Flexible Monetary Policy Rules

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Abstract

The thesis includes three independent essays that investigate the properties of monetary policy rules that modern Central Banks enact in response to different shocks. Chapter 2 considers the changes observed in the policy rules followed by the Bank of England after the financial crisis in 2007. Strong evidence indicates that the linear Taylor rule is not able to capture the behaviour of the Bank of England. Considering three different types of non-linear Taylor rules – in particular, the structural model, the threshold model and the opportunistic model – we obtain robust results showing that the Bank of England has changed its policy priorities after the crisis. In Chapter 3, we compare the endogenous switching rule, in which the weights change according to the macroeconomic conditions, with the "traditional" Taylor rule with fixed weights in the basic New Keynesian model. The results show that although the endogenous-switching rule outperforms the "original" Taylor rule, the economy could benefit from implementing the linear Taylor rule by increasing the weights of inflation and output gap. Chapter 4 evaluates different monetary policy rules in a small open economy. In this framework, there exists an optimal rule which may however be hard to implement in practice. Central Banks may thus consider alternative rules. In order to minimise the welfare loss with respect to the optimal rule, we consider discretionary rules, the Taylor rule and the Taylor rule with real exchange rate, finding that the ranking of welfare performance depends on intratemporal elasticity of substitution and the degree of openness.

Contents

List of Figures

List of Tables

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Chapter 1

Introduction

This thesis contains three essays on monetary policy which are motivated by the global financial crisis. It investigates the responses of monetary authority in the UK before and after the financial crisis, and exploits theoretical models to evaluate the performances of different monetary policy rules in response to different shocks.

After the global financial crisis, the UK has experienced consistent high inflation, large negative output gap and extremely low nominal interest rate. Among the many interesting research questions suggested by this experience, this thesis tackles three major issues: whether and how the Bank of England has changed behaviour, what are the consequences of switching monetary policy regimes, and which types of monetary rules are more likely to minimize welfare losses. In Chapter 2, we investigate empirically the possibility of changing behaviour in the Bank of England due to the financial crisis in 2007. The time paths of inflation, output and the Bank rate in the UK suggest the Bank of England has been more concerned about output in the post-crisis period, although it was strictly targeting inflation before 2007. Building on this observation, we consider the most recent formalizations of time-varying monetary policy rules that allow for regime change in response to changing macroeconomic conditions. We estimate non-linear Taylor rules using structural, threshold and opportunistic approaches to model the behaviour of the Bank of England. Strong empirical evidence suggests that the preferences of the Bank of England are time-varying. The parameters in the monetary policy rule appear to have changed over time, which suggests that exogenous Taylor rule parameters are not appropriate for modelling the Bank of England over the whole time horizon considered.

Chapter 3 studies policy change in a New Keynesian model by replacing the hypothesis of fixed policy rule with an endogenous-switching rule that allows monetary policy to change over time depending on observed shocks. Specifically, the Central Bank considers different Taylor rules and switches between them when the output gap breaches some threshold level. In addition, we evaluate the welfare performances of the "traditional" Taylor rules with different fixed weights. The endogenous-switching rule is shown to improve welfare in the event of a large negative demand shock comparing with the "original" Taylor rule. However, the "Crisis Regime" always outperforms other alternatives by showing the highest welfare although it is associated with high interest rate volatility. It suggests that a high social welfare could be achieved if Central Banks become more aggressive and raise both the weights attached to inflation and output stabilization.

An important difference between Chapters 2 and 3 is that Chapter 2 covers "positive" notions of the performance of monetary policy rules, while Chapter 3 covers "normative" notions. To be specific, in Chapter 2, a good performance of a monetary policy rule is taken to mean a close fit to the actual data, implying that the policy rule is an accurate representation of actual observed behaviour of policy-makers. In Chapter 3, in contrast, a good performance of a monetary policy rule is taken to mean that it results in a minimal welfare loss, and is hence the rule that we would advise policy-makers to use.

Chapter 4 employs a two-country dynamic general equilibrium model to examine the welfare effects of different policy responses to a Foreign demand shock in a small open economy. We evaluate monetary policy rules by using a utility-based loss function that depends on domestic inflation, output gap and real exchange rate. Using the model of

De Paoli (2009), we derive the optimal rule for a small open economy and re-express it as an interest-rate rule using the economy's equilibrium system. When the optimal rule cannot be implemented, alternative policy rules yield different welfare levels and their ranking depends on the values taken by some key parameters – in particular, the elasticity of substitution between home and foreign goods and the degree of openness. When domestic and foreign goods are close substitutes, a discretionary rule outperforms other simple rules, while the domestic Taylor rule shows the highest welfare when the intratemporal elasticity is low. For an intermediate degree of the elasticity of substitution, the performances of alternative policy rules depends on the degree of openness.

Chapter 2

Modelling Bank of England's Monetary Policy Rules before and during the Financial Crisis

2.1 Introduction

Before the financial crisis in 2007, inflation was successfully and strictly controlled by the Bank of England and there was no open letter to the Chancellor of the Exchequer. However, the post-financial crisis period is typified by slow growth, high unemployment, until recently, and relatively high inflation. High inflation led the Governor of the Bank of England to, for the first time, write open letters to the Chancellor, explaining why inflation was more than one percentage point above the inflation target. At the same time, interest rates were at record lows. The Bank of England could easily have raised rates to combat inflation but it did not, which might indicate the monetary authority have changed priorities when facing the severe shock. The research question we are interested in this Chapter is what type of the monetary policy rule could be considered as a proper way to model Bank of England behaviour when facing a severe shock. This chapter investigates various Taylor rules, modelling the Bank of England's behaviour since 1992.

In this Chapter, we model the behaviour of the Bank of England and find changes because of the financial crisis in 2007. Using the linear Taylor rule, we find evidence that the coefficients changed significantly after the crisis, which is also proved by the recursive estimation of the Taylor rule. The empirical results present evidence that the linear Taylor rule is not able to capture the behaviour of the Bank of England, therefore, the non-linear Taylor rule is considered instead. We employ three different ways to model the non-linear Taylor rule – that is, the structural model, the threshold model and the opportunistic model, and we find strong evidence which indicates that the Bank of England behaved differently following the financial crisis shock. This Chapter provides motivation and evidence for further work that will attempt to endogenise the Taylor rule weights which policymakers put on inflation and output gaps, rather than assuming that these are independent of the size of shocks to the economy.

2.2 Taylor Rules

2.2.1 General Discussion of Taylor Rules

In his seminal paper, John Taylor (1993) proposed a formal representation of monetary policy in terms of the following rule: the Central Bank influences the nominal interest rate so as to set its value equal to a pre-determined linear function of current inflation and output gaps. In this original formulation, the Taylor rule was a positive statement, an algebraic representation of how real-world interventions were being carried over by the monetary authorities. The Taylor rule gained wide popularity as the empirical evidence confirmed its general applicability to a different economies. One of the strands of literature generated by Taylor (1993) tackled the issue of the robustness of the original rule – which specified inflation gaps in terms of deviations between current and desired inflation – by considering alternative specifications including past (so-called backward rules) and expected (so-called forward rules) inflation gaps. In this respect, several contributions suggest that backward/forward rules may fit the data better (Svensson, 2003; Benhabib et al., 2001; Clarida et al., 1998; Carlstrom et al., 2000). However, the robustness of

Taylor's original rule may be assessed with respect to other characteristics: in the present Chapter, we do not consider the extensions to backward/forward rules and focus, instead, on possible non-linearities. More precisely, we investigate the possibility that a simple non-linear rule fits the data better than the traditional linear specification.

2.2.2 The Linear Taylor Rule

Taylor (1993) examined the performance of monetary policy across the G-7 countries, and found that the movements of the Federal fund rates from 1987 to 1992 were quite consistent with the estimates of the rule. Although the estimated period is just a short period, the good performance of the rule has made it to be one of the standard ways to describe monetary policy. Therefore, he suggests that a rule that moves the nominal interest rate in response to the deviation of inflation and output gap could provide a standard guidance for Central Banks to set monetary policy, which is the well-known "Taylor Rule". In the original Taylor (1993) paper, the policy rule reads

$$i_t = \pi_t + r_t^{natural} + h(\pi_t - \pi_t^T) + b(y_t - y_t^T) \tag{2.1}$$

where i_t is the short-term nominal interest rate, $r_t^{natural}$ is the natural interest rate, π_t is inflation, y_t is the logarithm of real GDP, and the π_t^T and y_t^T are policy targets. The coefficients h and b are the weights that Central Banks put on stabilising inflation and minimising the output gap respectively – a relatively higher h suggests a more inflation-averse Central Bank; while a higher b suggests a Central Bank more willing to loosen/tighten monetary policy when output is below/above potential.

Taylor (1993) points out that Central Banks could follow an exact rule when setting the monetary policy, which suggests that the natural interest rate and inflation target take constant values ($r^{natural} = 2$, $\pi_t^T = 2$), and the coefficients on the deviation of inflation from target and of output from its natural rate are constant over time and equal to 0.5. Taylor (1999b) tests the efficiency and robustness of the "original" linear rule and finds evidence that it could be used as a guideline for the European Central Bank. The good performance of the linear Taylor rule has been demonstrated by substantial studies, particularly, it shows that the Taylor rule works nearly as well as an optimal monetary policy rule (see e.g. Rudebusch and Svensson, 1999; Woodford, 2001; Ball, 2012). Also, Levin et al. (1999) state that the linear rule works better and perform more robustly than a complicated rule. It seems that the economic movements will be less unlikely to have a bad performance as long as the parameters in the Taylor rule are assigned appropriate weight. One advantage of the Taylor rule is that the rule is straightforward, it only requires current inflation and the inflation target and the output gap.

Although the Taylor rule could describe a monetary policy properly in normal times and over definite time periods, Taylor (1993) himself made it clear that the rule is too simple to capture the behaviour of Central Banks facing changing economic conditions that would likely induce monetary authorities to change the conduct of monetary policy. Österholm (2005) shows the Taylor rule is not able to fit the data completely, indicating Central Banks behave in a more sophisticated way than just following a simple instrument rule. Furthermore, Orphanides (2003) states that although a Taylor rule could be a useful tool for interpreting historical policy, strong evidence shows that it is not possible to have a stable monetary policy over time as policymakers would not have sufficient knowledge to assess the movement of the economy in the future. And Svensson (2003) argues that when setting the nominal interest rate, more variables should be taken into consideration than just inflation and the output gap.

2.2.3 Non-linear Taylor Rule

Asymmetric preferences in the behaviour of Central Banks have been argued by literature. Dolado et al. (2004) show that the Fed behaved more aggressively when there are positive inflation deviations from its target than negative ones, during the Volcker-Greenspan period. Lo and Piger (2005) find that output reacts more to policy actions during recessions than expansions, which is also supported by Peersman and Smets (2001), who show evidence that there are significantly stronger effects on output in recessions than in booms in the Euro area, indicating Central Banks implement asymmetric preferences towards prices and aggregate demand under different economic conditions. Similar results have been found in the UK and US by Cukierman and Muscatelli (2008) and in the US by Rudebusch and Svensson (1999). Therefore, non-linear interest rate reaction functions are considered in studies as it is required when modelling the asymmetry in Central Banks' behaviour (Taylor and Davradakis, 2006).

In the original paper of Taylor (1993), a short period in the US, from 1987 to 1992, was estimated and showed good performance of the Taylor rule. Not surprisingly, it is not sensible to assume the policymakers would follow a simple rule over time. Clarida et al. (2000) separate US data into three sample periods, Pre-Volcker, Volcker-Greenspan and Post-82, and then apply a forward-looking Taylor rule. And the results show that the estimated coefficients are sensitive to the sample estimation period employed. This is also confirmed by the results shown by Hamalainen et al. (2004) and Judd and Rudebusch (1998), where estimate Taylor rules for different periods and find the Fed reacted differently towards inflation and output over time.

Clearly, the linear Taylor rule is not able to capture the movement of nominal interest rates over long time periods. As much literature shows that the estimated weights on inflation and output may be sensitive to policy regime, it may suggest the possibility of structural breaks (Siklos and Wohar, 2005). This has been proved by Nelson (2001), who find evidence of structural breaks when estimating the UK monetary policy from 1972 to 1997, by separating the sample period into several regimes.

Cukierman and Muscatelli (2008) estimate a non-linear Taylor rule using a smooth-transition model. As the observed Bank rate in the UK drops from 5.25% to 0.5% within a year (from March 2008 to March 2009), the smooth-transition model is not a proper way to model the non-linear Taylor rule as it requires smooth changes in the nominal interest rate. Instead, a threshold model allows abrupt changes from one regime to another. Taylor and Davradakis (2006) point out that a simple way to describe nonlinearities is to estimate a threshold model, which has been applied to the UK from 1992 to 2003. Bunzel and Enders (2010) have argued that the loss function of Central Banks is likely to be asymmetric around inflation and output gaps. The Fed would be more aggressive when inflation is 1% above the target than 1% below target. Similarly, negative values of output gap are viewed as more problematic than the positive values of output gap. So these types of nonlinearities would imply that some threshold model is reasonable.

Cukierman and Muscatelli (2008) argue that the loss function of Central Banks could be asymmetric which refer to recession-avoidance preferences and inflation-avoidance preferences. Blinder (1999) states that in most situations Central Banks will take more political action when trying to avoid higher inflation than avoiding higher unemployment, suggesting that the negative output gap draws more attention than when there is a positive output gap. Taylor and Davradakis (2006) have modelled the Bank of England's behaviour using a non-linear Taylor rule and used threshold models to capture nonlinearities in policy behaviour. The policy rule switches when a certain value breaches the threshold. In their model, the interest rate is implemented under different Taylor rules when inflation is below or above threshold. The results show that a Taylor rule with a specified threshold is a proper way to describe the movement of the nominal interest rate.

Bunzel and Enders (2010) apply a non-linear Taylor rule using a threshold model to the US data and find strong evidence of threshold behaviour in most of the periods, either using the inflation rate or the output gap as a threshold. Then, the threshold model has been applied to several sub-periods of the US data using lagged inflation rate as threshold. The results suggest that the Fed responds much more strongly to the movement of inflation and output when inflation exceeds the threshold value.

Instead of using a deliberate inflation target, by implementing an "opportunistic strategy", opportunistic policymakers use an interim inflation target which may simply be the current inflation rate and use the effect of shocks to reduce inflation and eventually achieve the ultimate inflation target. The opportunistic rule allows monetary authorities to alter their priorities depending on movements of inflation and output gap. The advantage of opportunistic disinflation has been argued by Aksoy et al. (2006), who find that although it takes longer time to achieve disinflation using an opportunistic strategy, it requires smaller output losses than using a deliberate approach. Bomfim and Rudebusch (2000) estimates opportunistic monetary policy using the Taylor rule with an interim inflation target. The results show that as long as inflation is stable, where the current inflation is similar with a previous rate, the opportunistic policymaker will not take any action to reduce it.

The main insight of these empirical literature is that Central Banks react to changing economic conditions by "adjusting the weights" attached to output and inflation stabilization – that is, Central Banks update their priorities when the economy hit by severe shocks. However, the majority of these studies investigate the period before the financial crisis in 2007. In contrast, the present analysis includes both the period before and after 2007, and studies whether and how the policy rule has changed after the severe demand shock induced by the crisis.

2.3 Data

On 8^{th} October 1992, the UK monetary policy adopted a framework of inflation targeting and the first inflation target was announced. From that time the Bank of England started to target inflation at 2.5%, which is measured by RPIX (the Retail Price Index (RPI) excluding mortgage interest payments). The final decision on interest rates was not set by the Bank of England until May 1997, when the new Monetary Policy Committee (MPC) was given the power to set the official interest rate monthly. In December 2004, the inflation target changed from 2.5% RPIX to 2% CPI (Consumer Price Index).

Before the financial crisis, much of the interim period the Bank of England moved the nominal interest rates slightly, and was able to keep inflation close to the target. While, in response to the recent financial crisis the Bank of England reduced the policy interest rate to 0.5%, complemented by $\pounds 375bn$ of further monetary easing, popularly known as Quantitative Easing, which attempts to further stimulate the economy when conventional monetary policy becomes ineffective. The reduction of interest rates to such levels is unprecedented, having never previously been below 2% since the Bank of England was founded in 1694.

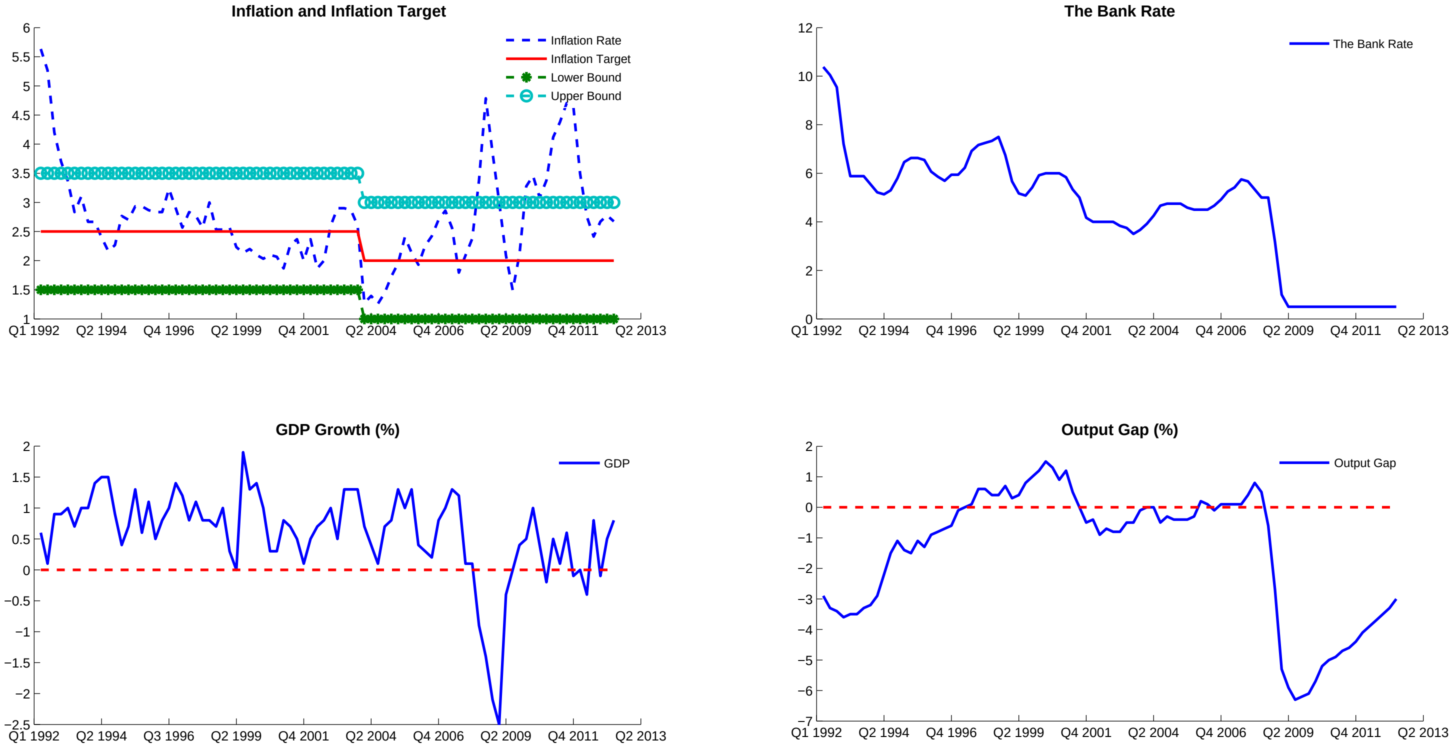
Meanwhile, the rate of inflation in the UK has spent much of recent years above target. If inflation is within 1% change in either direction, the deviation from the inflation target could be tolerated. Inflation more than 1% above the target has led to a series of open letters, from April 2007 to February 2012.

The data used to estimate the Taylor rule is quarterly data from the first quarter in 1992 until the second quarter in 2013. Figure 2.1 shows the quarterly data of the UK inflation, GDP, the output gap and the Bank Rate from 1992 to the second quarter in 2013. The monthly Bank rate for the whole period is available from the Bank of England. The inflation on a monthly basis (RPIX and CPI), which is measured by percentage change on the same period of the previous year, is available from the Bank of England. The output gap is constructed by the deviation of real output from potential output. The output gap used is published quarterly by the Office for Budget Responsibility.

The inflation gap is measured as the deviation of inflation from target. The first inflation target was 2.5% annually measured by RPIX which started in 1992. This situation continued for 12 years, until the new inflation target was announced in December 2003. Since then, the inflation measurement changed to CPI and the target changed to 2% annually. Throughout the entire period, the inflation rate is measured by the appropriate method (RPIX and CPI respectively), and along with the corresponding inflation target

(2.5% and 2% respectively), we are able to get the inflation gap.

Figure 2.1: The UK data



2.4 Methodology

2.4.1 The Linear Taylor Rule

The Taylor rule is a popular way of modelling Central Banks' behaviour. The rule assumes that the nominal interest rates are set in respect to some weighted combination of inflation and output from the trend. The linear Taylor Rule reads:

$$i_t = \alpha + h(\pi_t - \pi_t^T) + b(y_t - y_t^T) \tag{2.2}$$

The Taylor rule describes the change of nominal interest rate based on the deviation between the actual inflation rate and inflation target, and the divergence of actual GDP from potential GDP. The implication of the Taylor rule is that the parameters on these two gaps are decided directly by the policymakers. Central Banks can put their preferences into practice by choosing these parameters in order to attempt to improve economic performance.

2.4.2 Non-linear Taylor Rules

Expression (2.2) assumes constant weights attached to inflation and output gaps over time, which excludes the possibility of regime changes when Central Banks face unexpected events such as demand shocks of large magnitude. However, the behaviour of the Bank of England since the global financial crisis is strongly suggestive of the weights changing in response to a severe shock.

From the analytical point of view, abrupt changes in the policy rule may be formalized as "non-linearities" in the Taylor rule. In this Chapter, we consider non-linearities in Taylor-type rules allow a switch is different linear Taylor rules as determined by some condition such as a threshold or a structural break, which is commonly used in the literature. It is important to not confuse this terminology with a Taylor rule that is non-linear in the curved sense – e.g. being a polynomial of some degree higher than unity.

The Structural Model

The time paths of nominal interest rate, inflation and output gap are shown in Figure 2.1. In order to capture the features of the economy before and after the crisis, the possibility of structural break could be taken into consideration. Therefore, we estimated the Taylor rules for the entire period as well as two sub-periods based on the beginning of the economic crisis in the second quarter of 2007. The reason behind this is that March 2007 was the first time that inflation went over 1% above the target since the inflation target was introduced.

The structural model takes the form

$$i_t = \begin{cases} \alpha_1 + h_1(\pi_t - \pi_t^T) + b_1(y_t - y_t^T) & \text{if} \quad t < \bar{T} \\ \alpha_2 + h_2(\pi_t - \pi_t^T) + b_2(y_t - y_t^T) & \text{if} \quad t \geq \bar{T} \end{cases} \tag{2.3}$$

where $\bar{T}$ is the structural break date, which we choose the second quarter of 2007. Coefficients α_1, h_1 and b_1 are estimates when $t < \bar{T}$, while coefficients α_2, h_2 and b_2 are estimates otherwise.

The Threshold Model

The threshold regression was original developed by Hansen (1997), which allows two linear Taylor rules to switch as determined by the threshold value, $\hat{\tau}$.

Assume that the dependent variable is $Y_t = \begin{bmatrix} y_1 & y_2 & ... & y_n \end{bmatrix}$, a set of independent variable is $X_t = \begin{bmatrix} x_1 & x_2 & ... & x_n \end{bmatrix}$. The Taylor rule threshold regression model takes the form

$$i_t = [\alpha_1 + h_1(\pi_t - \pi_t^T) + b_1(y_t - y_t^T)]I(q_{t-d} \leq \hat{\tau}) + [\alpha_2 + h_2(\pi_t - \pi_t^T) + b_2(y_t - y_t^T)]I(q_{t-d} > \hat{\tau}) + \mu_t \tag{2.4}$$

where q_{t-d} is the threshold variable in period $t - d$, $I(\cdot)$ is the indicator function and the threshold parameter is $\hat{\tau}$.

Or, it could be written as

$$i_t = \begin{cases} \alpha_1 + h_1(\pi_t - \pi_t^T) + b_1(y_t - y_t^T) & \text{if} \quad q_{t-d} \leq \hat{\tau} \\ \alpha_2 + h_2(\pi_t - \pi_t^T) + b_2(y_t - y_t^T) & \text{if} \quad q_{t-d} > \hat{\tau} \end{cases} \tag{2.5}$$

Threshold Value Estimation

The threshold model allows the regression parameters to change depending on the threshold. Therefore, the threshold value needs to be estimated first. According to Hansen (1997), the appropriate estimation method is least squares. The practical approach of finding the threshold is as follows. First, form a potential threshold value set, Γ, using the value of q_{t-d} which eliminates the highest and lowest 15% of the sorted q_{t-d}, where $t = 1...n$. Then run Generalized Method of Moments (GMM) regressions of the form of equation (2.5), setting $\tau = q_{t-d}$, for each $\tau \in \Gamma$. And the sum of squared residuals obtained. The $\hat{\tau}$ can be found when the lowest sum of residuals variance appears, which can be expressed as

$$\hat{\tau} = argmin_{\tau \in \Gamma} \hat{\sigma}_n^2(\tau)$$

In this Chapter, we assume the output gap and inflation gap as threshold variables. Set $\tau = q_{t-1}$, where q_{t-1} is assumed to be y_{t-1} and π_{t-1} respectively. Then follow the method of Hansen (1997). The value of $\hat{\tau}$ is associated with the lowest sum of residuals variance.

Test for a Threshold

It is important to know whether the threshold model of the Taylor rule (2.5) is statistically significant from a linear model. So, next we discuss about the test for a threshold.

Define dummy variable $d_i(\tau) = \{q_i \leq \hat{\tau}\}$ and set $X_i(\tau) = X_i d_i(\tau)$.

The null hypothesis of the no threshold effect in the model is

$$H_0 : A_1 = A_2$$

where $A_1 = \begin{bmatrix} \alpha_1 & h_1 & b_1 \end{bmatrix}'$ and $A_2 = \begin{bmatrix} \alpha_2 & h_2 & b_2 \end{bmatrix}'$

The alternative hypothesis of threshold effect is

$$H_1 : A_1 \neq A_2$$

As the threshold value is an unknown parameter under the null hypothesis, a standard F-test is not suitable for this case. Instead, in order to test threshold behaviour, Hansen (1997) pointed out that it is necessary to bootstrap the F-statistic, and the procedure is as follows. In this Chapter, the bootstrap F-test is conducted in STATA.

First, the model is estimated by least squares for each $\tau \in \Gamma$ and the sum of squared residuals obtained. So, the pointwise F-statistic is formed:

$$F_n(\tau) = n \frac{[\tilde{\sigma}_n^2 - \hat{\sigma}_n^2(\tau)]}{\hat{\sigma}_n^2(\tau)}$$

where $\tilde{\sigma}_n^2$ is obtained from the linear Taylor rule, and $\hat{\sigma}_n^2(\tau)$ is obtained from the threshold model.

Then the bootstrap F-statistic is the largest value of these statistics, which can be expressed as:

$$F_n = sup_{\tau \in \Gamma} F_n(\tau)$$

In order to achieve the p-value for F-statistic, a bootstrap procedure has been proposed by Hansen (1997). Detailed process is let u_t^* be *i.i.d.* $N(0,1)$ random draws, and obtain $\hat{e}_t$ from the estimation of the threshold model. Then set $y_t^* = u_t^*$. Regress y_t^* on X_t to obtain the residual variance $\tilde{\sigma}_n^{*2}$, and regress y_t^* on $X_t(\tau)$ to obtain the residual variance $\hat{\sigma}_n^{*2}(\tau)$. Then form $F_n^* = n\frac{[\tilde{\sigma}_n^{2*} - \hat{\sigma}_n^{*2}(\tau)]}{\hat{\sigma}_n^{*2}(\tau)}$ and $F_n^* = sup_{\tau\in\Gamma} F_n^*(\tau)$ is obtained. Finally, the bootstrap p-value of the test is formed by counting the percentage of bootstrap samples for which F_n^* exceeds the observed F_n.

The Opportunistic Model

As the inflation in the UK was being consistent high after the financial crisis, it seems that the inflation target is not followed strictly by the Bank of England, which makes it interesting to investigate opportunistic monetary policy. According to Bunzel and Enders (2010), the opportunistic policy assumes the interim inflation target is changing over time, in this Chapter, we estimate the monetary rule using an interim target rate instead of using a fixed threshold. Assume that there is a regime change if the current inflation gap breaches the average rate of the last two years. The non-linear Taylor rule using opportunistic approach can be expressed as

$$i_t = \begin{cases} \alpha_1 + h_1(\pi_t - \pi_t^T) + b_1(y_t - y_t^T) & \text{if} \quad (\pi_{t-1} - \pi_{t-1}^T) \leq \frac{(\pi_{t-5} - \pi_{t-5}^T) + (\pi_{t-9} - \pi_{t-9}^T)}{2} \\ \alpha_2 + h_2(\pi_t - \pi_t^T) + b_2(y_t - y_t^T) & \text{if} \quad (\pi_{t-1} - \pi_{t-1}^T) > \frac{(\pi_{t-5} - \pi_{t-5}^T) + (\pi_{t-9} - \pi_{t-9}^T)}{2} \end{cases} \tag{2.6}$$

The opportunistic model allows policy regime changes between two linear Taylor rules based on a changing threshold. Furthermore, the full opportunistic model takes the output gap into consideration, it takes the same model shown in expression (2.6) but with a different indicator function. Specifically, the full opportunistic model reads:

$$i_t = \begin{cases} \alpha_1 + h_1(\pi_t - \pi_t^T) + b_1(y_t - y_t^T) & \text{if} \quad (\pi_{t-1} - \pi_{t-1}^T) \leq \frac{(\pi_{t-5} - \pi_{t-5}^T) + (\pi_{t-9} - \pi_{t-9}^T)}{2} \& (y_{t-1} - y_{t-1}^T) > 0 \\ \alpha_2 + h_2(\pi_t - \pi_t^T) + b_2(y_t - y_t^T) & \text{if} \quad (\pi_{t-1} - \pi_{t-1}^T) > \frac{(\pi_{t-5} - \pi_{t-5}^T) + (\pi_{t-9} - \pi_{t-9}^T)}{2} \& (y_{t-1} - y_{t-1}^T) \leq 0 \end{cases} \tag{2.7}$$

2.5 Empirical Results

2.5.1 Estimating a Non-linear Taylor Rule using a Structural Model

Before breaking the data into sub-periods, first we estimate the original Taylor rule for the whole sample. Table 2.1 reports the estimated linear Taylor rule since 1992 by using Ordinary Least Square (OLS).

