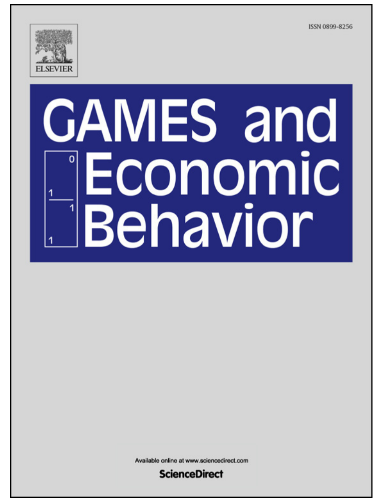

“Small, yet Beautiful”: Reconsidering the optimal design of multi-winner contests

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"Small, yet Beautiful": Reconsidering the Optimal Design of Multi-winner Contests *

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Abstract

We reconsider whether a grand multi-winner contest elicits more equilibrium effort than a collection of sub-contests. Fu and Lu (2009) employ a sequential winner-selection mechanism and find support for running a grand contest. We show that this result is completely reversed if a simultaneous winner-selection mechanism or a sequential loser-elimination mechanism is implemented. We then discuss the optimal allocation of players and prizes among sub-contests.

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JEL Classifications: C72; D72; D74

Keywords: Contest design; Multiple winner; Group-size; Selection mechanism

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1 Introduction

Contests, in which players exert costly and irreversible efforts to win a prize, are ubiquitous in day-to-day life. In cases such as war, terrorism or territorial conflicts, contests are not designed by an organizer. However, there are very many situations including sports, patent race, promotion tournament, crowd sourcing, legal battle etc. in which an organizer organizes the contest, and contest design issues become highly important. The topic of optimal contest design, hence, has been an active area of research. In the literature one of the most frequently attempted questions is how to maximize the total effort exerted in a contest. For a contest with noisy outcome it is a further important question whether arranging a grand contest elicits a higher equilibrium effort than arranging several sub-contests.

For a single-winner setting, Moldovanu and Sela (2001, 2006) show that under certain conditions a grand contest indeed elicits higher effort. Adding an important contribution to this area, Fu and Lu (2009) characterize the optimal structure for multi-winner contests.[1] They employ a nested winner-selection procedure as in Clark and Riis (1996) and show that a grand contest elicits greater equilibrium efforts than what a collection of mutually exclusive and exhaustive sub-contests does. This is an important finding since this extends the single-winner contest results of Moldovanu and Sela (2001, 2006) into multi-winner settings, and provides with clear policy design prescriptions.

In the specific mechanism employed above, the winners are selected sequentially. Players simultaneously exert their effort, and K winners are selected by K consecutive draws. Once a winner is selected through a Tullock (1980) contest success function, he/she is immediately removed from the pool of candidates up for the next draw. This procedure is repeated until all the prizes are exhausted.

In the field, however, the winner-selection procedure in a multi-winner contest is not always the one suggested above. Clark and Riis (1996) mention that when "*the imperfectly discriminating rent-seeking contest [...] ha(s) several winners, there is no unique method for selecting those winners*". Indeed, the very first winner-selection mechanism suggested in the multi-winner contest literature is by Berry (1993), who considers a one-shot winner-selection mechanism. Under this, the players exert effort and the set of winners are taken

[1] In these contests there are multiple prizes, but a contestant can win at most one prize.

out simultaneously. The probability of a player to win one of K prizes is the sum of efforts exerted by any combination of a K-player group that includes that specific player, divided by the sum of efforts exerted by any combination of a K-player group. There are different instances in which either a simultaneous (Berry, 1993) or a sequential (Clark and Riis, 1996) winner-selection mechanism is employed in the field.

