INTERFIRM BUNDLED DISCOUNTS AS A COLLUSIVE DEVICE*

Jong-Hee Hahn† Sang-Hyun Kim‡

June 13, 2014

Abstract

This paper investigates whether and how firms competing in price with homogeneous goods (i.e., Bertrand competitors) can achieve supernormal profits using interfirm bundled discounts. By committing to offering price discounts conditional on the purchase of a specific brand of other differentiated good, the homogeneous good suppliers can separate consumers into distinct groups. Such brand-specific discounts help the firms relax competition and attain a collusive outcome. Consumers become worse off due to higher effective prices. Our result shows that in oligopolies it is feasible to leverage other's market power without excluding rivals.

I. INTRODUCTION

It is a common practice for a firm to offer price discounts conditional on the purchase of another (related or unrelated) firm’s product. Credit card companies provide price discounts

---

*We gratefully acknowledge insightful comments and suggestions of an anonymous referee. We also thank Jay Pil Choi, Sanghoon Lee and seminar audiences at Yonsei University and University of East Anglia for useful comments and discussions on an earlier version of this paper. This work was supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2014-11-0395 and NRF-2013-11-1372).

†School of Economics, Yonsei University, Seoul 120-749, Korea; hahnjh@yonsei.ac.kr

‡School of Economics and the ESRC Centre for Competition Policy, University of East Anglia, Norwich NR4 7TJ, U.K.; sang-hyun.kim@uea.ac.uk
or (pseudo- or non-monetary) rewards to customers of a specific coffee chain, petrol station, telecommunication or Internet service provider, amusement park, airline, hotel chain, car rental company, motor insurance company, and so on. Many supermarkets and grocery stores offer discounts or reward points to consumers who buy from a specific petrol station, travel agent, Internet merchant or auction site. In the U.K., consumers can collect Nectar points every time they make a reservation via Expedia; these points can be redeemed at specific shops, supermarkets, cinemas, amusement parks, etc. Cross-market discounts between grocery retailers and gas stations (called fuelperks) are widely used all over the U.S., as reported by Goic et al. [2011].

There are some notable features of such discount schemes. First, there is brand-specific exclusivity in the sense that discounts are offered only to those who buy some designated brand of other products. Second, firms commit to discount schemes prior to choosing the headline price for their product.\(^1\) Obviously, these two elements create interlocking relations between the associated products. Third, such arrangements often involve different firms and products from otherwise unrelated markets.

This paper aims to investigate the competitive effect of such interfirm bundled discounts. In particular, we consider a situation where two firms competing in price with homogeneous products (i.e., Bertrand duopolists) can offer a price discount to consumers who purchase a specific brand of other differentiated product. We find that the brand specificity of the discounts, by creating a sort of artificial switching cost, segments otherwise homogeneous consumers into two groups. In this way, the bundled discounts relax price competition between the Bertrand duopolists and can even allow them to achieve the fully collusive outcome if the degree of market power in the differentiated market is sufficiently large relative to the consumers’ valuation of the homogeneous good.\(^2\) Consumers become worse off due to higher effective prices. Our result reveals the collusive nature of bundled discounts involving firms and products across seemingly unrelated markets.

\(^1\) Coupons, nation-wide advertisements, and interfirm contracts are probably the most popular commitment devices, which make cross-market discounts much more rigid than those of associated products.

\(^2\) This is similar to the ‘fat cat effect’ in Fudenberg and Tirole [1985]’s animal spirits taxonomy of business strategies.
Also, this paper demonstrates a way in which market power can be leveraged from one market to another without inducing exits of competitors. This contrasts with the standard theory on the leverage of monopoly power according to which a profitable bundling or tying usually requires exclusion of rivals. This observation suggests the need for caution when extending the results of the literature on monopoly leverage to oligopolistic environments.

The structure of the paper is as follows. Section II introduces a model of conditional discounts, which includes interfirm bundled discounts as a special case. We classify the discount condition into two categories depending on whether it is exogenously given prior to firms’ discount offers or endogenously determined through consumer purchasing decisions. In Section III, we examine the case of exogenous separation where Bertrand duopolists offer discounts conditional on a pre-determined separation device, such as purchasing history, age, gender, or occupation. Here, we show that the firms commit to bundled discounts large enough to attain the fully collusive outcome. Section IV addresses the case in which the firms commit to discounts conditional on the purchase of a specific brand of other unrelated and differentiated product, and consumers make their purchasing decisions in the differentiated good market after the bundled discounts are announced. In this case of endogenous separation, the market outcome is partially collusive. We conclude in Section V, briefly discussing the current stance of antitrust authorities with regard to the practice of cross-market bundled discounts.

I(i). Related Literature

Probably the most closely related to the present work is the study by Gans and King [2006]. Analyzing the Hotelling competition between two firms in each of two symmetric and differentiated markets, they show that two pairs of firms choose to jointly offer a discount on their product bundle even though no firm benefits from the discounts. That is, the firms may face the situation of a prisoners’ dilemma if there exist some transaction costs involved in arranging the discount scheme. The discount agreements reduce consumer surplus and total welfare because consumer choice is restricted due to the bundling nature of brand specificity.

---

3See, for instance, Whinston [1990], Choi and Stefanidis [2001], and Carlton and Waldman [2002].
4Brito and Vasconcelos [2013] extend Gans and King’s work to the case of vertical differentiation and find that only high quality sellers obtain higher profits relative to the no-bundling benchmark.
5Matutes and Regibeau [1992] derive somewhat similar results in the context of system markets consisting
In contrast, our model presents a case of jointly profitable and even fully collusive bundled discounts under asymmetric oligopolies. Also, we show that firms with no market power can achieve supernormal profits by leveraging other unrelated firms’ market power using unilateral (rather than bilateral) discount schemes. In this respect, our result shares some similarity with the recent work of Katz and Hermalin [2013], who show that relatively undifferentiated platforms can increase joint profits using exclusive contracts with relatively differentiated applications. Our analysis, however, differs from their model in the following aspects. First, the firms in our model use pricing schemes that are more flexible and easily enforceable than the exclusive contractual arrangements considered by Katz and Hermalin. Second, we consider unilateral discount schemes across unrelated markets rather than bilateral arrangements between suppliers of perfectly complementary goods in system markets.

