Introduction to the papers of TWG14: University mathematics education

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Research on university level mathematics education is a fast developing field as evident in the growth of the CERME *University Mathematics Education* (hereafter UME) Thematic Working Group. TWG14 was launched in CERME7 (Nardi, González-Martín, Gueudet, Iannone & Winsløw, 2011). After CERME8 (Nardi, Biza, González-Martín, Gueudet, & Winsløw, 2013), its leader team – in collaboration with TWG14 participants and others– worked towards a *Researchin Mathematics Education* Special Issue on *Institutional, sociocultural and discursive approaches to research in university mathematics education* (Nardi, Biza, González-Martín, Gueudet & Winsløw, 2014) which fo- cused on research that is conducted in the spirit of the following theoretical frameworks: *Anthropological Theory of the Didactic*, *Theory of Didactic Situations*, *Instrumental and Documentational Approaches*, *Communities of Practice and Inquiry* and *Theory of Commognition*.

The work of the group at CERME9 cemented and fur- thered this work but welcomed contributions from across the board of research approaches: the teaching and learning of advanced topics; mathematical rea- soning and proof; transition issues "at the entrance" to university mathematics, or beyond; challenges for, and novel approaches to, teaching (including the teaching of students in non-mathematics degrees); the role of ICT tools (e.g. CAS) and other resources (e.g. textbooks, books and other materials); assessment; the preparation and education of university math- ematics teachers; collaborative research between university mathematics teachers and researchers in mathematics education; and, theoretical approaches to UME research.

The critical – and growing – mass and quality of the work presented at TWG14 has led to the launch of an ERME Topic Conference, INDRUM2016, a confer- ence of the newly launched *International Networkfor Didactic Research in University Mathematics* (France, Montpellier, March 31 – April 2). Its two broad themes are *teaching and learning of specific topics in university mathematics* and *teachers' and students' practices at university level*. In anticipation of INDRUM2016, in this report, we outline briefly the main focal points of the 45 papers (31 long and 14 short contributions) that comprise the set of CERME9 TWG14 papers published in these proceedings in accordance with these two broad themes. We note that several papers fit both themes and that we have opted to classify the papers according to what we see as their main research focus and contribution.

TEACHING AND LEARNING OF SPECIFIC TOPICS IN UNIVERSITY MATHEMATICS

The 16 papers classified under this theme (8 long and 8 short papers) address a range of mathematical *topics*, elaborate discussions ofmathematical *reasoning, logic and proof* and introduce research into the teaching of mathematics to students in *other fields* (here: engi- neering and economics).

With regard to *mathematical topics*, contributions regarded topics in calculus and Complex Analysis. *Breen, Larson, O'Shea and Pettersson* analyse student data from Ireland and Sweden to discuss concept im- ages of inverse functions, particularly in relation to the predominance of the models of "swapping *x* and *y*", reflection and reversal. *Ghedamsi* offers a *Theory of*

Didactic Situations (TDS)-based analysis of a teaching session on sequence convergence in order to exam- ine the ways in which a university calculus teacher attends to students' prior knowledge in calculus and facilitates the transition from school to univer- sity mathematics. *Grønbæk and Winsløw* deploy an *Anthropological Theory of the Didactic* (ATD) lens to discuss the teaching of complex numbers using Maple sheets and demonstrate the institutional constraints – Maple sheets cannot create an appropriate media/ milieu dialectic – which lead to the development of disconnected practices. The short papers also covered a range of topic-specific research: the transition from informal to formal understanding of the concept of order in abstract mathematics (*Akdemir, Narlı and Kaşıkçı*); improper integrals (*Cortés and Velasco*); dif- ferential geometry (*Dana-Picard and Zehavi*); differ- ential equations (*Fardinpour*); linear independence of functions (*Wawro and Plaxco*); and, abstract algebra (*Mili and Ascah-Coallier*).

