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Privatization, Underpricing, and Welfare in the Presence of Foreign Competition

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Abstract

We analyze privatization in a differentiated oligopoly setting with a domestic public firm and foreign profit-maximizing firms. In particular, we examine pricing below marginal cost by the public firm, the optimal degree of privatization, and the relationship between privatization and foreign ownership restrictions. When market structure is exogenous, partial privatization of the public firm improves welfare by reducing public sector losses. Surprisingly, even at the optimal level of privatization, the public firm's price lies strictly below marginal cost, resulting in losses. Our analysis also reveals a potential conflict between privatization and investment liberalization (i.e., relaxing restrictions on foreign ownership) in the short run. With endogenous market structure (i.e., free entry of foreign firms), partial privatization improves welfare through an additional channel: more

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foreign varieties. Furthermore, at the optimal level of privatization, the public firm's price lies strictly above marginal cost and earns positive profits.

1. Introduction

In several industries such as energy, steel, airlines, and banking, public or state-owned firms co-exist and often compete with private firms (Matsumura and Matsushima 2004). Public firms represent up to 40% of value added and 10% of employment in some OECD countries (Long and Stähler 2009). These shares are even higher in developing and transition economies. Over the years however, the poor performance of public firms has prompted Canada, Europe, Japan, the United Kingdom, and more recently the developing economies in Asia and Latin America to embark on privatization (Gupta 2005, Dong, Putterman, and Unel 2006). Most developed Western economies were already open when they embarked on privatization. Mukherjee and Suetrong (2009) note that privatization and trade liberalization coincided in several developing and transition economies.

Privatization in a closed economy setting has been studied extensively in the literature on mixed oligopoly.[1] However, as the discussion above suggests, open economy considerations are at least equally important for understanding the welfare implications of privatization. This paper contributes to the small yet growing literature on privatization in an open economy setup where domestic public firms compete with foreign private firms (see, e.g., Fjell and Pal 1996, Pal and White 1998, Barcena-Ruiz and Garzon 2005, Long and Stähler 2009, Matsumura, Matsushima, and Ishibashi 2009, Mukherjee and Suetrong 2009).

We consider two interrelated questions. First, does privatization necessarily improve welfare in an open economy setting? Second, what are the effects of privatization on the financial health of public firms? We address these questions in a differentiated mixed oligopoly where a welfare-maximizing public firm competes with profit-maximizing foreign firms. As is standard in this literature (see Matsumura 1998, Fujiwara 2007, Long and Stähler 2009) we assume that a partially privatized firm maximizes a weighted average of its own profit and welfare where the weight attached to profit captures the extent of privatization.[2] Analyzing privatization in this environment, we find two fairly robust results:

[1] See, for example, De Fraja and Delbono (1990), Cremer, Marchand, and Thisse (1991), Anderson, de Palma, and Thisse (1997), and Matsumura (1998).

[2] Fershtman (1990) suggested an alternative approach to modeling privatization, in which the weights are assigned to reaction functions, instead of to the objective functions.

- Partial privatization to an "appropriate" extent always improves welfare. This is true both in the short run—with a fixed number of foreign firms—as well as in the long run—with free entry of foreign firms.
- The financial health of the public firm in the short run is quite different from that in the long run. Under optimal privatization, the public firm continues to make losses in the short run, while it makes positive profits in the long run.

The earlier works that have considered optimal privatization found privatization to be welfare-improving either in the short run or in the long run, but not necessarily in both. Generally, the welfare effect of privatization varies depending on whether the goods are homogenous or differentiated, and/or whether the economy is closed or open. In a closed economy with homogenous goods, privatization improves welfare in the short run, if the marginal cost of production is increasing (Matsumura 1998), but not in the long run (Matsumura and Kanda 2005). In such settings, the fully public firm produces more and sets price equal to its own marginal cost. Typically, private competitors produce less and the price remains well above the marginal cost causing allocative inefficiency. Privatization helps to reduce this inefficiency in the short run by altering the public firm's behavior. In the long run, free entry does the trick.