The present work is also related to the literature of endogenous switching costs. Banerjee and Summers [1987] show that, in a two-period Bertrand competition model, firms can earn monopoly profits by offering discounts to repeat buyers in period one. Repeat-purchase discounts induce consumer loyalty due to switching costs, which enables the firms to segment the market and charge higher prices in period two. Moreover, the firms resist price reduction in period one in order to insure a large install base of the rival in the later period. Caminal and Matutes [1990] examine two distinct types of loyalty-inducing strategies in differentiated product markets and show that equilibrium profits decrease with price commitment but increase with discount commitment. Also related in this line of research are the recent works on behavior-based price discrimination, which study intertemporal discriminatory pricing based on purchase histories.\(^6\) Even though the basic logic is similar, our paper is distinct from these studies in several aspects. First, we show that the collusive effect of discounts operates in a of two fully integrated suppliers of complementary components. They show that the firms choose to offer discounts to consumers who purchase all components from the same supplier, even though the firms would be better off if they could agree not to offer bundled discounts. Extending their model to more general preferences, Thanassoulis [2007] shows that the competitive effect of mixed bundling crucially depends on whether buyers’ tastes are firm-specific or product-specific.

one-shot framework as opposed to repeat-purchases. Second, consumer loyalty in our model is created by interfirm relations across different markets rather than intertemporal consumption relations of the same good. Third, we focus on the leverage of market power from one market to another. Some relevant works also exist in the bundling literature. Chen [1997] shows that a multiproduct firm producing a primary good in a duopoly market and another good under perfectly competitive conditions may wish to bundle its products in order to differentiate otherwise identical products and relax price competition in the duopoly market. Spector [2007], on the other hand, points out the possibility that a monopoly supplier of a primary good ties its monopoly good to a complementary good produced in an oligopolistic market for the purpose of facilitating collusion in the oligopoly market. In both papers, bundling or tying is pursued by a multiproduct firm with its own products, while bundled discounts in our model involve independent firms in unrelated markets. More recently, Armstrong [2013] has shown that independent sellers of two substitutes may wish to offer interfirm discounts in order to relax competition when they can coordinate on the size of the discount. Although this is quite similar to the finding of our analysis, his result is derived in a setting where two competitors in a single market jointly offer bundled products to customers, whereas we consider two unrelated oligopolies in which bundled discounts are offered across the markets.

II. MODEL

Consider a market where two firms, denoted $A_1$ and $A_2$, compete in price with homogeneous good $A$. There is a continuum of consumers of mass 1 with unit demands. Let $v_A$ denote the reservation price for the product, which is common to all consumers.\(^7\) For simplicity, the cost of production is assumed to be zero for both firms.

A crucial feature of the model is that individual firms can independently pre-commit, before announcing their headline price, to a discount that is available only to those who satisfy some pre-specified conditions.\(^8\) For instance, the firms can distribute coupons that discount a fixed

\(^7\) Allowing heterogeneous consumer preferences for good $A$ would not change the qualitative results.

\(^8\) Here we consider the case of additive discounts, deducting an absolute amount of money from the headline
amount off the headline price for consumers who present evidence of purchases of a specific brand of a product. Or they can offer other non-monetary benefits, such as free gifts, instead of price discounts.

Although our analysis is focused on discounts schemes arranged between otherwise unrelated firms, our model can be interpreted in a more flexible manner. In general, the conditions for discounts can take various forms; they can be intrinsic consumer characteristics such as age, gender, occupation, location of residence, etc., or they can be present or past consumer behavior such as the purchase of a particular brand of another (related or unrelated) product or a verifiable purchasing history. In any case, conditional discounts usually allow firms to separate consumers who are otherwise identical into different groups.

Exogenous separation, which is analyzed in Section III, is the case where the condition for discounts is permanently fixed or exogenously given before firms $A_1$ and $A_2$ take any action and is therefore totally independent of the events in market $A$. Discounts conditional on invariant consumer characteristics or purchasing history of a particular good or service correspond to this case.\(^9\) On the other hand, endogenous separation describes a situation where the condition for discounts is determined within the model according to consumer purchasing behavior. More specifically, as in the game considered in Section IV, firms can commit to a discount conditional on the purchase of a particular brand of another good, and consumers make purchasing decisions on the two products by simultaneously considering the conditional discounts.

Attention is restricted to the case where the homogeneous goods suppliers (firms $A_1$ and $A_2$) individually tie their discount ($d_i$) only to a single group of consumers (in the exogenous separation case) or a single brand of other product (in the endogenous separation case) and coordinate their discount schemes so that the two discounts are conditioned on different groups of consumers or brands. More generally, individual firm $i$ can offer discriminatory discounts ($d_{ij}$) to both groups, where $j(=1,2)$ denotes the consumer groups. However, given that price. However, the analysis can be easily extended to other cases where discounts are in the form of proportional discounts.

\(^9\)Notice that this differs from the standard third-degree price discrimination in that all consumers here have identical preferences for the good.
consumer purchasing decisions are made based on relative effective prices (i.e., nominal prices less discounts), it is without loss of generality to normalize the smaller discount to zero unless the firms intend to practice intragroup second-degree price discrimination.\footnote{Formally, we can redefine the real discount as $d_i = |d_{i1} - d_{i2}|$ and the corresponding headline price as $p_{iA}^* = p_{iA}^* - \min\{d_{i2}, d_{i2}\}$. Then the model is identical to the one analyzed in the next section.} In fact, casual observations in the real world market seem quite compatible with our assumption of single group discounts.\footnote{For example, two dominant mobile telecommunication companies in South Korea (SK Telecom and Korea Telecom) provide membership services that give discounts only to consumers who buy from a single designated brand of major bakery.}

III. EXOGENOUS SEPARATION

Consumers are exogenously divided into two groups $B_1$ and $B_2$ with proportions $\lambda$ and $1 - \lambda$, respectively, where $\lambda \in (0, 1)$. The two groups may represent people living in the southern and northern districts of a town, or installed bases of two brands of a particular product. Firms $A_1$ and $A_2$ independently commit to discounts conditional on group identity. Let $p_{iA}^*$ denote the headline price of firm $A_i$.

The timing of the game is as follows:

1. The firms in market $A$ independently and simultaneously pre-commit to conditional discounts.
2. A sequential pricing game between $A_1$ and $A_2$ follows, where the price leader and follower are exogenously determined.
3. Consumers make purchasing decisions, given the prices and discounts set in the early stages.

We model the second-stage pricing game as sequential moves in order to guarantee the existence of a pure-strategy Nash equilibrium. The interlocking relations induced by conditional discounts create endogenous switching costs, and it is well known that, with switching costs, pure-strategy equilibria may not exist in simultaneous pricing games. We avoid this
technical issue by assuming sequential moves in the pricing game, as in Banerjee and Summers [1987]. The identities of the price leader and follower are known to the firms before they commit to discounts. One may consider the case where the order of pricing is revealed only at the beginning of the second stage, which however does not affect our results, as will be shown below.

