Advances

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Systematic Bailout Guarantees and Tacit Coordination

Abstract: Both the academic literature and the policy debate on systematic bailout guarantees and Government subsidies have ignored an important effect: in industries where firms may go out of business due to idiosyncratic shocks, Governments may increase the likelihood of (tacit) coordination if they set up schemes that rescue failing firms. In a repeated-game setting, we show that a systematic bailout regime increases the expected profits from coordination and simultaneously raises the probability that competitors will remain in business and will thus be able to “punish” firms that deviate from coordinated behaviour. These effects make tacit coordination easier to sustain and have a detrimental impact on welfare. While the key insight holds across any industry, we study this question with an application to the banking sector, in light of the recent financial crisis and the extensive use of bailout schemes.

Keywords: competition policy, systematic bailout guarantees, collusion, banking, State aid

JEL Classification: D43, G21, K21, L41

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1 Introduction and motivation

The financial crisis and its aftermath have been affecting the global economy since 2007. This has involved the bankruptcies of a large number of global firms, including major financial institutions. State intervention in the form of bailouts

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has been playing a major role, with Governments rushing to rescue not only a large fraction of their financial industries but also other major industries like the automotive one. The banking sector has recently appeared to enjoy some degree of explicit bailout guarantees for systemic financial institutions that were deemed to be “too-big-to-fail”.¹

Research studying the economic effects of bailouts has focused on firms’ unilateral incentives, as opposed to their coordinated behaviour. Arguments against Government intervention typically revolve around the distortion of the competitive process and the moral hazard issue: if firms expect that the Government will intervene to help them in case of failure (or in adverse circumstances more generally), these may have the incentive to take excessive risks.² On the other hand, arguments in favour of intervention range from distributional considerations to the potential resolution of existing market failures.³

In this article, by contrast, we focus on the effect of bailouts on the incentives to engage in coordinated behaviour. In particular, we develop a simple (infinite-horizon) model that shows that a Government policy aimed at systematically bailing out firms in the presence of negative idiosyncratic shocks facilitates (tacit) coordination. To our knowledge, this result had not been previously identified.

Think of an oligopolistic industry where firms may receive random idiosyncratic shocks that would force them out of business, absent Government intervention. Now consider a Government policy that systematically bails out any firm that has been subject to such a shock. This policy would make tacit coordination easier to sustain in such industry because of two effects that work in the same direction.

First, the net present value of tacit coordination is higher if a firm knows it will stay in business forever: with systematic bailouts, a firm knows that it will earn a share of industry profits in the future for sure; absent intervention, after receiving a negative shock, it would go out of business and no longer earn any

¹ Apart from the fiscal burden imposed, bailouts and Government subsidies entail some thorny legal considerations, especially in the European Union (EU), where Article 107 of the Treaty on the Functioning of the EU defines and sets restrictions on “State aid” measures (or Government subsidies) that confer, through public resources, economic advantages to selected entities, affecting trade between EU Member States.
² Beck et al. (2010) discuss in a policy report state-supported schemes for financial institutions and their implications for competition, as well as European competition policy.
³ In the financial sector, for example, intervention (including that of a Central Bank) is often justified on the grounds of preserving the stability of the financial system. Bankruptcies of individual banks may trigger contagion effects across the sector (through the interbank and asset markets) and may also harm consumers directly through the loss of private deposits (subject to national deposit insurance schemes).
profit from that point on. Second, the consequences from deviating from a tacit coordination path are harsher in a scenario with systematic bailouts: absent such bailouts, a firm that has deviated may be “lucky” and face no punishment from its competitors in future periods because (with some probability) such competitors may have received a negative shock and may not be in business at the point when the punishment would take place; such a scenario could not occur in the presence of systematic bailouts, that is, competitors would be in business and would carry out the punishment strategy, according to a standard supergame framework.\(^4\)

We study this problem through an application to the banking sector, as it has recently received much attention due to large bailouts. However, we also show that the model and its implications are general and can be relevant to many industries. Several industry features are sometimes thought to potentially facilitate coordination, such as stable demand, homogeneous products, limited innovation, symmetric cost structures and market transparency.\(^5\) Whether the banking industry actually meets these criteria is an empirical question. Our main insight is that keeping all factors equal, a commitment to bail firms out may facilitate tacit coordination.

We proceed as follows: Section 2 briefly considers the relevant literature. Section 3 presents the main model. Section 4 shows and discusses the results, including the implications for welfare. Section 5 discusses the robustness of the results, after relaxing some of the assumptions made in the main model. Section 6 concludes and offers some possible extensions.

## 2 Related literature

There is not a vast literature on the competitive effects of bailout guarantees or on the economics of State aid as such, somewhat in contrast to the richness of

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4 The results from our main analysis rely on the provision of systematic bailouts. A bailout for systemically important institutions was not always guaranteed in the past. For example, while Bear Stearns received a bailout, Lehman Brothers had to file for bankruptcy in 2008. However, after the Lehman Brothers experience policy-makers became highly reluctant to let a systemically important bank fail. Symptomatically, the European Commission has departed from their principle of “one-time-last-time” aid a number of times (especially so in the latest financial crisis) where ex post this would have been detrimental for the economy (see European Commission (2004) for the principle and European Commission (2008, 2009) for its relaxation). In Section 5, in any event, we show that our results remain valid in the presence of a stochastic bailout regime (i.e. where firms are bailed out only with some probability).

5 A clear exposition of the economics of tacit coordination can be found in Ivaldi et al. (2003).
the literature on subsidies and trade. Bailout guarantees and Government subsidies are criticised by economists, as they may lead to a variety of inefficiencies. In the financial sector, bailout guarantees are sometimes believed to lead to excessive risk-taking and over-investment due to moral hazard problems. Trading off the costs and benefits of financial sector bailouts is complex (see Gale and Vives (2002) and Freixas, Parigi, and Rochet (2004)). However, ex post bailouts may sometimes be justified in the interest of financial stability.

In relation to Government subsidies and their competitive effects more generally, Besley et al. (1999) discuss two broad classes: externalities arising from aid (strategic trade policy, tax competition and economic geography considerations) and inefficient competition between Governments. Dewatripont and Seabright (2006) go beyond intergovernmental issues and build a model where local politicians invest in wasteful projects purely to show their diligence and win votes. Collie (2000) instead proposes an economic explanation of why individual States may have an incentive to subsidise firms with the aim of reducing oligopolistic distortions. He shows that a multilateral institution responsible for prohibiting subsidies can increase welfare.

Friedriszick, Roeller, and Verouden (2008) review the efficiency rationales for aid (tackling market failures such as externalities, public goods, asymmetric information and lack of coordination) as well as equity considerations. They also point towards cross-border (positive) externalities in the case of EU State aid. Their paper then highlights the potential costs of State aid (beyond the direct cost of intervention) such as anti-competitive effects, “picking wrong winners” and international spillover effects. Among the potential distortions of competition, they list the support of inefficient production; the distortion of dynamic (inter-temporal) incentives; the potential increase in market power; and the distortion of production and location decisions across EU countries. Finally, they propose an actual effects-based framework to assess whether particular State aid measures should be approved. Martin and Valbonesi (2008) develop a model of the impact of State aid on market structure and performance in an integrating market (i.e. a common market with increasing trade flows) and find that in equilibrium Governments grant State aid, reducing common market welfare. However, they do not consider tacit coordination.

Hainz and Hakenes (2012) compare the efficiency properties of five options to grant State aid to firms (some of which would also feed through to the banking system). The most efficient option is shown to depend on the tax

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6 An extensive review of the role and the effects of State aid can be found in Nitsche and Heidhues (2006). For an equally policy-oriented approach based on economic theory, the reader is also directed to OFT (2004), Buelens et al. (2007) and Spector (2009).
distortion and the informational cost needed to select the “good” firms. Finally, recent unpublished work by Schinkel and Randag (2012) suggests that tacit coordination may play a role in the banking industry. Schinkel and Randag consider the Dutch mortgage market after State aid was granted to several Dutch commercial banks in 2009. They observe a substantial increase in Dutch mortgage rates against a downward trend of mortgage rates in Europe. The authors argue that coordinated behaviour in the Dutch mortgage market was facilitated because the European Commission imposed price leadership bans on the affected banks as part of their restructuring conditions.\(^7\)

Instead, our model focuses on the effects of bailout guarantees on the incentives of firms to coordinate their behaviour. In the case of banking, it may be beneficial to sacrifice some level of competition in the interest of financial stability.\(^8\) However, this relationship is rather complex and both empirical and theoretical results are far from being clear-cut.\(^9,10\) This debate is nevertheless beyond the scope of our paper, since we only take banking as an example and our model is not sector-specific.

In industrial economics, models of tacit coordination have been applied extensively, but not in the context of systematic bailout guarantees. Our model adopts the standard framework of tacit coordination for repeated oligopoly interaction (originally due to Stigler (1964)).

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7 The effect of price leadership on the viability of coordination, however, is debated in the theoretical literature (see Mouraviev and Rey (2011) who show that price leadership can facilitate coordination).

8 See Carletti (2008) for an excellent survey on this trade-off.

9 Keeley (1990) was the first one to find a positive empirical relationship between more competition and more risk-taking but later studies came to mixed or even opposite results. The earlier theoretical literature mostly gave rationales for Keeley’s findings but also discussed why the opposite may be the case (most prominently Boyd and De Nicolò (2005)). For a theoretical discussion see Allen and Gale (2004). A more recent extensive review is given by Vives (2010).