There are both pros and cons of employing the simultaneous mechanism. Clark and Riis (1996) show that with this mechanism the very first prize is allocated according to the effort outlays whereas all the other prizes are implicitly allocated randomly - allowing for an incentive to free-ride. Chowdhury and Kim (2014), on the other hand, find an equivalence of the simultaneous mechanism to a mechanism in which the losers are sequentially taken out - essentially providing a microfoundation for the contest success function arising out of the simultaneous mechanism.[2] Since the loser-elimination mechanism is well implemented in real life, this helps one to reformulate those real life situations as well. Hence, it is important to understand whether the answer to the original question (of comparing grand contest with sub-contests) depends on the particular winner-selection mechanism implemented.

In this study we reconsider such comparison of a grand contest with a collection of mutually exclusive and exhaustive sub-contests from a design point of view. We employ the simultaneous winner-selection mechanism (Berry, 1993) and find that the result of Fu and Lu (2009) gets reversed, i.e., a collection of sub-contests elicit a higher level of equilibrium effort than what a grand multi-winner contest does. In such a situation we characterize the optimal allocation of players and prizes for the case with identical prizes. We further show that with the sequential loser-elimination mechanism (Chowdhury and Kim, 2014) the Fu and Lu (2009) result is again reversed. We then characterize the optimal contest structure when the number of sub-contests is limited to two and no prize can be wasted.

[2]Moreover, Chowdhury and Kovenock (2012) find effort equivalence between the simultaneous mechanism and a situation in which the players in a multi-winner contest are connected by a ring network. de Palma and Munshi (2013) find that the simultaneous mechanism has a probabilistic foundation.

2 Model and main result

2.1 Contest design under simultaneous winner-selection

Consider N identical players competing for K indivisible prizes with $N > K \geq 2$. The common values of the prizes are v_1, v_2, ..., v_K, and a player can win at most one prize. Without any loss of generality, assume that $v_1 \geq v_2 \geq ... \geq v_K$. The contest designer can run the grand contest by putting all the players and the prizes together or run M small contests by dividing the contestants and the prizes into mutually exclusive groups. Let n_g be the number of contestants in group g and $\mathbf{v}_g = (v_1^g, v_2^g, ..., v_{k_g}^g)$ be the vector of values of prizes allocated to group g where k_g be the number of prizes in group g, and $v_1^g \geq v_2^g \geq ... \geq v_{k_g}^g$. We also define a collection of contests $\mathbf{C} = \{c_g\}_{g=1}^M$ of which entry is $c_g = \{n_g, \mathbf{v}_g\}$. Since the groups are mutually exclusive and exhaustive, given the sets of contestants and of prizes, collection $\mathbf{C}$ defines a contest structure. Therefore, $\sum_{g=1}^M n_g = N$ and $\sum_{g=1}^M |\mathbf{v}_g| = \sum_{g=1}^M k_g = K$ where $|\mathbf{v}_g|$ is the number of elements in vector $\mathbf{v}_g$. We allow both a contest without a prize ($k_g = 0$ but $n_g > 0$) and that without a player ($k_g > 0$ but $n_g = 0$). In other words, the designer can throw prizes or players away. Throughout the paper, we assume linear cost function with unit marginal cost.

Let us first consider the problem of player i who is allocated to group g with $k_g \geq 1$. Berry (1993) proposes a simultaneous winner-selection mechanism according to which the probability of a player to win a prize is the sum of efforts expended by any combination of k_g players *that includes the specified player*, divided by the sum of efforts expended by any combination of k_g players. Hence, when $n_g > k_g$, the probability that player i wins a prize is:

$$P_i^{SM}(\underline{x}) = \frac{\overbrace{\sum_{j=1}^{k_g-1} x_j + x_i + \sum_{j=1,j\neq 2}^{k_g} x_j + x_i + \cdots + \sum_{j=n_g-k_g+1}^{n_g-1} x_j + x_i}^{\binom{n_g-1}{k_g-1}\text{times}}}{\underbrace{\sum_{j=1}^{k_g} x_j + \sum_{j=1,j\neq 2}^{k_g+1} x_j + \cdots + \sum_{j=n_g-k_g+1}^{n_g} x_j}_{\binom{n_g}{k_g}\text{times}}}$$

where $\underline{x}$ is a vector of efforts, and x_i is the effort of player i. If every player in the group other than player i expends the same amount of effort x_{-i}, the contest success function of the simultaneous winner-selection mechanism boils down to:

$$P_i^{SM}(x_i, x_{-i}) = \min\left\{\frac{(k_g-1)x_{-i}+x_i}{(n_g-1)x_{-i}+x_i}, 1\right\}$$