Note that the pricing game in this section is formally identical to the second-period pricing game in Banerjee and Summer repeat purchases model. This shows that conditional discount schemes are closely related to the intertemporal loyalty-inducing program analyzed by Banerjee and Summers [1987]. Our analysis is, however, more general than theirs in that we consider generic proportions of groups or installed bases \((\lambda \in (0, 1))\), while Banerjee and Summers solve for only two special cases of \(\lambda = 1, 1/2\).

We solve for subgame perfect equilibria of the dynamic game using backward induction. Let us first consider the sequential pricing game in the second stage. Assume without loss of generality that, in the first stage, the price leader (denoted \(l\)) has promised to offer discount \(d_l\) to group \(B_1\) consumers, and the follower (denoted \(f\)) committed to discount \(d_f\) to those in group \(B_2\). Then, the follower’s demand is

\[
D_f(p_l, p_f; d_l, d_f) = \begin{cases} 
1 & \text{if } p_f \leq p_l - d_l \\
1 - \lambda & \text{if } p_l - d_l < p_f \leq p_l + d_f \\
0 & \text{if } p_l < p_f - d_f
\end{cases}
\]

where \(p_l\) and \(p_f\) denote the headline prices of the leader and the follower, respectively. The leader’s demand is given by \(D_l = 1 - D_f\). With the conditional discounts, consumers will choose to buy from the firm offering a discount as long as the difference between the two headline prices does not exceed the amount of discount. In this case, the firms share the market according to the pre-determined proportions of two groups. When the difference in the headline prices is sufficiently large, on the other hand, the firm with a lower effective price

\[^{12}\text{See Banerjee and Summers [1987] for more detail on this issue in the context of intertemporal loyalty-inducing discounts. The qualitative result would remain the same even if we assume simultaneous moves. In that case, the prices would be above the marginal cost with positive probability at the mixed strategy equilibrium.}\]
captures the entire market. Obviously, the effective prices (headline prices less discounts) cannot be greater than the reservation value \( v_A \) in equilibrium, i.e., \( p_i - d_i \leq v_A, i = l, f \).

The follower’s profit is given by

\[
\pi_f(p_l, p_f; d_l, d_f) = \begin{cases} 
(1 - \lambda)(p_f - d_f) + \lambda p_f & \text{if } p_f \leq p_l - d_l \\
(1 - \lambda)(p_f - d_f) & \text{if } p_l - d_l < p_f \leq p_l + d_f \\
0 & \text{if } p_l < p_f - d_f 
\end{cases}
\]

which is piecewise continuous with discontinuities at \( p_f = p_l - d_l \) and \( p_f = p_l + d_f \). Note that the profit function is strictly increasing in \( p_f \) for \( p_f \leq p_l - d_f \), drops to \( (1 - \lambda)(p_f - d_f) \) at \( p_f = p_l - d_l \), and then increases again. Thus, the follower’s problem is simplified to determining whether to share the market with the leader by setting \( p_f^S = \min \{p_l, v_A\} + d_f \) or to capture the entire market by setting \( p_f^M = p_l - d_l \). So, we can write the maximum profit of the follower as

\[
\pi_f(p_l; d_l, d_f) = \max \left\{ (1 - \lambda)(p_f^S - d_f), (1 - \lambda)(p_f^M - d_f) + \lambda p_f^M \right\} = \max \left\{ (1 - \lambda) \min \{p_l, v_A\}, p_l - d_l - (1 - \lambda)d_f \right\}.
\]

We can easily see that the follower will corner the market if the leader’s effective price is sufficiently high and will share the market otherwise.

Let us now consider the leader’s pricing problem. Obviously, the leader will choose a price that induces the follower to later choose to share the market (the follower’s market-cornering means zero profits for the leader), i.e.,

\[
(1 - \lambda) \min \{p_l, v_A\} \geq p_l - d_l - (1 - \lambda)d_f. \tag{1}
\]

Given that the leader’s market share is fixed at \( \lambda \), the leader’s problem is to select the highest price satisfying condition (1). The largest possible price that the leader can charge is given by \( p_l - d_l = v_A \). Substituting \( p_l = v_A + d_l \) and rearranging terms, condition (1) reduces to

\[
d_f \geq \frac{\lambda}{1 - \lambda} v_A. \tag{2}
\]

So, if \( d_f \) is large enough to satisfy (2) the leader can extract the entire surplus \( v_A \) from the consumers in group \( B_1 \). Otherwise (i.e., for a a small \( d_f \)), the leader needs to lower the price
in order to induce the follower to later choose to share the market. In this case, the leader will set the price that makes the follower indifferent between market sharing and cornering, i.e.,

\[(1 - \lambda) \min \{p_l, v_A\} = p_l - d_f - (1 - \lambda)d_f,\]

and, therefore, the leader’s optimal price is given by

\[p^*_l = \begin{cases} 
\frac{d_l + (1-\lambda)d_f}{\lambda} & \text{if } \frac{d_l + (1-\lambda)d_f}{\lambda} \leq v_A \\
 d_l + (1 - \lambda)(v_A + d_f) & \text{if } \frac{d_l + (1-\lambda)d_f}{\lambda} > v_A .
\end{cases}\]

Thus, given discounts \(d_l\) and \(d_f\) the second-stage equilibrium profits of the two firms are

\[
\pi_l(d_l, d_f) = \begin{cases} 
\lambda v_A & \text{if } d_f \geq \frac{1}{1-\lambda} v_A \\
\lambda(p^*_l - d_l) & \text{if } d_f < \frac{1}{1-\lambda} v_A ,
\end{cases}
\]

\[
\pi_f(d_l, d_f) = \begin{cases} 
(1 - \lambda)v_A & \text{if } d_f \geq \frac{1}{1-\lambda} v_A . \\
(1 - \lambda) \min \{p^*_l, v_A\} & \text{if } d_f < \frac{1}{1-\lambda} v_A .
\end{cases}
\]

Let us now consider the first-stage discount game. Note that the follower’s profit is weakly increasing in \(d_f\) and reaches its maximum for \(d_f \geq \frac{1}{1-\lambda} v_A\). Thus the follower will optimally pre-commit to a sufficiently large discount \(d_f \geq \frac{1}{1-\lambda} v_A\). The leader’s profit, which is weakly increasing in \(d_l\) and \(d_f\), is also maximal for \(d_f \geq \frac{1}{1-\lambda} v_A\). In equilibrium, the leader will choose discount and headline prices such that \(p_l = v_A + d_l\). The market is divided into two exogenously separated groups in equilibrium. The firms achieve the fully collusive outcome, extracting the whole consumer surplus. Note that this result holds irrespective of the mass distribution of the two groups, provided that consumers are separated into two distinct groups, i.e., \(\lambda \in (0, 1)\). While \(\lambda\) does not affect the effective price in equilibrium, it does affect the size of discounts. The minimum discount that sustains this arrangement increases as \(\lambda\) increases. Intuitively, the follower’s incentive to capture the entire market gets stronger as \(\lambda\) increases. Hence, in order to ensure the collusive outcome, the follower needs to commit to a larger discount for a larger \(\lambda\). The discussion so far is summarized in the following proposition.