Table 2.1: Linear Taylor rule from Q1 1992 to Q2 2013

$i_t = \alpha + h(\pi_t - \pi_t^T) + b(y_t - y_t^T)$	
Coefficents	
α	5.4375^{***}
	(22.17)
h	0.6626^{**}
	(2.44)
b	0.8391^{***}
	(7.50)
R^2	0.4183
$F(2, 83)$	29.83
$Prob > F$	0.000
Durbin-Watson	0.0698
Breusch-Pagan test for heteroskedasticity	5.30
$Prob > chi^2$	0.0213

Notes: T-statistics are reported in parentheses. The superscript * indicates significance (p-value<0.1); ** indicates strong significance (p-value<0.05); *** indicates very strong significance (p-value<0.01). The model is estimated by OLS.

The results show coefficients obtained in the linear rule are quite consistent with the values suggested by Taylor (1993), which is confirmed by an F-test. However, the results are not efficient as the data suffers from a high degree of autocorrelation (Durbin-Watson statistic = 0.0698). And the heteroskedasticity causes the poor performance of the regression. A Chow test for structural break suggests that the parameters are not constant over time (F-statistic = 29.83 with a p-value of 0.000). Therefore, we spilt the data into two sub-periods and choose March 2007 as the break point.

As the data has the problem of autocorrelation and heteroskedasticity, OLS becomes inefficient and results are not the "best linear unbiased estimator". Therefore, instead of using OLS, we use Generalized Method of Moments (GMM) which can give robust results and the corrected standard errors are known as "heteroscedasticity-and autocorrelation-consistent" (HAC) standard errors. Table 2.2 reports the estimated Taylor rule before and after the financial crisis.

Table 2.2: Non-linear Taylor rule – the structural model

$$i_t = \begin{cases} \alpha_1 + h_1(\pi_t - \pi_t^T) + b_1(y_t - y_t^T) & \text{if} \quad t < \bar{T} \\ \alpha_2 + h_2(\pi_t - \pi_t^T) + b_2(y_t - y_t^T) & \text{if} \quad t \geq \bar{T} \end{cases}$$

Start	End	α_1	h_1	b_1	R^2
Q1 1992	Q1 2007	5.4287***	1.5419***	0.1276	0.4810
		(35.37)	(8.56)	(1.32)	
		α_2	h_2	b_2	R^2
Q2 2007	Q2 2013	4.8215***	-0.1386	0.8247***	0.8461
		(16.52)	(-0.87)	(14.89)	

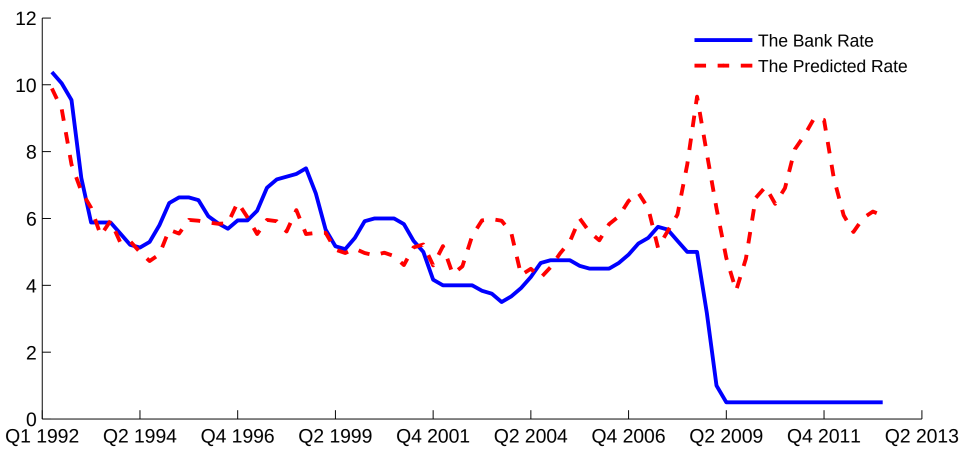
Notes: Z-statistics are reported in parentheses. The superscript * indicates mild significance (p-value<0.1); ** indicates significance (p-value<0.05); *** indicates strong significance (p-value<0.01). The model is estimated by GMM.

Generally, if a linear Taylor rule appropriately describes the behaviour of the Bank of England, the estimated parameters should be constant over time. However, the estimates on inflation become insignificant while output estimates become significant when moving to the post-crisis period. This suggests that the behaviour changed significantly after the crisis, indicating that the Bank of England does not follow a linear rule all the time. Table 2.2 shows that coefficients for the first period are both positive and the inflation gap has a very strong significant number (p-value=0.000) which suggests that the Bank of England cared more about inflation before the financial crisis started. Moreover, it seems that the Bank of England followed the "Taylor Principle" as the coefficient on the inflation gap is greater than 0.5. Interestingly, this coefficient changed to a negative but not significant number, so there is no evidence to show the Bank of England continues to put a strong weight on inflation since the crisis. Instead, a very significant positive coefficient is estimated on output gap (p-value=0.000), suggesting that the Bank of England changed preference and started to care more about output.

Figure 2.2 shows the path of nominal interest rates assuming the pre-2007 Taylor rule parameters presented in Table 2.2. The predicted rates are calculated using estimates obtained from the first period, shown as equation (2.8). The observable differences between these two rates give further evidence that the Bank of England behaved differently after the crisis.

$$i_t^{predict} = 5.4287 + 1.5419(\pi_t - \pi_t^T) + 0.1276(y_t - y_t^T) \tag{2.8}$$

Figure 2.2: The Bank rate and the predicted rate

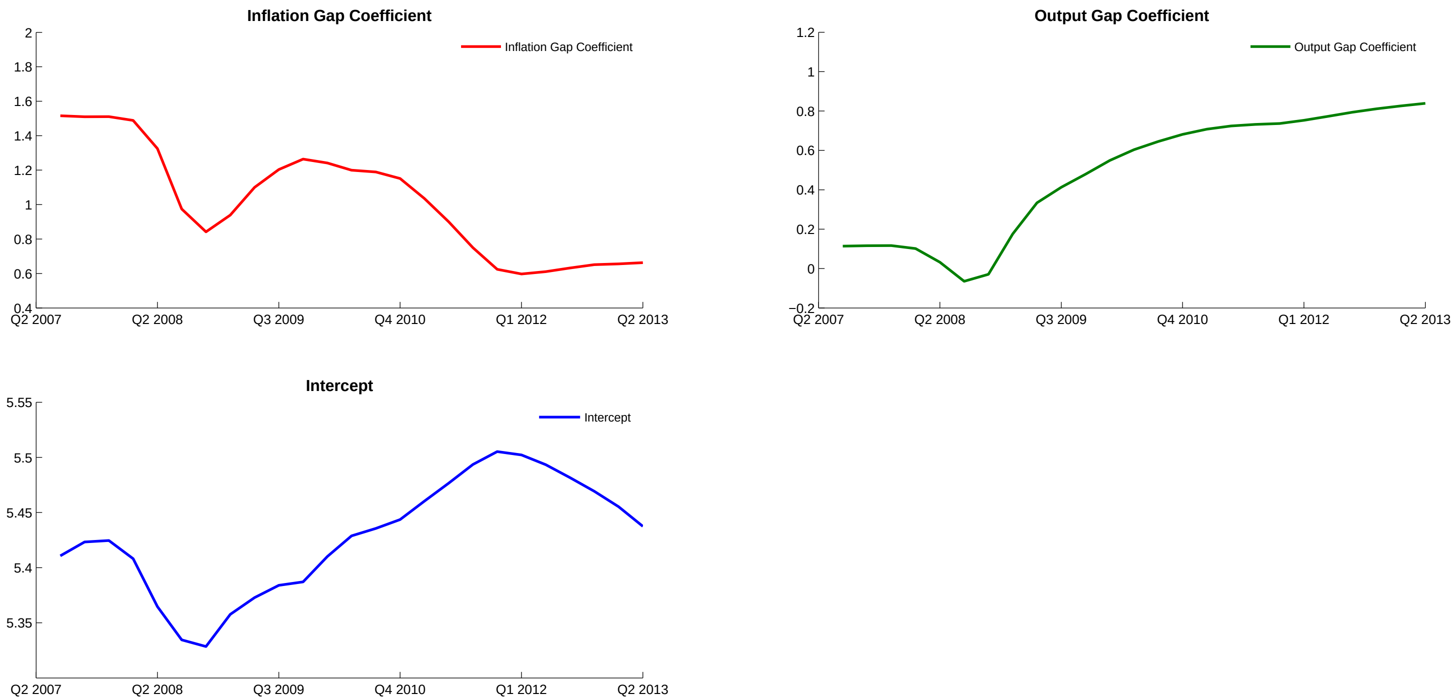


In order to test whether the coefficients of the estimated Taylor rule are constant over time statistically, a standard recursive estimation method has been used. To be precise, for each time T in the time period of Q2 2007 to Q2 2013, the linear Taylor rule is estimated using the observations from Q1 1992 to T. Therefore, there are 25 regressions and each has coefficients on inflation and output gap and an intercept. The results are shown in

Figure 2.3. Figure 2.3 shows the parameter movements when one single policy regime is used to describe the data from 1992 to 2013. If the parameters are constant over time, there should be no particular pattern observed in the coefficients. However, none of those coefficients are constant over time.

Specifically, there is a fall in all coefficients and intercept around 2008. After that the output gap coefficient increases gradually over time, while the coefficient of inflation gap decreased gradually since the last quarter of 2009. The results give further evidence that the policy priority of the Bank of England has changed over time – that is, a linear Taylor rule is not able to describe the behaviour of the Bank of England appropriately. Therefore, it is necessary to consider non-linear Taylor rules.

Figure 2.3: Recursive estimation of the Taylor rule



2.5.2 Estimating a Non-linear Taylor Rule using a Threshold Model

In the UK, the economy has experienced negative output gaps from 2008 to 2013. As shown from the recursive estimation, the Bank of England started to put more weight on output, relaxing the control of inflation at the same time. In order to investigate more about the monetary policy rule in the UK, the threshold model (2.3) will be applied to the UK economy next.

Table 2.3 reports the estimated non-linear Taylor rule using a threshold model which adopts the lagged output gap and inflation gap as threshold variables. No matter which threshold variable is chosen, there is strong evidence of threshold behaviour in the sample, indicating that a linear Taylor rule is not being employed. The two estimates of $\hat{\tau}$ are the estimation range of each threshold variable, indicating the Bank of England would follow a different rule when the lagged inflation gap (lagged output gap) crosses that value. The estimates show that when the lagged output gap is above the threshold value, it seems that the Bank of England followed the Taylor principle. However, when the lagged output gap is below the threshold, the model can not be estimated due to collinearity. This can be explained as most of the data in this sample falls into the post-crisis period which the structural model suggests. Specifically, most of the dependent variables are constant, having the nominal interest rate at 0.5%. There are more evidence that the regime has changed significantly when the lagged output gap is below the threshold value, but that this coincides with the post-financial crisis period.

Instead, if we assume the lagged inflation gap as the threshold variable, the coefficient on output gap has increased sharply when the lagged inflation gap is above the threshold value. Interestingly, this is opposite of what we might expect. When the inflation gap is above the threshold, the Central Bank is supposed to put more effort on controlling the rate of inflation, but it did not. The output-preferred behaviour may be triggered by recessions or shocks which could make the Bank of England focus on output in order to get the economy back on track. Correspondingly, when the lagged inflation gap is above the threshold, most of the data refer to the period after the financial crisis, when there is strong evidence from the recursive estimation that the Bank of England is putting more weight on output. It seems that the results are plausible.

Table 2.3: Non-linear Taylor rule – the threshold model

$$i_t = \begin{cases} \alpha_1 + h_1(\pi_t - \pi_t^T) + b_1(y_t - y_t^T) & \text{if} \quad q_{t-d} \leq \hat{\tau} \\ \alpha_2 + h_2(\pi_t - \pi_t^T) + b_2(y_t - y_t^T) & \text{if} \quad q_{t-d} > \hat{\tau} \end{cases}$$

Threshold Variable		Threshold Process Test					
$y_{t-1} - y_{t-1}^T$	$\hat{\tau}$	F-stat	P-value	α_1	h_1	b_1	R^2
	-3.7/-3.6	61.159	0.000	Collinearity			
				α_2	h_2	b_2	R^2
				5.3260^{***} (41.26)	1.0084^{**} (2.39)	0.3547^{*} (1.72)	0.1614
$\pi_{t-1} - \pi_{t-1}^T$	$\hat{\tau}$	F-stat	P-value	α_1	h_1	b_1	R^2
	0.8667/1.0059	7.25	0.000	5.3719^{***} (42.67)	0.2368 (0.85)	0.7039^{***} (7.15)	0.4512
				α_2	h_2	b_2	R^2
				9.8045^{**} (2.47)	0.2683 (0.29)	1.7326^{**} (2.50)	0.3851

Notes: Z-statistics are reported in parentheses. The superscript * indicates mild significance (p-value<0.1); ** indicates significance (p-value<0.05); *** indicates strong significance (p-value<0.01). F-stat and P-value are calculated using bootstrap procedure described in section 2.4.2. The model is estimated by GMM.

2.5.3 Estimating a Non-linear Taylor Rule using an Opportunistic Model

Instead of assuming Central Banks use a fixed threshold, next we will consider whether Central Banks could change their behaviour based on a changing threshold, which is commonly known as "Opportunism".

Table 2.4 shows the results if the Taylor rule for the UK is viewed as an opportunistic strategy process, which is shown as expression (2.6). When the lagged inflation gap is lower than the threshold, the Bank of England responds significantly to inflation and output. This may indicate that the Bank of England controls inflation strictly to its target rate when the economy behaves normally. Interestingly, the results indicate the Bank of England responds less to the inflation and coefficient of inflation gap becomes insignificant when the lagged inflation gap is above the average inflation rate of the last two years. This is quite unusual as it seems that the Bank of England should care more about inflation when it becomes higher than threshold. Instead, the results show a very strong significant effort has been put on output. The results are quite consistent with the threshold model.

Table 2.4: Non-linear Taylor rule – the opportunistic model

$$i_t = \begin{cases} \alpha_1 + h_1(\pi_t - \pi_t^T) + b_1(y_t - y_t^T) & \text{if} \quad (\pi_{t-1} - \pi_{t-1}^T) \leq \frac{(\pi_{t-5}-\pi_{t-5}^T)+(\pi_{t-9}-\pi_{t-9}^T)}{2} \\ \alpha_2 + h_2(\pi_t - \pi_t^T) + b_2(y_t - y_t^T) & \text{if} \quad (\pi_{t-1} - \pi_{t-1}^T) > \frac{(\pi_{t-5}-\pi_{t-5}^T)+(\pi_{t-9}-\pi_{t-9}^T)}{2} \end{cases}$$

Threshold Variable				
$\pi_{t-1} - \pi_{t-1}^T$	α_1	h_1	b_1	R^2
	5.6641***	1.7968***	0.8562***	0.4711
	(30.60)	(4.36)	(6.33)	
	α_2	h_2	b_2	R^2
	5.3537***	-0.0743	0.8964***	0.7763
	(22.88)	(-0.53)	(16.82)	

Notes: Z-statistics are reported in parentheses. The superscript * indicates mild significance (p-value<0.1); ** indicates significance (p-value<0.05); *** indicates strong significance (p-value<0.01). The model is estimated by GMM.

As there is a strong significant result estimated on output, it is reasonable to assume the output gap to be one of the thresholds. Therefore, we estimate the "full" opportunistic model (2.7), which implies policy changes rely on two thresholds: inflation gap and output gap.

After the financial crisis, the UK experienced consistently high inflation and negative output gaps, therefore, in this Chapter, we try to capture the UK experience using the full opportunistic model. In particular, we assume there will be a policy regime change when the inflation gap is above the threshold and the output gap is below zero in the full opportunistic model. Table 2.5 shows the estimates of the UK data under the full opportunistic model. When the inflation gap is above the threshold and the output is lower than zero, the coefficient of output gap raises sharply and becomes strongly significant, which indicates that the Bank of England cares much more about output than inflation when the inflation gap is less than threshold and output gap is more than threshold.

Table 2.5: Non-linear Taylor rule – the full opportunistic model

$$i_t = \begin{cases} \alpha_1 + h_1(\pi_t - \pi_t^T) + b_1(y_t - y_t^T) & \text{if} \quad (\pi_{t-1} - \pi_{t-1}^T) \leq \frac{(\pi_{t-5}-\pi_{t-5}^T)+(\pi_{t-9}-\pi_{t-9}^T)}{2} \& (y_{t-1} - y_{t-1}^T) > 0 \\ \alpha_2 + h_2(\pi_t - \pi_t^T) + b_2(y_t - y_t^T) & \text{if} \quad (\pi_{t-1} - \pi_{t-1}^T) > \frac{(\pi_{t-5}-\pi_{t-5}^T)+(\pi_{t-9}-\pi_{t-9}^T)}{2} \& (y_{t-1} - y_{t-1}^T) \leq 0 \end{cases}$$

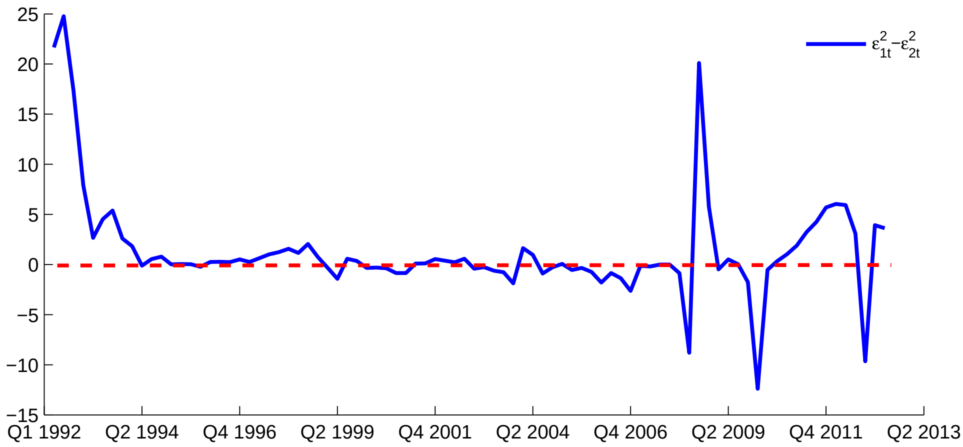
Threshold Variable				
$\pi_{t-1} - \pi_{t-1}^T \& y_{t-1} - y_{t-1}^T$	α_1	h_1	b_1	R^2
	6.6861***	0.1522	-0.9235	0.5923
	(14.34)	(0.20)	(-1.40)	
	α_2	h_2	b_2	R^2
	5.3209***	-0.1272	0.8695***	0.7664
	(17.74)	(-0.72)	(12.62)	

Notes: Z-statistics are reported in parentheses. The superscript * indicates mild significance (p-value<0.1); ** indicates significance (p-value<0.05); *** indicates strong significance (p-value<0.01). The model is estimated by GMM.

Based on the estimates obtained from the threshold and opportunistic models, significant coefficients are shown on output gap in high inflation regimes, which probably could be explained by the financial crisis. After the financial crisis in 2007, the Bank of England is not able to maintain the "normal" policy as they have to deal with the recession. Thus, the Bank of England has to put more weight on output in order to eliminate the negative output gap even if there is pressure on prices, which is consistent with the results obtained from the structural model.

In order to see whether the Bank of England follows a threshold process or tends to behave based on an opportunistic strategy, it is necessary to compare the performance of the two models. Figure 2.4 shows the differences between the threshold model and the opportunistic model. Let $\{\varepsilon_{1t}^2\}$ represents the sequence of squared variances from the threshold model and let $\{\varepsilon_{2t}^2\}$ be the sequence of squared variance from the opportunistic model. Figure 2.4 shows $\varepsilon_{1t}^2 - \varepsilon_{2t}^2$ during the period of 1992 to 2013 second quarter.

Figure 2.4: Differences of the squared residuals between the threshold and opportunistic model



Positive numbers occur in the period before 1994 and after 2008, indicating that the behaviour of the Bank of England is better described using an opportunistic model. After that, it is quite consistent for the two models until 2008. After 2008, it is complicated but the opportunistic model is better for most periods, which is also supported by the R-squared estimates in both models. Moreover, the opportunistic model seems more realistic as the policymakers will not take any action to inflation as long as inflation is stable even it stays beyond the band, which is quite consistent with the experience in the UK since 2008.

2.6 Conclusion

This chapter discusses ways of modelling the Bank of England's behaviour using different Taylor-type rules. We estimate a standard linear Taylor rule for the whole period, which shows strong evidence of a structural break. Due to the instability existing in the parameters, non-linear rules are taken into account. First, a non-linear Taylor rule has been estimated using structural approach. The significant results show that the Bank of England put more attention to output rather than inflation after the severe shock.

Instead of assuming Central Banks will change their attitude towards the reaction to inflation and output gap based on some structural point, we use the threshold model which allows the policy rule to switch depending on some threshold. Two threshold models have been estimated with a fixed threshold value using the inflation gap and output gap. Strong evidence of threshold behaviour has been found no matter which threshold is chosen. There is a clear sign of changing behaviour when the inflation gap is above the threshold or the output gap is below the threshold. This period happens to be the post-crisis period which is suggested by structural model, indicating further that the Bank of England cared more about output even when inflation was high after the crisis.

As the UK inflation keeps consistently high after the global financial crisis, it seems that the Bank of England accommodates the high inflation, which might indicate the Bank is implementing with an opportunistic disinflation strategy. Therefore, we estimate the non-linear Taylor rule using the opportunistic model and find that the Bank of England seems assign more weights on output rather than inflation even if inflation was relatively high.

To summarise, the estimated results of the three different models suggest that the Bank of England cares more about output rather than inflation after experiencing the severe financial crisis shock, indicating that a non-linear Taylor rule could be a better method to describe and model the behaviour of the Bank of England.

Chapter 3

Endogenous-switching Rules: An Evaluation

3.1 Introduction

After the financial crisis in 2007, the world has experienced the largest recession since 1930s. The slow recovery sparked a huge debate about optimal policy responses, and in particular, about how monetary policy should be conducted when facing severe shocks. This Chapter investigates the consequences of changing monetary policy regimes in a New Keynesian model where Central Banks switch between different Taylor rules as the output gap breaches some threshold level following an exogenous shock.

Many studies examined the response of US monetary authorities to the crisis (Clarida et al., 2000; Lubik and Schorfheide, 2004; van Binsbergen et al., 2008; Sims and Zha, 2006; Lo and Piger, 2005), showing that some parameters in the assumed policy rules are not stable and vary across periods. For example, Lo and Piger (2005) have found that monetary policy reacted significantly to negative variations in output in the US. Similar results have been found for the Euro area by Peersman and Smets (2001) and for the UK by Nelson (2001). In general, the robust finding of this empirical literature is that Central

Banks change their behaviour when facing severe crises. The theoretical literature began to address this issue only recently. A first strand of contributions focuses on monetary policy rules with time-varying coefficients that obey Markov processes (Valente, 2003; Chung et al., 2007; Liu et al., 2009; Farmer et al., 2011 and Davig and Doh, 2014). This modelling strategy does not however capture the link between changing economic conditions and the response of monetary authorities: policy changes occur randomly and are thus not related to the macroeconomic environment. Building on these critical considerations, Davig and Leeper (2008) provide a first analysis of "endogenous regime change" by adopting a flexible policy rule in which parameter changes are triggered when certain endogenous variables cross specified thresholds. This approach rationalizes the idea that a regime switch is indeed a calibrated policy response to changing economic conditions. The analysis in this Chapter exploits the framework built by Davig and Leeper (2008) using a New Keynesian model. More precisely, we assume that the Central Bank adopts an endogenous-switching rule, which contains two Taylor rules with different weights on inflation and output gap. The Taylor rule that is actually implemented in a give period depends on the size of the output gap, as determined by demand shocks and equilibrium conditions, in that period. Given a benchmark measure of "normal output gap", the relevant Taylor rule changes from the "normal times regime" to the "crisis regime" when the lagged output gap breaches specified threshold. In this framework, we conduct numerical simulations to assess the consequences of such endogenous-switching rule in dynamic general equilibrium. The simulations show that the endogenous-switching rule improves welfare in the event of a large negative demand shock comparing with the "original" Taylor rule which takes the weights suggested by Taylor (1993), but shows lower welfare comparing with the linear Taylor rule with higher weights on inflation and output gap. The results indicate that higher welfare could be achieved if Central Banks were to permanently adopt higher weights in the linear Taylor rule.

3.2 Literature Review

In a seminal contribution, Taylor (1993) proposed to model the behaviour of Central Banks as a "policy rule" whereby the monetary authority sets the level of the nominal interest rate in response to output gap and inflation. The Taylor rule has since been considered a standard way to model the monetary policy in the specialized literature (e.g. Taylor, 1999a,b; Rudebusch and Svensson, 1999; Levin et al., 1999; Woodford, 2001 and Orphanides, 2003). The benchmark Taylor rule takes the form

$$i_t = h\pi_t + b(y_t - y_t^T) \tag{3.1}$$

where i_t is the nominal interest rate, y_t^T is the output target. Parameters h and b represent the weights that the Central Bank put on controlling inflation or stabilising output.

In general, the Taylor rule should not be regarded as a "welfare-maximizing" rule adopted by benevolent policymakers. There are specific environments in which the Taylor rule may be labelled as "optimal" according to several underlying notions of social desirability (see e.g. Woodford, 2003a; Giannoni and Woodford, 2003a,b). But the conventional, perhaps most general legitimation is that Taylor Rules can be derived from the solution of an optimization problem in which the Central Bank minimises a loss function that depends on the joint deviations of inflation and output from specified target levels (see e.g. Carlin and Soskice, 2014). In this interpretation, the weights attached to inflation and output gap would represent the Central Bank's preferences towards the two objectives of stabilizing inflation and stabilizing output. Beyond the pros and cons of this justification for the use of Taylor rules, what is relevant to the present analysis is that expression (3.1) postulates that the weights attached to inflation and output gaps are constant over time. In other words, the simple Taylor Rule (3.1) excludes the possibility of regime changes when Central Banks face unexpected events such as demand shocks of large magnitude.

However, many recent contributions (Dolado et al., 2004; Clarida et al., 2000; Lubik and

Schorfheide, 2004; van Binsbergen et al., 2008; Sims and Zha, 2006; Lo and Piger, 2005), argue that the behaviour of Central Banks is typically not stable over time. Sims and Zha (2006) estimate different structural VAR models using US data from 1959 to 2003, and conclude that the best-fit model is one that allows the coefficients of monetary policy to change. Valente (2003) employs Markov-switching models to identify the existence of non-linearity in the nominal interest rate movements during 1979 and 1997 for G3 and E3 countries, and provides strong evidence that the Markov-switching VAR models fit the data better than a linear VAR. These results are relevant not least because they suggest new ways to model monetary policy as "modified Taylor rules" in which the coefficients can change according to Markov processes. For example, Davig and Leeper (2007) extended the New Keynesian model to include a Markov process that allows for monetary policy regimes change. In Davig and Leeper (2007), the monetary policy reads:

$$i_t = h(S_t)\pi_t + b(S_t)(y_t - y_t^T) \tag{3.2}$$

where $\pi_t = p_t - p_{t-1}$ is the rate of inflation between period t and $t-1$ (having defined $p_t = logP_t$) and y_t and y_t^T are the real output and the natural output respectively (similarly, $y_t = logY_t$). The state of nature S_t is a random variable following a two-regime Markov chain, which takes values $S_t \in (1, 2)$ depending on a list of predetermined – i.e., exogenously given – probabilities that appear in the associated transition matrix. Therefore, in this framework, the nominal interest rate i_t is determined by a Taylor rule whose weights take different values depending on which regime, $S_t = 1$ or $S_t = 2$, prevails at time t. Davig and Leeper (2007) assume that the active, or more aggressive regime, where the nominal interest rate is changed more than one percentage point to a change of expected inflation, corresponds to $S_t = 1$, while the passive, less aggressive regime where the interest rate is changed less than one percent, corresponds to $S_t = 2$.