With regard to *mathematical reasoning, logic and proof, Hausberger* introduces the innovation of the *banquet*, a pocket-size algebraic structure aimed at helping students reflect on mathematical structures and the axiomatic method. *Bridoux and Durand-Guerrier*, through an a-priori and a-posteriori analysis of two tasks in an exam paper taken by students of a Computing Sciences module that aimed at improv- ing students' proof production, find that the course did improve students' proof fluency, although they also observe that many difficulties remain. In their short paper concerning students' conceptions of logic, *Kazima, Eneya and Sawerengera* also highlight some of these difficulties, mainly focusing on issues of language.

With regard to research into the teaching of mathematics to students in *other fields*, a relatively novel strand, *Biehler and Kortemeier* analyse students' work with a typical electrical engineering task in relation to an expert solution and conclude that it is counterproductive to try to separate the mathematical and "real world" (engineering) parts of the problem. *Kürtenand Greefrath* report aspects of a "bridging" course aim- ing to reduce engineering students' difficulties with mobilizing school mathematical skills. *Mkhatshwa and Doerr* investigate economics students' reasoning about marginal change (instantaneous rate of change) and in her short paper *Selinski* explores student notic- ing of exponential and power functions in university financial mathematics.

TEACHERS' AND STUDENTS' PRACTICES AT UNIVERSITY LEVEL

The 29 papers classified under this theme (23 long and 6 short papers) also address a range of teaching and learning issues: *curriculum and assessment; innovative course design in UME; student approaches to study; relating research mathematicians' practices to student practices; views and practices of mathematics lecturers;* and, *methodological and theoretical contributions to UME research.*

In the cluster of papers on *curriculum and assessment, González-Martín* deploys a combination of theoretical frameworks (ATD and the *documentational* approach) to investigate the use of textbooks by pre-university teachers (particular focus: the topic concept of series of real numbers) and to observe that the textbook is a central tool for the teachers, who align with its presentation and organisation. *Dibbs* describes the outcomes of the use of formative assessment in a calculus class and concludes that regular participation in formative assessment is the best predictor of achievement. *Raen* compares the assessment of student competencies through closed book examination and talk aloud interviews. She concludes that different methods reveal different competencies and that therefore a mixture of assessment methods is desirable. *Thoma and Iannone* use two different frameworks, the MATH framework based on Bloom's taxonomy, and a framework based on functional linguistics and Sfard's *commognitive* approach, to analyse tasks from an examination in abstract algebra. They find both frameworks useful in highlighting different, and often complementa- ry, aspects of the tasks. In their short paper *Derouet, Henríquez, Menares and Panero* also deploy a priori analyses of examination tasks in order to compare final secondary assessments in different countries.

With regard to *innovative course design in UME, Biza and Vande Hey* deploy the *Communities of Practice* approach to study the process of – and the pedagogical benefits deriving from – involvement of two undergraduate students in a project of resource development for statistics. *Mesa and Cawley* report the 3-year implementation of Inquiry-Based Learning (IBL) in a range of courses. Drawing on data from teacher logs and a *Mathematical Knowledge for Teaching* (MKT)

framework, they discuss challenges of the IBL approach. *Nardi and Barton* present a *commognitive* analysis of a "low lecture" episode (student-led inquiry oriented discussion on open-ended problems) to illustrate crucial steps of student enculturation into mathematical ways of acting and communicating, including a shift away from the lecturer's 'ultimate substantiator' role. *Rämö, Oinonen and Vikberg* take a similar approach to report the shifting of an introduc- tory course on linear algebra from a "lecture based" format to a new "extreme apprenticeship" format.

In the growing area of *student approaches to study, Farah* investigates the role of students' personal work in mathematics and highlights the influence of institutional differences on student approach. *Gómez-Chacón, Griese, Roesken-Winter and Gónzalez-Guillén* report similarities in the learning strategies employed across two cohorts of engineering students, in Spain and Germany. *Liebendörfer and Hochmuth* identify different factors which support or hinder the autonomy of first year students and observe that student teachers are not convinced about the need of university mathematics for teaching at school. *Lehmann, Roesken-Winter and Schueler* reveal that mathematical competencies and beliefs about phys- ics are substantial for engineering students' success in technical mechanics. In their short papers in this area, *Griese, Lehmann and Roesken-Winter* focus on what obstructs or facilitates examination success in first year engineering and *Švecová, Kohanová and Drábeková* explore issues concerning the mathemat- ical literacy of firstyearstudents.