**Proposition 1.** With two exogenously separated groups of consumers, the homogeneous goods suppliers can achieve the fully collusive outcome by committing to sufficiently large conditional discounts. The firms extract the whole consumer surplus, and each firm’s profit
is proportional to the size of the group on which its discount is conditioned, i.e., \( \pi_l + \pi_f = \lambda v_A + (1 - \lambda) v_A = v_A \).

This result shows that price discounts conditional on exogenous segmentation can be used to facilitate collusion among competitors by letting them ‘mutually forbear’ intruding into each other’s market as in multimarket contact situations.\(^{13}\) It is noteworthy, however, that conditional discounts are more flexible in their applicability in the sense that they can exploit not only geographical differentiation, but also demographic characteristics of consumers and their purchasing histories. Also related is the work of Roy [2000], who demonstrates that competing firms can achieve a collusive outcome by targeting advertisement toward mutually exclusive consumer groups.

**Remark 1.** In order to correctly evaluate the competitive effect of conditional discounts, we need to clarify the benchmark equilibrium without discounts. Recall that there are infinitely many equilibria under sequential pricing in the absence of conditional discounts, where the leader sets a price greater than the marginal cost, and the follower makes all sales by slightly undercutting the leader’s price or charging the monopoly price. However, there are many reasons why the most reasonable equilibrium should be the static Bertrand outcome with marginal cost pricing and zero profits for both firms. First, the leader may prefer the equilibrium with positive sales to those with no sales at all (although zero profits are obtained in any case) because continuing production and operation is vital for maintaining customer relations for future businesses. Second, the subgame perfect equilibrium would involve marginal cost pricing and zero profits if there is a chance that the leader has slight cost advantages over the follower. Suppose that the follower’s marginal cost is \( c \) and the leader’s marginal cost is \( c \) with a large probability and \( c - \varepsilon \) with a small probability. Then, the follower’s best response is to undercut the leader’s price as long as it is larger than \( c \) and to set \( c \) otherwise. Anticipating this, the leader will optimally choose its price to be equal to \( c \).\(^{14}\) Third, the leader would want

\(^{13}\)For discussions of mutual forbearance in multimarket competition, see e.g., Bernheim and Whinston [1990] and Evans and Kessides [1994].

\(^{14}\)The leader would not choose a price less than \( c \) given the small probability that its marginal cost is less than \( c \).
to avoid supernormal profits to the follower, who is a potential rival in R&D races for other related products or technologies.

**IV. ENDOGENOUS SEPARATION**

Now suppose that there is no exogenous separation device in the market. Instead there is another product $B$ that is produced by two independent firms $B_1$ and $B_2$. Consumers have unit demands for product $B$. Firms $A_1$ and $A_2$ can individually pre-commit to a price discount for consumers who purchase a specific brand of product $B$ together with their own product. Consumer preferences for products $A$ and $B$ are independent. We assume that firms $B_1$ and $B_2$ do not offer their own brand-specific discounts, and focus on the strategic role of unilateral brand-specific discounts in leveraging market power across two otherwise unrelated markets.$^{15}$

Here we consider the case where consumer preferences for brands $B_1$ and $B_2$ are horizontally differentiated à la Hotelling.$^{16}$ So, consumers can be viewed as uniformly distributed on the unit interval $[0, 1]$. A particular consumer’s location on this line is denoted by $x$, with firms $B_1$ and $B_2$ being located at 0 and 1, respectively. If a consumer located at $x$ purchases from $B_j$, she gains the net utility of $v_B - p_{Bj}^i - t|x - \hat{x}_j|$ with $\hat{x}_j \in \{0, 1\}, j = 1, 2$, where $v_B$ is the consumer’s gross value of product $B$, $p_{Bj}^i$ is the price charged by firm $B_j$, and $t|x - \hat{x}_j|$ measures the disutility of the consumer due to the difference between the purchased product and her ideal product. It is assumed that the production cost is zero for both firms, and that $v_B$ is sufficiently large so that, in equilibrium, all consumers buy one unit of product $B$.

Then, the net utility gained by a consumer located at $x$ when purchasing product $A$ from $A_i$ and product $B$ from $B_j$ is given by

$$[v_A - (p_{Ai}^i - D_{ij})] + [v_B - t|x - \hat{x}_j| - p_{Bj}^j], \ i, j = 1, 2,$$

where $D_{ij} = d_i$ if $A_i$ is connected to $B_j$ via brand-specific discounts, and $D_{ij} = 0$ otherwise. Obviously, equilibrium effective prices for product $A$ cannot be larger than $v_A$ for all $i$ and

$^{15}$One may model this as two firms in different markets forming an alliance and cooperatively setting bundled discounts for consumers who buy from them, as in Gans and King [2006]

$^{16}$The analysis, however, would be easily extended to other duopoly models in which firms exercise non-trivial market power.
j. We initially consider the case where the degree of product differentiation in market $B$ (measured by $t$) is small in comparison with $v_A$ so that the firms in market $A$ can achieve only a partially collusive outcome (i.e., the market power to be leveraged is not strong enough for firms in market $A$ to achieve full collusion). Specifically, we assume that $v_A \geq 3t/2$. Later in subsection IV(iv), we discuss other cases where the product differentiation in market $B$ is sufficiently strong so that full collusion is attained in market $A$ via brand-specific discounts.

The timing of the game is the same as in the exogenous separation case, except that there exists another stage (stage 0) in which the firms in market $B$ set their prices independently and simultaneously before the firms in market $A$ set conditional discounts. We assume that firms $B_1$ and $B_2$ choose prices, not knowing who will be the price leader/follower in market $A$. So, in the eyes of the firms in market $B$ the price leader and follower in market $A$ are selected randomly. At the end of this section, we discuss how the results would change if the firms in market $B$ know the identity of the price leader/follower in market $A$.

We solve for a subgame perfect equilibrium using backward induction. Note that, in the absence of brand-specific discounts, the two markets are completely independent, giving rise to sequential Bertrand competition in market $A$ and standard Hotelling competition in market $B$. As before, we assume that marginal cost pricing prevails in the sequential pricing game in market $A$.