In his original paper, Berry considers only the case with identical prizes. Here, we assume that when prizes are heterogeneous, the prizes allocated to contest c_g are randomly assigned to the winners in c_g. Letting $\overline{v}^g$ denote the expected value of the prize, $\left(\sum_{j=1}^{k_g} v_j^g\right)/k_g$, we can write the objective function of player i as

$$\pi_i(x_i, x_{-i}|n_g, \mathbf{v}_g) = \overline{v}^g P_i^{SM}(x_i, x_{-i}) - x_i,$$

thus the symmetric equilibrium effort is

$$x_g^{SM}(n_g, \mathbf{v}_g) = \begin{cases} \overline{v}^g\left(n_g - k_g\right)/n_g^2 & \text{if } n_g \geq k_g \geq 1 \\ 0 & \text{otherwise} \end{cases}. \tag{1}$$

Let $T^{SM}(\mathbf{C})$ denote the total equilibrium effort with the simultaneous winner-selection mechanism, i.e.,

$$T^{SM}(\mathbf{C}) = \sum_{g=1}^{M} n_g \times x_g^{SM} \tag{2}$$

The following proposition states that the grand contest in which all the prizes and the players are put never maximizes the total effort if the simultaneous winner-selection mechanism is implemented.

Proposition 1 *Suppose the simultaneous winner-selection mechanism is employed. Then, the sum of the efforts elicited in a contest increases as the least-valued prize gets excluded from the list. Thus, in the optimal contest structure, a single prize is given to each group.*

Proof. It is clear from (1) that x_g^{SM} increases as k_g decreases if $n_g \geq k_g$. Suppose that we throw away the least valued prize from a contest with $k_g \geq 2$, and let $\overline{v}_{new}^g$ denote the new expected value of the prize in the contest. Then $\overline{v}_{new}^g \geq \overline{v}^g$, and the effort elicited by the new contest is

$$\frac{\overline{v}_{new}^g\left(n_g - k_g + 1\right)}{n_g} > \frac{\overline{v}^g\left(n_g - k_g\right)}{n_g} = \text{the effort from the old contest } c_g.$$

■

To characterize the optimal structure further, let us assume, following Berry (1993), that the prizes are identical (i.e., $v_1 = v_2 = ... = v_K = v$), and define $\overline{K} = \min\{K, \lfloor N/2 \rfloor\}$ where $\lfloor N/2 \rfloor$ is the largest integer not greater than $N/2$. Then we can show that with identical prizes, the total equilibrium effort is maximized by a symmetric structure.

Proposition 2 *Suppose that the prizes are identical and that the simultaneous winner-selection mechanism is implemented. In the optimal structure, $\overline{K}$ prizes are used, and N players are divided as symmetrically as possible into $\overline{K}$ groups.*

Proof. Proposition 1 shows that in order to maximize the total effort, for any competitive sub-contest, that is, one with $n_g > k_g$, k_g must be 1. In such a sub-contest, the elicited equilibrium effort is

$$e(n_g) = \frac{(n_g - 1)}{n_g} v.$$

Notice that $e(n_g)$ is increasing and concave in n_g, from which we infer the following. First, throwing away a player reduces the total effort because $e(n_g)$ is increasing in n_g, which implies that in the optimal structure the number of players who are in the competitive contests is N. Second, if we ignore the integer problem, then due to the concavity of $e(n_g)$, the total effort $\sum v(n_g - 1)/n_g$ is maximized when $n_i = n_j$ for all i, j.