10 Perotti and Suarez (2002) investigate the impact of competition on banks’ portfolio risk choices. In particular, they examine the relationship between the optimal portfolio risk and banking regulation (merger policy and market entry regulation) in an oligopoly context. The main mechanism in their model is the strategic substitutability between portfolio decisions of duopolistic banks. In particular, a given duopolist has an incentive to invest in the prudent asset if the competitor chooses a risky strategy (since she can expect large monopoly rents if the competitor fails).
3 Model

In this paper we develop a model on the relationship between systematic bailout guarantees and tacit coordination. Given the particular amount of attention received by the financial sector in relation to bailout schemes following the onset of the 2007–2008 financial crisis, our paper offers an application to the banking industry (Section 5.3 shows that the results can be easily transposed to any industry).

The basic building block of our model is Freixas and Rochet’s (2008) extension of the Klein–Monti model to Cournot oligopoly. The original monopolistic model features a single bank facing an upward-sloping demand for deposits and a downward-sloping demand for loans (as developed by Klein (1971) and Monti (1972)).

To shed light on the impact of systematic bailout guarantees on tacit coordination and consumer welfare, our model makes some simplifications with respect to Freixas and Rochet (2008). In our model, the banking industry is characterised by a duopoly competing in the deposit market over an infinite horizon. We consider discrete time. Production (management) costs are normalised to nil. Banks simultaneously set interest rates \( r_1 \) and \( r_2 \), where \( r_1, r_2 > 0 \). The (linear) demand function is given by:

\[
Q(r_1, r_2) = \min\{\bar{Q}, \max\{r_1, r_2\}\},
\]

where \( \bar{Q} > 0 \) is an upper bound on the demand. There is a single asset (project) in which a bank invests its funds. The return to the asset is stochastic. It is subject to the following idiosyncratic shock: it yields net return \( R_H \) (with probability \( p \)) and \( R_L \) (with probability \( 1 - p \)).\(^{11}\) We assume that \( R_H < \bar{Q} \) (this condition ensures that the upper bound on the demand never binds, which simplifies the analysis without qualitatively affecting our main results). For simplicity, and without loss of generality, we normalise \( R_L \) to \(-1\). This means that all funds are lost in the presence of a negative shock and nothing is returned to depositors.

Banks have discount factor \( 0 < \delta < 1 \) and the time line of our game, for each period \( t \in [1, \infty) \), is given in Figure 1. Discount factors can be justified by a positive market interest rate that discounts future payoffs or by a time preference for early payments.\(^{12}\)

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\(^{11}\) In another industry one could argue that firms can be hit with a certain probability in each period by a cost shock or a shock on their production technology that forces them to leave the market.

\(^{12}\) In industrial economics discount factors over infinite-horizon games are often interpreted as the probability that the game will actually be played in a given period, so as to implicitly relax the infinite-horizon interpretation. Note that this is different from the adverse shocks that we introduce (with probability \( 1 - p \)) as these are idiosyncratic.
Every period, bank $i = 1, 2$ maximises profits $\left( \frac{R_i}{C_0 r_i} \right)$ by choosing $r_i$, taking into account the possibility of tacitly coordinating behaviour for high enough discount factors (as in Stigler (1964)).\textsuperscript{13} We assume that the market (deposits and profits) is equally shared by the banks when they set the same interest rate. Importantly, when a bank receives a negative shock ($R_L$), it is forced out of the market (as in Perotti and Suarez (2002)). It goes bankrupt because it cannot repay depositors and its authorisation to operate is not renewed. We make the following assumption:

**Condition 1** $R_H > 4 \frac{1-p}{p}$.

This condition is necessary to ensure positive levels of consumer welfare.\textsuperscript{14} Banks will set interest rates only taking the good state of the world ($R_H$) into account. Let us adopt the superscript $M$ for monopoly (or coordination) and $C$ for competition and denote profits by $\pi$. If a bank is alone in the market, profit maximisation leads to $r^M = R_H/2$, $Q(r^M) = R_H/2$ and $\pi^M = (R_H/2)^2$. Competition, by contrast, leads to $r^C = R_H$, and $Q(r^C) = R_H$ and $\pi^C = 0$. Let us define $W^M = \frac{1}{2}(R_H/2)^2$ as the consumer surplus (or welfare) at the monopoly price level and $W^C = R_H^2/2$ as the consumer surplus under competition.

Another element of our model is a national deposit insurance scheme (NDIS), covering 100% of deposits. That is, when a bank goes bankrupt, depositors are returned their initial investments in full (but without interest). The NDIS

\textsuperscript{13} In general the game has multiple equilibria but we restrict ourselves to the best equilibrium in terms of profit maximisation, i.e. banks coordinate their behaviour for sufficiently high values of $\delta$ and both charge the monopoly interest rate.

\textsuperscript{14} Under reasonable parameter assumptions we have $R_H > 4 \frac{(1-p)}{p}$, e.g. for $p = 0.9$ and $R_H = 0.6$ we have $0.6 > 0.4$ (recall $R_L = -1$, so $R_H = 0.6$ is not particularly high).
is funded by flat-rate taxes. We use the subscript “C” for coordination and “NC” for “no coordination”. The *ex post* loss that has to be covered by the insurance scheme is $\Phi_{C1} = -R_H/4$ whenever one duopolist bank (which was tacitly coordinating with the other bank) fails; $\Phi_{C2} = -R_H/2$ whenever a monopolist bank fails or both banks in a duopoly with tacit coordination fail; $\Phi_{NC1} = -R_H/2$ whenever banks compete and one receives a negative shock; $\Phi_{NC2} = -R_H$ whenever banks compete and both receive a negative shock.

We assume the existence of a NDIS for three reasons. First, it reflects common practice in the banking sector across countries. Second, it enables us to isolate the effect of systematic bailout guarantees on collective competitive behaviour when we compare consumer welfare under a regime of systematic bailouts to that under a regime of no systematic bailouts (that is, we can focus on the welfare effects due to different levels of interest rates arising from different competitive conditions, as opposed to whether depositors get their initial investments back). This follows from the fact that depositors get their deposits fully refunded under both regimes whenever a bank fails, while the financing of the scheme is done by a non-distortionary tax in both regimes. Third, introducing NDIS (which adds and subtracts the same amount from a welfare perspective) makes our model readily comparable to applications to other industries where the failure of a firm does not cause an immediate loss in wealth to its customers.

Figure 2 summarises, graphically, the above discussion. Notice that as the demand schedule is upward-sloping, the various areas in the graph (profit $\pi$, consumer surplus and deadweight loss, or DWL) are inverted (horizontally) with respect
to a traditional diagrammatic analysis of linear demands. For simplicity, we have only depicted the potential loss from bankruptcy (Φ_{C2}) in the case of monopoly.\(^{15}\)

In our model, systematic bailout guarantees (which are financed through lump-sum taxes) operate as follows. First, the Government renews the bank’s authorisation to operate even when the bank has gone bankrupt. Second, the Government incurs a sunk cost \( \gamma \) per bailout. We do not explicitly model how this cost arises but it could be justified, for instance, as the cost of resources devoted by the financial regulator to examine the books of a failing bank and facilitate its rescue. In a non-banking industry this cost could arise through restructuring. Notice that we do not impose any assumption on \( \gamma \). Instead, in Section 4.2, we determine upper bounds on its value such that a systematic bailout policy is welfare-enhancing to consumers.\(^{16}\)

Policy-makers may want to commit to a “one-time-last-time” approach to bailouts or rescue (or restructuring) aid. But in practice there have been several exceptions to this rule, most notably during the financial crisis.\(^{17}\) Moreover, in the financial industry policy-makers often fear potentially strong negative externalities in the short-run (contagion effects, adverse impact on the real economy) if they do not grant bailouts to an insolvent (or illiquid) bank. For this reason policy-makers consider bailouts as being necessary \textit{ex post}.\(^{18}\) Consequently banks that are highly interconnected or “too-big-to-fail” can expect to be bailed out because policy-makers face a time-inconsistency problem. They can hardly make a credible commitment not to bail banks out. We therefore think that our approximation of a systematic bailout regime is adequate for this type of implicit or explicit guarantees in the financial sector.\(^{19}\) However, it can be argued that systemic bailout guarantees are also present in other industries such as the automotive or the airline industries.

\(^{15}\) Notice the analogy to a model of duopolistic competition in a non-banking industry with a downward-sloping demand curve (see also Section 5.3 for a detailed discussion of the applicability to other industries).

\(^{16}\) As noted earlier, the assumption of a 100\% deposit insurance allows us to isolate the effect of systematic bailout guarantees. In practice, typically less than 100\% of deposits are insured. Nevertheless, we rarely observe any losses by unsecured creditors of banks. In other words, bailouts are the norm and bail-ins are the exception. Hence, unsecured creditors enjoy an implicit insurance provided by systematic bailout guarantees. In this vein, we interpret systematic bailout guarantees in our model not as a pure rescue of a failing bank, but as a genuine bailout.

\(^{17}\) See for instance European Commission (2009).

\(^{18}\) See also Gale and Vives (2002) and Lyons (2009) for a discussion of this issue.

\(^{19}\) The existence of implicit guarantees is of course hard to prove empirically. Nevertheless there is some indirect evidence. For example, rating agencies publish “external support” ratings which reflect their expectations on the likelihood of a bailout. See Gropp, Hakenes and Schnabel (2011) for a paper that uses these data to estimate how bank risk-taking behaviour is affected by the presence of a guarantee.
Throughout the paper we focus on the minimal discount factor for which coordination can be sustained and assume that under coordination banks maximise joint profits and share them equally, i.e. banks set the monopoly interest rate. However, we acknowledge that other equilibria (including the competitive equilibrium) always exist. Furthermore, we posit a regime under tacit coordination in which firms adopt a simple trigger strategy by setting the profit-maximising interest rate as long as no competitor deviates and reverting to the competitive interest rate (forever) as soon as one bank has defected from the (tacitly) coordinated strategy. In Section 5 we show that our key insight is robust to a relaxation of this assumption.