IV(i). Pricing of the firms in market $B$

Here we derive some preliminary results regarding the pricing equilibrium in market $B$, which will be useful in solving the entire game. The following lemma says that, given incomplete information about the identity of the price leader/follower and the random nature of pricing sequence in market $A$, the pricing behavior of the firms in market $B$ is neutral to brand-specific discounts and pricing decisions of firms $A_1$ and $A_2$.

Lemma 1. Suppose that the firms in market $B$ have uniform beliefs about the identities of the price leader and follower in market $A$ and they are risk-neutral. Then, in equilibrium, the firms behave as duopolists in the standard Hotelling model, each setting the symmetric price $p^1_B = p^2_B = t$ and earning expected profits of $t/2$. 

13
See Appendix for proof.

It is as if the firms in market B are unaware of such discounts or even the presence of product A. Hence, firms B1 and B2 behave independently of the prices and discounts set by the firms in market A. Of course, as will be shown below, brand-specific discounts may inflict profit losses to individual firms in market B ex post. However, these losses, if they exist, could be compensated via monetary transfer (e.g., side payments) from the corresponding firm in market A. Given this neutrality result, we now proceed to analyze the remaining stages of the game.

IV(ii). Second-stage pricing game in market A

Again we assume that, in the first stage, the price leader has committed to a discount $d_l$ for consumers purchasing from firm $B_1$ and the follower has committed to a discount $d_f$ for those purchasing from firm $B_2$. Note that, however, the condition for discounts is endogenized by consumers’ purchasing decisions in market B, unlike in the previous exogenous separation case. Hence, consumers’ product choices are interrelated over two unrelated markets via brand-specific discounts.

IV(ii)(a). Follower’s optimal pricing

Given $p^1_B = p^2_B = t$ in market B at stage 0, the effective prices (including transportation costs) faced by a consumer located at $x \in [0, 1]$ for the two products are:

$$
P = \begin{cases} 
    t + tx + p_l - d_l, & \text{when buying from } B_1 \text{ and } l \\
    t + tx + p_f, & \text{when buying from } B_1 \text{ and } f \\
    t + (1 - x) + p_l, & \text{when buying from } B_2 \text{ and } l \\
    t + (1 - x) + p_f - d_f, & \text{when buying from } B_2 \text{ and } f 
\end{cases}
$$

The follower’s demand function is then given by\(^{17}\)

$$
D_f(p_l, p_f; d_l, d_f) = \begin{cases} 
    0 & \text{if } p_f > p_l + d_f + \min\{t - d_l, 0\} \\
    \frac{1}{2} + \frac{(p_l - d_l) - (p_f - d_f)}{2t} & \text{if } p_l - d_l + \max\{d_f - t, 0\} \leq p_f \leq p_l + d_f + \min\{t - d_l, 0\} \\
    1 & \text{if } p_f < p_l - d_f + \max\{d_f - t, 0\}
\end{cases}
$$

\(^{17}\)Here we assume that consumers, when they are indifferent, purchase the product with a discount.
For instance, the follower’s demand is zero if his effective price is higher than that of the leader for all consumers \( (p_f - d_f > p_l + t - d_l) \) or if his effective price is so high that even those purchasing from \( B_2 \) prefer the leader’s product to the follower’s \( (p_f - d_f > p_l) \), i.e., if \( p_f > p_l + d_f + \min \{t - d_l, 0\} \). The follower’s demands for other cases are derived similarly. Note that the demand is discontinuous at \( p_l + d_f \) if \( d_f < t \) and at \( p_l \) if \( d_l < t \).

The follower has two options in responding to the leader’s price. One is to share the market with the leader by setting a moderate price. The other is to corner the market by setting a sufficiently low price. Which tactic is more profitable depends on the sizes of the two discounts and the leader’s price. Intuitively, the follower chooses to corner the market for small discounts. For large discounts, however, cornering the market is too costly, and the follower finds it more profitable to share the market with the leader. Below, we compare the follower’s maximal profits under the two regimes.

The follower’s problem under market sharing is defined as follows:

\[
\max_{p_f} : \ (p_f - d_f) \left[ \frac{1}{2} + \frac{(p_l - d_l) - (p_f - d_f)}{2t} \right].
\]

The first-order condition gives a unique solution \( p_f^S = d_f + \frac{t + p_l - d_l}{2} \). In order for this price to yield market sharing, it is required that \( p_l - d_l + \max \{d_f - t, 0\} \leq d_f + \frac{t + p_l - d_l}{2} \leq p_l + d_f + \min \{t - d_l, 0\} \), which leads to the condition \( |t - d_l| \leq p_l \leq t + d_l + 2 \min \{d_f, t\} \). If \( p_l < |t - d_l| \), we have a corner solution with \( p_f^S = p_l + d_f + \min \{t - d_l, 0\} \), where the follower’s demand is \( D_f = \frac{1}{2} - \frac{d_f + \min \{t - d_l, 0\}}{2t} \). Then, the follower’s optimal price and profit under market sharing are respectively given by

\[
\begin{align*}
p_f^S &= \begin{cases} 
  d_f + \frac{t + p_l - d_l}{2}, & \text{if } |t - d_l| \leq p_l \leq t + d_l + 2 \min \{d_f, t\} \\ 
  p_l + d_f + \min \{t - d_l, 0\}, & \text{if } p_l < |t - d_l| \end{cases} \\
\pi_f^S &= \begin{cases} 
  \frac{(t + p_l - d_l)^2}{8t}, & \text{if } |t - d_l| \leq p_l \leq t + d_l + 2 \min \{d_f, t\} \\ 
  \frac{[p_l + \min \{t - d_l, 0\}] \max \{t - d_l, 0\}}{2t}, & \text{if } p_l < |t - d_l| \end{cases}.
\end{align*}
\]

On the other hand, if \( p_l > t + d_l + 2 \min \{d_f, t\} \), the follower optimally corners the market \( A \) by setting

\[
p_f^M = p_l - d_l + \max \{d_f - t, 0\}.
\]
and earns profits of

$$\pi_f^M = p_l - d_l - \min\{t, d_f\}.$$  

The timeline of the game indicates that the price leader will not select a price leading to monopolization of market $A$ by the follower. Since such a strategy will never be supported as a perfect equilibrium of the whole game, we can restrict our attention to subgames where both firms are active in market $A$. Suppose the follower chooses to share the market by setting price $p_f^S$ as in (3). Then, the leader’s profit is given by

$$\pi_l = (p_l - d_l) \left[ \frac{1}{2} - \frac{(p_l - d_l) - (p_f^S - d_f)}{2t} \right]$$

$$= \frac{(p_l - d_l)}{4t} \left[ 3t - \max\{p_l - d_l, t - 2d_l - 2\min\{t - d_l, 0\}\} \right],$$

which is continuous in $p_l$. Note that, for $p_l - d_l \leq t - 2d_l - 2\min\{t - d_l, 0\}$ (i.e. $p_l \leq |t - d_l|$), the profit function is monotonically increasing in $p_l$, which together with the continuity implies that the profit function is maximized at $p_l = p_l^*$ $\geq |t - d_l|$, where $p_l^*$ is characterized as below. This implies that the leader’s optimal price will be greater than or equal to $|t - d_l|$. Recall that the follower always corners the market for $p_l > t + d_l + 2\min\{d_f, t\}$. So, we restrict out attention to the case where $|t - d_l| \leq p_l \leq t + d_l + 2\min\{d_f, t\}$, for which the relevant follower profit is given by $\pi_f^S = \frac{(t + p_l - d_l)^2}{8t}$. Later, we will show that $(p_l^*, p_f^S)$ indeed constitutes an equilibrium.