The shortcoming of this approach is that, by assuming an exogenous transition matrix for variable S_t, the switching of the policy rule occurs randomly and is not determined by economic conditions – which is at odds with the idea that the monetary rule should capture Central Banks' behaviour in an economically meaningful way. Building on this observation, Davig and Leeper (2008) develop a monetary rule with endogenous switching regimes, where the weights associated to inflation and output gaps adjust automatically to the macroeconomic environment. Davig and Leeper (2008) use a simple threshold-style method for endogenising regime change and report the impulse responses of key variables when the lagged inflation breaches the inflation threshold. The results indicate that the endogenous-switching rule improves the effectiveness of monetary policy compared to the simple rule with fixed weights. In this Chapter, we will implement the approach of Davig and Leeper (2008) to define an "endogenous-switching rule" in the New Keynesian model and then perform numerical simulations to evaluate the welfare consequences of such monetary policy, drawing comparisons with the "original" Taylor rule with fixed weights. The specific form taken by the "endogenous-switching rule" will be discussed in detail in section 3.3.3.

3.3 The Model

We employ the New Keynesian model of dynamic general equilibrium proposed by Galí (2009). In this framework, firms produce differentiated goods and earn monopoly rents under Calvo pricing (Calvo, 1983). Random demand shocks introduce uncertainty in future inflation and give rise to the so-called New Keynesian Phillips Curve – where equilibrium employment depends on the level of labour supply chosen by homogeneous utility-maximizing households. In general, this framework postulates that monetary policy is a rule that establishes the level of the nominal interest rate. In the present context, we represent monetary policy according to the endogenous switching rule proposed by Davig and Leeper (2008). The following subsections describe various parts of the model in detail.

3.3.1 Households

The representative infinitely-lived household chooses consumption, C_t, and the level of labour supply, N_t, in order to maximise lifetime utility,

$$E_t \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{3.3}$$

where $\beta \in (0,1)$ is discount factor. Assume a continuum of firms indexed by $j \in [0,1]$. Each firm produces a different variety of consumption goods using labour supplied by households as an input. In view of these assumptions, the consumption index and the labour index are respectively given by

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.4}$$

$$N_t = \int_0^1 N_t(j) dj \tag{3.5}$$

where $C_t(j)$ is consumption for good j, ε is elasticity of substitution between goods, $N_t(j)$ is the amount of labour services supplied to the j firm.

Household choices are subject to the intertemporal budget constraint

$$P_t C_t + Q_t D_t = D_{t-1} + W_t N_t + T_t \tag{3.6}$$

where P_t represents the aggregate price level, D_t is the purchased quantity of one-period riskless discount bonds maturing in period $t+1$. Each of these bonds is purchased at price $Q_t = \frac{1}{1+i_t}$, where i_t is the nominal interest rate in period t. W_t is the nominal wage and T_t is additions to income in the form of dividends.

The paths of consumption and labour supply levels are chosen by maximizing lifetime utility. This problem can be solved by means of the Lagrangian function

$$L = E_t \sum_{t=0}^{\infty} \left\{ \beta^t U_t(C_t, N_t) - \lambda_t (P_t C_t + Q_t D_t - D_{t-1} - W_t N_t - T_t) \right\} \tag{3.7}$$

where λ_t is the Lagrangian multiplier. The utility function takes the specific form

$$E_t[U] = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{3.8}$$

where σ is intertemporal elasticity of substitution and φ is the elasticity of labour supply. The first order conditions for the consumer are given by

$$1 = \beta(1+i_t) E_t \left[\frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right] \tag{3.9}$$

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{C_t^{-\sigma}} \tag{3.10}$$

3.3.2 Firms

Each firm produces a differentiated good according to the production function

$$Y_t(j) = A_t N_t(j)^{1-\alpha} \tag{3.11}$$

where A is an exogenous measure of technology, assumed to be the same for all firms, $1-\alpha$ represents the production elasticity of labour.

Price stickiness is modelled as in Calvo (1983). A firm changes its price when a price-change signal is received but not all firms receive the same signal in a given period. The fraction of firms that do not receive the signal and, hence, keep their prices unchanged, is denoted by θ. The remaining $1-\theta$ firms, instead, receive the signal and change their prices. The aggregate price dynamics may thus be written as

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta) \left(\frac{\tilde{P}_t}{P_{t-1}} \right)^{1-\varepsilon} \tag{3.12}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate, $\tilde{P}_t$ is the re-optimised price set by firms that received the price-change signal in period t.

Firms choose prices each period to maximise their current profit. Because of price stickiness, firms realise that the price they decide now will affect profits over the next k periods. Firms re-optimising the price in period t will choose $\tilde{P}_t$ so as to maximise the current market value of the profits for the next k periods given a probability "θ" that the chosen price indeed remains effective over that horizon. Formally, the firm's maximisation problem is

$$Max \sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \left(\tilde{P}_t Y_{t+k|t}(j) - \Psi_{t+k}(Y_{t+k|t}(j)) \right) \right] \tag{3.13}$$

subject to the constraints represented by the demand schedule

$$Y_{t+k|t}(j) = \left(\frac{\tilde{P}_t}{P_{t+k}} \right)^{-\varepsilon} C_{t+k} \tag{3.14}$$

where $Q_{t,t+k} = \beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}}$, is stochastic discount factor for nominal payoffs over the interval $[t, t+k]$, $Y_{t+k|t}(j)$ is output in period $t+k$ for a firm re-optimise price in period t, $\Psi_{t+k}(Y_{t+k|t}(j))$ is cost function.

Substituting the firm's demand curve (3.14) and the stochastic discount factor in expression (3.13), the objuective function to be maximized becomes:

$$L = \sum_{k=0}^{\infty} \theta^k E_t \left[\beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \left(\tilde{P}_t Y_{t+k|t}(j) - \Psi_{t+k} \left(\frac{\tilde{P}_t}{P_{t+k}} \right)^{-\sigma} C_{t+k} \right) \right] \tag{3.15}$$

The first order condition with respect to $\tilde{P}_t$ reads

$$\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} Y_{t+k|t}(j) \left(\tilde{P}_t - \frac{\varepsilon}{\varepsilon - 1} \psi_{t+k|t}(j) \right) \right] = 0 \tag{3.16}$$

where $\psi_{t+k|t}(j) = \Psi'_{t+k} \left(Y_{t+k|t}(j) \right)$ is the marginal cost of a firm which last reset its price in period t.

Result (3.16) yields the following expression for the optimal price set by the firm:

$$\tilde{P}_t = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \psi_{t+k|t}(j) Y_{t+k|t}(j) \right]}{\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} Y_{t+k|t}(j) \right]} \tag{3.17}$$

The next subsection shows that, imposing the conditions of symmetric equilibrium across firms, the price setting strategy of firms combined with the labour supply choices of households gives rise to the so-called New Keynesian Phillips Curve.

3.3.3 Macroeconomic Equilibrium: Key Equations

The New Keynesian Phillips Curve

The first equation that we will use to characterize the macroeconomic equilibrium is the New Keynesian Phillips Curve. In a closed economy, the equilibrium in the goods market must satisfy

$$Y_t(j) = C_t(j) \tag{3.18}$$

For all $j \in [0, 1]$ for all t. Since aggregate output is $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$, the aggregate resource constraint of the economy is:

$$Y_t = C_t \tag{3.19}$$

We now exploit the notion of symmetric equilibrium across firms in order to reformulate the price-setting strategy of each firm – represented by equation (3.17) above – in terms of "relative optimal price", establishing a simple link between individual firms' prices and the general price level.

The total production cost in real terms for firm j is $TC_t(j) = \frac{W_t}{P_t} N_t(j)$. Substituting

$N_t(j)$ by means of the production function (3.11), and using the market clearing condition (3.19), the marginal cost of firm j can be written as

$$MC_t(j) = \left(\frac{1}{A_t}\right)^{\frac{1+\varphi}{1-\alpha}} \left(\frac{1}{1-\alpha}\right) \left(\frac{P_t(j)}{P_t}\right)^{-\frac{\alpha\varepsilon}{1-\alpha}} Y_t^{\sigma+\frac{\varphi+\alpha}{1-\alpha}} \tag{3.20}$$

Substituting the marginal cost function (3.20) in the price equation (3.17), we can define the equilibrium relative price $Z_t = \frac{\tilde{P}_t}{P_t}$ and write it as:

$$Z_t = \left[\frac{\varepsilon}{(1-\alpha)(\varepsilon-1)} \frac{E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\frac{1}{\prod_{j=1}^{k} \Pi_{t+j}}\right)^{-\frac{\alpha\varepsilon}{1-\alpha}-\varepsilon} Y_{t+k}^{(\sigma+\frac{\varphi+\alpha}{1-\alpha}+1)}}{E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\frac{1}{\prod_{j=1}^{k} \Pi_{t+j}}\right)^{1-\varepsilon} Y_{t+k}}\right]^{\frac{1}{\frac{\alpha\varepsilon}{1-\alpha}+1}} \tag{3.21}$$

where we have denoted $\frac{P_t}{P_{t+k}} = \frac{1}{\prod_{j=1}^{k} \Pi_{t+j}}$. The equilibrium relative price given by expression (3.21) allows us to derive the crucial relationship between inflation and output as a dynamic equilibrium condition. Linearizing equation (3.21) around zero inflation $\tilde{P}_t = P_{t-1} = P_t$ and substituting equation (3.12), we obtain the New Keynesian Phillips Curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t \tag{3.22}$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}(\sigma + \frac{\varphi+\alpha}{1-\alpha})$. Further details about the derivation of expression (3.22) are reported in Appendix A.1.

The Dynamic IS Curve

The second equation that we will use to characterize the macroeconomic equilibrium is the dynamic IS curve implied by households' intertemporal choices. Recalling the aggregate constraint in equation (3.19), the equality between aggregate consumption and output allows us to rewrite the Euler condition for consumption growth in terms of output levels: taking logarithms and substituting the definition of expected inflation rate, equation (3.9) yields the dynamic IS curve:

$$y_t = E_t y_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1}) + g_t \tag{3.23}$$

where y_t stands for the logarithm of output, and g_t represents a demand shock following the AR(1) process.

$$g_t = \rho_g g_{t-1} + \varepsilon_{gt} \tag{3.24}$$

where ε_{gt} is a stochastic term with mean 0 and variance $\sigma^2_{\varepsilon_{gt}}$.

Monetary Policy Rules

The third equation that we will use to characterize the macroeconomic equilibrium is the monetary policy rule. In this Chapter, we formalize monetary policy in terms of an endogenous switching rule with thresholds: the Central Bank considers two different Taylor rules, and chooses the specific Taylor rule to be implemented in a given period depending on whether the lagged output gap is above or below a pre-determined threshold level. This rule is taken to represent the behaviour of a Central Bank that obeys a "normal rule" in "normal times" but then switches to a "crisis rule" when a big shock hits the economy and drives output far away from its normal level.

Formally, the endogenous-switching rule reads

$$i_t = \begin{cases} h_1 \pi_t + b_1 (y_t - y_t^T) & \text{if} \quad (y_{t-1} - y_{t-1}^T) \geq \tau \\ h_2 \pi_t + b_2 (y_t - y_t^T) & \text{if} \quad (y_{t-1} - y_{t-1}^T) < \tau \end{cases} \tag{3.25}$$

where τ is the output gap threshold value, h_1 and b_1 are the coefficients that are relevant under regime 1, and h_2 and b_2 are the coefficients that are relevant under regime 2. In the present analysis, regime 1 is the "Normal Regime" whereas regime 2 is the "Crisis Regime". Following Davig and Leeper (2008), it is possible to rewrite rule (3.25) as follows:

$$i_t = h_{St}\pi_t + b_{St}(y_t - y_t^T) \tag{3.26}$$

where

$$h_{St} = \left(I[(y_{t-1} - y_{t-1}^T) \geq \tau]\right) h_1 + (1 - I)[(y_{t-1} - y_{t-1}^T) \geq \tau]h_2 \tag{3.27}$$

$$b_{St} = \left(I[(y_{t-1} - y_{t-1}^T) \geq \tau]\right) b_1 + (1 - I)[(y_{t-1} - y_{t-1}^T) \geq \tau]b_2 \tag{3.28}$$

where $I[\cdot]$ is the indicator function.

Compared with the exogenous Markov-switching monetary policy rule, the endogenous switching rule (3.26) may be interpreted as a special Markov-switching model with endogenous time-varying probabilities (Davig and Leeper, 2008). Specifically, the endogenous-switching rule will change to a different rule with probability of 1 as long as the lagged output gap crosses the output gap threshold. If the lagged output gap does not cross the threshold, then the monetary policy will switch with probability of 0.

3.4 Applying the Model

3.4.1 Social Welfare Loss Function

As shown by Woodford (2003a), a second-order approximation of the household's utility function typically yields a social welfare loss function that depends on inflation and output gap. This function will allow us to evaluate the welfare effects of different equilibrium values of inflation and output induced by monetary policy. In the present model, the welfare loss function is given by:

$$L_t = \pi_t^2 + \lambda_y \left(y_t - y_t^T\right)^2 \tag{3.29}$$

where the relative weight on output gap variability is given by $\lambda_y = \frac{\kappa}{\varepsilon}$, where κ is the coefficient on output gap in the new Keynesian Phillips curve and ε is the elasticity of substitution between different goods. It is clear that the weight on output gap (relative to inflation) in the loss function depends on model parameters, in particular, it is determined by the slope of the New Keynesian Phillips Curve that we derived in section 3.3.3 above. Detailed calculations are shown in Appendix A.2.

3.4.2 Solution Method

Equation (3.22), (3.23) and (3.26) form macroeconomic equilibrium in the New Keynesian model. To solve this model, we adopt the method developed by Blanchard and Kahn (1980), yielding the equilibrium system expressed in state-space form:

$$\bar{E}X_{t+1} = (\bar{A} + \bar{B}\bar{F})X_t + \bar{G}U_{t+1} \tag{3.30}$$

where X_{t+1} denotes the jump variables, $X_{t+1} = \begin{bmatrix} y_{t+1} & \pi_{t+1} \end{bmatrix}'$, and U_{t+1} denotes predetermined variable, $U_{t+1} = g_{t+1}$. And the coefficients are:

$$\bar{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \sigma^{-1} \\ 0 & 0 & \beta \end{bmatrix}, \bar{A} = \begin{bmatrix} -\rho_g & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -\kappa & 1 \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 \\ \sigma^{-1} \\ 0 \end{bmatrix}, \bar{G} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \bar{F} = \begin{bmatrix} 0 & h_{St} & b_{St} \end{bmatrix}$$

The above system expresses the jump variables as functions of the predetermined variable. In order to translate the state-space formulation into appropriate programming language, Blake and Fernandez-Corugedo (2010) use Scilab code to simulate the New Keynesian model. The simulations preformed in this Chapter are obtained using MATLAB by extending the original code by Blake and Fernandez-Corugedo (2010), to make it consistent with the theoretical model shown above.

3.4.3 Alternative Monetary Policy Rules

We have so far discussed our benchmark model in which monetary policy is represented by the endogenous-switching rule (3.26). In the remainder of the analysis, we will compare the equilibrium and welfare implications of rule (3.26) with the traditional Taylor rule with fixed weights shown in equation (3.1). To this aim, we will simulate the general equilibrium of the model by considering two variants – the benchmark variant in which monetary policy is given by (3.26), and the "Taylor" variant in which the nominal interest rate is determined by (3.1). Apart from the monetary rules, the key equilibrium equations remain the same and the welfare consequences of different rules will be evaluated using the same social "welfare loss function" derived in expression (3.29) above.

3.5 Numerical Analysis

3.5.1 Calibration: Structural Parameters in the Model

The model is calibrated using the previous studies of the UK data[1]. The discounted factor β is set as 0.99, implying the real interest rate is 4%. The elasticity of labour α is set to 0.31, which is the observed labour share during 1971 and 2009 in the UK (Faccini et al., 2011). Following in Smets and Wouters (2007), Gertler et al. (2008) and Faccini et al. (2011), the intertemporal substitution elasticity σ is 0.66. And the elasticity of labour supply is 0.43, in line with Harrison and Oomen (2010). The Calvo parameter on prices θ is set to 0.5, indicating that an average prices duration is about six months, following the estimates from Bunn and Ellis (2011) for the UK economy. Elasticity of substitution between different goods, is set to 11, as shown in Britton et al. (2000). Under the previous parameters calibration, it implies that $\kappa = 0.147$. Following the estimate found by Dixon et al. (2014), the value of ρ_g sets to 0.7, representing a high persistent shock.

Bank of England dropped the Bank rate to 2% after the fourth quarter in 2008, which was a more than 50% change compared with the same quarter in previous year. While the corresponding output gap in the fourth quarter in 2008 was -2.7. It seems that the Bank of England has changed policy after the financial crisis. Based on the empirical evidence,

[1]The weights used in this simulation do not have to match the empirically estimated weights of Chapter 2, as in Chapter 2 we do not use the New Keynesian model described in Galí (2009) to be determining the behaviour of the real economy.

we assume -2.7 to be the output gap threshold value.

3.5.2 Calibration: Monetary Rules

As explained above, the numerical analysis compares two different rules: the basic Taylor rule (3.1) and the endogenous-switching rule (3.26). The parameter values to be set are four. On the one hand, we set numerical values for h_1 and b_1 that characterize both the basic Taylor rule and the "Normal Regime" of the endogenous-switching rule. One the other hand, we need to set different values for h_2 and b_2 to characterize the "Crisis Regime" of the endogenous-switching rule.

Basic Taylor rule and normal regime. Regarding the first two parameters, we set $h_1 = 1.5$ and $b_1 = 0.5$. These are the values of the original Taylor rule (Taylor, 1993) that subsequent research has shown to be proper estimates of the behaviour of the UK nominal interest rate in the long run (Nelson, 2001). Therefore, parameters $h_1 = 1.5$ and $b_1 = 0.5$ will equally apply to the basic Taylor rule and to the "Normal Regime" of the endogenous-switching rule.

Crisis regime of the endogenous-switching rule. In choosing the parameters h_2 and b_2 for the crisis regime, we will consider three different scenarios. The idea is that Central Banks may differ in their preferences for settling the trade-off between stabilizing inflation and minimizing the output gap: facing intense shocks on economic activity, some Central Banks may be willing to preserve output (rather than price stability) more than other Central Banks, and vice versa. The three scenarios that we will consider reflect different cases in which the Central Bank is more or less averse to negative output gaps.

Crisis regime / Scenario 1. Foerstery (2014) argues that US monetary policy after the crisis has been more tolerant of inflation than in normal times, and the data seem to support a similar conclusion for the UK. Building on this consideration, our first scenario assumes $h_2 = 1.5$ and $b_2 = 1$. That is, compared to the normal regime, we keep the same weight on inflation whereas we double the value of the weight attached to the output gap. This scenario thus represents a Central Bank that, in the event of a crisis, becomes more averse to negative output gaps.

Crisis regime / Scenario 2. In order to evaluate more clearly the role of "output-gap aversion" in the crisis regime, we will compare the effects of conducting the monetary policy assumed in Scenario 1 with the opposite case in which $h_2 = 3$ and $b_2 = 0.5$. That is, compared to the normal regime, we double the value of the weight attached to inflation whereas we keep the same weight on the output gap.

Crisis regime / Scenario 3. The third Scenario combines the former cases and may be taken to represent a "more aggressive Central Bank" that, facing a crisis, increases both weights at the same time while attaching the same relative weight (h/b) to inflation versus output stabilization. By setting $h_2 = 3$ and $b_2 = 1$, we obtain a crisis regime that, compared to the normal regime, doubles the values of both weights appearing in the monetary policy rule. In practice, Scenario 3 assumes that a Central Bank facing a crisis will change interest rates more drastically than in normal times without changing its relative preferences towards the inflation-output tradeoff.

In order to assess the performances of the Taylor rules with different weights on inflation and the output gap, we assume when there are two shocks with different variances – a non-crisis shock with variance $\sigma_{\varepsilon_{gt}} = 5\%$ and a crisis shock with variance $\sigma_{\varepsilon_{gt}} = 10\%$. The calibrated parameters are summarized in Table 3.1.

Table 3.1: Parameter values

Parameters	Value	Description	Motivation
β	0.99	Discount factor	Assumption
σ	0.66	Intertemporal substitution	Smets and Wouters (2007)
φ	0.43	Elasticity of labour supply	Faccini et al. (2011)
α	0.31	Elasticity of labour	Faccini et al. (2011)
ε	11	Elasticity of substitution	Britton et al. (2000)
θ	0.5	Index of price stickiness	Bunn and Ellis (2011)
κ	0.147	Slope of the Phillips curve	Result
ρ_g	0.7	AR-coefficient of demand shock	Dixon et al. (2014)
$\sigma_{\varepsilon_{gt}}$	5%/10%	Non-crisis and crisis shock variance	Assumption
h_1	1.5	Inflation coefficient (Normal Regime)	Nelson (2001)
b_1	0.5	Output gap coefficient (Normal Regime)	Nelson (2001)
h_2	1.5/3	Inflation coefficient (Crisis Regime)	Assumption
b_2	1/0.5	Output gap coefficient (Crisis Regime)	Assumption
τ	-2.7	Output gap threshold	Assumption

3.5.3 Results

In this section, we analyse the consequences of different types of monetary rules, the Taylor rules with different fixed weights and the endogenous-switching rules, in a New Keynesian model developed by Galí (2009). Different shocks will be investigated, a non-crisis shock with a variance of 5%, a crisis shock with variance of 10% and a whole-period random shock follows normal distribution of $N(0, 10)$.

Rule comparisons under a Non-crisis Shock

As Central Banks alter their policy priorities when implementing different weighted Taylor rules, the nominal interest rate responds differently to the shock and leads to different behaviour of inflation and output gap. Suppose a non-crisis shock hits the economy at period 5, with a variance of 5%. Figure 3.1 shows impulse responses of inflation, the output gap and the nominal interest rate under different Taylor rules and the endogenous-switching rules respectively. When a non-crisis demand shock hits, price and aggregate demand fall, and the Central Bank responds to the shock by decreasing the nominal interest rate. When the nominal interest rate drops, the real interest rate falls and thus stimulates aggregate demand. The rising aggregate demand affects inflation through the New Keynesian

Phillips curve, and consequently inflation rises.

Table 3.2 shows the corresponding standard deviations[2] of the different weighted Taylor rules and their corresponding endogenous-switching rules. Among the basic Taylor rules, as the Central Bank responds aggressively by increasing both weights of inflation and output gap under the "Crisis Regime / scenario 3", it outperforms other "Crisis Regime" scenarios by showing less deflation and smaller negative output gap, and thus higher social welfare. Also, results show that if the Central Bank puts more weight on output, it achieves a higher social welfare, compared with an inflation-averse Central Bank. It indicates that when facing a demand shock, instead of placing more weight on inflation, assigning a higher weight on output could be effective for Central Banks to bring prices and aggregate demand back to equilibrium.

Comparing the "original" Taylor rule with the endogenous-switching rules, the results are identical as the shock is not big enough to drive the output gap below the threshold – that is, the endogenous-switching rules follow the "original" Taylor rule.

[2] In this Chapter, the standard deviations of the inflation, the output gap and the nominal interest rate, as well as the social welfare losses are all expressed in percentage terms.

Figure 3.1: Impulse responses to a non-crisis shock

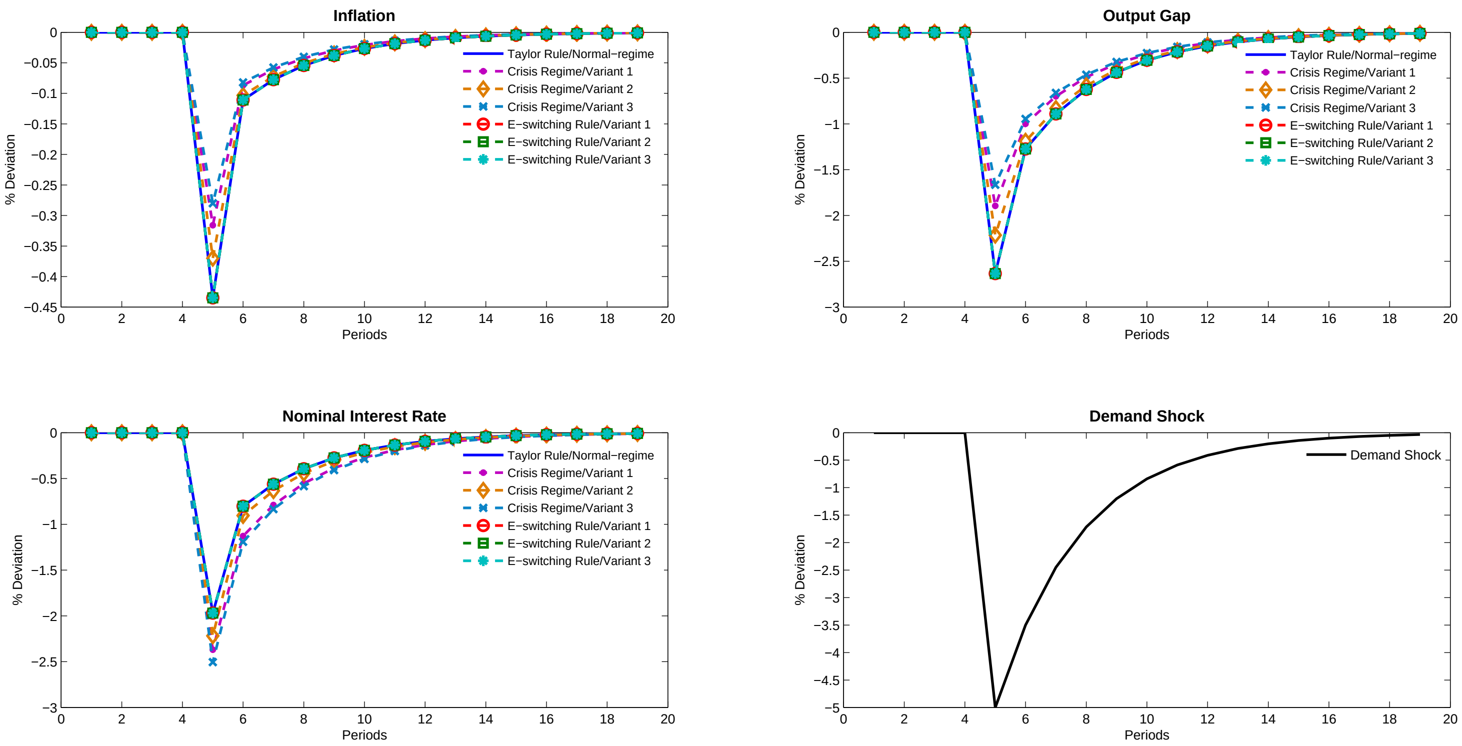


Table 3.2: Evaluation of different policy rules under a non-crisis shock

	$s.d.(\pi)$	$s.d.(y)$	$s.d.(i)$	Welfare Loss
Taylor Rule/Normal-regime ($h = 1.5; b = 0.5$)	0.0998	0.6519	0.4723	0.0156
Crisis Regime / Scenario 1 ($h = 1.5; b = 1$)	0.0728	0.4786	0.5844	0.0084
E-switching Rule / Scenario 1	0.0998	0.6519	0.4723	0.0156
Crisis Regime / Scenario 2 ($h = 3; b = 0.5$)	0.0854	0.5621	0.5320	0.0115
E-switching Rule / Scenario 2	0.0998	0.6519	0.4723	0.0156
Crisis Regime / Scenario 3 ($h = 3; b = 1$)	0.0648	0.4290	0.6176	0.0067^{***}
E-switching Rule / Scenario 3	0.0998	0.6519	0.4723	0.0156

Notes: The superscripts *** shows the lowest welfare loss which indicates the corresponding policy rule is the preferred rule.

Rule Comparisons under a Crisis Shock

We now investigate the economy responses when there is a "crisis shock", by increasing the variance of a demand shock to 10% ($\sigma_{\varepsilon_{gt}} = 10\%$). When implementing the endogenous switching rules, Central Banks observe shocks, revise their policy priorities and thus choose a regime. Under the assumption, if the output gap is below the threshold, one of the variants of the "Crisis Regimes" will be used, otherwise, the "Normal Regime" is implemented by Central Banks. Intuitively, when the output gap is above the threshold, Central Banks prefer to not move the interest rate aggressively to stabilize prices and demand. While monetary authorities tend to focusing on stabilizing output and inflation when the output gap is below its threshold. The advantage of the endogenous-switching rule is that it allows the monetary authorities to change their policy reactions in response to different macroeconomic conditions, so as to limit the deviations of prices and aggregate demand, and thus improve welfare. In this Chapter, we assume the Central Bank could either place more weight on output gap or more weight on inflation. If there is a inflation-averse

Central Bank, the "endogenous-switching / Scenario 1" rule is adopted. Instead, if the output stabilization is the Central Bank's priority, the Central Bank would like to lower the nominal interest rate more in order to push the economy back to equilibriums. The "endogenous-switching / Scenario 2" rule is implemented. Furthermore, the "endogenous-switching / Scenario 3" rule stands for the more aggressive Central Bank, which assigns more weights to stabilize the prices and aggregate demand.

Figure 3.2 shows movements of the output gap, inflation and nominal interest rate in response to a large negative aggregate demand shock, comparing the Taylor rules with different fixed weights and the endogenous-switching rules. After the crisis shock, the prices and aggregate demand drop significantly. In response to the shock, the Central Bank lowers the nominal interest rate to restore the social welfare. The endogenous-switching rules take the "Crisis Regime" when the output gap falls below the threshold. In general, the nominal interest rate drops significantly, causing less deviation of inflation and output, and thus welfare rises. When the output gap shifts up and above the threshold, the endogenous-switching rules switch to the "Normal Regime" which is identical to the basic Taylor rule. Comparing the endogenous-switching rules with the basic Taylor rule, the endogenous-switching rules show a lower welfare loss, although they are associated with a high deviations of the nominal interest rate.