With differentiated goods, however the story can be different. Using the constant elasticity of substitution (CES) utility function, Anderson *et al.* (1997) show that privatization only increases price in the short run and thus reduces welfare; but in the long run, privatization is accompanied by the entry of newer varieties. If the consumers' preference for variety is strong, then privatization improves welfare. However, Fujiwara (2007) has shown that the short-run privatization result is not necessarily robust. If the utility function is quadratic and privatization is optimally set, then, unlike Anderson *et al.* (1997), privatization can improve welfare.

We extend this line of enquiry to open economy settings and show that such welfare improvements occur not only in the short run, but also in the long run. We also show that privatization may be difficult in the short run. In an open economy where a public firm competes with foreign firms, it sets price *below* marginal cost which creates allocative inefficiency.[3] Privatization increases the price of domestic variety, cuts back losses, and reduces the inefficiency of the public firm. Counteracting these positive effects is price increase of the foreign variety which reduces consumer surplus. An appropriate level of privatization, determined by balancing these two effects, improves welfare. The welfare improvement result appears to be fairly robust,

[3] Fjell and Pal (1996) were the first to point out this underpricing problem. In a homogenous goods setting, they showed that the cost of the public firm must be increasing for underpricing to occur. We find that in a differentiated products setting, increasing marginal costs is not necessary for underpricing.

and it holds under at least two utility functions—quadratic (as in our paper) and CES (Matsumura *et al.* (2009). However, importantly, underpricing does not occur in Matsumura *et al.* (2009) and Anderson *et al.* (1997).[4]

Optimal privatization helps us to gain deeper insights into the underpricing problem of the public firm. Starting from a situation of full public ownership, a small order of privatization reduces the loss of the domestic public firm. However, it also increases the price of the foreign variety; consequently, the domestic expenditure on foreign goods increases causing an increase in the leakage from the national economy. To restrict this adverse effect of privatization, government does not privatize fully and tolerates some losses in the public firm, settling for partial privatization.

The phenomenon of underpricing leads to an important question: Who will buy a loss-making firm? One may argue that the government should pay for the losses. The payment can be financed by a lump-sum tax on consumers or by a tariff on the foreign good, but the idea of paying a private firm from the public exchequer may not be politically palatable. A way out is either to consider privatization with an explicit non-negative profit constraint (Vickers and Yarrow 1988), or to have supplementary policies to overcome the losses. For example, if the share of foreign ownership is restricted to a partial level, the losses of the privatized firm will disappear. We show that when the foreign firm is a joint venture, the degree of optimal privatization will be inversely related to the degree of foreign ownership. Therefore, the losses of the (privatized) public firm can be overcome by forcing the foreign firm to have a domestic partner (see Section 5). In a similar vein, if there are some domestic profit-maximizing firms in addition to the foreign firm, then the underpricing problem can also be mitigated (see Section 4.3.1).

In light of these findings, our underpricing result should be taken as a reflection of a short-run policy dilemma. If welfare is to be maximized, some loss of the public firm has to be tolerated; alternatively, if the loss is to be eliminated, then welfare will be less than maximal. There is also a short-run tradeoff between inviting foreign capital and privatizing loss-making public enterprises.[5]

Surprisingly, the underpricing problem and consequently the losses disappear in the long run with the free entry of foreign firms (see Section 6). The long-run scenario can be seen as a framework of comprehensive policy reform, in which not only is privatization possible, but also trade and investment reforms can be carried out simultaneously. To see the long-run effect, we need to understand how privatization encourages entry and competition in a fully liberalized environment. Starting from a situation of full

[4] The main reason for the lack of underpricing in their settings is reliance on price competition.

[5] In a different setup, Mukherjee and Suetrong (2009) show that privatization and foreign investment will be mutually reinforcing. However, critical to their argument is the assumption that the foreign firm has superior technology.

public ownership and a fixed number of foreign firms, a small order of privatization introduces three effects: it reduces the losses of the public firm, it puts an upward pressure on the foreign firms' price, and it encourages the entry of more foreign firms. Entry not only improves social welfare by offering more variety, but also reduces the upward pressure on the foreign price. Because of these entry-induced effects and the falling losses of the public firm, privatization can improve welfare. The government then privatizes up to a level at which the public firm earns strictly positive profit, and social welfare is also maximized. Previously, authors such as Anderson *et al.* (1997) and Matsumura *et al.* (2009) did not obtain the underpricing result; nor did they find an unconditional welfare improvement, possibly because they did not examine optimal privatization as we have done.