To show that exactly $\overline{K}$ must be used in the optimal structure, let us again ignore the integer problem. Suppose $k(\leq K)$ prizes are used and $K - k$ are thrown away. When N players are symmetrically allocated to k contests, $n_g = N/k$ for all g, and the total effort is

$$\sum_{g=1}^{k} \frac{(n_g - 1)v}{n_g} = k\frac{(N/k - 1)v}{N/k} = (N - k)k\frac{v}{N}$$

which is maximized when $k = N/2$. Therefore, exactly $\overline{K}$ prizes are used in the optimal structure. ■

When the prizes are identical, it is optimal to run as many symmetric sub-contests as possible. If the prizes are heterogeneous, however, it may be optimal to make more players compete for a more valuable prize for which each player is willing to expend more effort. To see this in a clearer manner, let us consider four players ($N = 4$) competing for two different prizes, 1 and 2 ($K = 2$), with values $v_1 \geq v_2$. Because according to (1), $n_g x_g^{SM}$, the effort elicited by contest c_g, increases in n_g, allocating a player to a contest without a prize (i.e., excluding a player) is never optimal. And we know that the grand contest does not maximize the total effort. Therefore, we only need to consider how to allocate the four players to two contests each of which has a single prize.

If the players are symmetrically allocated to the two contests ("2-2" structure), each of them is the standard Tullock contest. Therefore, the equilibrium effort of a player who competes for prize $k(=1,2)$ is $v_k/4$, and the total effort is $T^{2\text{-}2} = (v_1 + v_2)/2$. If, on the other hand, all four of them compete for the more valuable prize ("4-0" structure), according to (1), the total effort is $T^{4\text{-}0} = 3v_1/4$.[3] Thus, if the prizes are sufficiently heterogeneous (more precisely if $v_1 > 2v_2$), the most asymmetric contest structure, in which the low value prize is excluded, maximizes the total effort.

2.2 Sequential loser-elimination mechanism

In the previous subsection we examined whether the optimality of the grand contest remains valid if the simultaneous winner-selection mechanism is employed. Here we consider a sequential loser-elimination mechanism (Chowdhury and Kim, 2014). According to this mechanism, the players expend effort, then one player is selected as a loser, i.e., $n_g - 1$ players are selected as winners using Berry's (1993) contest success function. Then that player and his effort are taken out of the calculation, another contest is run among the remaining $n_g - 1$ players using their already expended effort, and another loser is taken out. After $n_g - k_g$ losers are taken out, the least-valued prize is given to the next "loser", and this procedure is repeated until all the prizes are distributed.

Formally, let Ω_k denote a set of $n_g - (k-1)$ players. In the loser-elimination mechanism, conditional on that player i is in Ω_k, the probability that player i is selected in the k^{th} draw is

$$q_i(\underline{x}|\Omega_k) = \frac{X(\Omega_k) - x_i}{\sum_{j\in\Omega_k}(X(\Omega_k) - x_j)} = \frac{X(\Omega_k) - x_i}{(n_g - k)X(\Omega_k)}$$

where $X(\Omega_k)$ is the sum of the efforts exerted by the players in Ω_k. This is the probability that, according to Berry (1993), player i is not in a group of $n_g - k$ players when the number of contestants is $n_g - k + 1$. When the number of players is n_g and that of prizes is k_g, the

[3] Structure "3-1" (making only three players compete for prize 1) is dominated by structure "4-0" in terms of the total elicited effort because in structure "3-1" one player is wasted.

probability that player i wins a prize can be written as

$$P_i^{LE}(\underline{x}) = \sum_{\forall \Omega_{k_g}} \left[\Pr\left(\Omega_{k_g}\right) I\left(i \in \Omega_{k_g}\right)\left(1 - q_i(\underline{x}|\Omega_{k_g})\right)\right].$$

where $\Pr(\Omega_k)$ is the probability that the set of the remaining contestants for the k^{th} draw is Ω_k, and $I(i \in \Omega_k)$ is the indicator function that takes value 1 if $i \in \Omega_k$ and 0 otherwise.