However, Table 3.3 shows that the crisis rules always give the highest welfare compared with the basic Taylor rule and the endogenous-switching rules. It indicates that the social welfare could be improved significantly if the Central Bank follows a linear Taylor rule but increases the weights on inflation or/and output gap. Different from the contribution of Davig and Leeper (2008), who believe the economy could be benefit from making the policy rule switches endogenously, our results show that the social welfare could be further improved by simply raising the weights on inflation and output gap in the linear Taylor rule.

Figure 3.2: Impulse responses to a crisis shock

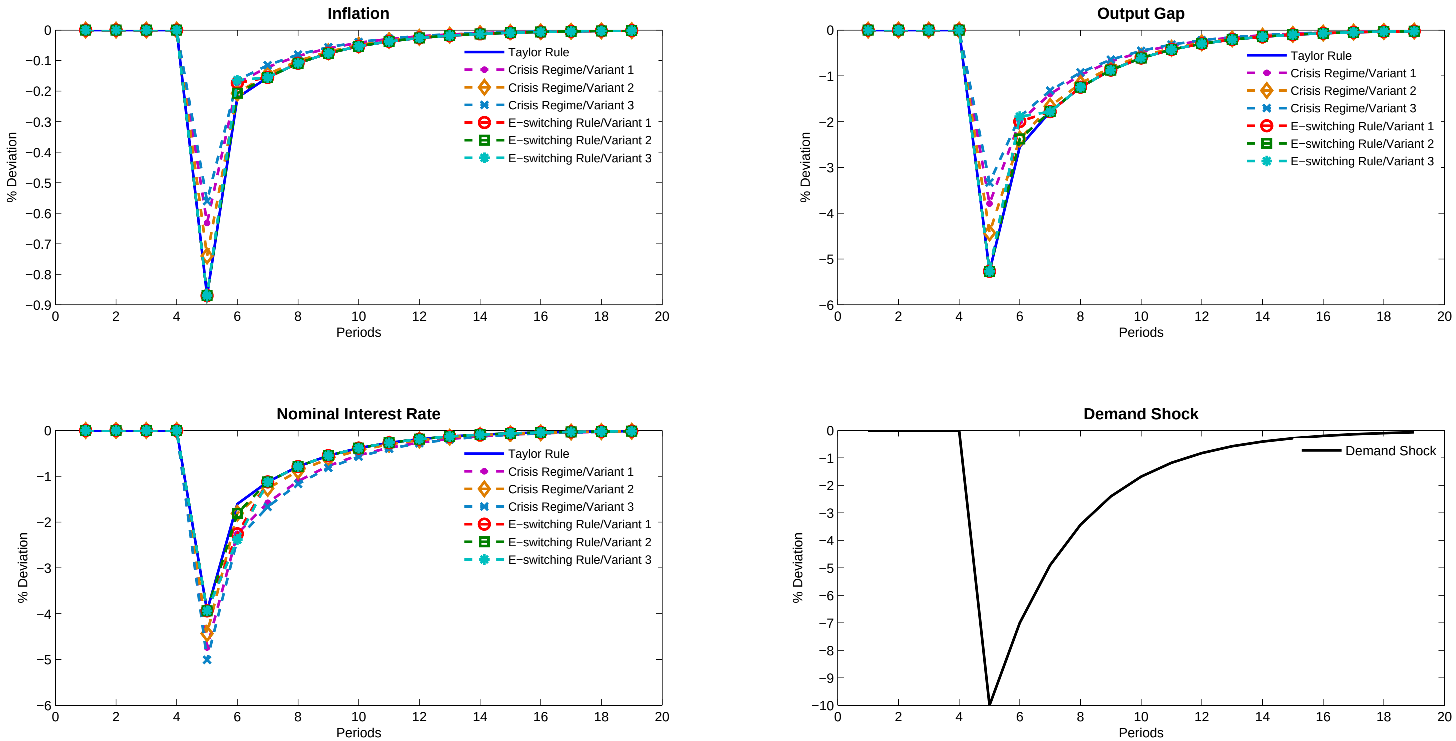


Table 3.3: Evaluation of different policy rules under a crisis shock

	$s.d.(\pi)$	$s.d.(y)$	$s.d.(i)$	Welfare Loss
Taylor Rule/Normal-regime ($h = 1.5; b = 0.5$)	0.1996	1.3038	0.9446	0.0626
Crisis Regime / Scenario 1 ($h = 1.5; b = 1$)	0.1456	0.9572	1.1688	0.0335
E-switching Rule / Scenario 1	0.1981	1.2665	0.9978	0.0607
Crisis Regime / Scenario 2 ($h = 3; b = 0.5$)	0.1708	1.1242	1.0639	0.0461
E-switching Rule / Scenario 2	0.1991	1.2910	0.9590	0.0619
Crisis Regime / Scenario 3 ($h = 3; b = 1$)	0.1296	0.8580	1.2353	0.0267^{***}
E-switching Rule / Scenario 3	0.1979	1.2606	1.0104	0.0604

Notes: The superscripts *** shows the lowest welfare loss which indicates the corresponding policy rule is the preferred rule.

A Model with Random Shocks

Instead of assuming a stochastic shock appears in one period, we assume random shocks in the whole period, which follows a normal distribution with mean 0 and standard deviation 10. Figure 3.3 shows impulse responses for key variables in response to random shocks. Comparing with the “original” Taylor rule, the endogenous-switching rules are welfare-improving as they show less deviation of inflation and the output gap. Specifically, if the random shock is not able to push the output gap below the threshold, the endogenous switching rules take the “Normal Regime”, therefore, inflation, the output gap and the nominal interest rate show identical paths with the “original” Taylor rule. Instead, when the output gap is below the threshold, the endogenous-switching rules follow the “Crisis Regime” which the nominal interest rate responds aggressively and causes a lower prices and aggregate demand deviations but higher volatility on nominal interest rate.

Although the endogenous-switching rules outperform the “original” Taylor rule, they show higher welfare losses compared with the “Crisis Regime”. It indicates that the Central

Bank could achieve a higher social welfare by implementing a linear Taylor rule with higher weights on inflation and output gap. Furthermore, the “Crisis Regime / Scenario 3” gives the lowest welfare loss among different “Crisis Regime” rules, as the Central Bank lowers the nominal interest rate significantly to restore prices and aggregate demand.

Figure 3.3: Impulse responses to random shocks

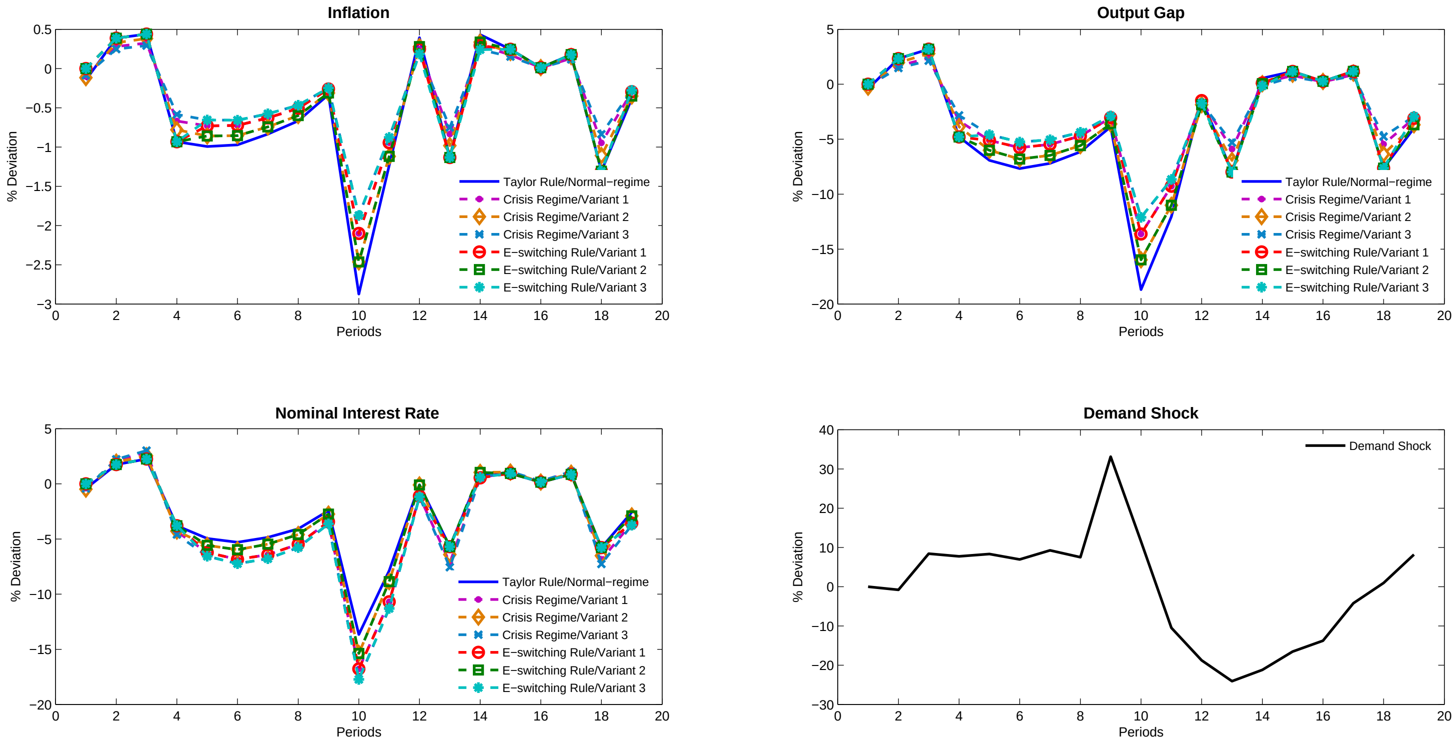


Table 3.4: Evaluation of different policy rules under a random shock

	$s.d.(\pi)$	$s.d.(y)$	$s.d.(i)$	Welfare Loss
Taylor Rule/Normal-regime ($h = 1.5; b = 0.5$)	0.8427	5.5273	3.9969	1.1191
Crisis Regime / Scenario 1 ($h = 1.5; b = 1$)	0.6148	4.0628	4.9541	0.5989
E-switching Rule / Scenario 1	0.6845	4.4325	4.8043	0.7315
Crisis Regime / Scenario 2 ($h = 3; b = 0.5$)	0.7210	4.7733	4.5016	0.8248
E-switching Rule / Scenario 2	0.7576	4.9600	4.4156	0.9034
Crisis Regime / Scenario 3 ($h = 3; b = 1$)	0.5472	3.6463	5.2356	0.4774***
E-switching Rule / Scenario 3	0.6409	4.1426	5.0417	0.6405

Notes: The superscripts *** shows the lowest welfare loss which indicates the corresponding policy rule is the preferred rule.

To summarise, the endogenous-switching rules enable Central Banks to adjust the policy reactions in response to economic conditions. To be precise, the level of the aggregate output determines Central Banks' priorities and thus the preferred policy rule. Specifically, when the economy faces a shock which pushes the lagged output gap below its threshold, the endogenous-switching rules adopt a "Crisis Regime" which interest rate responds aggressively in order to bring inflation and output back to equilibriums. While if the output gap stays above the threshold, the endogenous-switching rules implement a "Normal Regime" which takes the weights suggested in the original the Taylor rule. As shown in Figure 3.1, Figure 3.2 and Figure 3.3, the endogenous-switching rules are welfare-improving compared with the "original" Taylor rule. However, the "Crisis Regime" rules give the best welfare performances compared with other rules. A higher welfare could be achieved by simply adding weights to inflation and output gap in a linear Taylor rule. The results are inconsistent with the conclusion made by Davig and Leeper (2008), who claim that the welfare could be improved by adopting an threshold switching rule rather than a fixed policy rule.

Although the "Crisis Regime" rules show the highest social welfare, the rules are associated with the highest interest rate volatility. The shortcoming of these rules is that Central Banks have to accommodate the risk of high interest rate volatility to achieve high welfare.

3.6 Conclusion

This Chapter investigates different monetary policy rules in response to a demand shock in a basic New Keynesian model. In general, we consider two different types of policy rules, the linear Taylor rule and the endogenous-switching rule. In particular, we assume different weights to the linear Taylor rule to form the "Normal Regime" and different scenarios of "Crisis Regime". And following Davig and Leeper (2008), we assume monetary policy rules switch endogenously, depending on macroeconomic environment, instead of assuming the policy rules change exogenously. The endogenous-switching rule contains two different Taylor rules which switch when the output gap breaches the output threshold. Specifically, in order to combat the large deflation and negative output gap, we therefore assume Central Banks will adopt the rule which place more weight on inflation or output gap ("Crisis Regime") when there is a severe shock, but implement a rule with a moderate weights on inflation and output but more on stabilising inflation ("Normal Regime") if the output gap stays above the threshold.

Based on the social welfare function considered in this Chapter, we found the endogenous switching rule outperforms the "original" Taylor rule which takes the weights suggested by Taylor (1993). At the same time, if Central Banks were to follow a linear Taylor rule with higher weights on both the inflation and the output gaps – that is, a more aggressive counter-cyclical policy – then the welfare losses would be even lower. Our results show that if Central Banks follow a linear Taylor rule by increasing the weights on inflation and output gap, the social welfare can be improved significantly, which is inconsistent with the conclusion made by Davig and Leeper (2008), arguing that the economy benefits from allowing the policy rules switch endogenously.

However, we also found that although the “Crisis Regime” rule outperforms other alternative rules, it is associated with high interest rate volatility. Therefore, Central Banks face a trade-off between nominal interest rate stabilisation and prices and aggregate demand stabilisation. The empirical evidence indicate that in order to minimize social losses, Central Banks also seek to “smooth” the nominal interest rate. Even if the interest rate is not included in the “social welfare loss function”, a relative stable interest rate is desirable (see Goodhart, 1999; Woodford, 1999; Woodford, 2003b). Therefore, one fruitful extension of the analysis would be to evaluate the overall benefits to Central Banks taking the additional of objective of interest-rate stabilization explicitly into account – to assess whether the introduction of nominal interest-rate stabilization as one of Central Banks' objectives will change the implementation of monetary policy rule.

Two possible extensions of the present work would be (i) to include backward/forward specifications of inflation gaps, and (ii) to consider the zero-lower-bound (ZLB) problem. The first extension would be based on the strand of literature suggesting that introducing past and/or expected inflation gaps in the Taylor rule would improve welfare (or reduce further the welfare loss) – (see Svensson, 2003; Benhabib et al., 2001; Clarida et al., 1998; Carlstrom et al., 2000); The second extension would model the fact that many economies experience nearly zero short-term nominal interest rate after 2008, therefore, another interesting extension would be investigating the consequences if the nominal interest rate is constrained to the Zero Lower Bound. Due to the existence of the zero-lower-bound problem, Taylor and Williams (2010) point out that the monetary policy rule may be not sufficient to control inflation, which is further argued by McCallum (2000). If considering the zero-lower-bound problem in the presented model in this Chapter, it may cause variability on inflation and interest rates, although an aggressive response to output could reduce the effect of the zero-lower-bound. Also, Fernández-Villaverde et al. (2015) argue that nonlinearities should be considered when analysing the nominal interest rate with the zero lower bound in a New Keynesian model. Therefore, nonlinearities issues should be taken into account when adding the zero-lower-bound to the model.

Chapter 4

Monetary Policy Rules in a Small Open Economy

4.1 Introduction

A two-country dynamic general equilibrium model is employed to examine the welfare effects of different policy responses to a Foreign demand shock in a small open economy. We evaluate monetary policy rules by using a utility-based loss function that depends on domestic inflation, output gap and real exchange rate. When the optimal rule cannot be implemented, alternative policy rules yield different welfare levels and their ranking depends on the values taken by some key parameters. Our research question is motivated by the recent economic crisis. The financial crisis in 2007, which initially started in the US, hit many countries around the world. As a result, many economies reduced the nominal interest rates significantly in response to the crisis simultaneously. This Chapter investigates the monetary policy reactions in a small open economy, especially if there is a foreign shock and in particular, we simulate various policy rules in a small open economy model and examine the welfare performances under different scenarios.

The design of monetary policy rules in a small open economy has been studied by a large range of literature. Some studies argue that the policy response in closed and open economy are identical (Clarida et al., 2001; Clarida et al., 2002; Gali and Monacelli, 2005; Taylor, 2001), and suggest that even in small open economies, monetary authorities should aim at stabilising domestic inflation and output. However, Corsetti and Pesenti (2001) and Berger (2008) point out that targeting domestic variables only may be sub-optimal under some circumstances. Due to the expenditure switching effect associated with the elasticity of substitution between home and foreign goods, the Home demand is sensitive to the movement of the real exchange rate, so the real exchange rate should be one of the target variables. In this framework, the policy rule is not isomorphic to one that applies to closed economies.

In this Chapter, we use the model of De Paoli (2009) to derive the optimal rule for a small open economy as a function that depends on the domestic inflation, output gap and real exchange rate. This optimal rule can be re-expressed as an interest-rate rule in which the weights assigned to each target depend on the economy's structural parameters – in particular, the elasticity of intertemporal substitution, the elasticity of substitution between home and foreign goods, the degree of openness, and home bias in consumers' preferences. However, due to complex calculations of the weights in the optimal rule and the difficulty to observe the output target, monetary authorities may find it hard to implement the optimal rule in practice. This problem may force Central Banks to rely on alternative monetary rules which, albeit easier to implement, are sub-optimal. The relevant question thus becomes that of choosing an alternative policy rule that minimises the welfare loss with respect to the first-best rule. This chapter investigates this issue by studying the welfare effects of such alternative rules in order to identify which type of rules minimise losses under different combinations of structural parameters.

The main result of our analysis is the performances of alternative policy rules depend on the degree of openness and the elasticity of substitution between domestic and foreign goods. When the intratemporal elasticity of substitution is low, the domestic Taylor rule shows the best welfare, while the discretionary rule with a lower target output, is the preferred policy when the Home and Foreign goods are close substitutes. The degree of openness affects the welfare performances for an intermediate value of the elasticity of substitution between home and foreign goods. When the Home economy is relatively closed, the domestic Taylor rule works the best, while the discretionary rule with a lower output target is welfare-improving if the degree of openness becomes higher.

The intuition for these results is that the intratemporal elasticity of substitution and the degree of openness determine the ranking of alternative rules in Home country in response to a Foreign shock. Higher elasticity of substitution between home and foreign goods implies more elastic Home demand for Foreign goods. Therefore, a fall in Foreign production induces a significant rise in Home output so that a contractionary monetary policy could raise Home welfare. Instead, lower intratemporal elasticity of substitution implies less elastic Home demand, so a reduction in Foreign output only leads to a mild increase in Home demand, therefore, the Home country could benefit from focusing on stabilising domestic inflation. For an intermediate value of intratemporal elasticity of substitution, the performances of alternative policy rules depend on the degree of openness, as it alters the home bias which affects the goods demand to the change of real exchange rate. More precisely, keeping domestic inflation stable is preferred if the households have strong preferences for Home goods, while when households prefer Foreign products, the losses induced by more appreciated real exchange and the lower output is compensated by the consistently lower inflation, which in turn, raises the Home welfare.

In addition, we are interested in whether Central Banks implement the discretionary rules persistently or just for a short-period. We investigate the discretionary rules with the output only being sub-optimal for one-period. We find that the policy-performance ranking changes when we assume the discretionary output target only varies for one period. The discretionary rule with a higher output target outperforms other alternatives when the intratemporal elasticity of substitution is high and the Home economy is relatively open. That is, in a relatively open Home economy, when the Home and Foreign goods are good substitutes, temporary lower inflation is not sufficient to offset the losses induced by the lower output and more appreciated Home currency. Therefore, implementing a discretionary rule with a higher output target, leading to higher output, could help to raise the domestic welfare.

4.2 Literature Review

The vast literature on monetary policy comprises a strand of contributions that argue that optimal rules for small open economies are isomorphic to those obtained in closed-economy models. Clarida et al. (2001) develop a two-country model for a small open economy, although the openness does affect the parameters in the model, hence, the policy design, the monetary policy setting is isomorphic to the policy problem in closed economy, which is further confirmed by Clarida et al. (2002) and Gali and Monacelli (2005). Gali and Monacelli (2005), who have developed a small open economy with Calvo-type price setting, which are similar with its corresponding closed economy, but the dynamic equilibrium in a small open economy depends on parameters which are associated with an open economy setting. A welfare loss function has been derived to evaluate different types of monetary policy. As the welfare is measured by the fluctuations in domestic inflation and the output gap, it implies that the evaluation method is similar with the method used in a closed economy. Three different types of simple monetary regimes have been analysed by Gali and Monacelli (2005). Compared with a Taylor rule with CPI inflation targeting and an exchange rate peg, the results show that a Taylor rule which reacts to domestic inflation gives the highest welfare, due to the presence of a trade-off between the stabilization of exchange rate and the stabilization of domestic inflation and output gap. A similar result has been found by Taylor (2001) who states that temporary fluctuations in the exchange rate do not have much effect on expectations of inflation and output gap, so there is no need to include the exchange rate in the interest rate rule. If the monetary policy takes a direct response to exchange rate, it may cause unnecessary volatility in the interest rate.

He states that no evidence has been found that monetary policy reacts directly to the exchange rate outperforms other rules in stabilising inflation and real output.

However, different from the above contributions, a second strand of literature argues that monetary policy should respond to exchange rate movements. Corsetti and Pesenti (2001) show that in an open economy, an external distortion is associated with openness. That is a country could affect the terms of trade by influencing Home supply. A monetary expansion could stimulate the Home output, but in the meantime, the appreciation of the exchange rate reduces the terms of trade. Therefore, a country could benefit from monetary policies that influence the terms of trade in the desirable direction. A similar argument is put forward by Ball (1999), who shows that extended Taylor rules that include the real exchange rate as an additional target can indeed be welfare improving.

In a different model proposed by De Paoli (2009), the notion of "small open economy" is obtained by considering a two-country general equilibrium model and then limiting the size of the "Home country" to be asymptotically zero. Under the assumption of home bias, the real exchange rate influences the relative domestic price which in turn affects demand in the Home country. This is the channel through which the real exchange rate affects welfare. The consequence is that, if monetary authorities wish to minimize a utility-based loss function, the resulting optimal policy rule will be a function of domestic inflation, output gap and real exchange rate. This result indicates that monetary policy is not isomorphic to the one in the closed economy, as the Central Bank needs to take into account the variability of the real exchange rate. However, implementing such an optimal rule may be difficult in practice, which suggests considering alternative "simple rules" and assessing their performance by comparing the resulting welfare loss relative to the welfare levels that would be obtained under the first-best (yet unfeasible) optimal rule. In this respect, De Paoli (2009) shows that the performance of different rules varies depending on the values taken by structural parameters (intertemporal and intratemporal substitution and the degree of openness).

Similarly, Berger (2008) discusses the optimal choice of monetary policy rules in open economies. In an equal-sized two country model, an optimal monetary policy has been derived. However, due to it being highly complex and hard to implement an optimal rule in practice, therefore, some simple, non-optimal rules are considered, finding which is the closest in welfare compared to the optimal rule. He finds that the welfare performance of the alternative policy varies, and depends significantly on the elasticity of substitution of home and foreign goods. In this Chapter, we will evaluate the welfare performances of these different policy rules – in particular, the optimal rule, discretionary rules and alternative simple rules, based on the small open economy model developed by De Paoli (2009). The specific forms of different types of monetary policy rules will be discussed in detail in section 4.4.2.

4.3 Model

The model we adopt is the New Keynesian model by De Paoli (2009). The world comprises two countries, called Home and Foreign. After solving for the standard two-country general equilibrium, Home economy is transformed into a "small open economy" be letting the size of Home country to be asymptotically zero. Representative households consume goods produced in both Home and Foreign countries and supply labour to monopolistically competitive firms. In addition, the model assumes trade imbalances, no trade or financial frictions exist, that is, the law of one price holds and asset markets are complete.

Although the law of one price holds, purchasing power parity does not hold due to the home bias in consumption. The home bias depends on the degree of openness and the relative size of the economy (De Paoli, 2009). The world economy is populated with a continuum of agents, where an interval $(0, n]$ is located in Home country and the interval $(n, 1]$ lives in Foreign country. According to De Paoli (2009), the model is solved by finding the equilibrium of the two-country model and after taking the limit, the two countries, Home and Foreign, represent the small open economy and the rest of the world respectively.

Variables with an asterisk superscript $(*)$ represent the world economy.

4.3.1 Households

The representative infinitely-lived household in Home country chooses consumption, C_t, and the level of labour supply, N_t, in order to maximise lifetime utility,

$$E_t \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{4.1}$$

where $\beta \in (0,1)$ is discount factor. Assume a continuum of firms indexed by $j \in [0,1]$ and each firm produces a different variety of consumption goods using labour supplied by households as an input. In view of these assumptions, the consumption index and the labour index are respectively given by

$$C_t = \left[v^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-v)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{4.2}$$

$$N_t = \int_0^1 N_t(j)dj \tag{4.3}$$

where η is elasticity of substitution between home and foreign goods, and v is home bias, representing each Home consumer has an asymmetric preference for Home and Foreign goods. For $v = \frac{1}{2}$, the model assumes symmetric preferences across countries. $(1-v)$ stands for Home consumers' preferences for Foreign goods, which is a function of the relative size of the Foreign country $(1-n)$ and the degree of openness λ, specifically, $(1-v) = (1-n)\lambda$. $N_t(j)$ is the amount of labour services supplied to the j firm. $C_{H,t}$ and $C_{F,t}$ represent consumption in the Home country which is produced in the Home and Foreign country respectively, and which are defined as

$$C_{H,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad C_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{4.4}$$

where ε is elasticity of substitution across goods, and n is the size of Home economy. $C_{H,t}(j)$ is Home consumption for Home-produced good j and $C_{F,t}(j)$ is Home consumption for Foreign-produced good j.

Similar preferences of consumption are defined for the Foreign country,

$$C_t^* = \left[v^{*\frac{1}{\eta}} C_{H,t}^{*\frac{\eta-1}{\eta}} + (1-v^*)^{\frac{1}{\eta}} C_{F,t}^{*\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{4.5}$$

where $v^* = n\lambda$, which is Foreign consumers' preferences for goods produced in Home country, depending on the size of the small open economy and the degree of openness. $C_{H,t}^*$ and $C_{F,t}^*$ are consumption in the Foreign country which is produced in the Home and Foreign economy respectively, which is given by:

$$C_{H,t}^* = \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n C_{H,t}^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad C_{F,t}^* = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 C_{F,t}^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{4.6}$$

where $C_{H,t}^*(j)$ is Foreign consumption for Home-produced good j and $C_{F,t}^*(j)$ is Foreign consumption for Foreign-produced good j.

Correspondingly, the price index in the Home and Foreign country is given respectively by

$$P_t = \left[vP_{H,t}^{1-\eta} + (1-v)P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{4.7}$$

$$P_t^* = \left[v^* P_{H,t}^{*1-\eta} + (1-v^*)P_{F,t}^{*1-\eta} \right]^{\frac{1}{1-\eta}} \tag{4.8}$$

where $P_{H,t}$ and $P_{H,t}^*$ are the price of Home-produced goods expressed in Home and Foreign currency respectively, and $P_{F,t}$ and $P_{F,t}^*$ are the price for Foreign-produced good expressed in Home and Foreign currency.

$$P_{H,t} = \left[\left(\frac{1}{n}\right)\int_0^n P_{H,t}(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}} \qquad P_{F,t} = \left[\left(\frac{1}{n-1}\right)\int_n^1 P_{F,t}(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}} \tag{4.9}$$

$$P^*_{H,t} = \left[\left(\frac{1}{n}\right)\int_0^n P^*_{H,t}(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}} \qquad P^*_{F,t} = \left[\left(\frac{1}{n-1}\right)\int_n^1 P^*_{F,t}(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}} \tag{4.10}$$

Utility maximization is subject to the budget constraint

$$\int_0^n P_{H,t}(j)C_{H,t}(j)dj + \int_n^1 P_{F,t}(j)C_{F,t}(j) + E_t[Q_{t,t+1}D_{t+1}] = D_t + W_tN_t + T_t \tag{4.11}$$

where D_{t+1} is the nominal payoff of one-period riskless discount bonds maturing in period $t+1$. Each of these bonds is purchased at price $E_t[Q_{t,t+1}] = \frac{1}{1+i_{t+1}}$, where i_{t+1} is the nominal interest rate in period $t+1$. W_t is the nominal wage and T_t is additions to income in the form of dividends.

The optimal allocation of consumption for household j gives the demand functions:

$$C_{H,t}(j) = \frac{1}{n}\left(\frac{P_{H,t(j)}}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \tag{4.12}$$

$$C_{F,t}(j) = \frac{1}{n-1}\left(\frac{P_{F,t(j)}}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \tag{4.13}$$

And the optimal allocation of consumption between goods produced in the Home and Foreign countries is given by

$$C_{H,t} = v\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{4.14}$$

$$C_{F,t} = (1-v)\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{4.15}$$

Combining the previous equations and substituting into the budget constraint, gives

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + E_t[Q_{t,t+1}D_{t+1}] = D_t + W_tN_t + T_t \tag{4.16}$$

As the total consumption of households in Home economy is given by $P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$, the budget constraint can be written as

$$P_tC_t + E_t[Q_{t,t+1}D_{t+1}] = D_t + W_tN_t + T_t \tag{4.17}$$

Assume the utility function takes the form

$$E_t[U] = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{4.18}$$

where σ is the intertemporal elasticity of substitution and φ is the elasticity of labour supply. Then solving the maximisation problem, the optimal choices for the household are given by

$$1 = \beta(1+i_t)E_t\left[\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\right] \tag{4.19}$$

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{C_t^{-\sigma}} \tag{4.20}$$

Correspondingly, the Foreign economy's Euler equation and optimal condition is given by

$$1 = \beta(1+i_t^*)E_t\left[\frac{P_t^*}{P_{t+1}^*}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\right] \tag{4.21}$$

$$\frac{W_t^*}{P_t^*} = \frac{N_t^{*\varphi}}{C_t^{*-\sigma}} \tag{4.22}$$

where i_t^* is the nominal interest rate in the Foreign country. W_t^* and N_t^* are respectively nominal wage and labour supply in the Foreign country.