4 Results

In this section we first demonstrate that the introduction of systematic bailout guarantees decreases the minimal discount factor for which tacit coordination is viable. This implies that there is a range of values of the discount factor for which the implementation of systematic bailout guarantees can cause a change from a competitive outcome to one with tacit coordination. Second, we examine the welfare implications of this result. We demonstrate that for such range of values of the discount factor, systematic bailout guarantees, by generating tacit coordination, can thus lower consumer welfare as long as the probability of an adverse shock \((1-p)\) is not too high, while they have a positive effect for all other values of the discount factor if the intrinsic cost of bailouts \((\gamma)\) is not too high.

4.1 Systematic bailout guarantees and incentive to coordinate

The first step is to derive the critical discount factors above which tacit coordination is sustainable under each policy regime.

\textit{In the absence of systematic bailout guarantees}, the incentive compatibility constraint (ICC) for tacit coordination – which we fully derive in Appendix A.1 – is as follows:

\[\text{ICC} \]
From here onwards, we use the superscripts “NB” for “no bailouts” and “B” for “bailouts”. Noting that $\pi_C = 0$, tacit coordination can be sustained for all $\delta \geq \delta_{NB} = \frac{1}{2p}$, where $\delta_{NB} \leq 1$ requires $p \geq p = \frac{1}{\sqrt{2}}$.

In the presence of systematic bailout guarantees, by contrast, a bank that has encountered a negative shock is bailed out at no cost to it. There is no profit (nor loss) in the period of failure. Noting once again that the competitive profit is zero ($\pi_C = 0$) every period, the relevant ICC reads

$$\sum_{t=1}^{\infty} \delta^{t-1} \left( p^{2t-1} \frac{\pi_M}{2} + p^t (1 - p^{t-1}) \pi_M \right) \geq \frac{p \pi_M}{2} + \sum_{t=2}^{\infty} \delta^{t-1} \left( p^{2t-1} \pi_C + p^t (1 - p^{t-1}) \pi_M \right).$$

That is, tacit coordination can be sustained for all $\delta \geq \delta_B = \frac{1}{2}$. This is the traditional result obtained in a supergame where symmetric duopolists engage in price competition. The only difference is that both the profit from coordinated behaviour and the deviation profit have to be scaled by the probability $p$ of receiving a positive shock.

**Proposition 1 Systematic bailout guarantees and incentive to coordinate**

In the presence of idiosyncratic shocks, a policy of systematic bailout guarantees that keeps banks (firms) in business after a shock facilitates tacit coordination; that is, tacit coordination can be sustained for lower values of discount factors.

**Proof.** Note simply that $\frac{1}{2} = \delta_B < \delta_{NB} = \frac{1}{2p^2}$, $\forall \ p < 1$. ■

Figure 3 provides a graphical representation of this result. When banks place little value on the future (left half of the chart) tacit coordination cannot be sustained, regardless of whether there are systematic bailout guarantees. When banks care much about the future for a given likelihood of a positive shock, possibility that the deviating bank will actually be a monopolist for some time, as well as the possibility that the deviating firm goes out of business.
(δ ≥ δ^{NB}, i.e. the area at the top-right corner) tacit coordination can be sustained in either regime. Finally, for intermediate values of the discount factor (\frac{1}{2} ≤ δ < δ^{NB}, i.e. the bottom-right area) only the competitive outcome is sustainable in the absence of a systematic bailout policy, while tacit coordination is made possible by a systematic bailout policy.

The intuition for the mechanism at work in the bottom-right area (i.e. the reason for the change in the critical discount factor between the two regimes) is straightforward. In our model, a systematic bailout regime has two main effects on a bank (or firm), both working in the same direction. First, such a policy increases the present value of future profits under coordination. Banks know they will receive a share of monopoly profits forever: it can no longer be the case that they “miss out” because an adverse shock sends them out of business. Second, absent a systematic bailout regime a bank may have an incentive to deviate from the coordinated strategy in order to raise short-term profits in the hope that its competitor will (exogenously) go bankrupt and thus be unable to punish the deviant bank in future periods. But such an incentive would no longer exist in a regime where failing banks are systematically bailed out. Both effects make coordination more attractive than competition for banks (firms), all else equal.

### 4.2 Welfare impact of systematic bailout guarantees

In this section we perform a comparison of consumer welfare levels under the two regimes considered (systematic bailout guarantees and no systematic...
bailout guarantees). First, in Section 4.2.1, we compute the consumer welfare levels absent a systematic bailout guarantee (with and without coordination). Second, in Section 4.2.2, we do likewise, but in the presence of a systematic bailout regime. Third, in Section 4.2.3, we present our findings in Proposition 2 and provide a discussion.

Our key result is straightforward: if the introduction of a bailout regime does not make an industry switch from a competitive outcome to a coordinated one (either because the discount factor was “too low”, so it remains competitive, or because it was “too high”, so there was already coordinated behaviour), such a policy enhances consumer welfare, provided the exogenous bailout costs are not too high. This is because consumers (depositors) will benefit from the continued existence of the industry (as opposed to the banks going bankrupt, at some point). By contrast, for the intermediate range of discount factors identified in the previous section, whereby a systematic bailout regime triggers the incentive for competitive banks to coordinate, such a policy is detrimental for welfare as long as the probability of a negative shock is not too high.

In this section we focus on consumer welfare, as opposed to total welfare. We do so because consumer welfare is the standard typically used by competition authorities in their investigations and in policy-making. Furthermore, in a context with tacit coordination it may be controversial, from a public policy perspective, to include, as part of a welfare measure, the super-normal profits earned. In an extension in Section 5.3, we show that while our intuition was applied to a banking model, it holds in fact for a much more general industry setting with a downward-sloping demand curve. As part of that extension, we also show that a policy that guarantees systematic bailouts is also detrimental for total welfare as long as the probability of a negative shock is not too high.

4.2.1 No systematic bailout guarantees

Under coordination \((\delta \geq \frac{1}{\hat{p}})\): consumer welfare needs to be computed as the weighted average of consumer welfare levels under different scenarios (e.g. duopolistic coordination and monopoly, considering all the possible combinations of events, i.e. positive and negative shocks), where the weights are the probabilities under which each scenario occurs. We provide a full derivation in

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21 In Spector (2009; section 7.4.3) there was also a brief, informal, recognition of the trade-off between benefits consumers may derive from the continuation of a firm’s business and the need to raise tax revenues, as part of an assessment of the welfare implications of rescue or restructuring aid.
Appendix A.2. Present discounted (expected) consumer welfare (under a high enough discount factor to sustain tacit coordination) in the absence of bailout guarantees is given by:

\[ E(W^\text{NB}_C) = \frac{RH \left(1 - \frac{p}{R_H} R_H\right)}{2(1 - \delta p^2)} + \frac{RH \left(\frac{p}{R_H} R_H - 2(1 - p)\right)}{2(1 - \delta p)}, \]  

where subscript “C” stands for “coordination”. Note that Condition 1 ensures positive consumer welfare levels.\(^\text{22}\)

**Under no coordination** (\(\delta < \frac{1}{\delta p}\)): in this case, due to the lower discount factor, there is no tacit coordination by construction, and there can only be either a monopoly (if the competitor has received a negative shock) or a competitive duopoly. In either case, one needs to account for the possibility that positive or negative shocks have occurred. The present discounted (expected) consumer welfare in the absence of bailouts, under a low enough discount factor to guarantee competition,\(^\text{23}\) is given by:\(^\text{24}\)

\[ E(W^\text{NB}_{NC}) = \frac{p(R_H)^2}{4(1 - \delta p^2)} + \frac{RH \left(\frac{p}{R_H} R_H - 2(1 - p)\right)}{2(1 - \delta p)}, \]  

where subscript “NC” stands for “no coordination”.

### 4.2.2 Systematic bailout guarantees

We proceed in the same fashion as in Section 4.2.1. However, here, banks never exit from the market, as they are bailed out whenever they are hit by a negative shock. When consumers’ deposits are lost due to the negative shocks, the Government refunds them the original capital, as well as incurring the bailout costs \(\gamma\). To close the model, we compute the total expected stream of bailout costs and set up a corresponding lump-sum tax (which includes the financing of both the NDIS and the direct bailout costs \(\gamma\)) on consumers, thus reducing their welfare.

\(^\text{22}\) To see this notice that the terms in brackets are negative since \(R_H > 4 \frac{(1-p)}{p}\). Further \(R_H/\delta p^2 < \frac{R_H}{\delta p}\) whenever \(p < 1\). Eventually for expected welfare to be positive we need to have \(R_H/\delta p^2 \left[(1 - p) - \frac{p}{R_H} R_H\right] > -\frac{R_H}{\delta p} \left[\frac{p}{R_H} R_H - 2(1 - p)\right]\), which holds since \(1 - \delta p < 2(1 - \delta p^2)\).

\(^\text{23}\) Of course competition takes place so long as both banks are in business. If only one remains in business, it is assumed to behave monopolistically until it is hit by a negative shock and thus forced out of the market.

\(^\text{24}\) See Appendix A.2 for the derivation. Again, Condition 1 ensures positive consumer welfare levels.
Under coordination \((\delta \geq \frac{1}{2})\): in the presence of systematic bailout guarantees, the present discounted (expected) consumer welfare under a high enough discount factor to sustain tacit coordination is

\[
E(W^B_C) = \frac{R_H (\frac{p}{\delta} R_H - (1 - p))}{2(1 - \delta)} - \frac{2(1 - p) \gamma}{1 - \delta}.
\] [6]

Under no coordination \((\delta < \frac{1}{2})\): in the presence of bailouts, the present discounted (expected) consumer welfare under a discount factor low enough to guarantee competition is

\[
E(W^B_{NC}) = \frac{R_H (pR_H - (1 - p))}{2(1 - \delta)} - \frac{2(1 - p) \gamma}{1 - \delta}.
\] [7]

4.2.3 Welfare results and discussion

Proposition 2 summarises the results on the welfare effects of systematic bailout guarantees.