Three papers documented the *interplay between research mathematicians' pedagogical and mathematical practices and the influence of these on learner practices*. *Cooper* proposes a commognitive configuration of MKT (MDT, *Mathematical Discourse for Teaching*) as a tool to identify – and make optimal pedagogical use of – differences in the student teachers' and a mathematician's discourses. *Ouvrier-Buffet* presents a model of how research mathematicians practise the construction of formal mathematical definitions and highlights the pedagogical potency of epistemological analyses of mathematicians' practices. *Kondratieva* also favours epistemological analyses and discuss- es the pedagogical potential of exposing students to mathematical problems with different, more or less advanced, solutions to problems as opportunities for buildingmathematicalconnections.

In the populated area of studies on the *views and practices of mathematics lecturers* (6 long and 3 short papers), *Bergsten and Jablonka* investigate the views of mathematics lecturers on the transition problem for engineering students and observe that, despite the engineering context, lecturers see this transition as apprenticeship into becoming a mathematician, namely able to produce mathematics. *Hernandes Gomes and González-Martín* highlight differences in how teach- ers in engineering and in mathematics address rigor, approximation and modelling differently and how these views influence their teaching. *Gueudet* deploys the *documentational* approach to study teacherpreparation and communication practices. She traces the interaction of teachers with resources in a goal-oriented activity that produces *documentation systems* (structured set of all the documents they develop) and identifies features of these systems. *Mali* studies how teachers with different disciplinary backgrounds use examples and representations in their teaching. *Petropoulou, Jaworski, Potariand Zachariades* deploy the *Teaching Triad* construct to investigate lecturer practices and rationales. They illustrate a case of a lecturer who shows sensitivity to students' needs and draws students into mathematical culture through mathematical challenge. *Viirman* offers *commognitive* analyses of how lecturers' epistemological and ontological positions on mathematics are articulated in their teaching discourse. The three short papers in this area touch on ways to enable student mean- ing making (*Didis and Jaworski*), UME conceptuali- sations of pedagogical content knowledge (*Khakbaz*) and tackling the difficulties of the transition from school to university mathematics (*Kouvela, Biza and Zachariades*).

Finally, *Kaspersen, Pepin and Sikko* propose a *methodologicaladvance* in the study of the transition from higher education to the world of work through proposing an approach to purposeful sample selection for measuring student teachers' beliefs and practic- es. An advance of a *methodological as well as theoret- ical* character is put forward by *Tabach, Rasmussen, Hershkowitz and Dreyfus* who use a transcript of an excerpt of four undergraduate students' interaction while working on a specific initial value problem, to demonstrate a local integration of two theoretical and methodological perspectives on knowledge construction, namely *Abstraction in Context* (focusing on individuals) and *Documenting Collective Activity*.

IN CLOSING

While our presentation of CERME7 and CERME8 pa- pers was in accordance with slightly different themes – for example in CERME8: *transitions*, *affect*, *teacher practices* and *mathematical topics* – some comparative observations across the three sets of papers are apt. As we noted in the Editorial of the RME Special Issue (Nardi et al., 2014), there is a clear surge of sociocul- tural and discursive approaches – and the number of papers using ATD and TDS is also remarkable. An emerging focus seems to be also on systematic inves- tigations of innovative course design and implemen- tation and there is certainly a rise in the number of studies that examine the teaching and learning of mathematics in the context of disciplines other than mathematics, such as engineering and economics. Furthermore, this time we welcomed more colleagues from outside Europe and also noted the rise in the number of papers on assessment and examination. We also observed the further strengthing, maturity and increasingly more robust theorizing of studies into teaching practices. Finally, we noticed in several papers the establishing of promising liaisons across different theoretical perspectives. We now look for- ward to cementing these developments further in future CERME conferences, in the rich presence of UME at the upcoming ICME13 and EMF2015 confer- ences – and of course INDRUM2016!

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