Domestic Inflation, CPI Inflation, the Real Exchange Rate and the Terms of Trade

Under the assumption of the law of one price, the relationships between Home and Foreign prices are:

$$P_{H,t} = \xi_t P^*_{H,t} \text{ and } P_{F,t} = \xi_t P^*_{F,t}$$

where ξ is the nominal exchange rate, denoting the price of Foreign currency in terms of domestic currency.

Define the real exchange rate as

$$\mathcal{Q}_t = \frac{\xi_t P^*_t}{P_t} \tag{4.23}$$

Linearising the price index in Home economy (4.7) around zero inflation steady state, yields[1]

$$p_t = v p_{H,t} + (1-v) p_{F,t} \tag{4.24}$$

The terms of trade is defined as $S_t = \frac{P_{F,t}}{P_{H,t}}$, measuring the relative price of the imported goods produced in Foreign economy in terms of the domestic goods in the Home market. Linearizing the terms of trade in terms of the real exchange rate, yields

$$s_t = -\frac{1}{v} q_t \tag{4.25}$$

The relationship between CPI inflation and domestic inflation is given by:

$$\pi_t = \pi_{H,t} + \frac{\lambda}{1-\lambda}(q_t - q_{t-1}) \tag{4.26}$$

[1] Lower-case letters denote the natural logs of the corresponding variable, such as $x_t = logX_t$.

where $\pi_t = p_t - p_{t-1}$ and $\pi_{H,t} = p_{H,t} - p_{H,t-1}$

International Risk Sharing

Under the assumption of complete international market, combining the Euler equation in Home and Foreign country (4.19) and (4.21), and using the definition of real exchange rate (4.23), the optimal risk sharing agreement between agents in Home and Foreign country is

$$C_t = \left(\frac{C_{t+1}}{C^*_{t+1} \mathcal{Q}_{t+1}^{\frac{1}{\sigma}}} \right) C^*_t \mathcal{Q}_t^{\frac{1}{\sigma}} \tag{4.27}$$

As the symmetric initial condition is assumed, $\left(\frac{C_{t+1}}{C^*_{t+1} \mathcal{Q}_{t+1}^{\frac{1}{\sigma}}} \right) = 1$, so the international risk sharing condition reads:

$$C_t = \mathcal{Q}_t^{\frac{1}{\sigma}} C^*_t \tag{4.28}$$

Linearising equation (4.28) gives:

$$c_t = c^*_t + \frac{1}{\sigma} q_t \tag{4.29}$$

4.3.2 Firms

Each firm produces a differentiated good based on the production function

$$Y_t(j) = A_t N_t(j)^{1-\alpha} \tag{4.30}$$

where A stands for exogenous measure of technology, assuming to be the same across firms. $1 - \alpha$ represents output elasticity of labour.

Following De Paoli (2009), price stickiness is modelled as in Calvo (1983) – in each period, only a fraction of $1 - \theta$ of firms could receive price-changing information and are able to choose their prices optimally. The remaining fraction of firms, given by θ, do not receive the signal and keep the prices unchanged. Thus, the aggregate price dynamics can be written as

$$\Pi_{H,t}^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{\tilde{P}_{H,t}}{P_{H,t-1}}\right)^{1-\varepsilon} \tag{4.31}$$

where $\Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$ is the gross inflation rate, $\tilde{P}_{H,t}$ is the re-optimised price by firms which received price-change signal in period t.

In the Home country, firms choose prices to maximise their current profit each period. Due to the constraint of price stickiness, firms realise that the price they decide now will affect profits over the next k periods. Firms re-optimising the price in period t will choose $\tilde{P}_{H,t}$ so as to maximise the current market value of the profits for the next k periods given a probability "θ" that the chosen price indeed remains effective over that horizon. Formally, the firm's maximisation problem is

$$Max \sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \left(\tilde{P}_{H,t} Y_{t+k|t}(j) - \Psi_{t+k}(Y_{t+k|t}(j)) \right) \right] \tag{4.32}$$

subject to demand constraints

$$Y_{t+k|t}(j) = \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} Y_{t+k} \tag{4.33}$$

where $Q_{t,t+k} = \beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_{H,t}}{P_{H,t+k}}$, is a stochastic discount factor for nominal payoffs over the interval $[t, t+k]$, $Y_{t+k|t}(j)$ is output in period $t+k$ for a firm re-optimise price in period t, $\Psi_{t+k}(Y_{t+k|t}(j))$ is cost function.

The first order condition for the optimal price for firms in the Home economy gives:

$$\tilde{P}_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \psi_{t+k|t}(j) Y_{t+k|t}(j) \right]}{\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} Y_{t+k|t}(j) \right]} \tag{4.34}$$

where $\psi_{t+k|t}(j) = \Psi'_{t+k}(Y_{t+k|t}(j))$ is the marginal cost of a firm which last reset its price in period t.

Similarly, the optimal price for firms in the Foreign economy is given by

$$\tilde{P}_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \psi_{t+k|t}(j) Y_{t+k|t}^*(j) \right]}{\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} Y_{t+k|t}^*(j) \right]} \tag{4.35}$$

4.3.3 Equilibrium

Goods Market Equilibrium

Under the assumption of no investment and no government spending, in both the Home and Foreign country, aggregate demand depends on consumption only.

Combining demand constraints for Home and Foreign goods (4.12) and (4.13), the optimal allocations of consumption between differential goods in two countries (4.14) and (4.15), and the definition of real exchange rate (4.23), we have the total demand for good j, which is produced in the Home country.

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} \left[vC_t + \frac{1-n}{n} v^* \left(\frac{1}{\mathcal{Q}_t} \right)^{-\eta} C_t^* \right] \tag{4.36}$$

Following De Paoli (2009), using the definition of home bias v and $v*$ and take the limit $n \to 0$, the market clearing for a small open economy can be written as

$$Y_t = \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} \left[(1-\lambda) C_t + \lambda^* \left(\frac{1}{\mathcal{Q}_t} \right)^{-\eta} C_t^* \right] \tag{4.37}$$

Taking the first-order Taylor approximation of equation (4.37), the linearised aggregate demand function becomes

$$y_t = (1-\lambda) c_t + \lambda y_t^* + \gamma q_t \tag{4.38}$$

Similarly, the total demand for Foreign-produced good j takes the form:

$$Y_t(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[\frac{n}{1-n}(1-v)C_t + (1-v^*)\left(\frac{1}{\mathcal{Q}_t}\right)^{-\eta}C_t^*\right] \tag{4.39}$$

Then the market clearing for the rest of the world is

$$Y_t^* = C_t^* \tag{4.40}$$

Labour Market Equilibrium

Considering the optimal allocation for households in the Home country, shown in equation (4.22), and combining the demand constraint (4.33), the real marginal cost for firms located in the Home country is given by

$$MC_t(j) = \left(\frac{1}{A_t}\right)^{\frac{1+\varphi}{1-\alpha}}\left(\frac{1}{1-\alpha}\right)C_t\left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\frac{\alpha\varepsilon}{1-\alpha}}Y_t^{\frac{\varphi+\alpha}{1-\alpha}}\lambda\frac{P_{F,t}}{P_H,t} \tag{4.41}$$

where $C_t = \mathcal{Q}_t^{\frac{1}{\sigma}}C_t^*$ according to the international risk sharing condition. That is, the real marginal cost in a small open economy depends directly on the real exchange rate.

Substitute the real marginal cost in the Home country (4.41) and Home demand constraint (4.33) into the optimal price in the Home economy, and define $Z_{H,t} = \frac{\tilde{P}_{H,t}}{P_{H,t}}, \frac{P_{H,t}}{P_{H,t+k}} = \frac{1}{\prod_{j=1}^{k}\Pi_{H,t+j}}$, the optimal relative price for a small open economy can be derived as:

$$Z_{H,t} = \left[\frac{\varepsilon}{(1-\alpha)(\varepsilon-1)}\frac{E_t\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}C_t^{\sigma}\lambda\frac{P_{F,t}}{P_{H,t}}\left(\frac{1}{\prod_{j=1}^{k}\Pi_{H,t+j}}\right)^{-\frac{\alpha\varepsilon}{1-\alpha}-\varepsilon}Y_{t+k}^{\frac{\varphi+\alpha}{1-\alpha}+1}\lambda\frac{P_{F,t}}{P_H,t}}{E_t\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}\left(\frac{1}{\prod_{j=1}^{k}\Pi_{H,t+j}}\right)^{1-\varepsilon}Y_{t+k}}\right]^{\frac{1}{\frac{\alpha\varepsilon}{1-\alpha}+1}} \tag{4.42}$$

Applying the same method to the Foreign economy, the real marginal cost for the rest of the world is given by $MC_t^*(j) = (\frac{1}{A_t})^{\frac{1+\varphi}{1-\alpha}}(\frac{1}{1-\alpha})(\frac{P_t^*(j)}{P_t^*})^{-\frac{\alpha\varepsilon}{1-\alpha}}Y_t^{*(\sigma+\frac{\varphi+\alpha}{1-\alpha})}$, and the optimal relative price for the rest of the world is given by

$$Z_t^* = \left[\frac{\varepsilon}{(1-\alpha)(\varepsilon-1)} \frac{E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\frac{1}{\prod_{j=1}^{k} \Pi_{F,t+j}^*} \right)^{-\frac{\alpha\varepsilon}{1-\alpha}-\varepsilon} Y_{t+k}^{*(\sigma+\frac{\varphi+\alpha}{1-\alpha}+1)}}{E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\frac{1}{\prod_{j=1}^{k} \Pi_{F,t+j}^*} \right)^{1-\varepsilon} Y_{t+k}^*} \right]^{\frac{1}{\frac{\alpha\varepsilon}{1-\alpha}+1}} \tag{4.43}$$

Equilibrium in a Small Open Economy

After applying the first order Taylor approximation to the model around the zero inflation[2], the equilibrium in a small open economy could be expressed using two structural equations:

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_a \left[\left(\frac{\alpha+\varphi}{1-\alpha} + \frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)} \right) y_t + \left(\sigma - \frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)} \right) y_t^* \right] \tag{4.44}$$

$$y_t = E_t y_{t+1} - (1-\lambda) \frac{1-\lambda+\sigma\gamma}{\sigma} (i_t - E_t \pi_{H,t+1}) + [\sigma\gamma - \lambda - \lambda(1-\lambda+\sigma\gamma)] \Delta E_t y_{t+1}^* \tag{4.45}$$

where $\kappa_a = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$, $\gamma = \frac{\eta\lambda(2-\lambda)}{1-\lambda}$. The relationship between CPI inflation (π_t) and domestic inflation ($\pi_{H,t}$) is shown in equation (4.26). Note that when the model is assumed to be a closed economy, which means the degree of openness is zero ($\lambda = 0$), the equations collapse to the standard New Keynesian Phillips curve and the IS curve respectively which are identical to the ones commonly used in the closed economy.

Now we have derived the two equations to characterize the macroeconomic equilibrium in a small open economy: equation (4.44) is the New Keynesian Phillips curve and equation (4.45) is the dynamic IS curve. It shows that Home output and inflation are functions of the Foreign output gap and the exchange rate. Since the Foreign Central Bank determines the size and time path of Foreign output, the Foreign monetary policy has a direct influence

[2]Detailed calculations are shown in Appendix B

on the Home economy.

Under the assumption that Home and Foreign goods are not perfect substitutes for Home consumers, the "terms of trade externality" arises. It is an external distortion which leads to inefficient fluctuations in the terms of trade. That is the small open economy retains some market power in its terms of trade (De Paoli, 2009). According to Haberis and Lipinska (2012), the Home economy is able to affect the supply of Home goods on world markets to improve welfare by using its monopolistic power, due to the existence of the "terms of trade externality". Therefore, unlike the closed economy, Central Banks are no longer able to seek inflation and output gap stabilisation at the same time, even when there are no cost-push shocks.

Equilibrium in Foreign Economy

Following De Paoli (2009), the Foreign economy represents the rest of the world, which behaves like a closed economy as it cannot be affected by a small open economy. Therefore, a standard New Keynesian model in the closed economy is used to describe the Foreign economy. The non-policy block of the model is described using the standard New Keynesian Phillips curve and the IS curve.

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa y_t^* \tag{4.46}$$

$$y_t^* = E_t y_{t+1}^* - \sigma^{-1} \left(i_t^* - E_t \pi_{t+1}^* \right) + g_t^* \tag{4.47}$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}(\sigma + \frac{\varphi+\alpha}{1-\alpha})$, π_t^* is Foreign CPI inflation, y_t^* is Foreign output gap, which is the deviation between output and its natural level. $g_t^* = \rho_g g_{t-1}^* + \varepsilon_{gt}^*$ represents a demand shock, where ε_{gt}^* is a stochastic term with mean 0 and variance $\sigma_{\varepsilon_{gt}}^2$.

4.3.4 Welfare

Following Woodford (2003a) and De Paoli (2009), the welfare function can be derived by solving a second-order approximation of the households' utility function. In Appendix B.2, the detailed calculation is shown to derive a welfare function using a second-order Taylor approximation. The welfare function for a small open economy can be written as a quadratic function of output gap, domestic inflation and real exchange rate,

$$L = \frac{1}{2}\Phi_\pi \pi_{H,t}^2 + \frac{1}{2}\Phi_y \left(y_t - y_t^T\right)^2 + \frac{1}{2}\Phi_q \left(q_t - q_t^T\right)^2 \tag{4.48}$$

where $\Phi_\pi = \frac{1}{(1-\lambda+\sigma\gamma)}\frac{\varepsilon}{\kappa_a}$, $\Phi_y = \frac{1}{(1-\lambda+\sigma\gamma)}\left(\frac{\alpha+\varphi}{1-\alpha} + \frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right)$ and
$\Phi_q = \frac{1-\lambda+\sigma\gamma}{\sigma^2}\left(1 + \frac{(\lambda-\lambda\sigma-1)}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right)$, and y_t^T and q_t^T are policy targets.

In the welfare function, the relative weight of inflation with respect to output gap is determined by the structural parameters in the model. When the Home economy becomes closed ($\lambda = 0$), the welfare function collapses to the standard welfare in a closed economy.

Like the closed economy, welfare loss is affected by two economic distortions, price stickiness and monopolistic competition in domestic production. Therefore, the Central Bank in the Home country has an incentive to keep inflation stable and reduce output fluctuations.

In addition, in a small open economy, there is an external distortion which the is terms of trade externality, when Home and Foreign goods are not perfect substitutes. As the imported goods are not perfect substitutes to domestic goods, welfare could be improved through firms' monopolistic power. Firms in the Home economy could gain from improving terms of trade by fluctuating real exchange rate. When goods are substitutes between countries, an appreciating real exchange rate could increase welfare, while a depreciating real exchange rate could be welfare improving when domestic and foreign goods are complements. When goods are substitute, an appreciating real exchange rate will raise consumption of the Foreign production, so firms could reduce production in order to reduce losses. While a depreciated real exchange rate could help to increase welfare when goods are complements, as there will be higher consumption and production in Home goods.

4.4 Monetary Policy Rules

4.4.1 Monetary Policy Rule in Foreign Country

In the two-country dynamic equilibrium model, Foreign country represents the rest of the world, behaving like a closed economy. So we assume the monetary policy conducted in Foreign country follows a standard Taylor rule, which is proposed by John Taylor in 1993.

$$i_t^* = h\pi_t^* + b\left(y_t^* - y_t^{*T}\right) \tag{4.49}$$

where h and b are the weights the the Foreign Central Bank assigned to stabilizing inflation and minimizing output gap.

4.4.2 Monetary Policy Rules in Home Country

Optimal Monetary Policy Rule

According to De Paoli (2009), the optimal monetary policy can be obtained by minimizing the welfare loss function (4.48) subject to the constraints of the policy problem, which are the Phillips curve, as well as the small open economy aggregate demand equation (4.38) and the international risk-sharing condition (4.29).

Rewriting the constraints in terms of the output gap and real exchange rate, the linear constraints become

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_a \left[\left(\frac{\alpha + \varphi}{1-\alpha} + \frac{\sigma}{1-\lambda} \right) \left(y_t - y_t^T\right) - \sigma \frac{\lambda}{1-\lambda} y^* + \frac{\lambda - \sigma\gamma}{1-\lambda} \left(q_t - q_t^T\right) \right] \tag{4.50}$$

$$y_t - y_t^T = \frac{1-\lambda+\sigma\gamma}{\sigma}\left(q_t - q_t^T\right) + y_t^* \tag{4.51}$$

Based on De Paoli (2009), the optimal plan is under an assumption that policymakers can commit to maximize the economy's welfare. The multipliers associated with the linear constraints are φ_1 and φ_2 respectively, and the first order conditions with respect to $\pi_{H,t}$, x_t, and q_t are

$$\kappa_a \Phi_\pi \pi_{H,t} = \varphi_1 - \varphi_{1,t-1} \tag{4.52}$$

$$\Phi_y\left(y_t - y_t^T\right) = \varphi_2 - \left(\frac{\alpha+\varphi}{1-\alpha} + \frac{\sigma}{1-\lambda}\right)\varphi_1 \tag{4.53}$$

$$\Phi_q\left(q_t - q_t^T\right) = -\frac{\lambda-\sigma\gamma}{1-\lambda}\varphi_1 - \frac{1-\lambda+\sigma\gamma}{\sigma}\varphi_2 \tag{4.54}$$

Combining equations (4.52), (4.53) and (4.54), an optimal targeting rule in a small open economy can be obtained:

$$\Phi_\pi^{'}\pi_{H,t} + \Phi_y^{'}\left(y_t - y_t^T\right) + \Phi_q^{'}\left(q_t - q_t^T\right) = 0 \tag{4.55}$$

where $\Phi_\pi^{'} = \sigma(\lambda-\sigma\gamma)+(1-\lambda)(1-\lambda+\sigma\gamma)\left(\frac{\alpha+\varphi}{1-\alpha}+\frac{\sigma}{1-\lambda}\right)\kappa_a\Phi_\pi$, $\Phi_y^{'} = (1-\lambda)(1-\lambda+\sigma\gamma)\Phi_y$ and $\Phi_q^{'} = \sigma(1-\lambda)\Phi_q$

The previous equation describes the small open economy optimal monetary policy rule, in response to domestic inflation, the output gap and the real exchange rate. As a small open economy retains some power to affect the terms of trade, so the real exchange rate is one of the target variables in a social welfare function. It indicates that Central Banks may tolerate some fluctuations in domestic inflation, in order to deal with costly changes in output and real exchange rates.

Reformulation as an Optimal Interest Rate Rule

Clarida et al. (2002) argue that the monetary policy problem for a small open economy is to choose a path for the nominal interest rate to achieve the optimal condition. In order to obtain an expression for nominal interest rate, it is necessary to transfer the optimal targeting rule to an instrument rule. According to Svensson (1999) and McCallum (1999), the optimal interest rate rule could be written as an endogenous optimal reaction function expressing the instrument as a function of the optimal condition. The practical approach is to derive the interest rate from the IS curve, and combine the optimal condition, so that the optimal interest rate rule can be obtained. The approach is consistent with the statement made by De Paoli (2009), that the interest rate could be retrieved by using households' intertemporal choice.

Based on the dynamic IS curve, the nominal interest rate depends on expected future output gap, current output gap and expected domestic inflation. Substituting the optimal condition into the IS curve, the nominal interest rate is expressed as an endogenous reaction function of expected future output gap, current domestic inflation and real exchange rate, therefore, an interest-rate rule which is corresponds to the optimal condition (4.55) can be written as

$$\begin{aligned} i_t = & \left[\frac{1}{\beta} + \frac{\Phi_\pi'}{\Phi_y'}\left(\frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right) + \frac{\kappa_a}{\beta}\left(\frac{\alpha+\varphi}{1-\alpha} + \frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right)\right]\pi_{H,t} \\ & + \left[\frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right]\left(y_{t+1} - y_{t+1}^T\right) \\ & + \frac{\Phi_q'}{\Phi_y'}\left[\frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)} + \frac{\kappa_a}{\beta}\left(\frac{\alpha+\varphi}{1-\alpha} + \frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right)\right]\left(q_t - q_t^T\right) \end{aligned} \tag{4.56}$$

Where $y_{t+1}^T = y_{t+1}^n$, meaning target output equals natural output

Svensson (2003) states that as the optimal policy is too complex to be practicable, the calculation is complex and the weights of structural parameters are complex functions.

Even if a small error in the presented optimal rule, it will lead to a large variance on welfare. In addition, De Paoli (2009) mentions that the policy targets in the optimal policy are difficult to monitor. Therefore, we will henceforth consider scenarios in which monetary authorities are bound to implement alternative (i.e., suboptimal) interest rate rules.

Alternative Rules / 1: Discretionary Rule

Since the natural level of output is difficult to observe, Central Banks may decide to implement a rule that takes exactly the same form as the optimal rule but in which the natural level of output is replaced by a discretionary target output level. Such policy rules are sometimes called "discretionary rule" in the literature. In the present model, discretionary and optimal rules take the same functional form but differ in the specification of the output gap that is relevant for the policymaker.

Formally, the discretionary rule reads

$$\begin{aligned}\tilde{i}_t = &\left[\frac{1}{\beta} + \frac{\Phi'_\pi}{\Phi'_y}\left(\frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right) + \frac{\kappa_a}{\beta}\left(\frac{\alpha+\varphi}{1-\alpha} + \frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right)\right]\pi_{H,t} \\ &+ \left[\frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right]\left(y_{t+1} - y^T_{t+1}\right) \\ &+ \frac{\Phi'_q}{\Phi'_y}\left[\frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)} + \frac{\kappa_a}{\beta}\left(\frac{\alpha+\varphi}{1-\alpha} + \frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right)\right]\left(q_t - q^T_t\right) \quad (4.57)\end{aligned}$$

Where $y^T_{t+1} \neq y^n_{t+1}$, representing the target output chosen by the Central Bank. In the next section, we will exploit expression (4.57) to calculate welfare gaps between optimal and discretionary policies.

Alternative Rules / 2: Simple Interest Rate Rules

Berger (2008) believes simple policy could be considered as it could give a guideline to policy-makers to assess and prepare policy decisions. In order to evaluate welfare performances of simple policy rules in a small open economy, we consider two different types of policy rules, the Taylor rule, proposed by John Taylor in 1993, and the Taylor rule with real exchange rate, introduced by Laurence Ball in 1999. Also, two types of inflation targeting have been considered, domestic inflation-targeting and CPI inflation-targeting. The specification of the simple policy rules are summarised in Table 4.1. Specifically, domestic inflation-targeting Taylor rule (D. Taylor Rule) takes the form

$$i_t = h_1 \pi_{H,t} + b_1 \left(y_t - y_t^T \right) \tag{4.58}$$

where h_1 and b_1 are coefficients under the "original" Taylor rule

While CPI inflation-targeting Taylor Rule (CPI Taylor Rule) is

$$i_t = h_1 \pi_t + b_1 \left(y_t - y_t^T \right) \tag{4.59}$$

Ball (1999) modified the Taylor rule by adding a real exchange rate and its lagged term. Therefore, following the method developed by Ball (1999), if the domestic Central Bank sets nominal interest rate in response to domestic inflation, the rule (D. Taylor rule with q) follows

$$i_t = h_2 \pi_{H,t} + b_2 \left(y_t - y_t^T \right) + c q_t + d q_{t-1} \tag{4.60}$$

where parameters h_2, b_2, c and d are coefficients under the Taylor-type rule with real exchange rate.

Instead, if the Central Bank reacts to CPI inflation, the CPI Taylor Rule with exchange rate takes the form

$$i_t = h_2 \pi_t + b_2 \left(y_t - y_t^T \right) + c q_t + d q_{t-1} \tag{4.61}$$

Table 4.1: Simple policy rules

	Target Inflation	Monetary Rule
D. Taylor Rule	$\pi_{H,t}$	$i_t = h_1\pi_{H,t} + b_1\left(y_t - y_t^T\right)$
D. Taylor Rule with q	$\pi_{H,t}$	$i_t = h_2\pi_{H,t} + b_2\left(y_t - y_t^T\right) + cq_t + dq_{t-1}$
CPI Taylor Rule	π_t	$i_t = h_1\pi_t + b_1\left(y_t - y_t^T\right)$
CPI Taylor Rule with q	π_t	$i_t = h_2\pi_t + b_2\left(y_t - y_t^T\right) + cq_t + dq_{t-1}$

Notes: The coefficients h_1, b_1, h_2, b_2, c and d are not set optimally in the simulations. The method for setting values for these parameters is empirical and will be discussed in the following section.

4.5 Simulation Results

4.5.1 Simulation Approach

In a small open economy model, the size of Home country limits to zero, therefore, the economic movement in Foreign country could be solved independently, which represents the rest of the world. Therefore, following Haberis and Lipinska (2012), we solve the dynamics in the Foreign country, then simulate dynamic movements of the endogenous movements in the small open economy, feeding the relevant Foreign variables as exogenous variables.

The small open economy model can be summarised by the linearised dynamic equation, the New Keynesian Phillips curve (4.44), the dynamic IS (4.45) curve and the risk sharing equation (4.29). In order to transfer the model into a readable Matlab language, it is useful to use the method developed by Blanchard and Kahn (1980) and Blake and Fernandez-Corugedo (2010), putting the model into state-space formulation which helps to solve and program the simulations.

The model can be written in a state-space form:

$$EX_t = AX_{t-1} + GU_t \tag{4.62}$$

where $X_t = \begin{bmatrix} q_t & \pi_t & i_t & y_{t+1} & \pi_{H,t+1} \end{bmatrix}'$ and $U_t = \begin{bmatrix} y^*_{t+1} & y^*_t & u_t & 0 & 0 \end{bmatrix}'$, where u_t represents an exogenous term in the interest rate rule. Also E, A and G are 5×5 matrices which are formed by the structural parameters in the model, where matrices E and A depends on the type of interest rate rule used in the model.

4.5.2 Calibration

Table 4.2 summarises the parameters that are calibrated to the simulation of the model. Following the literature, the discount factor β is set to 0.99, which implies the annual real interest rate is 4%. The intertemporal elasticity of substitution is equal to 0.66, in line with Smets and Wouters (2007), Gertler et al. (2008), Faccini et al. (2011) and Harrison and Oomen (2010). The elasticity of labour α is set to 0.31, which is the observed labour share during 1971 and 2009 in the UK (Faccini et al., 2011). The Calvo parameter on prices θ is set to 0.5, which indicates that an average prices duration is about six months, following the results from Bunn and Ellis (2011) for the UK economy. Elasticity of substitution between different goods, is set to 11, as shown in Britton et al. (2000).

Following Harrison and Oomen (2010), we assume the elasticity of labour supply (φ) is equal to 0.43. The value of elasticity of substitution between home and foreign goods is taken from Harrison and Oomen (2010) and Mumtaz and Theodoridis (2015), which is 1.77. The openness of degree, λ, is assumed to be 0.3, implying a 30% import share of the GDP in the UK in 2014.

We assume the Taylor rule used by Foreign Central Bank (h, b) takes the weights suggested by Taylor (1993). And as Nelson (2001) found that the UK nominal interest rate in response to the inflation and output gap during 1992 and 1997, are remarkably close to the suggestion by John Taylor in 1993, the coefficients of the "original" Taylor rule will apply to the basic Taylor rule in the Home country and the Taylor rule in the Foreign country. Coefficients in the Taylor rule with exchange rate (Ball's rule), h_2, b_2, c and d take value of 4.8420, -0.0296, -0.1104 and -0.0377 respectively, which is in line with

Batini et al. (2003). It indicates that if there is an appreciation of real exchange rate by 10%, leads a cut in the interest rate by 1.104%, which is further appreciated by 0.377%, meaning that the interest rate will drop by 1.481% in the long run.

Furthermore, in order to evaluate the performances of the discretionary rule when a Central Bank pushes the target output above or below the natural output level, we assume the Central Bank will push the output target by 2% in either direction.

Table 4.2: Parameter values

Parameters	Value	Description	Source
β	0.99	Discount factor	Assumption
σ	0.66	Intertemporal elasticity of substitution	Smets and Wouters (2007)
φ	0.43	Elasticity of labour supply	Harrison and Oomen (2010)
α	0.31	Elasticity of labour	Faccini et al. (2011)
ε	11	Elasticity of substitution	Britton et al. (2000)
θ	0.5	Index of price stickiness	Bunn and Ellis (2011)
λ	0.3	Degree of openness	World Bank
v	0.7	Home bias	$v = 1 - \lambda$
η	1.77	Intratemporal elasticity of substitution	Mumtaz and Theodoridis (2015)
h	1.5	Inflation coefficient in Foreign Taylor rule	Taylor (1993)
b	0.5	Output gap coefficient in Foreign Taylor rule	Taylor (1993)
h_1	1.5	Inflation coefficient in the Taylor rule	Nelson (2001)
b_1	0.5	Output gap coefficient in the Taylor rule	Nelson (2001)
h_2	4.8420	Inflation coefficient	Batini et al. (2003)
b_2	-0.0296	Output gap coefficient	Batini et al. (2003)
c	-0.1104	Real exchange rate coefficient	Batini et al. (2003)
d	-0.0377	Lagged real exchange rate coefficient	Batini et al. (2003)

4.5.3 Rules Comparison: Discretionary Rules with Persistent Output Gaps

Benchmark

This section shows the impulse responses of Home economy in response to a negative demand shock to the Foreign economy at period 5 under different rules. At the beginning, the Home and Foreign economy are both in their equilibrium situation. Before analysing the response of a small open economy to a Foreign shock, we describe the behaviour of the Foreign economy when hit by a demand shock.