**Proposition 2 Systematic bailout guarantees and consumer welfare**

(i) In the range of discount factors such that there is competition in absence of systematic bailout guarantees but tacit coordination with systematic bailout guarantees, such a policy reduces present discounted (expected) consumer welfare if the probability of a positive shock is sufficiently high \((p \geq \frac{1}{\sqrt{\gamma}})\). That is, \(E(W^B_C) < E(W^B_{NC}), \forall \frac{1}{2} \leq \delta < \frac{1}{2p}\). This is true even in the absence of direct bailout costs.

(ii) In an environment where duopolistic banks (firms) compete regardless of whether there are systematic bailout guarantees \((\delta < \frac{1}{2})\), such a policy reduces present discounted (expected) consumer welfare if and only if the direct costs of bailing out a bank (firm) exceed \(\gamma_C\).

(iii) In an environment where duopolistic banks (firms) can sustain tacit coordination regardless of whether there are systematic bailout guarantees \((\delta \geq \frac{1}{2p})\), such a policy reduces present discounted (expected) consumer welfare if and only if the direct costs of bailing out a bank (firm) exceed \(\gamma_{NC}\).

**Proof.** The proof of (i) is in Appendix A.4. To see why (ii) and (iii) hold, we compute the upper limits on \(\gamma\) such that the present discounted (expected) consumer welfare under systematic bailout guarantees is larger than under no systematic bailout guarantees, in the cases where public intervention does not
affect the competitive state of the industry. Call $\gamma_{NC}$ this limit value of $\gamma$ for the case of no coordination in both policy regimes, i.e. for $\delta < \frac{1}{2}$:

$$E(W^B_{NC}) \geq E(W^{NB}_{NC}) \iff \gamma \leq \gamma_{NC} = \frac{1 - \delta}{2(1 - p)} \left( \frac{R_H(pR_H - 2(1 - p))}{2(1 - \delta)} - E(W^{NB}_{NC}) \right)$$

[8]

In Appendix A.3 we show that $\gamma_{NC} \geq 0$. Next we compute the upper limit $\gamma_C$ corresponding to the case of coordination in both regimes, i.e. for $\delta \geq \frac{1}{2p}$:

$$E(W^B_C) \geq E(W^{NB}_C) \iff \gamma \leq \gamma_C = \frac{1 - \delta}{2(1 - p)} \left( \frac{R_H(pR_H - (1 - p))}{2(1 - \delta)} - E(W^{NB}_C) \right)$$

[9]

Again we show in Appendix A.3 that $\gamma_C \geq 0$. ■

Proposition 2 introduces a simple dichotomy between two scenarios that summarises the effect of systematic bailout guarantees on consumer welfare. In one scenario (capturing both cases ii and iii above), systematic bailout guarantees do not influence the competitive state of the industry and are therefore welfare-improving as long as their intrinsic cost is not too large. Indeed, in such a case, the only effect of systematic bailout guarantees is to preserve the very existence of the market, which is beneficial to both firms and consumers. In the other scenario (case i), where systematic bailout guarantees affect the competitive state of the industry by triggering coordination, we find that its overall effect on consumer welfare is guaranteed to be negative if the probability of a positive shock is sufficiently high. A sufficient condition is given by $p \geq \frac{R_H}{\sqrt{7}} \approx 0.76 > p \approx 0.71$. This means that in such a scenario, the adverse coordination-facilitating effect of bailouts dominates its beneficial market-preserving effect. The dominance of the first force over the second force does not appear a priori self-evident. We attribute this feature to the fact that this scenario corresponds to relatively low values of the discount factor. For such values of the discount factor and as long as the probability of failures $1 - p$ is not too high, the adverse price effect of coordination thus dominates the long-run positive benefits related to the preservation of the market.

---

25 Recall from Section 4.1 that $p \geq \frac{1}{\sqrt{7}}$ is required for $\delta^{NB} \leq 1$. The sufficient condition, though more demanding than needed, is not very restrictive as it allows for relatively high failure probabilities due to a negative shock.
Figure 4: Effect of a systematic bailout policy on consumer welfare, by discount factor

Figure 4 presents our main welfare result graphically (for simplicity, it abstracts from the cost $\gamma$ of bailing out a bank). The horizontal axis corresponds to the discount factor $\delta$, and the vertical axis measures present discounted (expected) consumer welfare. Three relevant regions can be identified. In the left region ($\delta < \frac{1}{2}$) banks compete with each other regardless of whether there are systematic bailout guarantees. In the right region ($\delta \geq \frac{1}{2}$) tacit coordination can be sustained in either regime. However, in the central region, tacit coordination can only be sustained in the presence of systematic bailout guarantees. In the left and right regions, systematic bailout guarantees can be consumer welfare-enhancing, and this depends on whether the vertical distance between the broken and the solid line (for any given discount factor) exceeds the (expected tax bill due to the) direct costs of bailing out banks $\gamma$. In the central region, by contrast, systematic bailout guarantees decrease consumer welfare.

5 Discussion and robustness

Our main insights are robust to several important model variations. In Section 5.1 we demonstrate that the intermediate range of discount factors underlying the results in Propositions 1 and 2 increases for important modifications of the model (modified trigger strategies and interest rate under coordination). Next, Section 5.2
analyses the robustness of the intermediate range of discount factors to several other variations of the model, showing that the range continues to exist (though it may decrease). Finally, in Section 5.3, we jointly demonstrate the applicability of our model to a more general industry setting with a downward-sloping demand curve, as well as the robustness of our key insight when considering total welfare instead of consumer welfare by revisiting the results of Proposition 2.

5.1 Model variations where the intermediate range of discount factors increases

The intermediate range of discount factors where our result arises is expanded in the case of two variations of our model: when considering trigger strategies with a finite punishment phase of \( T \geq 1 \) periods and when relaxing the assumption that the interest rate under tacit coordination is set at the monopolistic level. Proposition 3 summarises such findings.

**Proposition 3 Robustness of the existence of a range of discount factors under which a systematic bailout regime generates an incentive to coordinate**

The intermediate range of discount factors expands when: (i) the length \( T \geq 1 \) of the punishment phase is reduced:

\[
\frac{d(\delta^{NB}(T) - \delta^B(T))}{dT} < 0, \tag{10}
\]

or when (ii) the interest rate on deposits under tacit coordination is higher than the monopoly interest rate, i.e. \( r = r^M + \varepsilon \) where \( \varepsilon \in (0, \sqrt{(2p^2 - 1)(R^M)^2}) \):

\[
\frac{d(\delta^{NB}(\varepsilon) - \delta^B(\varepsilon))}{d\varepsilon} > 0. \tag{11}
\]

**Proof.** See Appendix A.5. \( \blacksquare \)

Intuitively, coordination is harder to sustain when considering different trigger strategies where the punishment phase is not infinite, but lasts for \( T \geq 1 \) periods. As a result, the critical discount factor increases under both regimes. However, \( \delta^{NB} \) increases by more than \( \delta^B \) and, hence, the intermediate range of discount factors where our qualitative result persists increases. Instead, a relaxation of the assumption that the interest rate under tacit coordination is equal to the monopolistic rate leads to a reduction of the present value of expected profits under coordination and makes coordination harder to sustain. Again,
the intermediate range of discount factors for which our result arises increases (the upper bound on $\varepsilon$ merely ensures that $\delta^{NB}(\varepsilon) < 1$).

5.2 Robustness of the existence of the intermediate range of discount factors

In what follows, we consider three variations of our model and prove that while the precise range of intermediate discount factors $[\delta^B, \delta^{NB}]$ decreases, it is never an empty set. Our qualitative result thus survives.

First, consider a variant of the model where the profit from coordinated behaviour is asymmetrically shared. In this scenario coordination is only sustainable if also the firm with the lower profit share from coordinated behaviour does not have an incentive to deviate. Let $f \leq \frac{1}{2}$ be the fraction of profits from coordinated behaviour going to the firm with the lower profit share, then $\delta^B = f < \delta^{NB} = \frac{f}{p}$. If $f$ decreases (firms become more asymmetric) and the intermediate range of discount factors decreases:

$$\frac{d(\delta^{NB}(f) - \delta^B(f))}{df} > 0.$$  \[12\]

Second, consider a variant of the model where bailouts are stochastic. Our qualitative results continue to hold. The only difference is that the range of discount factors where our result arises becomes narrower, because the regime with bailouts becomes more similar to the regime without bailouts. Suppose that banks are bailed out with probability $0 \leq q \leq 1$ conditional on failing. Then $\delta^B(q) = \frac{1}{2(p+(1-p)q^2)} \in [\delta^B, \delta^{NB}]$ and:

$$\frac{d\delta^B(q)}{dq} < 0.$$  \[13\]

See Appendix A.6 for the derivations.

Third, consider relaxing the assumption that bailouts are unconditional. In particular, assume that the bailout cost has to be financed by firms who have to pay a fixed proportion of their future profits in order to be eligible for a bailout. This leads to a reduction of the expected profits under coordination, while expected profits in the punishment phase stay unaltered (assuming that firms with zero profits cannot be taxed).\textsuperscript{26} As a result, with bailouts, coordination is

---

\textsuperscript{26} We are aware that such a policy rule could not work in practice or at least should not be interpreted literally. If a policy-maker believed in this framework, knew the model parameters and observed payment of the tax, she would deduce that firms have been coordinating their behaviour.
now harder to sustain than before (i.e. $\rho^{B}$ increases). Still, as long as the bailout cost $\gamma$ is not too high our key insight prevails. Interestingly, bailout guarantees can be re-interpreted as a market-based insurance mechanism. In other words, firms may find it profitable to create an insurance fund that supports ailing firms as long as the bailout costs are not too high. Our paper suggests that such endogenous insurance arrangements can facilitate coordination.