Then, comparing the follower’s profits under the two regimes, $\pi_f^S$ and $\pi_f^M$, leads to the following result.

**Lemma 2.** Given $|t - d_l| \leq p_l \leq t + d_l + 2\min\{d_f, t\}$, the follower prefers market sharing if

$$d_f \geq t$$  

(5)

or

$$d_f \leq t \text{ and } p_l - d_l \leq 3t - \sqrt{8t(t - d_f)},$$  

(6)

and prefers market cornering otherwise.

See Appendix for proof.
IV(ii)(b). Leader’s optimal pricing

The leader’s problem is

$$\max_{p_l} \left( \frac{(p_l - d_l)}{4t} \right) \left[ 3t - \max\{p_l - d_l, t - 2d_l - 2 \min\{t - d_l, 0\}\} \right]$$

subject to \( \pi_f^S(d_l, d_f) \geq \pi_f^M(d_l, d_f) \). Note that \( p_l - d_l > t - 2d_l - 2 \min\{t - d_l, 0\} \) must be true given \( p_l \geq |t - d_l| \) at equilibrium.

If we ignore the constraint for the moment, the solution of the problem is given by

$$p_l^* = \frac{3}{2} t + d_l.$$  

Then, from the previous analysis, we obtain the following:

$$p_f^* = p_f^S = \frac{5}{4} t + d_f,$$

$$\pi_l^* = \frac{9}{16} t,$$

$$\pi_f^* = \pi_f^S = \frac{25}{32} t.$$  

Note that the condition \(|t - d_l| \leq p_l\) is satisfied at \( p_l^* = \frac{3}{2} t + d_l \) \((|t - d_l| \leq p_l^* = 3t/2 + d_l)\).\(^{18}\) Then, the constraint \( \pi_f^S(d_l, d_f) \geq \pi_f^M(d_l, d_f) \) reduces to the condition for market sharing in Lemma 2, which are satisfied at \( p_l^* = \frac{3}{2} t + d_l \) if

$$d_f \geq \frac{23t}{32}. \quad (7)$$

So, the prices \((p_l^*, p_f^*)\) indeed constitute an equilibrium of the second-stage pricing game if the discount set by the follower is large enough (precisely if \( d_f \geq 23t/32 \)), irrespective of the leader’s discount. We do not explicitly characterize other possible equilibria of this pricing game for different values of \( d_l \) and \( d_f \) since the corresponding profits of the leader and follower cannot be greater than \( \pi_l^* \) and \( \pi_f^* \), respectively, as will be shown later. Note that, given the equilibrium prices, consumers are divided into two groups at the indifferent type \( x = 3/8 \).

IV(iii). First-stage discounting game

Let us now consider the first-stage discounting game where firms \( A_1 \) and \( A_2 \) independently and simultaneously choose their brand-specific discounts. The following proposition establishes

\(^{18}\)The condition \( p_l \leq t + d_l + 2 \min\{d_f, t\} \) is also satisfied under condition \((7)\).
that the firms in market A can use brand-specific discounts to achieve (partially) collusive outcomes, making strictly positive profits at equilibrium.

**Proposition 2.** In the subgame perfect equilibrium, the price follower commits to a brand-specific discount larger than or equal to $23t/32$ (i.e., $d_f > 23t/32$), the follower earns profits of $\pi_f^* = 25t/32$ and the leader earns profits of $\pi_l^* = 9t/16$.$^{19}$

See Appendix for proof.

We can show that $\pi_l^*$ and $\pi_f^*$ are indeed the maximal profits obtainable by the leader and the follower, respectively, for all values of $d_l$ and $d_f$. With a large discount, the follower finds it very costly to cut prices for market cornering since it has to set its headline price sufficiently low in order to capture all consumers, including those who purchase from firm $B_2$ and are therefore entitled to the discount offered by the leader. Offering a large discount is a commitment to less aggressive pricing in the later stage, which in turn induces the rival’s friendly behavior.

Consumers become worse off with brand-specific discounts since they face higher effective prices in market A compared with the benchmark equilibrium with the standard Bertrand outcome:

$$p_f^* - d_f = \frac{5}{4}t > 0 \text{ and } p_l^* - d_l = \frac{3t}{2} > 0.$$ 

Social welfare is also lower under brand-specific discounts because consumer gross utilities decrease due to the increase in transportation costs. Note that consumers with $x \in [3/8, 1/2]$ end up buying their less preferred brand of product $B$, bearing larger transportation costs compared with the benchmark case where market $B$ is symmetrically split.$^{20}$ Note that the adverse welfare effect would be more serious if we allow for elastic demands and partial participation.$^{19}$

$^{19}$If the price leader and follower are selected randomly, each firm’s problem is to choose a discount to maximize expected profits $\pi_l/2 + \pi_f/2$ or, equivalently, joint profits $(\pi_l + \pi_f)$. It is immediate from the previous analysis that firms $A_1$ and $A_2$ will both commit to brand-specific discounts $d_i \geq 23t/32$ in order to obtain the maximal joint profit $(43t/32)$, each expecting one-half of it.

$^{20}$Note that, however, this is an artifact caused by sequential pricing in the second stage and would disappear in simultaneous pricing.
The result above shows that brand-specific discounts can be used as a collusion-facilitating device. However, given that \( v_A \geq 3t/2 \), the equilibrium prices fall short of the full collusive price. That is, the collusive effect of brand-specific discounts is limited by the degree of monopoly power in the market to which discounts are tied. This contrasts with the exogenous separation case where the firms in market \( A \) achieve the fully collusive outcome, extracting the whole consumer surplus.

IV(iv). Full collusion with large \( t \)

Suppose that \( t \) is large in comparison with \( v_A \) (i.e. \( v_A < 3t/2 \)). Then, the previously derived effective prices \( 3t/2 \) and \( 5t/4 \) are not feasible (consumers would not buy a unit of product \( A \) at these prices), and the constraint that \( p_i - d_i \leq v_A \) is binding for some \( i \).