Under the assumptions of the model, the Foreign economy represents the rest of the world, which behaves like a closed economy. So any change in the Home variables will not affected the Foreign inflation, output and nominal interest rate. Assuming the Foreign Central Bank adopts a simple Taylor rule as monetary policy, Figure 4.1 shows the impulse responses of the Foreign inflation, output and nominal interest rate following a large negative demand shock. When a Foreign demand shock hits, the Foreign price and aggregate demand fall, and the Central Bank reacts to the shock by changing the nominal interest rate. When the nominal interest rate drops, the real interest rate falls, stimulating aggregate demand. Then the rising aggregate demand starts to affect inflation through the Phillips curve, and inflation rises.

Figure 4.1: Foreign economy follows a negative demand shock

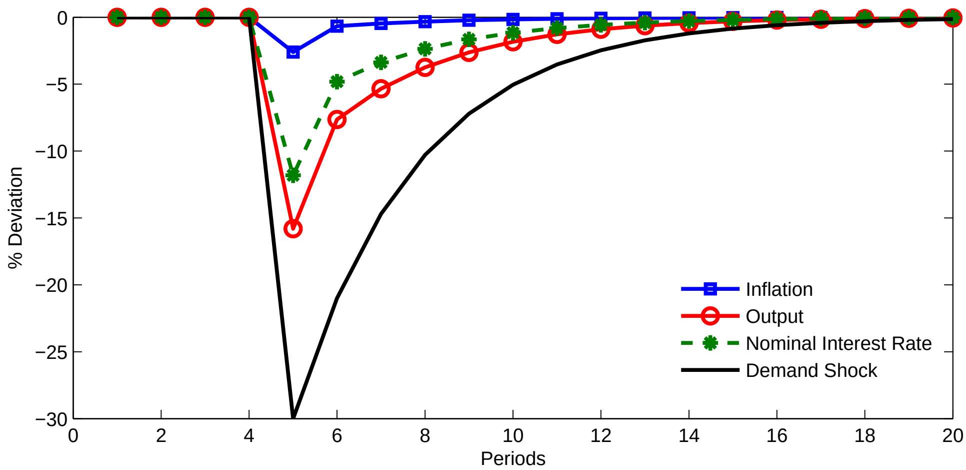


Figure 4.2 shows the behaviour of the Home domestic inflation, output, the nominal interest rate, the real exchange rate and the CPI inflation in response to a Foreign demand shock under the optimal rule, domestic Taylor rule and the discretionary rules. Under the optimal policy, when a negative demand shock hits the Foreign economy, the Home aggregate output is pushed down as the Foreign shock reduces the demand for Home-produced goods. However, as the Home and Foreign goods are substitutes, the reduction in Foreign output increases the marginal utility of Home goods, therefore, the

Home consumption tends to rise. Under the optimal rule, the depreciation of the real exchange rate causes consumption switching towards Home goods, pushing output back to equilibrium. A positive output gap appears when using the domestic Taylor rule due to the depreciation of the real exchange rate. The real exchange rate experiences an appreciation after it depreciates initially, which enables Home consumers to enjoy more consumption at a given labour supply. Therefore, consumers would like to decrease their labour supply, which causes the real wages to rise, and consequently domestic inflation rises.

Comparing the two discretionary policy rules, the rule with a higher discretionary output target outperforms the one with a lower target output. Under the assumption of the benchmark, the substitutability between the Home and Foreign products are low, so the Home economy will benefit less from the improvement of terms of trade, which is the appreciation of the real exchange rate. When the Central Bank pushes the output target below the optimal level, it causes the Home real exchange rate to appreciate more, leading Foreign products are relatively cheaper for Home consumers. So Home consumption switches towards to the Foreign goods, and as a result, Home output drops. Although the discretionary rule with a lower output target generates lower inflation, the welfare loss increases due to the more appreciated real exchange rate and the decreased output.

Figure 4.2: Home economy in response to a foreign shock

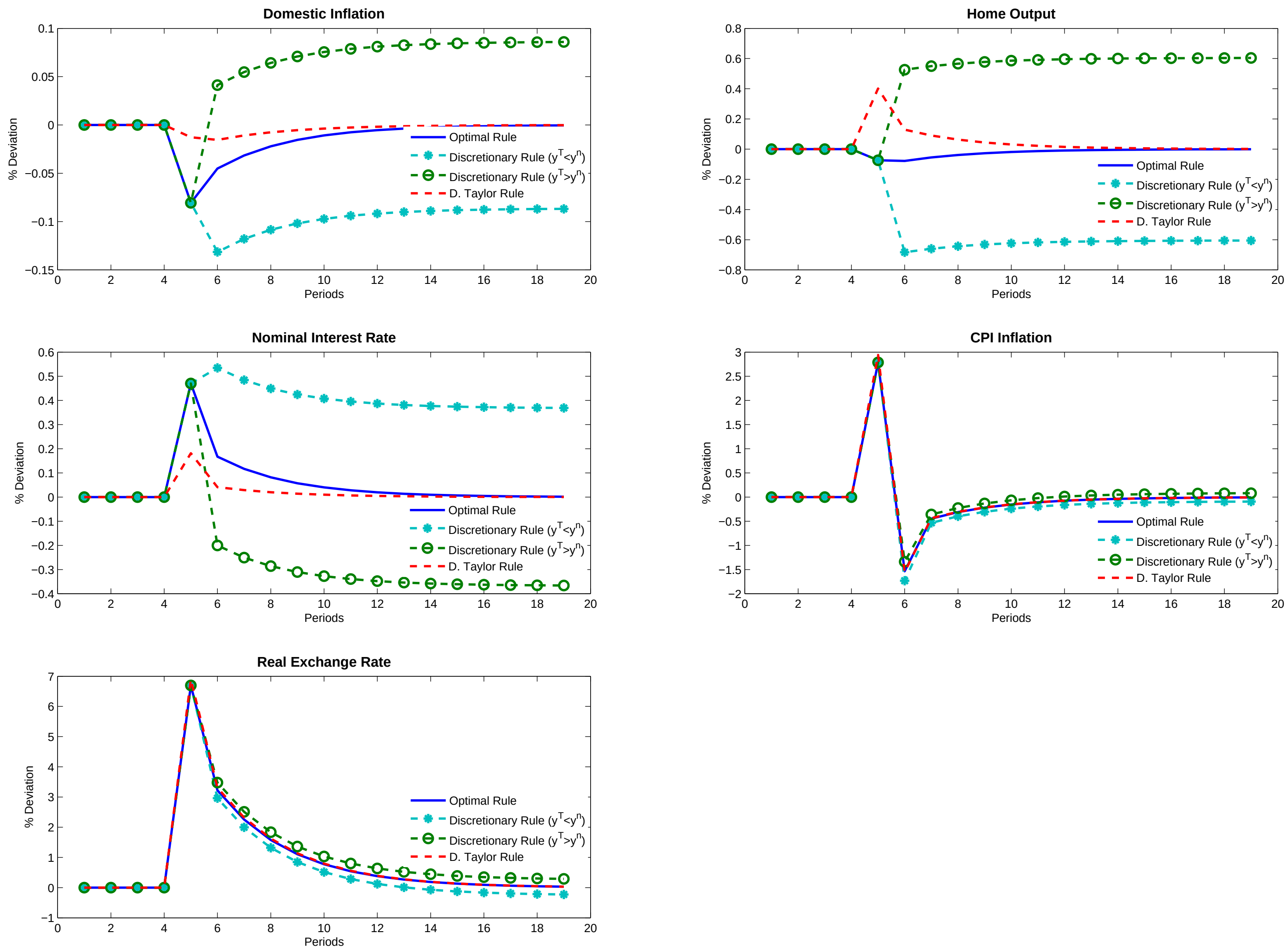


Table 4.3 reports the standard deviations of key variables in the Home economy. Like Figure 4.2, Table 4.3 confirms that the domestic Taylor rule shows the lowest welfare loss among alternative (non optimal) rules, which allows fluctuation in the real exchange rate but focuses on controlling domestic inflation and output. The domestic Taylor rule with exchange rate gives the highest volatility of real exchange rate, however, it is the second-best rule among different simple rules. Comparing the two discretionary rules, the rule with a higher discretionary target output is a preferred rule, the less appreciated real exchange rate causes less volatility on output which outweighs more fluctuation on Home inflation. Furthermore, CPI inflation is more stable under policy rules which uses CPI inflation as a target variable than other policy rules. Under the calibration of the benchmark, the Home economy is relatively closed, therefore, improving the terms of trade contributes less to the Home welfare as the Home products are preferred by consumers. Consistently, although volatility of real exchange rate is low under the CPI inflation-targeting rules, they show the largest volatility on domestic inflation, the output and the nominal interest rate, which in turn have the lowest welfare. It might indicate that when the intratemporal elasticity of substitution is low ($\eta = 1.77$), Home welfare gains from stabilising domestic prices rather than improving the terms of trade.

Table 4.3: Standard deviations of the Home variables

	$s.d.(\pi_{H,t})$	$s.d.(y_t)$	$s.d.(i_t)$	$s.d.(\pi_t)$	$s.d.(q_t)$	Welfare Loss
Optimal Rule	0.0207	0.0254	0.1107	0.7643	1.6263	0.8611^{***}
Discretionary Rule ($y^T < y^n$)	0.0420	0.2764	0.1781	0.7953	1.6769	1.1545
Discretionary Rule ($y^T > y^n$)	0.0463	0.2730	0.2193	0.7353	1.6402	1.1352
D. Taylor Rule	0.0048	0.0936	0.0415	0.7974	1.7035	0.8993^{**}
D. Taylor Rule with q	0.0415	0.3559	0.0716	0.8524	1.8046	1.1399
CPI Taylor Rule	0.1610	0.3277	0.7672	0.6525	1.6444	1.9723
CPI Taylor Rule with q	0.2617	0.6522	1.7675	0.3845	1.2653	3.5808

Notes: The superscripts *** shows the lowest welfare loss which indicates the corresponding policy rule is the optimal rule, and ** shows the second-lowest welfare loss and the corresponding rule is the first-best policy rule among alternative rules.

Evaluating the Different Interest Rate Rules

In order to evaluate the performance of different policy rules more generally, as well as which measure of inflation Central Banks should target, some sensitivity analysis will be conducted. Rules comparisons will be done to evaluate the performance of discretionary rules compared with other simple policy rules for different parameter values.

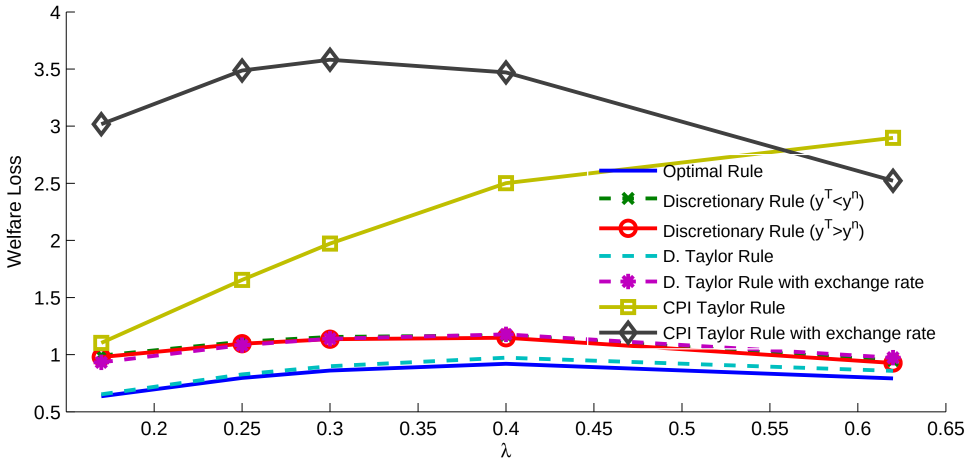
We compute welfare of different policy rules, specifically, the optimal rule, discretionary rules, domestic Taylor rule, domestic Taylor rule with exchange rate, CPI Taylor rule and CPI Taylor rule with exchange rate. We start by varying the degree of openness, λ, while keep the elasticity of substitution between home and foreign goods fixed, $\eta = 1.77$. Alternatively, we vary η, while fixing $\lambda = 0.3$. And finally, we vary η and λ at the same time, in order to analyse different scenarios.

Following the literature, the value of λ starts from 0.17, which in line with Matheson (2010), who uses it to evaluate the degree of openness in Australia. Then λ increases to 0.25, 0.3 and 0.4, which are estimates of the degree of openness in the Euro Zone (Adolfson et al., 2007), the UK (World Bank) and Canada (Gali and Monacelli, 2005) respectively. The highest value of λ is 0.62, indicating the import share of the GDP was 62% in Thailand in 2014 (World Bank). Figure 4.3 shows the welfare performances of different rules, varying the value of parameter λ. In general, the deviation from the optimal allocation is relatively small if the elasticity of substitution between home and foreign goods is low and if the Central Bank uses domestic inflation as target variable. Specifically, rules that target domestic inflation outperform any CPI inflation-targeting regime. The lowest welfare loss is given by the Central Bank using a simple Taylor rule and targeting domestic inflation, for the whole selected range of λ.

Table 4.4 reports that the volatility of the real exchange rate is decreasing in the degree of openness λ, which is consistent with the results found by Faia and Monacelli (2008).

When the Home economy is relatively closed (lower value of λ), consumers prefer the Home-produced goods, causing the Home country to benefit from stabilising domestic inflation rather than improving the terms of trade. So, overstabilising domestic inflation while allowing fluctuations in the real exchange rate could improve welfare performance. As a result, the domestic Taylor rule shows the best performance and the domestic Taylor rule with exchange rate is the second-best simple policy when λ is small. However, when λ becomes bigger, the Foreign products are preferred by Home households, but the effect of terms of trade is reduced because the intratemporal substitution of elasticity is low. Therefore, the discretionary policy with a higher output target shows a better welfare, as it generates a less appreciated real exchange rate and induces a less deviation of output, which dominates the losses caused by higher inflation. It might indicate that when the Home economy is relatively open and the intratemporal elasticity of substitution is low, the economy benefits from adopting a more expansionary monetary policy (by setting a higher output target). However, the welfare gains from adopting an expansionary discretionary policy rule are not significant, as it has a similar welfare volatility with the discretionary rule with lower output target and the domestic Taylor rule with exchange rate, as shown in Figure 4.3 and Table 4.4.

Figure 4.3: Welfare performance of policy rules, varying the degree of openness



The welfare performances of alternative rules could be evaluated by measuring the deviation of welfare under the alternative rules from welfare under the optimal rule (Berger, 2008). To be precise, for example, the welfare of the domestic Taylor rule with respect to the optimal rule is measured as $\frac{W^{D.Taylorrule}-W^{optimalrule}}{W^{optimalrule}}$. Figure 4.4 shows the welfare performance of each alternative policy rules relative to the welfare under the optimal rule, varying the degree of openness.

Figure 4.4: Alternative rules compared to the optimal rule, varying the degree of openness

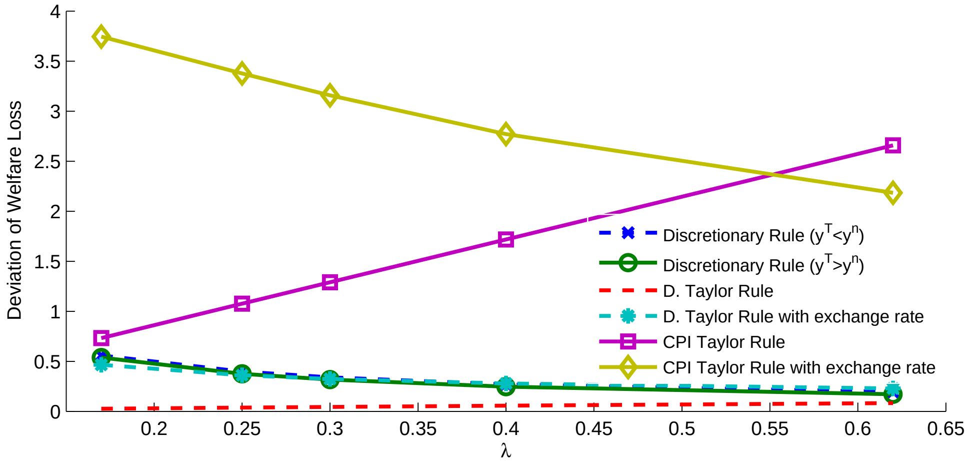


Table 4.4: **Standard deviations of the Home variables, varying the degree of openness**

	$s.d.(\pi_{H,t})$	$s.d.(y_t)$	$s.d.(i_t)$	$s.d.(\pi_t)$	$s.d.(q_t)$	Welfare Loss
$\lambda = 0.17$						
Optimal Rule	0.0125	0.0127	0.0677	0.4477	2.0305	0.6360^{***}
Discretionary Rule ($y^T < y^n$)	0.0405	0.2789	0.1717	0.4747	2.0581	0.9918
Discretionary Rule ($y^T > y^n$)	0.0433	0.2756	0.2001	0.4238	2.0129	0.9785
D. Taylor Rule	0.0031	0.0579	0.0256	0.4615	2.0658	0.6527^{**}
D. Taylor Rule with q	0.0485	0.3433	0.0901	0.5037	2.1943	0.9324^{*}
CPI Taylor Rule	0.0935	0.2130	0.4783	0.4046	2.0252	1.1021
CPI Taylor Rule with q	0.2208	0.2737	1.3208	0.3061	1.7732	3.0175
$\lambda = 0.25$						
Optimal Rule	0.0177	0.0202	0.0950	0.6447	1.7952	0.7966^{***}
Discretionary Rule ($y^T < y^n$)	0.0412	0.2773	0.1743	0.6741	1.8195	1.1131
Discretionary Rule ($y^T > y^n$)	0.0451	0.2738	0.2115	0.6175	1.7796	1.0961
D. Taylor Rule	0.0042	0.0807	0.0357	0.6696	1.8401	0.8264^{**}
D. Taylor Rule with q	0.0442	0.3522	0.0785	0.7199	1.9494	1.0838^{*}
CPI Taylor Rule	0.1355	0.2890	0.6625	0.5623	1.7870	1.6543
CPI Taylor Rule with q	0.2504	0.5008	1.6291	0.3607	1.4412	3.4866
$\lambda = 0.3$						
Optimal Rule	0.0207	0.0254	0.1107	0.7643	1.6263	0.8611^{***}
Discretionary Rule ($y^T < y^n$)	0.0420	0.2764	0.1781	0.7953	1.6769	1.1545
Discretionary Rule ($y^T > y^n$)	0.0463	0.2730	0.2193	0.7353	1.6402	1.1352^{*}
D. Taylor Rule	0.0048	0.0936	0.0415	0.7974	1.7035	0.8993^{**}
D. Taylor Rule with q	0.0415	0.3559	0.0716	0.8524	1.8046	1.1399
CPI Taylor Rule	0.1610	0.3277	0.7672	0.6525	1.6444	1.9723
CPI Taylor Rule with q	0.2617	0.6522	1.7675	0.3845	1.2653	3.5808
$\lambda = 0.4$						
Optimal Rule	0.0263	0.0370	0.1396	0.9967	1.3861	0.9204^{***}
Discretionary Rule ($y^T < y^n$)	0.0439	0.2748	0.1887	1.0310	1.4047	1.1687
Discretionary Rule ($y^T > y^n$)	0.0491	0.2723	0.2357	0.9643	1.3741	1.1474^{*}
D. Taylor Rule	0.0059	0.1163	0.0517	1.0488	1.4393	0.9729^{**}
D. Taylor Rule with q	0.0364	0.3592	0.0591	1.1122	1.5166	1.1768
CPI Taylor Rule	0.2105	0.3928	0.9560	0.8168	1.3723	2.5008
CPI Taylor Rule with q	0.2744	0.9317	1.9660	0.4174	0.9702	3.4705
$\lambda = 0.62$						
Optimal Rule	0.0370	0.0685	0.1922	1.4859	0.8431	0.7923^{***}
Discretionary Rule ($y^T < y^n$)	0.0492	0.2732	0.2183	1.5270	0.8543	0.9476
Discretionary Rule ($y^T > y^n$)	0.0557	0.2764	0.2709	1.4464	0.8358	0.9276^{*}
D. Taylor Rule	0.0075	0.1530	0.0683	1.5903	0.8906	0.8575^{**}
D. Taylor Rule with q	0.0255	0.3482	0.0373	1.6654	0.9299	0.9737
CPI Taylor Rule	0.3126	0.4999	1.2973	1.1194	0.8249	2.8983
CPI Taylor Rule with q	0.2814	1.4174	2.2067	0.4542	0.5015	2.5222

Notes: The superscripts *** shows the lowest welfare loss which indicates the corresponding policy rule is the optimal rule; ** shows the second-lowest welfare loss and the corresponding rule is the first-best policy rule among alternative rules; * indicates the corresponding rule is the second-best policy rule among alternative rules.

Alternatively, instead of varying λ, Figure 4.5 and Table 4.5 summarise the welfare losses of different rules varying η. The range of the value of the elasticity of substitution between home and foreign goods is ambiguous in the literature. Obstfeld and Rogoff (2001) believe η is in the range between 3 and 6, while Anderson and Van Wincoop (2004) think it could be between 5 and 10, while it is considered in the range between 1 and 6 by Berger (2008). Here, we consider the value of 1.77 which is the estimate of the intratemporal elasticity of substitution in the UK (Mumtaz and Theodoridis, 2015), and the range between 3 and 6.

The volatility of the real exchange rate affects the terms of trade, which in turn induces a change in international demand and consumption between Home and Foreign goods in the Home country. As the effects of the terms of trade is reduced when the intratemporal elasticity of substitution is low, welfare improves when the Central Bank concentrates on stabilising domestic inflation and output when η is small. Therefore, as shown in Table 4.5 along with Figure 4.5, the domestic Taylor rule is the best alternative when intratemporal elasticity of substitution is relatively small. Instead, policies which target CPI inflation perform worst, which give the highest welfare loss compared with other policy rules, when the domestic and foreign products are not good substitutes.

Because of the terms of trade externality, an improvement of terms of trade can raise domestic welfare when the Home and Foreign goods are close substitutes. For intermediate and high degrees of substitutability between Home and Foreign goods, the discretionary policy with a lower output target becomes the best alternative rule. By implementing a discretionary rule with an output target below the natural output level, the Central Bank could improve the terms of trade via the more appreciated real exchange rate, which means the households benefits from consuming more imported goods without increasing labour supply.

Due to the expenditure switching effect, the effectiveness of terms of trade is increasing with the level of the substitutability of Home and Foreign goods, an appreciation of the real exchange rate can improve domestic welfare. Also, we found that the more appreciated real exchange rate is normally associated with a less volatile real exchange rate. So, stabilising the real exchange rate becomes more desirable when the intratemporal elasticity of substitution is high. In general, the results show that welfare losses under the alternative rules are increasing, while the welfare loss in the CPI Taylor rule with exchange rate is decreasing with the increasing value of η. To be precise, the domestic Taylor rule with exchange rate shows the the highest volatility of the real exchange rate, the welfare loss associated with this rule increases significantly with η. Also, when η is high enough, policies which use the CPI inflation as a target variable become the best policy rules if the Central Bank chooses from the alternative simple rules, indicating that targeting CPI inflation could be welfare improving when Home and Foreign goods are close substitutes.

In addition, comparing two discretionary rules, the performances of the two rules is determined by the elasticity of substitution between home and foreign goods. As ability of terms of trade is affected by the degree of substitutability of Home and Foreign products. When the elasticity is low, the effects of terms of trade is limited. Instead, the Home country would benefit from an appreciation of the real exchange rate if the elasticity is high. Figure 4.5, Figure 4.6 and Table 4.5 show when the intratemporal elasticity of substitution is low, the rule with a higher discretionary output target shows a lower welfare loss, as more output has been generated by a less appreciated real exchange rate. However, with increasing η, the improvement of terms of trade is welfare-improving, so the rule with a lower output target outperforms the rule with higher target. That is, when the Home demand is highly elastic to the Foreign products, the Home demand is more sensitive to the change of prices, a more appreciated real exchange rate can improve the terms of trade, which in turn rises the domestic welfare.

Figure 4.5: Welfare performance of policy rules, varying the elasticity of substitution between home and foreign goods

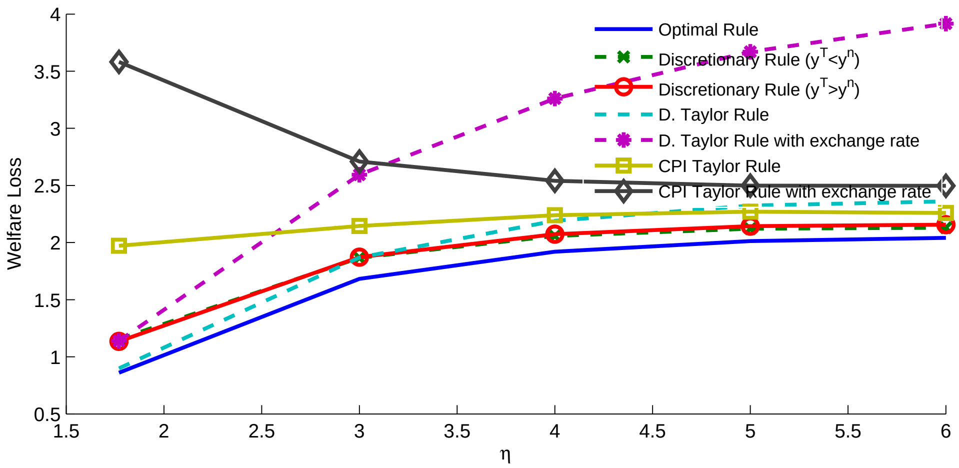


Figure 4.6 shows the welfare performance of each alternative policy rules relative to the welfare under the optimal rule, varying the intratemporal elasticity of substitution.

Figure 4.6: Alternative rules compared to the optimal rule, varying the elasticity of substitution between home and foreign goods

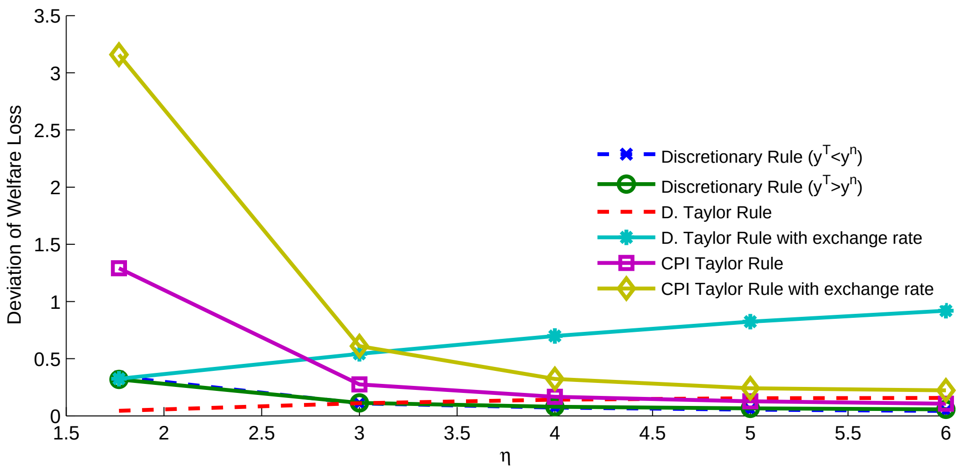


Table 4.5: Standard deviations of the Home variables, varying the elasticity of substitution of home and foreign goods

	$s.d.(\pi_{H,t})$	$s.d.(y_t)$	$s.d.(i_t)$	$s.d.(\pi_t)$	$s.d.(q_t)$	Welfare Loss
$\eta = 1.77$						
Optimal Rule	0.0207	0.0254	0.1107	0.7643	1.6263	0.8611^{***}
Discretionary Rule ($y^T < y^n$)	0.0420	0.2764	0.1781	0.7953	1.6769	1.1545
Discretionary Rule ($y^T > y^n$)	0.0463	0.2730	0.2193	0.7353	1.6402	1.1352^{*}
D. Taylor Rule	0.0048	0.0936	0.0415	0.7974	1.7035	0.8993^{**}
D. Taylor Rule with q	0.0415	0.3559	0.0716	0.8524	1.8046	1.1399
CPI Taylor Rule	0.1610	0.3277	0.7672	0.6525	1.6444	1.9723
CPI Taylor Rule with q	0.2617	0.6522	1.7675	0.3845	1.2653	3.5808
$\eta = 3$						
Optimal Rule	0.0448	0.2337	0.3115	0.5856	1.2745	1.6828^{***}
Discretionary Rule ($y^T < y^n$)	0.0541	0.3956	0.3031	0.6110	1.2901	1.8669^{**}
Discretionary Rule ($y^T > y^n$)	0.0578	0.2979	0.3585	0.5620	1.2639	1.8725
D. Taylor Rule	0.0182	0.4605	0.2120	0.6375	1.3447	1.8692^{*}
D. Taylor Rule with q	0.0569	0.9495	0.0818	0.7309	1.4858	2.5945
CPI Taylor Rule	0.1117	0.6144	0.7352	0.5187	1.2870	2.1446
CPI Taylor Rule with q	0.2292	0.3584	1.5100	0.3291	1.0448	2.7093
$\eta = 4$						
Optimal Rule	0.0535	0.4039	0.4210	0.4985	1.0829	1.9196^{***}
Discretionary Rule ($y^T < y^n$)	0.0604	0.5294	0.4071	0.5207	1.0952	2.0583^{**}
Discretionary Rule ($y^T > y^n$)	0.0628	0.4138	0.4527	0.4778	1.0742	2.0733^{*}
D. Taylor Rule	0.0232	0.6844	0.3207	0.5524	1.1539	2.1899
D. Taylor Rule with q	0.0712	1.3404	0.1456	0.6657	1.3113	3.2613
CPI Taylor Rule	0.0893	0.8024	0.7411	0.4495	1.1014	2.2391
CPI Taylor Rule with q	0.2173	0.4451	1.3938	0.3044	0.9267	2.5403
$\eta = 5$						
Optimal Rule	0.0572	0.5572	0.5018	0.4386	0.9472	2.0128^{***}
Discretionary Rule ($y^T < y^n$)	0.0632	0.6630	0.4879	0.4585	0.9572	2.1217^{**}
Discretionary Rule ($y^T > y^n$)	0.0642	0.5447	0.5255	0.4201	0.9398	2.1431^{*}
D. Taylor Rule	0.0261	0.8631	0.4093	0.4894	1.0127	2.3235
D. Taylor Rule with q	0.0849	1.6698	0.2129	0.6171	1.1780	3.6696
CPI Taylor Rule	0.0749	0.9561	0.7571	0.3991	0.9659	2.2701
CPI Taylor Rule with q	0.2115	0.6630	1.3183	0.2886	0.8382	2.4989
$\eta = 6$						
Optimal Rule	0.0579	0.6983	0.5641	0.3953	0.8456	2.0409^{***}
Discretionary Rule ($y^T < y^n$)	0.0637	0.7916	0.5514	0.4133	0.8540	2.1294^{**}
Discretionary Rule ($y^T > y^n$)	0.0634	0.6747	0.5829	0.3785	0.8393	2.1562^{*}
D. Taylor Rule	0.0280	1.0091	0.4828	0.4407	0.9035	2.3610
D. Taylor Rule with q	0.0974	1.9514	0.2758	0.5791	1.0717	3.9157
CPI Taylor Rule	0.0650	1.0837	0.7769	0.3604	0.8620	2.2586
CPI Taylor Rule with q	0.2090	0.8900	1.2665	0.2779	0.7685	2.4967

Notes: The superscripts *** shows the lowest welfare loss which indicates the corresponding policy rule is the optimal rule; ** shows the second-lowest welfare loss and the corresponding rule is the first-best policy rule among alternative rules; * indicates the corresponding rule is the second-best policy rule among alternative rules.