5.3 Applicability to other industries and total welfare standard

In this section we simultaneously discuss two results. First, we demonstrate the applicability of our model to a more general industry setting with a downward-sloping demand curve. Second, we show the robustness of our key insight when considering total welfare instead of consumer welfare in the more general industry setting.

In Section 3, we argued that the mechanism we describe can readily be transposed to other industries where systematic Government intervention is prevalent. In a non-banking industry, the idiosyncratic shocks to firms could be negative shocks to their production technology that force them to leave the market. A few differences arise with respect to the banking setup. First, the computations are slightly altered because a non-financial firm can gain the full market share in the period where the rival fails. Second, the welfare levels associated with some of the events differ because of the absence of immediate losses in wealth for consumers after a failure of a non-financial firm.

We discuss the following general industry setting. Consider a non-banking industry with a duopoly, idiosyncratic shocks, zero marginal costs and a linear downward-sloping demand that is the equivalent of Figure 2 (i.e. rotate the demand curve in Figure 2 by $90^\circ$ anti-clockwise, swap the DWL and the consumer surplus areas, set the intercept to $Q=R_H$ and replace $r$ with $p$, $r^C=R_H$ with $p^C=0$, and $r^M=R_H/2$ with $p^M=R_H/2$). The result of Proposition 1 is unaltered. However, the result of Proposition 2 is modified. In particular, for the non-banking industry case (i) of Proposition 2 changes. Let $TW$ denote total welfare and define $p_0 = \sqrt[5]{3/5} \approx 0.77$ and $p'' \approx 0.87$.

---

27 This was not possible in the banking industry, where the deposits are collected at the beginning of the period.
Proposition 4 Systematic bailout guarantees in a non-banking industry and total welfare

In the intermediate range of discount factors, systematic bailout guarantees reduce:

(a) present discounted (expected) consumer welfare if the probability of a positive shock is sufficiently high \( p \geq p' \). That is, \( E(W_C^B) < E(W_{NC}^{NB}) \), \( \forall \frac{1}{2} \leq \delta < \frac{1}{2p^0} \).

(b) present discounted (expected) total welfare if the probability of a positive shock is sufficiently high \( p \geq p'' \). That is, \( E(TW_C^B) < E(TW_{NC}^{NB}) \), \( \forall \frac{1}{2} \leq \delta < \frac{1}{2p^0} \).

This is true even in the absence of direct bailout costs.

Proof. The proof is in Appendix A.7. ■

Intuitively, we need that the probability of failure, \( 1 - p \), is not too high. Otherwise, the adverse coordination-facilitating effect of bailouts is dominated by its beneficial market-preserving effect. Different to our main analysis in Section 4, the market-preserving effect is stronger in the total welfare analysis. Now the market-preserving effect shows up not only as preserving future consumer surplus but also as preserving future producer surplus in the event of negative shocks to both firms. Hence, the sufficient condition on the lower bound for the probability of a positive shock is more restrictive when considering total welfare.

6 Conclusion

The literature on Government subsidies and the related policy debate have typically focused on the adverse efficiency effects of such policies (misallocation of resources, moral hazard) and on countervailing arguments typically (though not exclusively) based on social policy.

This paper has developed a simple infinite-horizon model that sheds light on a result that to our knowledge had not been identified before: a Government policy aimed at systematically bailing out firms in the presence of negative idiosyncratic shocks facilitates (tacit) coordination. This is because expected future profits from coordination increase (since firms are guaranteed to be in business in future periods); and because the guaranteed presence of competitors in the next periods makes the (expected) punishment phase harsher than in an
environment where competitors may exit the market due to an exogenous shock (which would leave the deviant firm unpunished).

Examining the implications of this result on the welfare effects of systematic bailout guarantees, our main result is the identification of a range of discount factors for which a systematic bailout policy is coordination-facilitating. In this range coordination resulting from a systematic bailout policy is always detrimental for welfare. In the real world, though, this link would need to be examined empirically and regulators, policy-makers and courts would have to assess this on a case-by-case basis.

As shown in Section 5, the main mechanism can be generalised in several ways and is robust to a number of alternative assumptions about the industry environment and the Government policy chosen. This paper can set the stage for interesting extensions. One possible direction is to devise more complex bailout policies (e.g. with repayments or limited to the last failing firm) or to consider a more complex competitive setup (e.g. introducing asymmetries or a richer menu of contracts). Another possible extension would be to embed a genuine banking model within our framework. For instance one could endogenise portfolio choice and model an interbank (wholesale) market. Or one could examine whether the traditional result that bailout guarantees typically induce banks to take excessive risk would still hold in an environment where excessive risk-taking may be mitigated by coordinated behaviour (since higher profits may reduce moral hazard).

Appendix

A.1 Derivation of the critical discount factors

A.1.1 No systematic bailout guarantees

We start by deriving the expected profit from coordinated behaviour (LHS of the ICC). In each period $t$, a bank gets profits from coordinated behaviour $\delta_t \pi_M$ with probability $p_1(t)$; monopoly profits $\delta_t \pi_M$ with probability $p_2(t)$; and 0 with probability $p_3(t)$. The respective probabilities can be written as follows:

\[ p_1(t) = \frac{C_0}{C_0^*} \]

\[ p_2(t) = \frac{\delta_t \pi_M}{C_0^*} \]

\[ p_3(t) = 1 - p_1(t) - p_2(t) \]

Notably the probability of the competitor being in the market in period $t$ can be computed as $p^{t-1}$. The fact that the competitor might have to leave the market in period $t$ does not affect the profits of the other bank in period $t$.28
\[ p_1(t) = p'(p^{t-1}) = p^{2t-1} \]

\[ p_2(t) = p'[1 - p(1 - p) + p^2(1 - p) + \ldots + p^{t-2}(1 - p)] \]

\[ = p'(1 - p) \frac{1 - p^{t-1}}{1 - p} = p'(1 - p^{t-1}) \]

\[ p_3(t) = 1 - p^{2t-1} - p'(1 - p^{t-1}) = 1 - p'. \]

This yields

\[ \text{LHS} = \sum_{t=1}^{\infty} \delta^{t-1} \left( p^{2t-1} \frac{\pi^M}{2} + p'(1 - p^{t-1}) \pi^M \right) \]

\[ = \left( \frac{p}{2(1 - \delta p^2)} + \frac{p}{1 - \delta p} - \frac{p}{1 - \delta p^2} \right) \pi^M = \left( \frac{p}{1 - \delta p} - \frac{p}{2(1 - \delta p^2)} \right) \pi^M. \]

Next, we turn to the right-hand side of ICC, i.e. the immediate deviation profit (obtained with probability \( p \) since the shock occurs after the interest rate decision) plus the expected punishment stream from the following period onwards. The former profit is simply \( p\pi^M \). As for the latter, there are four possible events at each time \( t \geq 2 \):

1. Both banks are in the market at the beginning of the period, and the deviating bank has a positive shock in that period: \( p_4(t) = p^{2(t-1)} \ast \pi \).

2. Both banks are in the market at the beginning of the period, and the deviating bank has a negative shock in that period: \( p_5(t) = p^{2(t-1)} \ast (1 - p) \).

3. The deviating bank will be in the market at time \( t \geq 2 \) and earn monopoly profit alone:

\[ p_6(t) = p'[1 - p(1 - p) + p^2(1 - p) + \ldots + p^{t-2}(1 - p)] \]

\[ = p'(1 - p) \frac{1 - p^{t-1}}{1 - p} = p'(1 - p^{t-1}). \]

4. The deviating bank will not be in the market: \( p_7(t) = 1 - p^{2(t-1)} - p'(1 - p^{t-1}). \)

However, it is only \( p_6(t) \) that is associated with a non-zero payoff (\( p_4(t) \) and \( p_5(t) \) are associated with \( \pi^C = 0 \) and \( p_7(t) \) to previous exit). Thus, adding over \( t \):

\[ \text{RHS} = p\pi^M + \sum_{t=2}^{\infty} \delta^{t-1} p'(1 - p^{t-1}) \pi^M = p\pi^M + \frac{\delta p^2}{1 - \delta p} \pi^M - \frac{\delta p^3}{1 - \delta p^2} \pi^M. \]
Constructing the overall ICC by comparing LHS against RHS (i.e. eq. [2]), solving for $\delta$ and noticing that $\pi^M$ falls through, one gets that tacit coordination is sustainable if:

$$\left(\frac{1}{1 - \delta p} - \frac{1}{2(1 - \delta p^2)}\right)p\pi^M \geq \left(1 + \frac{\delta p}{1 - \delta p} - \frac{\delta^2 p^2}{1 - \delta^2 p^2}\right)p\pi^M$$

i.e. $\delta \geq \frac{1}{2p^2}$.

### A.1.2 Systematic bailout guarantees

With systematic bailout guarantees a bank that has received a negative shock is rescued and allowed to operate in the following period. There is no profit (nor actual loss) in the period of failure. Setting up the ICC and solving for the critical discount factor, we obtain the traditional supergame result in a symmetric price-setting duopoly. Coordination is sustainable if:

$$p \sum_{t=1}^{\delta t-1} \frac{\pi^M}{2} + (1-p) \sum_{t=1}^{\delta t-1} * 0 \geq p\pi^M + (1-p) * 0 + p \sum_{t=2}^{\delta t-1} * \pi^E + (1-p) \sum_{t=2}^{\delta t-1} * 0$$

i.e. $\delta \geq \frac{1}{2}$.