A number of papers have allowed foreign competition in mixed oligopolies (with homogenous goods), such as Pal and White (1998), Barcena-Ruiz and Garzon (2005), Chao and Yu (2006), and Long and Stähler (2009), but the main concerns of these papers are subsidies, optimal trade policies, or cost asymmetry, rather than optimal privatization. Similarly, there are papers like ours which have studied product variety in mixed oligopolies (with domestic firms), but they have taken a spatial approach, which leads to a different set of concerns. More commonly, their concerns are the impact of privatization on equilibrium locations (i.e., choice of variety), and in some cases on technological improvements. See, for example, Cremer *et al.* (1991), Matsumura and Matsushima (2004), Heywood and Ye (2009), and Kumar and Saha (2008).[6]

Closest to our work is Matsumura *et al.* (2009), who examine privatization in the presence of foreign competition. While our paper is complementary to theirs, there are two key differences. First, we focus on quantity competition while they consider price competition. This difference is important since the issue of underpricing and loss-making public firms, which are of central concern in our paper, does not arise in price competition. Second, the degree of privatization is endogenous in our model, whereas, in Matsumura *et al.* (2009), the firm is either fully public or fully private. Treating privatization as endogenous has important implications. For example, in the presence of free entry, a public firm can make losses for certain levels of privatization, whereas under optimal privatization that never happens.

In sum, we bring together differentiated products, foreign competition, and endogenous privatization, and look at both short and long runs. We find that, in all regimes, privatization improves welfare, but does not necessarily eliminate the losses of the public firm. To eliminate losses, privatization needs to be complemented with both trade and entry reforms.

[6] Cremer *et al.* (1991) were the first to analyze privatization in a differentiated products setting.

2. Preliminaries

Consider a simple open economy with two sectors: a competitive sector producing the numéraire good y and an imperfectly competitive sector with two firms, one domestic (denoted d) and one foreign (denoted f), each producing a distinct differentiated product for a home market. Let p_i and q_i, respectively, denote firm i's price and quantity where $i = d, f$. The representative consumer maximizes $V(q, y) \equiv U(q) + y$ subject to $p_d q_d + p_f q_f + y \leq I$ where $q \equiv (q_d, q_f) \in \Re^2_+$ and I denotes income. The utility function $U(q)$ is continuously differentiable as many times as required on $\Re^2_+$. Furthermore, the following holds:

ASSUMPTION 1: *For $i, j \in \{d, f\}$, $i \neq j$, (i) $U_i(q) \equiv \frac{\partial U(q)}{\partial q_i} > 0$, (ii) $U_{ii}(q) \equiv \frac{\partial^2 U(q)}{\partial {q_i}^2} < 0$, (iii) $U_{ij}(q) \equiv \frac{\partial^2 U(q)}{\partial q_i \partial q_j} < 0$, (iv) $U_{ij}(q) = U_{ji}(q)$, and (v) $|U_{ii}(q)| > |U_{ij}(q)|$.*

These assumptions are standard in the differentiated duopoly literature. See, for example, Section 5 in Singh and Vives (1984).

Since $V(q, y)$ is separable and linear in y, there are no income effects. Consequently, for a large enough income, the representative consumer's optimization problem is reduced to choosing q to maximize $U(q) - p_d q_d - p_f q_f$. Utility maximization yields the inverse demands: $p_i = \frac{\partial U(q)}{\partial q_i} \equiv P_i(q)$ for $q_i > 0$. Then, applying Assumption 1 gives the following. For all $i, j \in \{d, f\}$, $i \neq j$,

$$\frac{\partial P_i(q)}{\partial q_i} < 0, \quad \frac{\partial P_i(q)}{\partial q_j} < 0, \text{ and } \frac{\partial P_i(q)}{\partial q_j} = \frac{\partial P_j(q)}{\partial q_i},$$

which respectively imply that the inverse demand slopes downward, domestic and foreign varieties are substitutes, and that the cross effects are symmetric. Following Singh and Vives (1984), we make an additional assumption.