Chowdhury and Kim (2014) show that if everyone but player i expends the same amount of effort x_{-i}, then the probability for i to win a prize under this mechanism is:

$$P_i^{LE}(x_i, x_{-i}) = \min\left\{\frac{(k_g-1)x_{-i} + x_i}{(n_g-1)x_{-i} + x_i}, 1\right\}$$

Provided that $n_g > k_g$, after $n_g - k_g$ losers are eliminated, the one who will get the lowest-value prize is drawn and eliminated from the pool. This repeats until only one player is left in the pool. Therefore, the expected payoff without the effort cost for player i in contest c_g is:

$$\begin{aligned}
&\frac{(k_g-1)x_{-i} + x_i}{(n_g-1)x_{-i} + x_i}\left[v_{k_g}^g + \frac{(k_g-2)x_{-i} + x_i}{(k_g-1)x_{-i} + x_i}\left[\left(v_{k_g-1}^g - v_{k_g}^g\right) + \dots + \frac{x_i}{x_{-i} + x_i}\left(v_1^g - v_2^g\right)\right]\right] \\
&= \frac{(k_g-1)x_{-i} + x_i}{(n_g-1)x_{-i} + x_i}v_{k_g}^g + \frac{(k_g-2)x_{-i} + x_i}{(n_g-1)x_{-i} + x_i}\left(v_{k_g-1}^g - v_{k_g}^g\right) + \dots + \frac{x_i}{(n_g-1)x_{-i} + x_i}\left(v_1^g - v_2^g\right) \\
&= \frac{\left(\sum_{l=2}^{k_g} v_l^g\right)x_{-i} + v_1^g x_i}{(n_g-1)x_{-i} + x_i}
\end{aligned}$$

from which the symmetric equilibrium effort with the loser-elimination mechanism is derived as

$$x_g^{LE}(n_g, \mathbf{v}_g) = \begin{cases} v_1^g/n_g - \left(\sum_{l=1}^{k_g} v_l^g\right)/(n_g)^2 & \text{if } n_g > \overline{k}_g \geq 1 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where $\overline{k}_g = \min\{k < k_g | v_k^g > v_{k+1}^g\}$ if not all the prizes in contest c_g are identical, and $\overline{k}_g = k_g$ if the prizes are identical. In words, $n_g > \overline{k}_g$ means that the (more valuable) prizes are not enough to be shared by everybody, so the players have to compete. Thus, when a contest structure $\mathbf{C}$ is given, the total equilibrium effort is

$$T^{LE} = \sum_{g=1}^{M} n_g \times x_g^{LE}(n_g, \mathbf{v}_g).$$

The following proposition states that the grand contest never maximizes the total effort if the sequential loser-elimination mechanism is implemented.

Proposition 3 *Suppose the sequential loser-elimination mechanism is employed. Then, the sum of the efforts elicited in a group increases as the least-valued prize is excluded from the list. Thus, in the optimal contest structure, a single prize is given to each group.*

Proof. It is clear from (3) that x_g^{LE} increases as we keep throwing away the least valued prize provided that $n_g \geq k_g$. This does not affect the efforts elicited from the other sub-contests. Thus, the total effort increases. ■

Just as Proposition 1, this proposition shows that in the optimal contest structure, there should not be a sub-contest with more than one prize. Recall that when the prizes are identical, the loser-elimination mechanism and the simultaneous mechanism yield the exactly same outcome.[4] This implies that when the prizes are identical, the optimal structure characterized in Proposition 2–that is, the symmetric allocation of prizes and players into $\overline{K} = \min\{K, \lfloor N/2 \rfloor\}$ contests–is also optimal when the loser-elimination mechanism is implemented.