It turns out that including the price ceiling does not drastically alter the incentives for pre-commitment to price discounts. Intuitively, since the follower’s profit is (weakly) increasing in the leader’s effective price, the follower still wishes to commit to sharing the market by choosing a high discount. Consequently, the cross-market bundled discounts prevail even with the price ceiling.

Proposition 3. When \( v_A \in [t, 3t/2] \), the price follower commits to a brand-specific discount greater than or equal to \( t-(3t-v_A)^2/8t \), and the follower and leader earn profits of \( (v_A + t)^2 / 8t \) and \( v_A(3t - v_A)/4t \), respectively. When \( v_A < t \), the follower commits to a discount greater than or equal to \( v_A/2 \), and both firms earn \( v_A/2 \) (i.e., half of the fully collusive joint profit).

See Appendix for proof.

Notice that, if the degree of product differentiation is sufficiently large compared to \( v_A \) (i.e., \( v_A < t \)), the firms can attain the fully collusive outcome as in the exogenous separation case analyzed in Section III. Thus, this proposition clearly shows that bundled discounts for unrelated products can be a blatant collusion device, and the practice can be abused by firms as a disguise to antitrust investigations.

Remark 2. We made a simplifying assumption that the firms in the differentiated goods market do not know who will be the price leader in the homogenous goods market. Here
we briefly discuss whether our main result would remain valid even without the assumption. Suppose the identities of the price leader/follower in market \( A \) are known to the firms in market \( B \). The simplest setup we can imagine is the case where two alliances are exogenously formed with respect to brand-specific discounts. Then the model is the same as before, except that the prices set by the firms in market \( B \) are now asymmetric. The analysis would be more complicated due to the strategic pricing behavior of the firms in market \( B \). However, the previous analysis of exogenous separation indicates that, no matter how the market is separated, the firms in market \( A \) would have incentive to offer large discounts in order to make supernormal profits at equilibrium. So, without solving for the analytic solution, we can see that the qualitative result would continue to hold even if the identities of the price leader and follower in market \( A \) are known to the firms in market \( B \). The only difference is that the partner firm of the leader would decrease its price (compared with the symmetric case) in order to preemptively respond to the follower’s later undercutting of the leader’s price, and the opposite holds for the partner firm of the follower. (This can be verified by inspecting the reaction functions of the firms in market \( B \).) Obviously, however, the price leader in market \( A \) will need to compensate (maybe through side payments) its partner firm in market \( B \).

V. CONCLUSION

This paper investigated the competitive effect of interfirm bundled discounts, the marketing practice of offering discounts conditional on the purchase of a particular brand of other (related or unrelated) products. The central finding is that firms with no (or less) market power can use the interfirm bundled discount to leverage market power of other unrelated firms.

We have shown that interfirm bundled discounts, by creating interlocking relations between otherwise unrelated products, act to relax price competition. Furthermore, using the discount scheme, firms with no market power may achieve the fully collusive outcome when the difference in market power between the associated markets is sufficiently large. Obviously, consumers are worse off since the effective prices for the goods increase in the presence of such discount schemes.

Our analysis provides important implications for public policy toward bundled discounts.
Specifically, the above results suggest that competition policy needs to address the collusive effect of such discount practices, especially when the market tied by discounts is subject to a high degree of market power. However, thus far antitrust case involving bundled discounts has been rare. An exception is the case of the ‘shopper docket’ scheme tying petrol discounts with grocery purchases, which has been reviewed by the Australian Competition and Consumer Commission since 2004.\textsuperscript{21} The ACCC has recently accepted undertakings from two major supermarket chains to voluntarily limit the bundled discounts to a maximum of 4 cents per liter of fuel.\textsuperscript{22} The chairman of the ACCC warned that bundled discounts "could have long-term effects on the structure of the retail fuel markets, as well as the short-term effects of increasing general pump prices," which is in line with the analysis of this paper.\textsuperscript{23}

It is fortunate that bundled discount schemes are getting more attention in the antitrust arena. Gans and King [2006] argue that bundled discounts of unrelated products should be regarded with suspicion. Organizations such as Master Grocers Australia [2012] insist that the practice remains anti-competitive in effect and is not in the public interest, and that ACCC should revoke their decision. Also, the Korea Fair Trade Commission recently announced that it will introduce credit cards that offer reward points for all, as opposed to particular, petrol stations in order to discourage potentially anti-competitive bundled discounts between credit card companies and petrol stations.

APPENDIX

\textit{Proof of Lemma 1}

\textsuperscript{21}\textsuperscript{21} The main shopper docket schemes under review relate to discounts offered by a subsidiary of Coles Myer Ltd (Coles), resulting from an alliance between Coles and The Shell Company of Australia Ltd (Shell), and by Woolworths Limited (Woolworths) and one of its subsidiaries, Australian Independent Retailers Pty Ltd (AIR.)' according to the Australian Competition and Consumer Commission [2004, p.2]

\textsuperscript{22}\textsuperscript{22} In April 2014, the Federal Court found one of the supermarkets, Woolworths, to have breached this undertaking.

\textsuperscript{23}\textsuperscript{23} The official media release is available in: www.accc.gov.au/media-release/coles-and-woolworths-undertake-to-cease-supermarket-subsidised-fuel-discounts
In general, equilibrium market shares and profits of firms $B_1$ and $B_2$ depend on not only their own prices, but also the prices and brand-specific discounts set by the firms in market $A$. However, since firms $B_1$ and $B_2$ do not know which firm in market $A$ it will be connected to via brand-specific discounts and who will later be the price leader in market $A$, they do not know in advance how brand-specific discounts will affect their profits. Nevertheless, we know that, given that all consumers buy one unit of product $B$, if one firm gains $\xi$ in terms of market share from brand-specific discounts, the other firm will lose exactly the same amount. Thus, we can write firm $B_1$’s market share as

$$ x^+ = \frac{t - p_B^1 + p_B^2}{2t} + \xi (p_A^i, d_i) $$

if it benefits from brand-specific discounts and as

$$ x^- = \frac{t - p_B^1 + p_B^2}{2t} - \xi (p_A^i, d_i) $$

if it loses from brand-specific discounts, where $\xi (p_A^i, d_i)$ denotes the sole effect of brand-specific discounts on market shares. Firm $B_2$’s market share will be then $1 - x^+$ or $1 - x^-$, respectively. Given the uniform belief about the identity of the price leader in market $A$, the risk-neutral firms $B1$ and $B2$ will choose prices in order to maximize their expected profits:

$$ E[\pi_B^1] = p_B^1 \frac{x^+ + x^-}{2} = p_B^1 \frac{t - p_B^1 + p_B^2}{2t}, $$

$$ E[\pi_B^2] = p_B^2 \frac{(1 - x^+) + (1 - x^-)}{2} = p_B^2 \frac{t - p_B^2 + p_B^1}{2t}, $$

which are the profit functions obtained in the standard Hotelling model. Therefore, we expect that they will behave just like Hotelling duopolists.