Table 4.6 shows the policy rule which is associated with the highest level of welfare under different scenarios, following a Foreign demand shock. With the increasing of the value of the elasticity of substitution between home and foreign goods, the elasticity of demand to the change of real exchange rate rises. The domestic welfare benefits from improving the terms of trade by appreciating the real exchange rate. Therefore, the discretionary rule with a lower target output becomes the best alternative choice when η is high, by inducing lower inflation, which compensates the losses of the output by the more appreciated real exchange rate. Instead, if the Home and Foreign goods are less substitutable, the expenditure switching effects caused by the terms of trade are diminishing, welfare gains from stabilising domestic inflation outweigh the gains from more appreciation of the real exchange rate. As a result, the domestic Taylor rule is the preferred simple policy rule.

With the intermediate value of intratemporal elasticity of substitution, the welfare performances of alternative rules depend on the degree of openness. When the Home economy is relatively closed (lower λ), Home consumers prefer the Home-produced goods, the welfare gains from stabilising domestic prices. As a result, the domestic Taylor rule leads to the highest welfare compared with other policies which put weight to controlling real exchange rate. On the other hand, when the Home country is more open, Foreign products are preferred, causing the improvement of the terms of trade to improve welfare, as the agents could consume more imported goods. So, the discretionary policy with a lower output target, which has lower domestic prices, and is associated with a more appreciated real exchange rate, shows the highest welfare.

Table 4.6: Preferred policy rule, varying the degree of openness and the intratemporal elasticity of substitution

η/λ	0.17	0.25	0.3	0.4	0.62
1.77	D. Taylor Rule	D. Taylor Rule	D. Taylor Rule	D. Taylor Rule	D. Taylor Rule
3	D. Taylor Rule	D. Taylor Rule	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$
4	D. Taylor Rule	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$
5	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$
6	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$	Dis. $\mathrm{R}^{y^T<y^n}$

Notes: Dis. $\mathrm{Rule}^{y^T<y^n}$ represents the discretionary rule with an output target is below the natural output level.

4.5.4 Rules Comparison: Discretionary Rules with One-period Output Gap

In order to evaluate the performances of the discretionary rules, we next assume a discretionary output target only being suboptimal for one period ($y^T_{t+1} \neq y^n_{t+1}$), which is chosen by the Home Central Bank.

Figure 4.7 and Table 4.7 show the impulse responses of key variables in the Home country under the discretionary policies with output target being suboptimal only for one period, compared with the optimal rule and the domestic Taylor rule. Although the domestic Taylor rule shows the highest welfare, the discretionary rules are the second-best policy rules. To be precise, the rule with a lower output target shows a lower welfare loss. The intuition for this result is that when the Central Bank implements the discretionary rule with a lower target output, the lower inflation compensates the losses generated by the lower output and the more appreciated real exchange rate. After the shock vanishes, the Central Bank moves the target output back to the optimal level, which is the natural output.

Figure 4.7: Home economy in response to a foreign shock

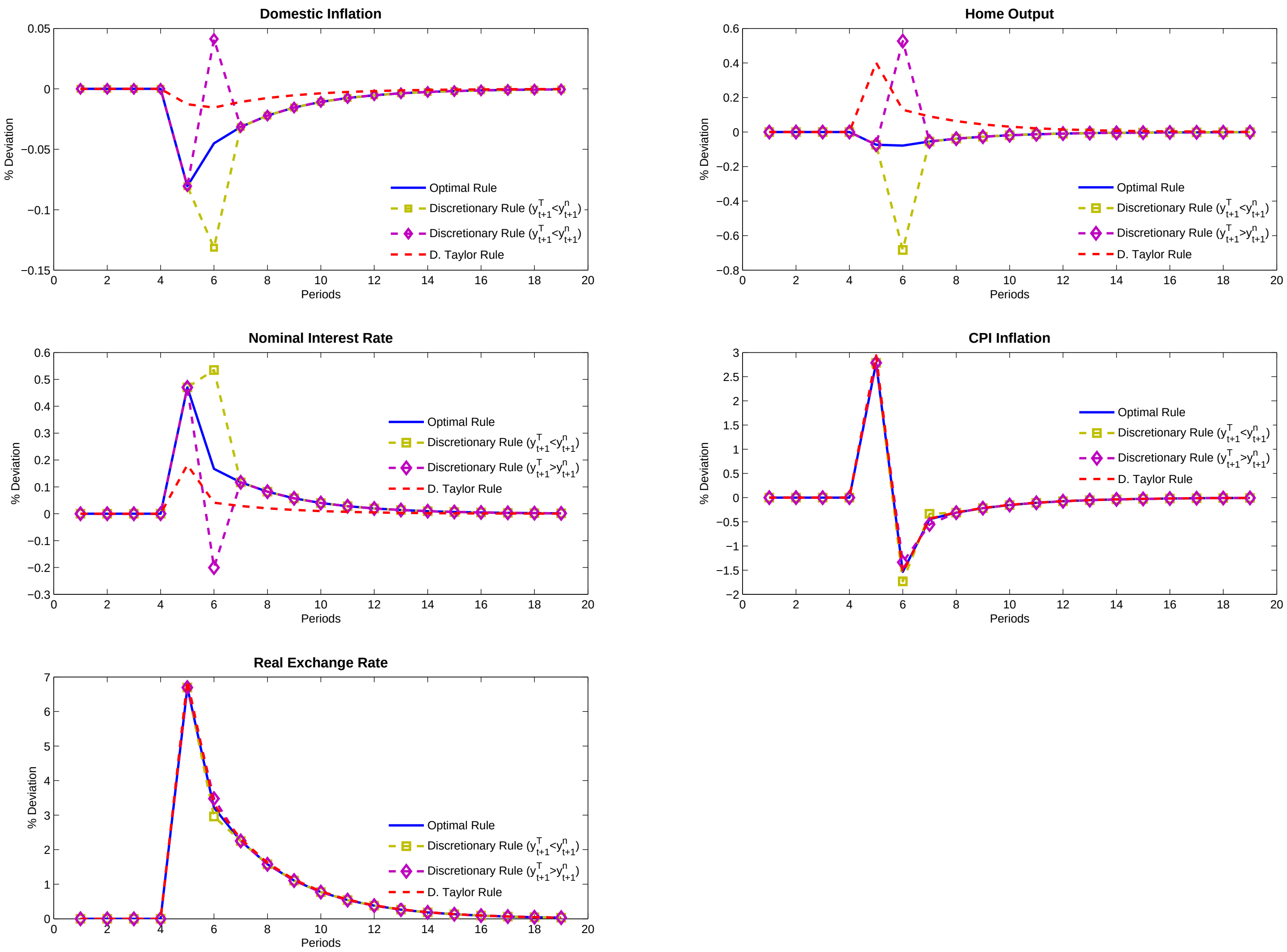


Table 4.7: Standard deviations of the Home variables

	$s.d.(\pi_{H,t})$	$s.d.(y_t)$	$s.d.(i_t)$	$s.d.(\pi_t)$	$s.d.(q_t)$	Welfare Loss
Optimal Rule	0.0207	0.0254	0.1107	0.7643	1.6263	0.8611^{***}
Discretionary Rule ($y^T_{t+1} < y^n_{t+1}$)	0.0337	0.1548	0.1549	0.7841	1.6356	0.9219
Discretionary Rule ($y^T_{t+1} > y^n_{t+1}$)	0.0224	0.1257	0.1214	0.7476	1.6754	0.9462
D. Taylor Rule	0.0048	0.0936	0.0415	0.7974	1.7035	0.8993^{**}
D. Taylor Rule with q	0.0415	0.3559	0.0716	0.8524	1.8046	1.1399
CPI Taylor Rule	0.1610	0.3277	0.7672	0.6525	1.6444	1.9723
CPI Taylor Rule with q	0.2617	0.6522	1.7675	0.3845	1.2653	3.5808

Notes: The superscripts *** shows the lowest welfare loss which indicates the corresponding policy rule is the optimal rule; ** shows the second-lowest welfare loss and the corresponding rule is the first-best policy rule among alternative rules.

Sensitivity analysis is used to analyse the welfare performances of policy rules under different value of the degree of openness and the elasticity of substitution between home and foreign goods. Figure 4.8 and Table 4.8 shows the welfare performances of the Home variables by varying the degree of openness, λ. When the Home economy is relatively closed, the Home consumers prefer Home goods, and stabilising domestic prices could be welfare-improving. As a result, the domestic Taylor rule is the best alternative when the degree of openness is small. As the ability of terms of trade is reduced because of the low elasticity of substitution between home and foreign goods, the Home country is less likely to benefit from Home currency appreciations. However, the appreciation of the real exchange rate is normally associated with low output and low inflation. Although the discretionary rule with a temporary lower output target generates a temporary more appreciated real exchange rate, the low inflation compensates the losses induced by the low output. Therefore, the discretionary rule with a lower output target outperforms other rules. However, the welfare gains are not significant as the welfare performances are quite similar among the domestic Taylor rule and the two discretionary rules.

Figure 4.8: Welfare performance of simple policy rules, varying degree of openness

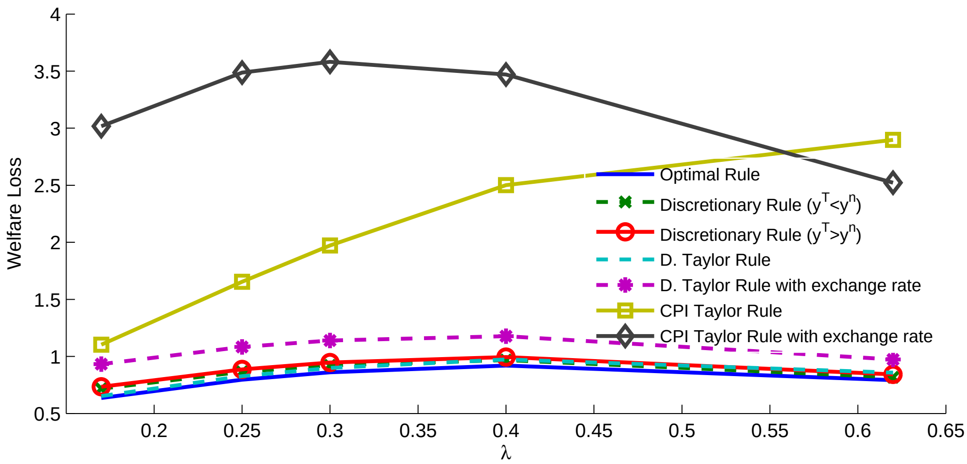


Figure 4.9: Alternative rules compared to the optimal rule, varying the degree of openness

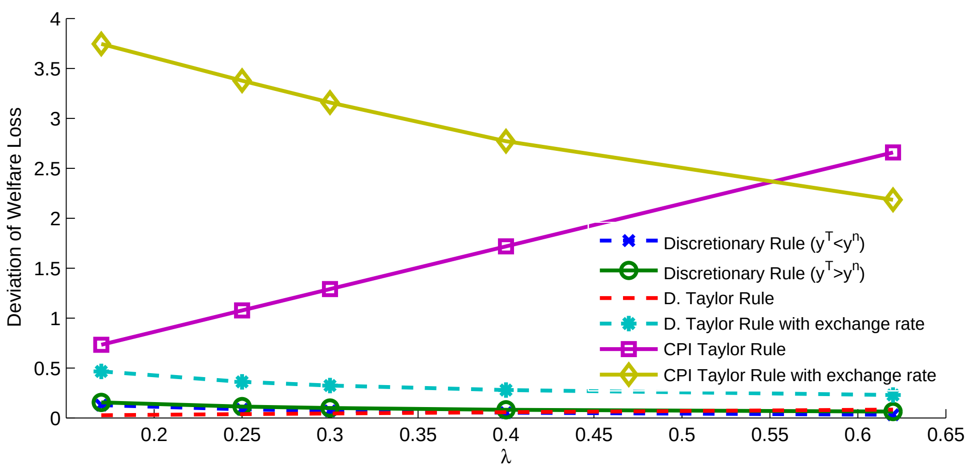


Table 4.8: Standard deviations of the Home variables, varying the degree of openness

	$s.d.(\pi_{H,t})$	$s.d.(y_t)$	$s.d.(i_t)$	$s.d.(\pi_t)$	$s.d.(q_t)$	Welfare Loss
$\lambda = 0.17$						
Optimal Rule	0.0125	0.0127	0.0677	0.4477	2.0305	0.6360***
Optimal Rule ($y^T_{t+1} < y^n_{t+1}$)	0.0277	0.1487	0.1236	0.4641	2.0070	0.7173*
Optimal Rule ($y^T_{t+1} > y^n_{t+1}$)	0.0193	0.1329	0.0970	0.4343	2.0564	0.7345
D. Taylor Rule	0.0031	0.0579	0.0256	0.4615	2.0658	0.6527**
D. Taylor Rule with q	0.0485	0.3433	0.0901	0.5037	2.1943	0.9324
CPI Taylor Rule	0.0935	0.2130	0.4783	0.4046	2.0252	1.1021
CPI Taylor Rule with q	0.2208	0.2737	1.3208	0.3061	1.7732	3.0175
$\lambda = 0.25$						
Optimal Rule	0.0177	0.0202	0.0950	0.6447	1.7952	0.7966***
Optimal Rule ($y^T_{t+1} < y^n_{t+1}$)	0.0314	0.1523	0.1427	0.6632	1.7745	0.8647*
Optimal Rule ($y^T_{t+1} > y^n_{t+1}$)	0.0209	0.1283	0.1111	0.6292	1.8179	0.8868
D. Taylor Rule	0.0042	0.0807	0.0357	0.6696	1.8401	0.8264**
D. Taylor Rule with q	0.0442	0.3522	0.0785	0.7199	1.9494	1.0838
CPI Taylor Rule	0.1355	0.2890	0.6625	0.5623	1.7870	1.6543
CPI Taylor Rule with q	0.2504	0.5008	1.6291	0.3607	1.4412	3.4866
$\lambda = 0.3$						
Optimal Rule	0.0207	0.0254	0.1107	0.7643	1.6263	0.8611***
Optimal Rule ($y^T_{t+1} < y^n_{t+1}$)	0.0337	0.1548	0.1549	0.7841	1.6356	0.9219*
Optimal Rule ($y^T_{t+1} > y^n_{t+1}$)	0.0224	0.1257	0.1214	0.7476	1.6754	0.9462
D. Taylor Rule	0.0048	0.0936	0.0415	0.7974	1.7035	0.8993**
D. Taylor Rule with q	0.0415	0.3559	0.0716	0.8524	1.8046	1.1399
CPI Taylor Rule	0.1610	0.3277	0.7672	0.6525	1.6444	1.9723
CPI Taylor Rule with q	0.2617	0.6522	1.7675	0.3845	1.2653	3.5808
$\lambda = 0.4$						
Optimal Rule	0.0263	0.0370	0.1396	0.9967	1.3861	0.9204***
Optimal Rule ($y^T_{t+1} < y^n_{t+1}$)	0.0383	0.1608	0.1786	1.0192	1.3703	0.9680**
Optimal Rule ($y^T_{t+1} > y^n_{t+1}$)	0.0260	0.1431	0.9778	1.4034	0.3431	0.9949
D. Taylor Rule	0.0059	0.1163	0.0517	1.0488	1.4393	0.9729*
D. Taylor Rule with q	0.0364	0.3592	0.0591	1.1122	1.5166	1.1768
CPI Taylor Rule	0.2105	0.3928	0.9560	0.8168	1.3723	2.5008
CPI Taylor Rule with q	0.2744	0.9317	1.9660	0.4174	0.9702	3.4705
$\lambda = 0.62$						
Optimal Rule	0.0370	0.0685	0.1922	1.4859	0.8431	0.7923***
Optimal Rule ($y^T_{t+1} < y^n_{t+1}$)	0.0475	0.1790	0.2250	1.5139	0.8336	0.8171**
Optimal Rule ($y^T_{t+1} > y^n_{t+1}$)	0.0345	0.1174	0.1881	1.4623	0.8535	0.8421*
D. Taylor Rule	0.0075	0.1530	0.0683	1.5903	0.8906	0.8575
D. Taylor Rule with q	0.0255	0.3482	0.0373	1.6654	0.9299	0.9737
CPI Taylor Rule	0.3126	0.4999	1.2973	1.1194	0.8249	2.8983
CPI Taylor Rule with q	0.2814	1.4174	2.2067	0.4542	0.5015	2.5222

Notes: The superscripts *** shows the lowest welfare loss which indicates the corresponding policy rule is the optimal rule; ** shows the second-lowest welfare loss and the corresponding rule is the first-best policy rule among alternative rules; * indicates the corresponding rule is the second-best policy rule among alternative rules.

Instead of changing λ, Figure 4.10 and Table 4.9 report the welfare performances of different policy rules from varying the elasticity of substitution between home and foreign goods. The effectiveness of terms of trade is increasing with the intratemporal elasticity of substitution. As a result, when the Home economy is relatively closed ($\eta = 0.3$), the domestic Taylor rule has the lowest welfare loss when the elasticity of substitution between home and foreign goods is low. When the elasticity starts to increase, the discretionary rule with a lower output target outperforms other rules. Generally, when Central Banks adopt the discretionary rules, lower volatility on domestic inflation and output are generated, so rules help to stabilise the domestic price level and unemployment, and improve domestic welfare.

Figure 4.10: Welfare performance of alternative policy rules, varying the elasticity of substitution between home and foreign goods

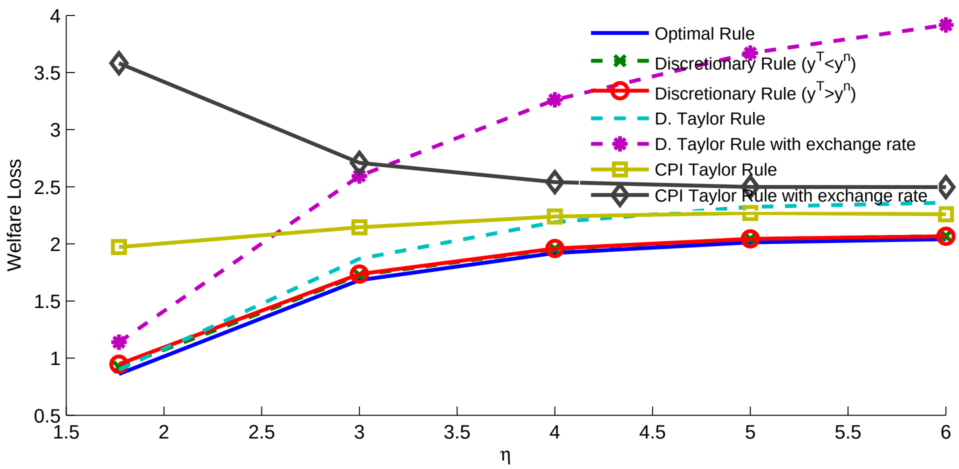


Figure 4.11: Alternative rules compared to the optimal rule, varying the elasticity of substitution between home and foreign goods

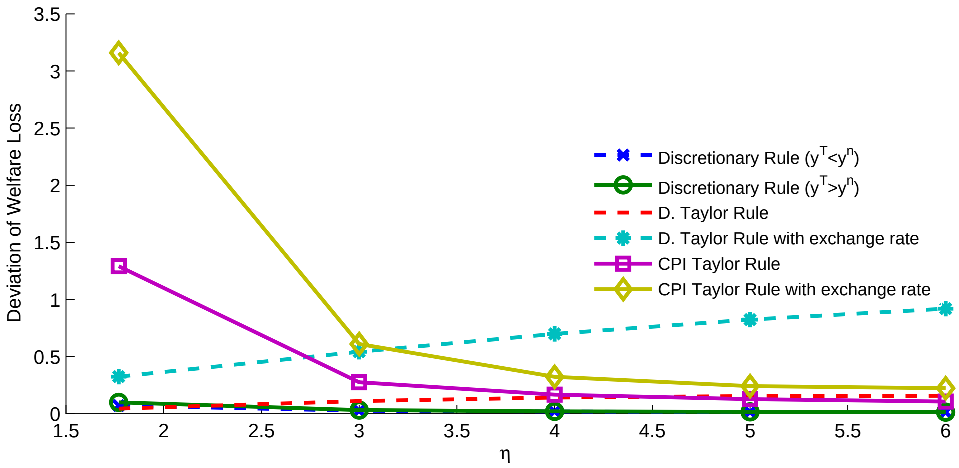


Table 4.9: Standard deviations of the Home variables, varying the elasticity of substitution of home and foreign goods

	$s.d.(\pi_{H,t})$	$s.d.(y_t)$	$s.d.(i_t)$	$s.d.(\pi_t)$	$s.d.(q_t)$	Welfare Loss
$\eta = 1.77$						
Optimal Rule	0.0207	0.0254	0.1107	0.7643	1.6263	0.8611***
Optimal Rule ($y^T_{t+1} < y^n_{t+1}$)	0.0337	0.1548	0.1549	0.7841	1.6356	0.9219*
Optimal Rule ($y^T_{t+1} > y^n_{t+1}$)	0.0224	0.1257	0.1214	0.7476	1.6754	0.9462
D. Taylor Rule	0.0048	0.0936	0.0415	0.7974	1.7035	0.8993**
D. Taylor Rule with q	0.0415	0.3559	0.0716	0.8524	1.8046	1.1399
CPI Taylor Rule	0.1610	0.3277	0.7672	0.6525	1.6444	1.9723
CPI Taylor Rule with q	0.2617	0.6522	1.7675	0.3845	1.2653	3.5808
$\eta = 3$						
Optimal Rule	0.0448	0.2337	0.3115	0.5856	1.2745	1.6828***
Optimal Rule ($y^T_{t+1} < y^n_{t+1}$)	0.0545	0.2491	0.3280	0.6021	1.2617	1.7263**
Optimal Rule ($y^T_{t+1} > y^n_{t+1}$)	0.0402	0.2866	0.3054	0.5714	1.2885	1.7353*
D. Taylor Rule	0.0182	0.4605	0.2120	0.6375	1.3447	1.8692
D. Taylor Rule with q	0.0569	0.9495	0.0818	0.7309	1.4858	2.5945
CPI Taylor Rule	0.1117	0.6144	0.7352	0.5187	1.2870	2.1446
CPI Taylor Rule with q	0.2292	0.3584	1.5100	0.3291	1.0448	2.7093
$\eta = 4$						
Optimal Rule	0.0535	0.4039	0.4210	0.4985	1.0829	1.9196***
Optimal Rule ($y^T_{t+1} < y^n_{t+1}$)	0.0623	0.4016	0.4321	0.5131	1.0730	1.9548**
Optimal Rule ($y^T_{t+1} > y^n_{t+1}$)	0.0482	0.4441	0.4147	0.4856	1.0937	1.9597*
D. Taylor Rule	0.0232	0.6844	0.3207	0.5524	1.1539	2.1899
D. Taylor Rule with q	0.0712	1.3404	0.1456	0.6657	1.3113	3.2613
CPI Taylor Rule	0.0893	0.8024	0.7411	0.4495	1.1014	2.2391
CPI Taylor Rule with q	0.2173	0.4451	1.3938	0.3044	0.9267	2.5403
$\eta = 5$						
Optimal Rule	0.0572	0.5572	0.5018	0.4386	0.9472	2.0128***
Optimal Rule ($y^T_{t+1} < y^n_{t+1}$)	0.0654	0.5485	0.5102	0.4518	0.9392	2.0424**
Optimal Rule ($y^T_{t+1} > y^n_{t+1}$)	0.0516	0.5914	0.4961	0.4269	0.9558	2.0445*
D. Taylor Rule	0.0261	0.8631	0.4093	0.4894	1.0127	2.3235
D. Taylor Rule with q	0.0849	1.6698	0.2129	0.6171	1.1780	3.6696
CPI Taylor Rule	0.0749	0.9561	0.7571	0.3991	0.9659	2.2701
CPI Taylor Rule with q	0.2115	0.6630	1.3183	0.2886	0.8382	2.4989
$\eta = 6$						
Optimal Rule	0.0579	0.6983	0.5641	0.3953	0.8456	2.0409***
Optimal Rule ($y^T_{t+1} < y^n_{t+1}$)	0.0658	0.6867	0.5708	0.4073	0.8391	2.0676*
Optimal Rule ($y^T_{t+1} > y^n_{t+1}$)	0.0521	0.7288	0.5591	0.3844	0.8527	2.0669**
D. Taylor Rule	0.0280	1.0091	0.4828	0.4407	0.9035	2.3610
D. Taylor Rule with q	0.0974	1.9514	0.2758	0.5791	1.0717	3.9157
CPI Taylor Rule	0.0650	1.0837	0.7769	0.3604	0.8620	2.2586
CPI Taylor Rule with q	0.2090	0.8900	1.2665	0.2779	0.7685	2.4967

Notes: The superscripts *** shows the lowest welfare loss which indicates the corresponding policy rule is the optimal rule; ** shows the second-lowest welfare loss and the corresponding rule is the first-best policy rule among alternative rules; * indicates the corresponding rule is the second-best policy rule among alternative rules.

In order to compare different scenarios, Table 4.10 shows the preferred policy rule under different values of λ and η. Compared with the discretionary rule where the output target varies for the whole period, the ranking of different policies have changed, but not significantly. When the elasticity of substitution between domestic and foreign goods is low, the performances of welfare depend on the degree of openness. If the Home economy is relatively closed, the domestic Taylor rule works the best, while the discretionary rule with a lower output target is the best alternative when λ is bigger. For intermediate levels of η, the discretionary policy with a lower output target gives the best welfare, which could imply that a contractionary monetary policy is welfare-improving.

If the Home economy is relatively open, and the Home and Foreign products are close substitutes to one another (corresponding the lower right corner of Table 4.10), the discretionary rule with a one-period higher output target shows the lowest welfare loss, which is different from the results obtained in Table 4.6. Intuitively, high substitutability implies more elastic demand to the change of the real exchange rate, so if the discretionary output target is only higher than the natural output for one period, the less appreciated real exchange rate and the higher output compensate the higher inflation, and as a result, the Home welfare rises. Instead, the domestic welfare drops if adopting the discretionary rule with higher output level for the whole period, inflation kept at a low level dominates the losses caused by the consistently appreciating real exchange rate and the low output.

Table 4.10: Preferred policy rule, varying the degree of openness and the intratemporal elasticity of substitution

η/λ	0.17	0.25	0.3	0.4	0.62
1.77	D. Taylor Rule	D. Taylor Rule	D. Taylor Rule	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$
3	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$
4	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$
5	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$
6	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}>y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}>y^n_{t+1}}$	Dis. $\mathrm{R}^{y^T_{t+1}>y^n_{t+1}}$

Notes: Dis. $\mathrm{R}^{y^T_{t+1}<y^n_{t+1}}$ represents the discretionary rule with an output target is below the natural output level, where the output target only varies for period $t+1$, while Dis. $\mathrm{R}^{y^T_{t+1}>y^n_{t+1}}$ stands for the discretionary rule with a higher discretionary output target which varies at period $t+1$.

4.6 Conclusion

We have formed a small open economy model, as a limiting case of a two-country equilibrium model, and derived a utility-based welfare loss function which contains domestic inflation, the output gap and the real exchange rate, according to De Paoli (2009). Unlike other existing literature (see e.g. Clarida et al., 2001; Gali and Monacelli, 2005), the optimal policy rule in a small open economy is not isomorphic to a closed economy.