ASSUMPTION 2: *For $i, j \in \{d, f\}$, $i \neq j$, we have (i) $\frac{\partial^2 P_i(q)}{\partial q_i^2} \leq 0$ and (ii) $\frac{\partial^2 P_i(q)}{\partial q_i \partial q_j} \geq 0$.*

Both (i) and (ii) hold for the standard linear inverse demand system given by $P_i(q) = a - q_i - bq_j$, where $b \in (0, 1)$.[7]

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[7] The assumption is stronger than necessary. For example, instead of (i), we could use $\frac{\partial P_i(q)}{\partial q_i} + \frac{\partial^2 P_i(q)}{\partial q_i^2} q_i \leq 0$. General demand functions are rarely used in differentiated oligopolies. So, we adopt the assumption made in Section 5 of Singh and Vives (1984), which is one of the very few analyses on differentiated duopoly with general demand functions.

Each firm has a symmetric constant marginal cost of production, $m > 0$. There are no fixed costs or trade costs. Later, we will allow cost asymmetry and tariff.

Cournot competition: Corresponding to a quantity vector $q \equiv (q_d, q_f)$ the profits of firm i, consumer surplus, and home country's welfare denoted by $\pi_i(q)$, $cs(q)$, and $w(q)$ respectively are

$$\begin{aligned}
\pi_i(q) &= (P_i(q) - m)\, q_i, \\
cs(q) &= U(q) - P_d(q) q_d - P_f(q) q_f, \\
w(q) &= cs(q) + \pi_d(q) \equiv U(q) - P_f(q) q_f - m q_d.
\end{aligned}$$

Note that since firm f is foreign, $\pi_f(q)$ is not accounted for in $w(q)$.

Firms compete in quantities. The foreign firm chooses q_f to maximize $\pi_f(q) \equiv (P_f(q) - m) q_f$. The domestic firm, which may be partially privatized, chooses q_d to maximize a weighted combination of national welfare and own profits:

$$(1 - \theta) w(q) + \theta \pi_d(q) \equiv R(q; \theta), \tag{1}$$

where $\theta \in [0, 1]$ is the extent of privatization. For a fully public firm, $\theta = 0$. For a profit-maximizing private firm, $\theta = 1$. More generally, we can interpret θ as a fraction of privately held shares in firm d and consider $R(q; \theta) \equiv (1 - h(\theta)) w(q) + h(\theta) \pi_d(q)$ where $h(0) = 0$, $h(1) = 1$ and $h'(\theta) > 0$ for all $\theta \in (0, 1)$. While any such $h(.)$ serves our purpose, we choose $h(\theta) = \theta$ for analytical convenience. Matsumura (1998) provides a similar interpretation of θ in a closed economy setting. Also see Fershtman (1990) in this regard.[8]

As is standard in most analyses of Cournot competition with profit-maximizing firms, we assume that q_d and q_f are strategic substitutes.

ASSUMPTION 3: *For all* $i, j \in \{d, f\}$, $i \neq j$, $\frac{\partial^2 \pi_i(q)}{\partial q_i \partial q_j} < 0$.

From Assumption 2 it is straightforward to show that $\frac{\partial^2 w(q)}{\partial q_d \partial q_f} \leq 0$. Together with Assumption 3 this implies that $\frac{\partial^2 R(q;\theta)}{\partial q_d \partial q_f} < (\leq) 0$ for all $\theta > (\geq) 0$. This ensures that the public firm's reaction function is downward sloping, irrespective of the degree of privatization.