2.3 Comparison with sequential winner-selection mechanism

The results above contrast sharply with the result of Fu and Lu (2009) who employs the sequential winner-selection mechanism à la Clark and Riis (1996) for each contest. In this mechanism, players are selected over multiple rounds of contests as in the sequential loser-elimination mechanism, but instead of a loser is selected out in each round, a winner is selected in.

Formally, the probability that player i is selected in the k^{th} draw is:

$$P_{ik}^{SQ}(\underline{x}) = \sum_{\forall \Omega_k} \left[\Pr(\Omega_k)\, I(i \in \Omega_k)\, p_i(\underline{x}|\Omega_k) \right]$$

where Ω_k is again a set of $n_g - (k-1)$ players, $\Pr(\Omega_k)$ is the probability that the set of the remaining contestants for the k^{th} draw is Ω_k, $I(i \in \Omega_k)$ is the indicator function that takes value 1 if $i \in \Omega_k$ and 0 otherwise, and $p_i(\underline{x}|\Omega_k) = x_i / \sum_{j \in \Omega_k} x_j$. And, the probability that player i wins a prize is $P_i^{SQ}(\underline{x}) = \sum_k P_{ik}^{SQ}(\underline{x})$. It can be shown that in the symmetric equilibrium, this mechanism elicits effort as much as

$$x_i^{SQ}(n_g, \mathbf{v}_g) = \frac{1}{n_g} \sum_{k=1}^{k_g} \left[v_k^g \left(1 - \sum_{l=0}^{k-1} \frac{1}{n_g - l} \right) \right]. \tag{4}$$

[4] See $P_i^{SM}(x_i, x_{-i})$ and $P_i^{LE}(x_i, x_{-i})$ above. For details, see Chowdhury and Kim (2014).

Suppose, as before, there are four players ($N = 4$) competing for two different prizes, 1 and 2 ($K = 2$), with values $v_1 \geq v_2$. As shown above, the maximized total effort with two small contests is

$$T^{small} = \max\left\{\frac{v_1 + v_2}{2}, \frac{3}{4}v_1\right\}.$$

If all the prizes and the players are put in one grand contest and the simultaneous winner-selection mechanism is implemented, then according to (2) the total equilibrium effort is

$$T^{SM} = \frac{\overline{v}\,(N-K)}{N} = \frac{v_1 + v_2}{4},$$

and if the sequential loser-elimination mechanism is implemented, according to (3) the total effort is

$$T^{LE} = v_1 - \frac{v_1 + v_2}{N} = \frac{3v_1 - v_2}{4}.$$

On the other hand, when the sequential winner-selection mechanism is implemented, then according to (4) the total effort is

$$T^{SQ} = v_1\left(1 - \frac{1}{4}\right) + v_2\left(1 - \frac{1}{4} - \frac{1}{3}\right) = \frac{3v_1}{4} + \frac{5v_2}{12}.$$

This example clearly shows that $T^{SQ} > T^{small} > T^{LE} > T^{SM}$.[5]

3 Further analysis: limit in the number of contests

Thus far, we have assumed that there is no additional cost for the designer to organize more contests, and showed that dividing a grand contest into smaller ones can increase the total effort. Let us now suppose that there exist operational costs for running these contests, which increases in the number of groups. Because of the costs, one cannot run more than two contests, i.e., $M \leq 2$.[6] Furthermore, suppose that the designer must not waste any prize even though doing so may increase the total elicited effort.[7] This requirement is comparable to

[5]The result illustrated by this example can be easily generalized. See the discussion in Section 4.

[6]One may generalize this with a generic convex cost function that considers the number of contests. But, to provide a simple and clear example, here we consider the case in which the cost is zero for up to two contests and then it becomes infinity.