### A.2 Derivation of the consumer welfare equations

#### A.2.1 No systematic bailout guarantees

**Under coordination** ($\delta \geq \frac{1}{2p}$): From a consumer welfare perspective, there are six possible states at time $t$: (I) there is a duopoly, banks coordinate their behaviour, and the deposits are returned with interest by both banks; (II) there is a duopoly, banks coordinate their behaviour, and all deposits are lost because of the negative shocks to both duopolists; (III) there is a duopoly, banks coordinate their behaviour, and only one bank receives a negative shock; (IV) there is a monopoly and the deposits are returned with interest; (V) there is a monopoly and the deposits are lost because of a negative shock; (VI) there is no
market at all (banks have exited and consumer welfare is nil). The respective probabilities are

$$p_I(t) = p^{2t}$$

$$p_{II}(t) = p^{2(t-1)}(1-p)^2$$

$$p_{III}(t) = 2\left(p^{2(t-1)}p(1-p)\right) = 2\left(p^{2(t-1)}(1-p)\right)$$

$$p_{IV}(t) = 2*\left[p^{t-1}((1-p) + p(1-p) + p^2(1-p) + \ldots + p^{t-2}(1-p))\right] * p = 2p^t(1-p^{t-1})$$

$$p_{V}(t) = 2*\left[p^{t-1}((1-p) + p(1-p) + \ldots + p^{t-2}(1-p))\right] * (1-p) = 2p^{t-1}(1-p)(1-p^{t-1})$$

$$p_{VI}(t) = 1 - (p_I(t) + p_{II}(t) + p_{III}(t) + p_{IV}(t) + p_{V}(t)).$$

The next step is to assign consumer welfare values to each state:

**I:**  $$W_C = W^M = \frac{1}{2}\left(\frac{R_H}{2}\right)^2$$

**II:**  $$\Phi_{C2} = -\frac{R_H}{2}$$

**III:**  $$\frac{W^M}{2} + \Phi_{C1} = \frac{1}{4}\left(\frac{R_H}{2}\right)^2 - \frac{R_H}{4}$$

**IV:**  $$W^M = \frac{1}{2}\left(\frac{R_H}{2}\right)^2$$

**V:**  $$\Phi_{C2} = -\frac{R_H}{2}$$

**VI:**  0.

Next, we simply sum up these welfare levels (adjusted by the probabilities) over time, accounting for the discount factors. Notice that the sum of the losses is the same as the total size of the NDIS and thus the expected present discounted value of total taxes in the economy, which thus enter as negative terms ($\Phi < 0$):

$$E(W_C^{NB}) = \sum_{t=1}^{\infty} \delta^{t-1}\left\{p^{2t} * W^C + p^{2(t-1)}(1-p)^2 * \Phi_{C2} + 2(p^{2t-1}(1-p)) * \left(\frac{W_C^M}{2} + \Phi_{C1}\right)\right\} +$$

$$+ \sum_{t=1}^{\infty} \delta^{t-1}\left\{2p^t(1-p^{t-1}) * W^M + 2p^{t-1}(1-p)(1-p^{t-1}) * \Phi_{C2}\right\}$$

$$= \frac{R_H/2}{1 - \delta p^2} \left[1 - p\right] + \frac{R_H/2}{1 - \delta p} \left[p R_H - 2(1-p)\right].$$

**Under no coordination ($\delta < \frac{1}{2p}$):** From a consumer welfare perspective, there are again six possible states at time $t$: (I) there is a competitive duopoly and the deposits are returned with interest by both banks; (II) there is a competitive duopoly and all deposits are lost because of the negative shocks to both duopolists; (III) there is a competitive duopoly and only one bank receives a negative
shock; (IV) there is a monopoly and the deposits are returned with interest; (V) there is a monopoly and the deposits are lost because of a negative shock; (VI) there is no market at all (banks have exited and consumer welfare is nil). These events occur, respectively, with the same probabilities $p_I(t)$ through $p_{VI}(t)$ that we discussed above; it is just that “duopoly, banks coordinate their behaviour” has to be replaced with “competitive duopoly”. However, the welfare levels associated with each state are different:

- $I: W_{NC}^C = \frac{R_H^2}{2}$
- $II: \Phi_{NC2} = -R_H$
- $III: \frac{W_{NC}^C}{2} + \Phi_{NC1} = \frac{R_H^2}{4} - \frac{R_H}{2}$
- $IV: W^M = \frac{1}{2} \left( \frac{R_H}{2} \right)^2$
- $V: \Phi_{C2} = -\frac{R_H}{2}$
- $VI: 0$

We sum again these probability-adjusted welfare levels over time, to obtain:

$$E(W_{NC}^{NB}) = \sum_{t=1}^{\infty} \delta^{t-1} \left\{ p^{2t} \cdot W_{NC}^C + p^{2(t-1)}(1-p)^2 \cdot \Phi_{NC2} + 2(p^{2t-1}(1-p)) \cdot \left( \frac{W_{NC}^C}{2} + \Phi_{NC1} \right) \right\}$$

$$+ \sum_{t=1}^{\infty} \delta^{t-1} \left\{ 2p^t(1-p^{t-1}) \cdot W^M + 2p^{t-1}(1-p)(1-p^{t-1}) \cdot \Phi_{C2} \right\}$$

$$= \frac{R_H}{2} \left( \frac{p}{2} R_H \right) + \frac{R_H}{1 - \delta p} \left[ \frac{p}{2} R_H - 2(1-p) \right].$$

### A.2.2 Systematic bailout guarantees

We proceed in the same fashion as before. However, here, banks never exit. When consumers’ deposits are lost due to the negative shocks, the Government refunds them the original capital, as well as paying the bailout costs $\gamma$.

**Under coordination ($\delta \geq \frac{1}{2}$):** There are three scenarios that can characterise the economy at any period $t$: (I) both banks have a positive shock and return deposits with interest (which occurs with probability $\hat{p}_I(t) = p^2$); (II) only one bank receives a negative shock (probability $\hat{p}_{II}(t) = 2p(1-p)$); (III) both banks receive a negative shock ($\hat{p}_{III}(t) = (1-p)^2$). This is true every period, and each state is associated with the following welfare levels:
We can therefore sum this stream of expected payoffs and then subtract the present discounted value of the total tax bill (NDIS and rescue costs):

\[
E(W_C^B) = \sum_{t=1}^{\infty} \delta^{-1} \left\{ p^2 W_M^2 + 2p(1-p) \left( \frac{W_M^2}{2} + \Phi_{C1} - \gamma \right) + (1-p)^2 \left( \Phi_{C2} - 2\gamma \right) \right\} = \frac{R_H/2}{1-\delta} \left[ pR_H + p - 1 \right] - \frac{2(1-p)\gamma}{1-\delta}. 
\]

**Under no coordination** \((\delta < \frac{1}{2})\): We proceed exactly as in the case of coordination. The probabilities are the same as those derived above, but the associated welfare levels are different:

\[
I : \quad W_{NC}^C = \frac{R_H^2}{2} \\
II : \quad \frac{W_{NC}^C}{2} + \Phi_{NC1} - \gamma = \frac{R_H^2}{4} - \frac{R_H^2}{2} - \gamma \\
III : \quad \Phi_{NC2} - 2\gamma = -R_H - 2\gamma.
\]

Summing up over time:

\[
E(W_{NC}^B) = \sum_{t=1}^{\infty} \delta^{-1} \left\{ p^2 W_{NC}^C + 2p(1-p) \left( \frac{W_{NC}^C}{2} + \Phi_{NC1} - \gamma \right) + (1-p)^2 \left( \Phi_{NC2} - 2\gamma \right) \right\} = \frac{R_H/2}{1-\delta} \left[ pR_H - 2(1-p) \right] - \frac{2(1-p)\gamma}{1-\delta}.
\]

### A.3 On the direct bailout costs

In this section we show the non-negativity of \(\gamma_{NC}\) and \(\gamma_C\). As for the threshold \(\gamma_{NC}\):

\[
\gamma_{NC} = \frac{1 - \delta}{2(1-p)} \left\{ \frac{R_H}{1-\delta} \left[ pR_H - 2(1-p) \right] - \frac{R_H}{1-\delta} \left[ pR_H \right] - \frac{R_H}{1-\delta} \left[ pR_H - 2(1-p) \right] \right\} \geq 0
\]

or

\[
pR_H \left( \frac{1}{1-\delta} - \frac{1/2}{1-\delta p^2} - \frac{1/2}{1-\delta p} \right) + 2(1-p) \left( \frac{1}{1-\delta} + \frac{1}{1-\delta p} \right) \geq 0.
\]
It can be shown that the first term is larger in absolute terms. As a result the threshold is positive. To see this remember (Condition 1) that $RH > 4 \frac{1-p}{p}$. Consequently we have that $pRH > 2(1-p)$. Further notice that
\[
\frac{1}{1-\delta} - \frac{1/2}{1-\delta p^2} - \frac{1/2}{1-\delta p} > \left(- \frac{1}{1-\delta} + \frac{1}{1-\delta p}\right)
\]
which gives us the result that $\gamma_{NC}$ is positive. As for the threshold $\gamma_C$:
\[
\gamma_C = \frac{1-\delta}{2(1-p)} \left\{ \frac{RH}{2} \left(\frac{1}{1-\delta} - \frac{p}{4} R_H - (1-p)\right) - \frac{RH}{2} \left(\frac{1}{1-\delta p^2} - \frac{p}{4} R_H\right) - \frac{RH}{2} \left(\frac{p}{2} R_H - 2(1-p)\right) \right\} \geq 0
\]
or
\[
\frac{p}{4} R_H \left[ \frac{1}{1-\delta} + \frac{1}{1-\delta p^2} - \frac{1}{1-\delta p} \right] + (1-p) \left[ \frac{1}{1-\delta} - \frac{1}{1-\delta p} + \frac{2}{1-\delta p} \right] \geq 0.
\]
For the same argument as before we have $\frac{p}{4} R_H > (1-p)$. Moreover
\[
\frac{1}{1-\delta} + \frac{1}{1-\delta p^2} - \frac{1}{1-\delta p} > \left[- \frac{1}{1-\delta} - \frac{1}{1-\delta p} + \frac{2}{1-\delta p}\right]
\]
which is true since $\delta p < 1$, hence $\gamma_C$ is positive.

