs. Note that expression (6) in Jang et al. (2018) is no less than the reduction in i's material payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ iff $Y \geq Y^*$, which is false.

Thus i deviates at h. ■

References

[1] Jang, D., Patel, A. and M. Dufwenberg (2018) "Agreements with Reciprocity: Co-financing and MOUs" mimeo.