[7]If the designer can throw away prizes, only one prize will be used in each sub-contest in the optimal structure.

the budget balance requirement in the mechanism design literature. An interesting question is what the optimal contest structure looks like when these constraints are imposed. In the following analysis, for the sake of simplicity we assume that the prizes are identical ($v_1 = v_2 = ... = v_K = v$), in which case the sequential loser-elimination mechanism is best-response equivalent to the simultaneous winner selection mechanism.

The following proposition, ignoring the integer problem, characterizes the optimal contest structure given the cost constraint.

Proposition 4 *Suppose that the simultaneous winner-selection mechanism or the sequential loser-elimination mechanism is employed. Furthermore, suppose that M cannot be greater than 2 and that any prize must not be wasted. Then the total effort is maximized by* $\mathbf{C}$ *such that* $k_1^* = 1$, $k_2^* = K - 1$,

$$n_1^* = \begin{cases} \frac{N\left(\sqrt{K-1}-1\right)}{K-2} & \text{if } K > 2 \\ N/2 & \text{if } K = 2 \end{cases},$$

and $n_2^* = N - n_1^*$.

Proof. Without loss of generality, suppose $n_1 \leq N/2$. Because the prizes are identical, the total effort can be written as

$$n_1 x_1 + n_2 x_2 = v\left[\frac{n_1 - k_1}{n_1} + \frac{n_2 - k_2}{n_2}\right].$$

As shown in the proof of Proposition 2, throwing away a player never increases the total effort. Thus, $n_1 + n_2 = N$. And due to the no-waste requirement, $k_1 + k_2 = K$. Therefore, we obtain the following.

$$\begin{aligned}\frac{n_1 x_1 + n_2 x_2}{v} &= \frac{n_1 - k_1}{n_1} + \frac{N - n_1 - K + k_1}{N - n_1} \\ &= \frac{2n_1(N - n_1) - n_1 K - k_1(N - 2n_1)}{n_1(N - n_1)}\end{aligned}$$

Notice that $n_1 x_1 + n_2 x_2$ is maximized when k_1 is the minimized because $N \geq 2n_1$. Therefore $k_1^* = 1$. Given $k_1^* = 1$, the expression is maximized at $n_1^* = N\left(\sqrt{K-1} - 1\right)/(K-2)$ which satisfies the assumption that $n_1 \leq N/2$ and converges to $N/2$ when K goes to 2. ■

This proposition shows that if the number of contests cannot be as big as the number of prizes and no prize can be wasted, the total effort can be maximized by an asymmetric contest structure. Observe that $N\left(\sqrt{K-1}-1\right)/(K-2)$ is larger than N/K, meaning that when $K>2$, only a single prize is allocated to group 1, but there are disproportionately many contestants in the group. So, in the optimal structure, two small contests are organized. In one of these contests players face a fierce competition, while in the other, players compete in a more relaxed manner.

4 Discussion

In this study we reconsider the design of multi-winner contests. Fu and Lu (2009) employ a sequential winner-selection mechanism (Clark and Riis, 1996) and find that a grand contest always elicits higher equilibrium effort than a collection of sub-contests. We show that the result is completely reversed if a simultaneous winner-selection or a sequential loser-elimination mechanism is implemented. This result is obtained because the simultaneous winner-selection mechanism (Berry, 1993) and the sequential loser-elimination mechanism (Chowdhury and Kim, 2014) suffer with the issue of free-riding. Free-riding is not a problem in the sequential winner-selection that is implemented in the analysis of Fu and Lu (2009) since winners are selected in each sequence depending on the amount of effort they spent in the start.

In the simultaneous winner-selection mechanism, however, winners are selected in combinations. A player's winning probability is the sum of effort of all k-players group that he is in, as a ratio of the sum of the efforts of all possible k-players group. Hence, it is possible that a player does not exert any effort, but wins a prize when a combination of k-players, that includes him, gets selected. Due to this possibility, it is optimal for the players to exert less effort than in the sequential winner-selection mechanism.