A.4 Proof of (i) in Proposition 2

We show that:
\[
E(W_C^B) = \frac{RH}{2} \left[\frac{1}{1-\delta} R_H + p - 1\right] - \frac{2(1-p)\gamma}{1-\delta} < 0
\]
\[
E(W_{NC}^B) = \frac{RH}{2} \left[\frac{p}{2} R_H\right] + \frac{RH}{2} \left[\frac{p}{2} R_H - 2(1-p)\right]
\]
or
\[
\frac{p}{4} R_H - (1-p) < \frac{1-\delta}{1-\delta p^2} \left[\frac{p}{2} R_H\right] + \frac{1-\delta}{1-\delta p} \left[\frac{p}{2} R_H - 2(1-p)\right].
\]

Note that RHS of inequality [14] is continuous and decreasing in $\delta$ if $\frac{p}{4} R_H + p - 1 > 0$, which holds by assumption. To see this take the derivatives:
\[
\frac{\partial}{\partial \delta} \left(\frac{1-\delta}{1-\delta p^2}\right) = -\frac{(1-\delta p^2) + (1-\delta)p^2}{(1-\delta p^2)^2} = \frac{p^2 - 1}{(1-\delta p^2)^2} < 0
\]
\[
\frac{\partial}{\partial \delta} \left(\frac{1-\delta}{1-\delta p}\right) = -\frac{(1-\delta p) + (1-\delta)p}{(1-\delta p)^2} = \frac{p^2 - 1}{(1-\delta p)^2} < 0.
\]
Yielding:

$$\frac{\partial \text{RHS}}{\partial \delta} = \frac{p^2 - 1}{1 - \delta p^2} \left[ \frac{p - 1}{2} \frac{R_H}{\delta} \right] + \frac{p - 1}{1 - \delta p^2} \left[ \frac{p - 1}{2} \frac{R_H + 2p - 2}{\delta} \right] < 0.$$  

Consequently the RHS of eq. [14] is smallest for the largest value of $\delta$ in the given range, which is $\delta^{NB} = \frac{1}{2p^2}$. Furthermore, note that inequality [14] holds for $R_H$ close to its minimum permissible value from Condition 1: $R_H > 4 \frac{1-p}{p}$.

Next, the LHS and RHS of inequality [14] are continuous and increasing in $R_H$. Given,

$$\frac{1 - \delta}{1 - \delta p^2} < \frac{1 - \delta}{1 - \delta p},$$

the RHS of inequality [14] is guaranteed to increase faster in $R_H$ than the LHS if:

$$\frac{p}{4} R_H - (1 - p) < \frac{1 - \delta^{NB}}{1 - \delta^{NB} p^2} \left[ \frac{p}{2} R_H \right] + 2 \frac{1 - \delta^{NB}}{1 - \delta^{NB} p^2} \left[ \frac{p}{4} R_H - (1 - p) \right],$$

which is guaranteed to hold if:

$$\frac{1}{4} < \frac{1 - \frac{1}{2p^2}}{1 - \frac{1}{2p^2} p^2} \iff p > \frac{2}{\sqrt{7}}.$$

As a result, inequality [14] is guaranteed to hold for all permissible values of $R_H$ if the probability of a positive shock is sufficiently high. We thus reach the result stated in Proposition 2(i): $E(W_B^C) < E(W_{NB}^N), \forall p \in (\frac{2}{\sqrt{7}}, 1)$ and $\frac{1}{2} \leq \delta < \frac{1}{2p^2}$.

### A.5 Proof of Proposition 3

Part (i) of Proposition 3 is proven in Section A.5.1 and part (ii) in Section A.5.2.

#### A.5.1 Different trigger strategies

From the modified ICCs we can derive $\delta^B(T)$:

$$p \sum_{t=1}^{\infty} \frac{1 - \delta^{t-1} \pi^M}{2} \geq p \pi^M + p \left( \sum_{t=2}^{T+1} \delta^{t-1} \delta \left( \sum_{t=T+2}^{\infty} \frac{1 - \delta^t \pi^M}{2} \right) \right)$$

$$\delta^B(T) \text{ solves } \frac{1 - \delta^{T+1}}{1 - \delta} = 2$$

and $\delta^{NB}(T)$:
\[
\sum_{t=1}^{\infty} \delta^{t-1} \left( \frac{p^{2t-1} \pi^M}{2} + p'(1 - p^{t-1}) \pi^M \right) \geq p\pi^M + \sum_{t=2}^{\infty} \delta^{t-1} \left( p'(1 - p^{t-1}) \pi^M + p^{2t-1} \pi^M \right)
\]

\[
\delta^N(T) \quad \text{solves} \quad \frac{1 - (\delta^p)^{T+1}}{1 - \delta^p} = 2.
\]

[16]

It can be proven in three steps that \( \frac{d(\delta^N(T) - \delta^B(T))}{dT} < 0 \).

**Step 1:** Using the implicit function theorem:

\[
\frac{d\delta^B(T)}{dT} = \frac{(\delta^p)^{\ln(\delta)}}{-T(1 - \delta^p) - 1 + \delta^{-T}}
\]

and

\[
\frac{d\delta^N(T)}{dT} = \frac{(\delta^p)^{\ln(\delta^p)}}{-T(1 - \delta^p) - 1 + (\delta^p)^{-T}}.
\]

Given the conjecture that \( \delta^p > \frac{1}{2} \) and, hence, \( \ln(\delta^p) > -\frac{1}{2} \), we have that \( \frac{d\delta^B(T)}{dT} < 0 \) and \( \frac{d\delta^N(T)}{dT} < 0 \). This is because the nominators of eqs [17] and [18] are negative, and the denominators are positive. The former is immediate, and the latter is proven below. As a result, the conjecture that \( \delta^p > \frac{1}{2} \) holds is confirmed because \( \delta^N(T) > \frac{1}{2} \delta^p \quad \forall \ T \in [1, \infty) \). The proof that the denominators of eqs [17] and [18] are positive follows from a continuity argument. First, notice that:

\[
(-T(1 - \delta^p) - 1 + (\delta^p)^{-T})|_{T=1} = -2 + \delta^p + (\delta^p)^{-1} > 0 \quad \text{since} \quad 0 < \delta^p < 1.
\]

Second:

\[
\frac{d(-T(1 - \delta^p) - 1 + (\delta^p)^{-T})}{dT} = -1 + \delta^p - (\delta^p)^{-T} \ln(\delta^p) > 0
\]

because:

\[
\frac{d(-1 + \delta^p - (\delta^p)^{-T} \ln(\delta^p))}{dT} = (\delta^p)^{-T} (\ln(\delta^p))^2 > 0
\]

and:

\[
(-1 + \delta^p - (\delta^p)^{-T} \ln(\delta^p))|_{T=1} = -1 + \delta^p - (\delta^p)^{-1} \ln(\delta^p) > 0 \quad \forall \frac{1}{2} \leq \delta^p < 1
\]

since:

\[
(-1 + \delta^p - (\delta^p)^{-1} \ln(\delta^p))|_{(\delta^p) \to \frac{1}{2}} > 0
\]
Hence, by continuity \((-1 + \delta p^2 - (\delta p^2)^{-1} \ln(\delta p^2)) = 0\)
\[
\frac{d(-1 + \delta p^2 - (\delta p^2)^{-1} \ln(\delta p^2))}{d(\delta p^2)} = 1 - \frac{1 - \ln(\delta p^2)}{(\delta p^2)^2} < 0.
\]

Step 2:
\[
\frac{d\Delta(NB)(T)}{dp} = 2\delta p \ln(\delta p^2)(-T - 1 + 2(\delta p^2)^{-T}) + (-T(1 - \delta p^2) - 1 + (\delta p^2)^{-T}) > 0
\]
\[
(-T(1 - \delta p^2) - 1 + (\delta p^2)^{-T})^2
\]
\[\text{[19]}\]
given that \(\delta p^2 < 1\) and, hence, \(\ln(\delta p^2) < 0\). This is because \((-T(1 - \delta p^2) - 1 + (\delta p^2)^{-T}) > 0 \ \forall \ T \in [1, \infty)\), which was proven in Step 1.

Step 3: Notice that \(\delta^B(T)\) is only a special case of \(\delta^{NB}(T)\), i.e. 
\[
\lim_{p \to 1} \delta^{NB}(T) = \delta^B(T).
\]
As a result:
\[
\frac{d(\delta^{NB}(T) - \delta^B(T))}{dT} < 0
\]
\[\text{[20]}\]
because of eq. [19]. In other words, the intermediate range of discount factors where our result arises increases if the length of the punishment phase \((T)\) is reduced.