**Proof of Lemma 2**

For $|t - d_l| \leq p_l \leq t + d_l + 2 \min \{d_f, t\}$, the follower’s optimal choice depends on the leader’s discounted price. The follower’s profit under market sharing is greater than or equal to that under monopolization if and only if

$$ \Delta \pi \equiv \pi_f^S - \pi_f^M = \frac{(t + p_l - d_l)^2}{8t} - (p_l - d_l - \min \{d_f, t\}) \geq 0. $$
Note that $\Delta \pi$ is a convex quadratic function of $z \equiv p_l - d_l$. Suppose first that $d_f \geq t$ (i.e., $\min \{d_f, t\} = t$). Then, we have $\Delta \pi(z) = \frac{(t+z)^2}{8t} - (z-t) \geq 0$ with equality only when $z = 3t$. In this case, the follower always chooses to share the market. Suppose next that $d_f \leq t$ (i.e., $\min \{d_f, t\} = d_f$). Then, among the two solutions of equation $\Delta \pi(z) = \frac{(t+z)^2}{8t} - (z-d_f) = 0$, the one that satisfies the condition $p_l \leq t + d_l + 2 \min \{d_f, t\}$ is $z^* = 3t - \sqrt{8t(t-d_f)}$. Thus, the follower prefers market sharing to cornering if $p_l - d_l \leq z^* = 3t - \sqrt{8t(t-d_f)}$ for the case of $|t-d_l| < d_l + z^*.$

**Proof of Proposition 2**

For the proof, it will suffice to show that $\pi^*_l$ and $\pi^*_f$ at the market sharing equilibrium with $d_f \geq 23t/32$ are truly the maximal profits obtainable by the leader and the follower respectively for all values of $d_l$ and $d_f$. First note that $\pi^*_f$ is the leader’s overall maximal profit under market sharing since it is the unconstrained solution of its profit-maximization problem, and the leader would obtain zero profit under monopolization. Next we show that the follower cannot earn profits greater than $\pi^*_f$ with $d_f < 23t/32$. Suppose that $d_f < 23t/32$. Then, the constraint $\pi^*_f \geq \pi^*_M$ in the leader’s profit maximization problem will be binding, and different equilibrium prices, denoted by $(p^*_l, p^*_f)$, will be obtained. Note from (6) that the constraint $\pi^*_f \geq \pi^*_M$ sets an upper bound on $p_l - d_l$. From (7), we know that the constraint is binding at $d_f = 23t/32$, and in this case $p^*_l - d_l = \frac{3t}{2}$. The binding constraint (i.e., $\pi^*_f = \pi^*_M$) for $d_f < 23t/32$ is given by $p^*_l - d_l = 3t - \sqrt{8t(t-d_f)}$. Since the upper bound is increasing in $d_f$, it must be true that

$$p^*_l - d_l = 3t - \sqrt{8t(t-d_f)} < \frac{3t}{2} = p^*_l - d_l \implies p^*_l - d_l < p^*_l - d_l.$$  

From (4), we can see that $\pi^*_f$ is increasing in $p_l - d_l$, independent of $d_f$. This implies that the follower’s profit with $(p^*_l, p^*_f)$ cannot be larger than the equilibrium profit with $(p^*_l, p^*_f)$ and $d_f \geq 23t/32$.

**Proof of Proposition 3**

Let us first consider the case of $t < v_A < 3t/2 = p^*_l - d_l$, i.e. the case where the constraint binds only for the leader’s price. When discounts are large enough for the follower to decide
to share the market, the equilibrium prices are given by

\[
\hat{p}_l = v_A + d_l \\
\hat{p}_f = \frac{v_A + t}{2} + d_f,
\]

and the corresponding profits are

\[
\hat{\pi}_l = \frac{v_A (3t - v_A)}{4t} \\
\hat{\pi}_f = \frac{(v_A + t)^2}{8t}.
\]

By the same logic developed in the proofs of Lemma 2 and Proposition 2, it is straightforward to show that both firms wish to commit to large brand-specific discounts. The logic is that, since the follower’s profit is (weakly) increasing in the leader’s effective price, the follower wishes to commit to market sharing by choosing a high discount. The cut-off value of the follower’s discount ensuring market sharing is derived from condition (6) in Lemma 2 as follows. The follower chooses to share the market if

\[
\hat{p}_l - d_l \leq 3t - \sqrt{8t(t - d_f)},
\]

which, after substituting \( \hat{p}_l - d_l = v_A \), reduces to

\[
d_f \geq t - \frac{(3t - v_A)^2}{8t}.
\]

Second, if \( v_A < t \), the follower is also constrained to an upper price limit of \( v_A \) (\( \hat{p}_f - d_f = \frac{v_A + t}{2} > v_A \)). Then the equilibrium prices and profits are given as

\[
\hat{p}_i = v_A + d_i, \\
\hat{\pi}_i = \frac{v_A}{2}
\]

for \( i = l, f \). Given the leader’s effective price \( v_A \), the follower’s profit is \( v_A/2 \) when sharing the market and \( v_A - \min\{d_f, t\} \) when monopolizing the market (see the proof of Lemma 2 for details). Therefore, in order for the follower to choose market sharing in the pricing game, the discount \( d_f \) should be greater than or equal to \( v_A/2 \).
REFERENCE


nomic Theory, 148, pp. 448-472.

Banerjee, A. and Summers, L. H., 1987, ‘On Frequent Flyer Programs and Other Loyalty-
inducing Economic Arrangements,’ Discussion paper No. 1337, Harvard Institute of Economic
Research..

Bernheim, B. and Whinston, M., 1990, ‘Multimarket Contact and Collusive Behavior,’

Brito, D. and Vasconcelos, H., 2013, ‘Inter-firm Bundling and Vertical Product Differen-

International Journal of Industrial Organization, 8, pp. 353-373.


Rand Journal of Economics, 32, pp. 52-71.

Evans, W. and Kessides, I., 1994, ‘Living by the “Golden Rule”: Multimarket Contact in

tomer Recognition,’ in T. Hendershott (eds), Economics and Information Systems, Vol. 1,
(Emerald Group Publishing Limited, Bingley BD16 1WA U.K.)


Fudenberg, D. and Tirole, J., 2000, ‘Customer Poaching and Brand Switching,’ RAND