In the sequential loser-elimination mechanism, the logic for free-riding similar to the simultaneous winner selection mechanism also applies. In each sequence of loser-elimination a combination of survivors are selected using the Berry (1993) mechanism, until only k survivors are left to be awarded prizes. Hence, again it is possible for a player not to exert any effort but still get selected for a prize when the combinations of players, to which he is included, are selected as survivors in each sequence. As a result, it is again optimal for the players to free-ride on each other and exert less effort than they would have exerted under the sequential winner-selection mechanism.

An increase in the number of prizes increases the effort exerted in the sequential winner-selection mechanism, and this leads to the "beauty of bigness" result of Fu and Lu (2009). However, in both the simultaneous winner-selection and the sequential loser-elimination mechanisms the free-riding opportunity also increases when the number of prizes increases. Moreover, in both Berry's and Chowdhury and Kim's mechanisms, the marginal effort of the "last" player is greater when the number of contestants is smaller. This is because, as in most contest mechanisms, the probability that an individual wins a prize is more sensitive to an individual's effort when the number of players is smaller. So, in the grand contest in which the number of prizes and the number of players both are maximized, players exert less effort than in a collection of sub-contests under the simultaneous winner-selection or the sequential loser-elimination mechanisms.

These results are of importance for several reasons. First, they show that the optimal design of a multi-winner contest depends crucially on the type of winner-selection mechanism. Hence, depending on the objective of the designer, a combination of a winner-selection mechanism and a grand or sub contest should be employed.

Furthermore, under symmetric prize values, Clark and Riis (1996) show that the equilibrium effort exerted under the sequential winner-selection mechanism is (weakly) greater than the equilibrium effort exerted under the simultaneous winner-selection mechanism. Hence, the sum of equilibrium effort exerted in the sub-contests with the sequential winner-selection mechanism is greater than or equal to the sum of equilibrium effort exerted in the sub-contests with the simultaneous winner-selection mechanism. Moreover, Fu and Lu (2009) show that for the sequential winner-selection mechanism, the equilibrium effort exerted in the grand contest is strictly greater than the sum of equilibrium effort exerted in the sub-contests. Finally, the current study shows that for the simultaneous winner-selection mechanism, the sum of equilibrium effort exerted in the sub-contests is strictly greater than the equilibrium effort exerted in the grand contest. Combining these, we get a clear ranking among the different mechanism as follows: *Grand contest with sequential winner selection* $>$ *Collection of sub-contests with sequential winner-selection* $\geq$ *Collection of sub-contests* *with simultaneous winner-selection* > *Grand contest with simultaneous winner-selection*.[8]

If a contest designer faces an unconstrained choice of which mechanism to be employed, this ranking clearly shows that running a grand contest with sequential winner selection mechanism would be preferred - triumphing the 'Beauty of Bigness'. In the field, however, a simultaneous mechanism might already be in place and would be costly to replace. The current study prescribes that in such a case it is preferred to implement a collection of small sub-contests, as in terms of total equilibrium effort, they are 'Small, yet Beautiful'.

Moreover, this study also indicates that if it is possible to employ the loser-elimination mechanism, then it might elicit more effort than the simultaneous winner-selection mechanism, and can be a compromise if a sequential winner-selection mechanism cannot be employed.

Finally, it is well known that in the collective rent-seeking contests (*à la* Nitzan, 1991), a part of the prize is allocated according to the effort outlays and the rest is allocated randomly. Since that is also the case for both the simultaneous winner-selection and the sequential loser-elimination mechanisms multi-winner contests, the current result indicates that in such collective contests it might be possible to elicit higher effort by splitting the prize from a grand prize into several small prizes.

[8]Here, "simultaneous winner-selection" can be replaced by "sequential loser-elimination". Moreover, it can be easily shown that given a sub-contest, the sequential loser-elimination mechanism is weakly preferred to the simultaneous mechanism. What is unclear is whether the grand contest with the sequential loser-elimination elicits a greater total effort than a collection of sub-contests with the simultaneous mechanism.

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