### A.5.2 Different interest rates on deposits under coordinated behaviour

Consider an interest rate under coordinated behaviour on deposits that is higher than the monopoly interest rate and that leads to lower, but positive, joint profits \(\pi^M = \frac{R_H}{2} - \epsilon(\frac{R_H}{2} + \epsilon) = \pi^M - \epsilon^2\), where \(\pi^M > \epsilon^2 > 0\). Let the corresponding critical discount factors be denoted by \(\delta^B(\epsilon)\) and \(\delta^{NB}(\epsilon)\). From the modified ICCs we can derive \(\delta^B(\epsilon)\):
\[
p \sum_{t=1}^{\infty} \delta^{t-1} \frac{\pi^M}{2} \geq \pi^M \Rightarrow \delta^B(\epsilon) = \frac{1}{2} + \frac{\epsilon^2}{2\pi^M}
\]
\[\text{[21]}\]
and \(\delta^{NB}(\epsilon)\):
\[
\sum_{t=1}^{\infty} \delta^{t-1}(p^{2t-1} - \frac{\pi^M}{2} + p^t(1 - p^{t-1})\pi^M) \geq \pi^M + \sum_{t=2}^{\infty} \delta^{t-1}(p^t(1 - p^{t-1})\pi^M)
\]
\[
\Rightarrow \delta^{NB}(\epsilon) = \frac{1}{2p^2} + \frac{\epsilon^2}{2\pi^M} \times \frac{1}{p^2}.
\]
\[\text{[22]}\]
Hence, \( \frac{d(\delta^{MN}(\epsilon) - \delta^{B}(\epsilon))}{d\epsilon} > 0 \) because \( p < 1 \). Further, \( \delta^{NB}(\epsilon) < 1 \) if \( \epsilon^2 < (2p^2 - 1)\pi^M = (2p^2 - 1)(\frac{R_H}{2})^2 \).

### A.6 Stochastic bailouts

The critical discount factor can be derived following the same steps as in Appendix A.1. Let \( \hat{p} = p + (1 - p)q \) denote a bank’s probability of continuation. We have

\[
p_1(t) = \hat{p}^{t-1}p(\hat{p}^{t-1}) = \hat{p}^{2t-2}p
\]

\[
p_2(t) = \hat{p}^{t-1}p(1 - \hat{p} + \hat{p}(1 - \hat{p}) + \hat{p}^2(1 - \hat{p}) + \ldots + \hat{p}^{t-2} + (1 - \hat{p}))
\]

\[
\text{LHS} = \sum_{t=1}^{\infty} \delta^{t-1}(\hat{p}^{2t-2}p \frac{\pi^M}{2} + \hat{p}^{t-1}p(1 - \hat{p}^{t-1})\pi^M) = \left( \frac{p}{1 - \delta \hat{p}} - \frac{p}{2(1 - \delta \hat{p}^2)} \right) \pi^M
\]

and

\[
p_6(t) = \hat{p}^{t-1}p(1 - \hat{p} + \hat{p}(1 - \hat{p}) + \hat{p}^2(1 - \hat{p}) + \ldots + \hat{p}^{t-2} + (1 - \hat{p})) = \hat{p}^{t-1}p(1 - \hat{p}^{t-1})
\]

\[
\text{RHS} = p\pi^M + \sum_{t=2}^{\infty} \delta^{t-1}(\hat{p}^{t-1}p(1 - \hat{p}^{t-1}))\pi^M = \left( p + \frac{p\delta \hat{p}}{1 - \delta \hat{p}} - \frac{p\delta \hat{p}^2}{(1 - \delta \hat{p})^2} \right) \pi^M.
\]

The critical discount factor can be computed as \( \delta^B(q) = \frac{1}{2\hat{p}^2} = \frac{1}{2(p + (1-p)q)^2} \).

### A.7 Proof of Proposition 4

We prove part (a) and part (b) of Proposition 4 in turn. The proof of part (a) consists of three steps.

**Step 1:** As explained in Section 5.3 some of the probabilities and consumer welfare levels associated with the different events from Appendix A.1 and A.2 need to be modified for a non-banking industry setting. We now have \( p_1(t) = p^{2t}, \) \( p_2(t) = p (1 - p^t), \) \( p_3(t) = 1 - p^t, \) \( p_4(t) = p^{2(t-1)}p, \) \( p_5(t) = p^{2(t-1)}(1 - p), \) \( p_6(t) = p (1 - p^t), \) \( p_7(t) = 1 - p^{2(t-1)} - p^t(1 - p^t) \). An examination of the ICCs shows that \( \delta^{NB} = \frac{1}{2\hat{p}} \) while \( \delta^B = \frac{1}{2} \) and, hence, the result of Proposition 1 prevails in a more general industry setting with a downward-sloping demand curve.

**Step 2:** Consumer welfare levels without bailouts under coordination and no coordination are, respectively, given by:

\[ \text{State I} \ (w. \ prob. \ p_1(t)) : \quad W_C = \frac{1}{2} \left( \frac{R_H}{2} \right)^2 \quad \text{and} \quad W_{NC}^C = \frac{R_H^2}{2} \]
State II (w. prob. \( p_{II}(t) \)):
\[ \Phi C2 = \Phi NC2 = 0 \]

State III (w. prob. \( p_{III}(t) \)):
\[ W^M + \Phi C1 = \frac{1}{2} \left( \frac{R_H}{2} \right)^2 \quad \text{and} \quad W^C_{NC} + \Phi NC1 = \frac{R_H^2}{2} \]

State V (w. prob. \( p_V(t) \)):
\[ \Phi C2 = 0 \]

and with systematic bailout guarantees:

State II (w. prob. \( \hat{p}_{II}(t) \)):
\[ W^M + \Phi C1 - \gamma = \frac{1}{2} \left( \frac{R_H}{2} \right)^2 - \gamma \quad \text{and} \quad W^C_{NC} + \Phi NC1 - \gamma = \frac{R_H^2}{2} - \gamma \]

State III (w. prob. \( \hat{p}_{III}(t) \)):
\[ \Phi C2 = 2\gamma = \Phi NC2 - 2\gamma = -2\gamma. \]

Furthermore, we can derive

* Step 3: For \( \gamma = 0 \), \( E(W^B_C) < E(W^N_{NC}) \) holds if:
\[ \frac{2p - p^2}{1 - \delta} < 3 \frac{2p - p^2}{1 - \delta p^2} + \frac{2p}{1 - \delta p} - \frac{p^2}{1 - \delta p^2}. \]  

A simple sufficient condition for inequality [23] to hold is that \( p > p' = \sqrt{3/5} \approx 0.77 \). Hence, the result of Proposition 2 prevails in a more general industry setting with a downward-sloping demand curve if the probability of a negative shock is not too high. This concludes the proof of part (a).

The proof of part (b) of Proposition 4 consists of two steps:

* Step 1: The total welfare levels (TW) associated with the different events from Appendix A.1 and A.2 without bailouts are under coordination and no coordination, respectively, given by:

  State I (w prob. \( p_I(t) \)):
  \[ TW_C = \frac{3}{2} \left( \frac{R_H}{2} \right)^2 \quad \text{and} \quad TW^C_{NC} = \frac{R_H^2}{2} \]

  State III (w prob. \( p_{III}(t) \)):
  \[ TW_C = \frac{3}{2} \left( \frac{R_H}{2} \right)^2 \quad \text{and} \quad TW^C_{NC} = \frac{R_H^2}{2} \]

  State IV (w prob. \( p_{IV}(t) \)):
  \[ TW^M = \frac{3}{2} \left( \frac{R_H}{2} \right)^2 \quad \text{and} \quad TW^M = \frac{3}{2} \left( \frac{R_H}{2} \right)^2 \]

and with systematic bailout guarantees:

State I (w prob. \( \hat{p}_I(t) \)):
\[ TW_C = \frac{3}{2} \left( \frac{R_H}{2} \right)^2 \quad \text{and} \quad TW^C_{NC} = \frac{R_H^2}{2} \]
State II (w prob. \( \hat{p}_{II}(t) \)): \[ TW_C - \gamma = \frac{3}{2} \left( \frac{R_H}{2} \right)^2 - \gamma \] and \( TW_{NC}^{C} - \gamma = \frac{R_H^2}{2} - \gamma \).

State III (w prob. \( \hat{p}_{III}(t) \)): \[ \Phi_{C2} - 2\gamma = \Phi_{NC2} - 2\gamma = -2\gamma. \]

We can derive
\[
E(TW_{C}^{B}) = \frac{3}{2} \left( \frac{R_H}{2} \right)^2 \frac{2p - p^2}{1 - \delta} - \frac{2(1 - p)}{1 - \delta} \gamma
\]
\[
E(W_{NC}^{NB}) = \left( \frac{\frac{2}{3}p^2}{1 - \delta p} + \frac{\frac{2}{3}p}{1 - \delta p^2} - \frac{2p}{1 - \delta p} \right) \frac{3}{2} \left( \frac{R_H}{2} \right)^2.
\]

Step 2: For \( \gamma = 0 \), \( E(TW_{C}^{B}) < E(W_{NC}^{NB}) \) holds if:
\[
\frac{2p - p^2}{1 - \delta} < \frac{1}{3} \left( \frac{2p}{1 - \delta p} - \frac{p^2}{1 - \delta p^2} \right) + \frac{2p - p^2}{1 - \delta p},
\]

Notice that the derivative of the left-hand side of inequality [24] with respect to \( \delta \) is positive and larger than the derivative of the right-hand side of inequality [24] with respect to \( \delta \), whenever inequality [24] holds. As a result, inequality [24] is guaranteed to hold if:
\[
\frac{2p - p^2}{1 - \delta^{NB}} < \frac{1}{3} \left( \frac{2p}{1 - \delta^{NB} p^2} - \frac{p^2}{1 - \delta^{NB} p^3} \right) + \frac{2p - p^2}{1 - \delta^{NB} p}.
\]

or
\[
\frac{6p - 3p^2}{1 - \frac{1}{2p^2}} < \frac{6p - 4p^2}{1 - \frac{1}{2p}} + 4p.
\]

The above inequality holds for \( p \to 1 \). Furthermore, there exists a lower bound \( p' > p \) such that inequality [26] holds for all \( p \in (p', 1] \). Hence, the result of Proposition 2 prevails in a more general industry setting for both the consumer welfare and the total welfare standard, provided the probability of a negative shock is not too high. A sufficient condition is given by \( p > p'' \approx 0.87 \). This concludes the proof of part (b).

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