However, the optimal rule may be difficult to implement, therefore we investigate several different policy rules. Under the benchmark calibration, the domestic Taylor rule shows the best welfare among alternative rules when facing a Foreign negative demand shock. The sensitivity analysis exercise shows that the performances of policy rules depend on the different combinations of key parameters. The results indicate that the domestic Taylor rule is the preferred policy if the elasticity of substitution between domestic and foreign goods are low, while a discretionary rule with an output target below the natural output level is the best policy choice when the Home demand is sensitive to the exchange rate movement. However, the openness of the economy modifies the optimal responses when there are intermediate values of intratemporal substitution elasticity. The domestic Taylor rule is the best alternative if the Home economy is relatively closed. If the Home country becomes more open, the Home economy benefits from adopting a discretionary policy with a lower target output. Intuitively, the elasticity of substitution between home and foreign goods determines the elasticity of Home demand and effects of the terms of trade, and therefore, performances of alternative policy rules. For a intermediate value of intratemporal elasticity of substitution, the degree of home bias affects the Home demand to the change of real exchange rate, and therefore, the ranking of different policy rules.

Instead, if we assume the Central Bank lets the output target be sub-optimal for only one period when implementing the discretionary rules, the ranking of different alternative policy rules has changed slightly. The substitution of elasticity between home and foreign goods determines the effects of terms of trade. As a result, if the elasticity of substitution between home and foreign goods is low and the Home country is relatively closed, the domestic Taylor rule is the preferred policy. While if the Home goods are highly elastic to the Foreign products in the relatively closed Home country, the discretionary rule with a lower output target is welfare-improving.

Although the short-term interest rate is commonly used as a tool to achieve stable inflation and output, some small economies choose the exchange rate as the instrument variable for monetary policy, e.g. Singapore. Since exports relative to GDP in Singapore is exceptionally high compared to other small open economies, targeting the exchange rate may be a sensible choice for the monetary authorities of this and similar countries (McCallum, 2007). In this respect, considering the exchange rate in monetary policy could be effective for economies which has very high ratios of trade to domestic production. In this Chapter, we consider different types of interest-rate rules, and we find that simple rules which take the exchange rate into consideration along with inflation and output sometimes are associated with larger welfare losses. For future research, when considering small and extremely open economies, it would be interesting to investigate welfare effects of a rule which takes the exchange rate as the instrument variable (Gali and Monacelli, 2005), or a rule considers the exchange rate and the output gap only.

Chapter 5

Conclusion

The thesis investigated the scope of regime change in monetary policy and evaluated the performance of different monetary policy rules in response to exogenous shocks. Chapter 2 provided strong empirical evidence that the parameters in the policy rules appear to be changing over time, and a non-linear Taylor rule could be an appropriate method to model the behaviour of the Bank of England. In order to capture the link between changing economic conditions and the response of monetary authorities, Chapter 3 studied the welfare consequences of implementing endogenous-switching rules in a New Keynesian model. The results show the endogenous-switching rule provides a better social welfare compared with the "original" Taylor rule which takes the weights suggested by John Taylor. However, when increasing the weights assigned to inflation and output gap in a linear Taylor rule, it outperforms the corresponding endogenous-switching rule. Our results suggest that the economy could benefit from implementing a linear Taylor rule but raising the weights on inflation and output gap. Chapter 4 explored the welfare effects of the optimal policy rule and different alternative rules in a small open economy. As the optimal rule is difficult to implement, it forces Central Banks to consider alternative rules in order to maximise the welfare with respect to the optimal rule. We found the ranking of alternative rules depends on the degree of openness and the intratemporal elasticity of substitution.

Several extensions could be considered in the future research. First, in the thesis, we evaluated Taylor-type rules which respond to the current inflation and output gap. Several contributions argue that responding to the past/expected inflation in monetary policy rules might improve welfare (see e.g. Benhabib et al., 2001; Clarida et al., 1998). So an investigation on improving welfare could be conducted by introducing backward/forward specifications of inflation gaps to interest-rate rules. Second, it is worth to introducing the Zero Lower Bound on nominal interest rates when analysing performance of monetary policy rules. As point out by Taylor and Williams (2010) and McCallum (2000), the zero-lower-bound problem might cause monetary policy inefficiency, leading variability on inflation and aggregate demand. An interesting analysis would be investigating how the zero-lower-bound problem affects ranking across different monetary policy rules considered in this thesis. The third extension would be, instead of using interest-rate rules, it would be interesting to consider the exchange rate exclusively in a monetary policy rule. The exchange rate-based monetary policy would be extremely useful for small and extremely open to trade economies (McCallum, 2007) – therefore, explore the potential benefit of rules which take the exchange rate as a targeting variable could be one possible future research.

The welfare loss function we used in the thesis includes inflation and output volatility, however, Woodford (2003b) introduces the interest rate variability to a welfare loss function as he argues that it would be desirable to achieve stability of inflation and output gap without generating much volatility on interest rates. Furthermore, several studies find welfare benefits when Central Banks consider interest-rate smoothing objective (see e.g. Woodford, 2003b; Blake et al., 2011). Therefore, a further interesting extension would be to evaluate the overall benefits to Central Banks when considering an additional of objective of interest-rate stabilization explicitly in welfare loss function. In this respect, the switching monetary policy regimes might outperform as the rules generate more stable inflation and output and without requiring much volatility on interest rate.

Appendix A

Linearisation of New Keynesian Model and Welfare around Zero Inflation Steady State

A.1 Linearisation of the New Keynesian Model

A.1.1 Linearisation of the New Keynesian Phillips Curve around Zero Inflation Steady State

In order to obtain a dynamic equilibrium between inflation and output, we linearise the equilibrium relative price (3.21) around zero inflation. Before the linearisation, it is necessary to re-write the optimal relative price:

$$Z_t^{\frac{\alpha\varepsilon}{1-\alpha}+1}(1-\alpha)(1-\varepsilon)E_t\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}\left(\frac{1}{\prod_{i=1}^{k}\Pi_{t+i}}\right)^{1-\varepsilon}Y_{t+k}=\varepsilon E_t\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}\left(\frac{1}{\prod_{i=1}^{k}\Pi_{t+i}}\right)^{-\frac{\alpha\varepsilon}{1-\alpha}-\varepsilon}Y_{t+k}^{\sigma+\frac{\varphi+\alpha}{1-\alpha}+1} \tag{A.1.1}$$

Linearising equation (A.1.1) around zero inflation. The left hand side (LHS) and right hand side (RHS) approximation steps are shown separately.

$$
\begin{aligned}
LHS = & \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Z^{\frac{\alpha\varepsilon}{1-\alpha}+1}(1-\alpha)(\varepsilon-1)Y \\
& + \left(\frac{\alpha\varepsilon}{1-\alpha}+1\right) Z^{\frac{\alpha\varepsilon}{1-\alpha}+1} \frac{Z_t - Z}{Z}(1-\alpha)(\varepsilon-1)E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y \\
& + Z^{\frac{\alpha\varepsilon}{1-\alpha}+1}(1-\alpha)(\varepsilon-1)Y E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k}(\varepsilon-1) \sum_{i=1}^{k} \frac{\Pi_{H,t+i}-\Pi}{\Pi} \\
& + Z^{\frac{\alpha\varepsilon}{1-\alpha}+1}(1-\alpha)(\varepsilon-1))E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y \frac{Y_{t+k}-Y}{Y} \\
= & (1-\alpha)(\varepsilon-1) \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Z^{\frac{\alpha\varepsilon}{1-\alpha}+1} Y \left[1 + \left(\frac{\alpha\varepsilon}{1-\alpha}+1\right) z_t + (\varepsilon-1)\sum_{i=1}^{k} \pi_{H,t+i} + \tilde{y}_{t+k}\right]
\end{aligned}
\tag{A.1.2}
$$

$$
\begin{aligned}
RHS = & \varepsilon \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y^{\sigma+\frac{\varphi+\alpha}{1-\alpha}+1} \\
& + \varepsilon \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right) \sum_{i=1}^{k} \frac{\Pi_t - \Pi}{\Pi} Y^{\sigma+\frac{\varphi+\alpha}{1-\alpha}+1} \\
& + \varepsilon \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}+1\right) Y^{\sigma+\frac{\varphi+\alpha}{1-\alpha}+1} \frac{Y_{t+k}-Y}{Y} \\
= & \varepsilon \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y^{\sigma+\frac{\varphi+\alpha}{1-\alpha}+1} \left[1 + \left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right) \sum_{i=1}^{k} \pi_{t+i} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}+1\right) y_{t+k}\right]
\end{aligned}
\tag{A.1.3}
$$

Then we can have the linearised optimal relative price reads:

$$
\begin{aligned}
z_t = &-\frac{1-\theta\beta}{\frac{\alpha\varepsilon}{1-\alpha}+1} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[1 + (\varepsilon - 1) \sum_{i=1}^{k} \pi_{t+i} + y_{t+k} \right] \\
&+ \frac{1-\theta\beta}{\frac{\alpha\varepsilon}{1-\alpha}+1} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[1 + \left(\frac{\alpha\varepsilon}{1-\alpha} + \varepsilon \right) \sum_{i=1}^{k} \pi_{t+k} + \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} + 1 \right) y_{t+k} \right]
\end{aligned}
\tag{A.1.4}
$$

To replace z_t, linearising the the aggregate price index (3.12) and substituting the definition of the equilibrium relative price $Z_t = \frac{\tilde{P}_t}{P_{t-1}}$ and $\Pi_t = \frac{P_t}{P_{t-1}}$, it yields:

$$
z_t = \frac{\theta}{1-\theta} \pi_t \tag{A.1.5}
$$

Using equation (A.1.4) and (A.1.5), we have

$$
\begin{aligned}
\pi_t = &-\frac{1-\theta}{\theta} \frac{1-\theta\beta}{\frac{\alpha\varepsilon}{1-\alpha}+1} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k}^k \left[1 + (\varepsilon - 1) \sum_{i=1}^{k} \pi_{t+i} + y_{t+k} \right] \\
&+ \frac{1-\theta}{\theta} \frac{1-\theta\beta}{\frac{\alpha\varepsilon}{1-\alpha}+1} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k}^k \left[1 + \left(\frac{\alpha\varepsilon}{1-\alpha} + \varepsilon \right) \sum_{i=1}^{k} \pi_{t+i} + \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} + 1 \right) y_{t+k} \right]
\end{aligned}
\tag{A.1.6}
$$

Now we lead the equation (A.1.6) one period ahead in order to isolate the next period's expected inflation and the current output gap in previous equation, so get

$$
\begin{aligned}
E_t \pi_{t+1} = &-\frac{1-\theta}{\theta} \frac{1-\theta\beta}{\frac{\alpha\varepsilon}{1-\alpha}+1} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k}^k \left[1 + (\varepsilon - 1) \sum_{i=1}^{k} \pi_{t+1+i} + y_{t+1+k} \right] \\
&+ \frac{1-\theta}{\theta} \frac{1-\theta\beta}{\frac{\alpha\varepsilon}{1-\alpha}+1} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k}^k \left[1 + \left(\frac{\alpha\varepsilon}{1-\alpha} + \varepsilon \right) \sum_{i=1}^{k} \pi_{t+1+i} + \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} + 1 \right) y_{t+1+k} \right]
\end{aligned}
\tag{A.1.7}
$$

And use the following identities

$$\sum_{k=0}^{\infty}(\theta\beta)^k(\varepsilon-1)\sum_{i=1}^{k}\pi_{t+1}=(\varepsilon-1)\left[\frac{\theta\beta}{1-\theta\beta}E_t\pi_{t+1}+\theta\beta\sum_{k=0}^{\infty}(\theta\beta)^k\sum_{i=1}^{k}\pi_{t+1+i}\right] \tag{A.1.8}$$

$$\sum_{k=0}^{\infty}(\theta\beta)^k\left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right)\sum_{i=1}^{k}\pi_{t+i}=\left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right)\left[\frac{\theta\beta\pi^{\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon}}{1-\theta\beta}E_t\pi_{t+1}+\theta\beta\sum_{k=0}^{\infty}(\theta\beta)^k\sum_{i=1}^{k}\pi_{t+1+i}\right] \tag{A.1.9}$$

Finally, using the equation (A.1.6) and equation (A.1.7) together with the right hand side of equation (A.1.8) and equation (A.1.9), we can get the new Keynesian Phillips curve (3.22):

$$\pi_t=\beta E_t\pi_{t+1}+\kappa y_t$$

where $\kappa=\frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\left(\sigma+\frac{\varphi+\alpha}{1-\alpha}\right)$.

A.1.2 Linearisation of IS Curve around Zero Inflation Steady State

Substituting the equality of the aggregate consumption and output (3.19), the Euler equation could be re-written as

$$1=\beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}}\frac{Y_{t+1}^{-\sigma}}{Y_t^{-\sigma}}\right) \tag{A.1.10}$$

Take a first Taylor approximation around zero inflation steady state, then the IS curve is given by

$$y_t=E_ty_{t+1}+\frac{1}{\sigma}(i_t-E_t\pi_{t+1}) \tag{A.1.11}$$

A.2 Derivation the Social Welfare Loss Function

The welfare function equation (3.29) is derived by a second-order Taylor series approximation of the expected utility of the representative household.

Take the second-order Taylor approximation of the households' utility

$$\frac{u_t - u}{u_c C} = c_t + \frac{1-\sigma}{2} c_t^2 + \frac{u_n}{u_c} \frac{N}{C} \left(n_t + \frac{1+\varphi}{2} n_t^2 \right) \tag{A.2.1}$$

where $\frac{u_n}{u_c}$ is the marginal product of labour. Using the production function (3.11), the marginal labour product could be re-write as

$$u_n = -u_c(1-\alpha)\frac{C}{N} \tag{A.2.2}$$

The expression for n_t could be derived using the labour index (3.5) and production function (3.11), that is

$$n_t = \frac{1}{1-\alpha}(y_t - a_t + d_t) \tag{A.2.3}$$

where $d_t = \frac{\varepsilon}{2} \frac{1-\alpha+\alpha\varepsilon}{1-\alpha} varp_t(i)$

The expression of a_t could be derived from the marginal costs of firms, which is

$$mc_t = \left(\sigma + \frac{\alpha+\varphi}{1-\alpha} \right) y_t^n - \frac{1+\varphi}{1-\alpha} a_t - ln(1-\alpha)$$

So a_t is:

$$a_t = \frac{\sigma(1-\alpha)+\varphi+\alpha}{1+\varphi} y_t^n \tag{A.2.4}$$

Substituting expression (A.2.2), (A.2.3), (A.2.4) and the market clearing condition (3.19) into equation (A.2.1), it gives the welfare function:

$$L_t = \pi_t^2 + \lambda_y y_t^2$$

where $\lambda_y = \frac{\kappa}{\varepsilon}$, where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$ is the coefficient on output gap in the new Keynesian Phillips curve.

Appendix B

Linearisation of the Small Open Economy Model and Welfare

B.1 Linearisation of the Small Open Economy Model

B.1.1 Aggregate Demand and Real Exchange Rate

Take the logarithm of the market clearing for the rest of the world, it gives:

$$y_t^* = c_t^* \tag{B.1.1}$$

then substitute into the aggregate demand function in Home economy (4.38):

$$c_t = \frac{1}{1-\lambda}[\tilde{y}_t - \lambda\tilde{y}_t^* - \gamma q_t] \tag{B.1.2}$$

In order to find an equation for the Home output which depends on world output and the real exchange rate only, we log-linearise the international risk sharing condition, which is shown in equation (4.28),

$$c_t = c_t^* + \frac{1}{\sigma}q_t \tag{B.1.3}$$

Therefore, a relationship between the Home output and the Foreign output is derived as

$$y_t = y_t^* + \left(\frac{1-\lambda}{\sigma} + \gamma\right) q_t \tag{B.1.4}$$

B.1.2 Linearisation of the New Keynesian Phillips Curve around Zero Inflation Steady State

Using the standard procedure to obtain the new Keynesian Phillips curve, the Home optimal relative price (4.42) is linearised around zero inflation steady state.

In terms of linearisation procedure, the optimal relative price in Home economy (4.42) can be re-written as

$$Z_t^{\frac{\alpha\varepsilon}{1-\alpha}+1}(1-\alpha)(\varepsilon-1)E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\frac{1}{\prod_{i=1}^{k} \Pi_{t+i}}\right)^{1-\varepsilon} Y_{t+k} = \varepsilon E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} C_t^{\sigma} \left(\frac{1}{\prod_{i=1}^{k} \Pi_{t+i}}\right)^{-\frac{\alpha\varepsilon}{1-\alpha}-\varepsilon} Y_{t+k}^{\sigma+\frac{\varphi+\alpha}{1-\alpha}+1} \tag{B.1.5}$$

Apply a first order Taylor approximation around zero inflation to previous equation. The left hand side (LHS) and right hand side (RHS) approximation steps are shown separately.

$$\begin{aligned}
LHS = & \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Z^{\frac{\alpha\varepsilon}{1-\alpha}+1}(1-\alpha)(\varepsilon-1)Y \\
& + \left(\frac{\alpha\varepsilon}{1-\alpha}+1\right) Z^{\frac{\alpha\varepsilon}{1-\alpha}+1} \frac{Z_t - Z}{Z}(1-\alpha)(\varepsilon-1)E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y \\
& + Z^{\frac{\alpha\varepsilon}{1-\alpha}+1}(1-\alpha)(\varepsilon-1)Y E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k}(\varepsilon-1) \sum_{i=1}^{k} \frac{\Pi_{H,t+i}-\Pi}{\Pi} \\
& + Z^{\frac{\alpha\varepsilon}{1-\alpha}+1}(1-\alpha)(\varepsilon-1)E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y \frac{Y_{t+k}-Y}{Y} \\
= & (1-\alpha)(\varepsilon-1) \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Z^{\frac{\alpha\varepsilon}{1-\alpha}+1} Y \left[1 + \left(\frac{\alpha\varepsilon}{1-\alpha}+1\right) z_t + (\varepsilon-1) \sum_{i=1}^{k} \pi_{H,t+i} + \tilde{y}_{t+k}\right]
\end{aligned} \tag{B.1.6}$$

$$
\begin{aligned}
RHS = &\varepsilon \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} C^{\sigma} Y^{\frac{\varphi+\alpha}{1-\alpha}+1} \\
&+ \varepsilon\sigma \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right) C^{\sigma} \frac{C_t - C}{C} Y^{\frac{\varphi+\alpha}{1-\alpha}+1} \\
&+ \varepsilon \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} C^{\sigma} \left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right) \sum_{i=1}^{k} \frac{\Pi_{H,t}-\Pi}{\Pi} Y^{\frac{\varphi+\alpha}{1-\alpha}+1} \\
&+ \varepsilon \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} C^{\sigma} \left(\frac{\varphi+\alpha}{1-\alpha}+1\right) Y^{\frac{\varphi+\alpha}{1-\alpha}+1} \frac{Y_{t+k}-Y}{Y} \\
= &\varepsilon \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} C^{\sigma} Y^{\sigma+\frac{\varphi+\alpha}{1-\alpha}+1} \left[1 + \sigma c_t + \left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right) \sum_{i=1}^{k} \pi_{H,t+i} + \left(\frac{\varphi+\alpha}{1-\alpha}+1\right) y_{t+k}\right]
\end{aligned} \tag{B.1.7}
$$

Equal the LHS and RHS, then the linearised optimal relative price derived

$$
\begin{aligned}
z_{H,t} = &-\frac{1-\theta\beta}{\frac{\alpha\varepsilon}{1-\alpha}+1} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[1 + (\varepsilon-1) \sum_{i=1}^{k} \pi_{H,t+i} + y_{t+k}\right] \\
&+ \frac{1-\theta\beta+\varepsilon}{\frac{\alpha\varepsilon}{1-\alpha}+1} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[1 + \sigma c_t + \left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right) \sum_{i=1}^{k} \pi_{t+k} + \left(\frac{\varphi+\alpha}{1-\alpha}+1\right) y_{t+k}\right]
\end{aligned} \tag{B.1.8}
$$

In order to replace $z_{H,t}$ in previous equation, linearise the aggregate price index, shown in equation (4.31), using the $Z_{H,t} = \frac{\tilde{P}_{H,t}}{P_{H,t}}$ and $\Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$, the aggregate price index can be re-written as

$$
1 = \theta \left(\frac{1}{\Pi_t}\right)^{1-\varepsilon} + (1-\theta) Z_t^{1-\varepsilon} \tag{B.1.9}
$$

and apply a first order Taylor approximation

$$
(1-\theta)Z^{1-\varepsilon} + (1-\theta)(1-\varepsilon)Z^{1-\varepsilon}\frac{Z_t - Z}{Z} = -\theta(1-\varepsilon)\frac{\Pi_t - \Pi}{\Pi} + \theta \tag{B.1.10}
$$

then it gives the linearised aggregate price level around zero inflation:

$$z_{H,t} = \frac{\theta}{1-\theta}\pi_t \tag{B.1.11}$$

Using equation (B.1.8) and (B.1.11), the New Keynesian Phillips curve is derived

$$\begin{aligned}
\pi_{H,t} = & -\frac{1-\theta}{\theta}\frac{1-\theta\beta}{\frac{\alpha\varepsilon}{1-\alpha}+1}E_t\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}^k\left[1+(\varepsilon-1)\sum_{i=1}^{k}\pi_{H,t+i}+y_{t+k}\right] \\
& +\frac{1-\theta}{\theta}\frac{1-\theta\beta}{\frac{\alpha\varepsilon}{1-\alpha}+1}E_t\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}^k \\
& *\left[1+\sigma c_t+\left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right)\sum_{i=1}^{k}\pi_{H,t+i}+\left(\frac{\varphi+\alpha}{1-\alpha}+1\right)y_{t+k}\right]
\end{aligned} \tag{B.1.12}$$

Then lead the equation (B.1.8) one period ahead in order to isolate the next period's expected inflation and the current output gap in previous equation, we have

$$\begin{aligned}
E_t\pi_{H,t+1} = & -\frac{1-\theta}{\theta}\frac{1-\theta\beta}{\frac{\alpha\varepsilon}{1-\alpha}+1}E_t\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}^k\left[1+(\varepsilon-1)\sum_{i=1}^{k}\pi_{H,t+1+i}+y_{t+1+k}\right] \\
& +\frac{1-\theta}{\theta}\frac{1-\theta\beta}{\frac{\alpha\varepsilon}{1-\alpha}+1}E_t\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}^k \\
& *\left[1+\sigma c_t+\left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right)\sum_{i=1}^{k}\pi_{H,t+1+i}+\left(\frac{\varphi+\alpha}{1-\alpha}+1\right)y_{t+1+k}\right]
\end{aligned} \tag{B.1.13}$$

Then use the following identities

$$\begin{aligned}
\sum_{k=0}^{\infty}(\theta\beta)^k(\varepsilon-1)\sum_{i=1}^{k}\pi_{H,t+i} &= (\varepsilon-1)\sum_{k=0}^{\infty}(\theta\beta)^{k+1}\left(E_t\pi_{H,t+i}+E_t\sum_{i=1}^{k}\pi_{H,t+1+i}\right) \\
&= (\varepsilon-1)\left[\frac{\theta\beta}{1-\theta\beta}E_t\pi_{H,t+1}+\theta\beta\sum_{k=0}^{\infty}(\theta\beta)^k\sum_{i=1}^{k}\pi_{H,t+1+i}\right]
\end{aligned} \tag{B.1.14}$$

$$\sum_{k=0}^{\infty}(\theta\beta)^k\left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right)\sum_{i=1}^{k}\pi_{H,t+i}=\left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right)\sum_{k=0}^{\infty}(\theta\beta)^{k+1}\left(E_t\pi_{H,t+i}+E_t\sum_{i=1}^{k}\pi_{H,t+1+i}\right)$$
$$=\left(\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon\right)\left[\frac{\theta\beta}{1-\theta\beta\pi^{\frac{\alpha\varepsilon}{1-\alpha}+\varepsilon}}E_t\pi_{H,t+1}+\theta\beta\sum_{k=0}^{\infty}(\theta\beta)^k\sum_{i=1}^{k}\pi_{H,t+1+i}\right] \tag{B.1.15}$$

Finally, using the equation (B.1.8) and equation (B.1.13) together with the right hand side of equation (B.1.14) and equation (B.1.15), and substitute c_t using the linearised international risk sharing condition (4.29), the Phillips curve in Home country could be written as

$$\pi_{H,t}=\beta E_t\pi_{H,t+1}+\kappa_a\left[\left(\frac{\alpha+\varphi}{1-\alpha}+\frac{\sigma}{1-\lambda}\right)\tilde{y}_t-\sigma\frac{\lambda}{1-\lambda}\tilde{y}^*-\sigma\gamma q_t\right]$$

Processing the similar method which is used to derive the dynamic IS curve, substitute q_t using the equation between real exchange rate and the output gap, so the Phillips curve in a small open economy reads:

$$\pi_{H,t}=\beta E_t\pi_{H,t+1}+\kappa_a\left[\left(\frac{\alpha+\varphi}{1-\alpha}+\frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right)y_t+\left(\sigma-\frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right)y_t^*\right]$$

B.1.3 Linearisation of IS Curve around Zero Inflation Steady State

Take the first order Taylor approximation of the Euler equation in Home economy shown in equation (4.19), it gives

$$c_t=E_tc_{t+1}-\frac{1}{\sigma}\left(i_t-E_t\pi_{t+1}\right) \tag{B.1.16}$$

In order to write the linearised Euler equation, equation (B.1.16), in terms of the Home output gap, we use the linearised the goods' market clearing condition in Home and Foreign economy (4.38) and (B.1.1), then the dynamic IS curve could be written as,

$$y_t = E_t y_{t+1} - \frac{1-\lambda}{\sigma}(i_t - E_t\pi_{t+1}) - \lambda\Delta E_t y^*_{t+1} - \gamma E_t \Delta q_{t+1} \tag{B.1.17}$$

where $E_t\Delta q_{t+1}$ could be replace using the relationship between the real exchange rate and the output (B.1.4), and π_t could be expressed in terms of domestic inflation based on equation (4.26), so

$$y_t = E_t y_{t+1} - (1-\lambda)\frac{1-\lambda+\sigma\gamma}{\sigma}(i_t - E_t\pi_{H,t+1}) + [\sigma\gamma - \lambda - \lambda(1-\lambda+\sigma\gamma)]\,\Delta E_t y^*_{t+1}$$

B.2 Welfare Approximation in a Small Open Economy

In this appendix, we derive a second-order Taylor approximation of utility function, shown in equation (4.18), to obtain a welfare loss function in a small open economy.

Take a second order approximation of utility,

$$\frac{u_t - u}{u_c C} = \hat{c}_t + \frac{1-\sigma}{2}\hat{c}_t^2 + \frac{u_n}{u_c}\frac{N}{C}\left(\hat{n}_t + \frac{1+\varphi}{2}\hat{n}_t^2\right) \tag{B.2.1}$$

In order to obtain a expression for aggregate demand in the Home country c_t, combine the international risk sharing (4.29) and the relationship between home and foreign output (B.1.4),

$$c_t = \frac{1}{1-\lambda+\sigma\gamma}y_t + \left(1 - \frac{1}{1-\lambda+\sigma\gamma}\right)y^*_t \tag{B.2.2}$$

Similar with the closed economy, the linearised labour index could be written as

$$\hat{n}_t = \frac{1}{1-\alpha}\left[\hat{y}_t - a_t + \frac{\varepsilon}{2}\frac{1-\alpha+\alpha\varepsilon}{1-\alpha}varp_{H,t}(i)\right] \tag{B.2.3}$$

According to the labour index (4.30), the marginal product of labour in a small open economy could be derived as:

$$-\frac{U_n}{U_c} = \frac{1-\alpha}{1-\lambda+\sigma\gamma}\frac{C}{N} \tag{B.2.4}$$

Now substitute the previous results into equation (B.2.1),

$$\frac{u_t - u}{u_c C} = -\frac{1}{2(1-\lambda+\sigma\gamma)}\left[\left(\frac{1+\varphi}{1-\alpha} - \frac{(1-\sigma)(1+\lambda-\sigma\gamma)}{1-\lambda+\sigma\gamma}\right)y_t^2 - 2\frac{1+\varphi}{1-\alpha}a_t y_t + \frac{(1-\sigma)(\lambda-\sigma\gamma)}{1-\lambda+\sigma\gamma}(y_t - y_t^*)^2 + \varepsilon\frac{1-\alpha+\alpha\varepsilon}{1-\alpha}varp_{H,t}(i)\right] + t.i.p. \tag{B.2.5}$$

where $t.i.p.$ stands for terms of independent of policy.

Log-linearise the marginal cost in the Home economy,

$$mc_t = \left(\frac{\alpha+\varphi}{1-\alpha} + \frac{\sigma(1-\lambda)+\lambda\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right)y_t - \frac{1+\varphi}{1-\alpha} \tag{B.2.6}$$

Therefore,

$$a_t = \frac{1-\alpha}{1+\varphi}\left[\frac{\alpha+\varphi}{1-\alpha} + \frac{\sigma}{(1-\lambda)(1-\lambda+\sigma\gamma)}\right]y_t \tag{B.2.7}$$

Insert a_t into equation (B.2.5), and using the discounted value of price index, $\sum_{t=0}^{\infty}\beta^t varp_{H,t}(i) = \frac{\theta}{(1-\theta)(1-\beta\theta)}\sum_{t=0}^{\infty}\beta^t\pi_{H,t}^2$, then welfare loss function, equation (4.48) could be obtained

$$L = \frac{1}{2}\Phi_\pi\pi_{H,t}^2 + \frac{1}{2}\Phi_y\left(y_t - y_t^T\right)^2 + \frac{1}{2}\Phi_q\left(q_t - q_t^T\right)^2$$

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