[8] One may argue that at a formal level privatization is just another way of altering incentives of the firm and influencing the Cournot outputs, as is done in the managerial incentives literature (Fershtman and Judd 1987). We agree that there are plenty of examples of incentivizing public sector managers. For example, one may recall Groves *et al.* (1994) for a study of managerial incentives in Chinese state-owned firms in the 1980s. However, privatization is widely regarded as a more committed and effective approach to seeking efficiency than managerial incentives. The recent experiences of China and India confirm this view. See Dong *et al.* (2006) and Gupta (2005) for privatization in China and India, respectively.

Next we characterize the Cournot equilibrium (Section 3) and then move on to analyze endogenous privatization (Section 4), joint ventures (Section 5), and finally privatization in the presence of free entry (Section 6).

3. Cournot Equilibrium

Let $q(\theta) = (q_d(\theta), q_f(\theta))$ denote the equilibrium quantity vector. Assume that $q_i(\theta) > 0$ for $i = d, f$. Then the following first-order conditions must hold:

$$\frac{\partial R(q(\theta), \theta)}{\partial q_d} = (1-\theta)\frac{\partial w(q(\theta))}{\partial q_d} + \theta\frac{\partial \pi_d(q(\theta))}{\partial q_d} = 0, \tag{2}$$

$$\frac{\partial \pi_f(q(\theta))}{\partial q_f} = P_f(q(\theta)) - m + q_f(\theta)\frac{\partial P_f(q(\theta))}{\partial q_f} = 0. \tag{3}$$

Define $p_i(\theta) = P_i(q(\theta))$ and $\Pi_i(\theta) = \pi_i(q(\theta))$ to be the equilibrium price and profit of firm i, respectively. Given that $U(q)$ is continuously differentiable, $q_i(\theta)$, $p_i(\theta)$, and $\Pi_i(\theta)$ are also continuously differentiable in θ.

From (3) it is clear that the foreign firm's price is strictly higher than its marginal cost. If $\theta = 1$, that is, if the domestic firm maximizes profit, then the same holds for the domestic firm as well. By the standard continuity argument, $p_d(\theta) > m$ provided θ is close to unity. However, if θ is small, that is, if the extent of privatization is low then $p_d(\theta) > m$ no longer holds.

PROPOSITION 1: *If the level of privatization is lower than a certain threshold then the domestic firm's price is strictly lower than marginal cost and it incurs losses. More formally, there exists* $\theta^o \in (0, 1)$ *such that for all* $\theta \in [0, \theta^o)$, $p_d(\theta) < m$ *and* $\Pi_d(\theta) < 0$.[9]

To understand the above *underpricing* result, consider the output choice by a welfare-maximizing domestic firm. In the absence of foreign firms, the domestic firm chooses output such that price (marginal benefit) equals marginal cost. However in the presence of foreign firms, the public firm perceives expenditure on foreign goods as leakage from the domestic economy. Recognizing that an increase in its own output lowers consumer spending on foreign goods and thereby reduces leakage, the public firm produces more than it would without the foreign firms. This mercantile reasoning drives the price of the welfare-maximizing domestic firm below marginal cost; consequently the public firm incurs losses.[10] Clearly the same holds for a partially privatized firm provided the extent of privatization is low.

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[9] Proposition 1 establishes the existence of the threshold θ^0. A sufficient condition for uniqueness of θ^0 is $\frac{dp_d(\theta)}{d\theta} > 0$. Proposition 3 shows that this condition is met.

[10] Pricing below marginal cost necessarily inflicts losses because of our constant marginal cost assumption.

The next proposition records the effect of privatization on equilibrium outputs for both firms.

PROPOSITION 2: *An increase in the level of privatization reduces the domestic firm's output but raises the foreign firm's output. More formally, for all* $\theta \in (0, 1)$, $\frac{dq_d(\theta)}{d\theta} < 0$, *and* $\frac{dq_f(\theta)}{d\theta} > 0$.

From Proposition 1 we know that a welfare-maximizing public firm incurs losses. As the extent of privatization increases, the domestic firm becomes more profit-oriented and cuts back q_d to lower losses. Since outputs are strategic substitutes, lower q_d, in turn leads to higher q_f.

Now consider the effect of privatization on equilibrium prices. We have that

$$\frac{dp_i(\theta)}{d\theta} = \frac{\partial P_i(q(\theta))}{\partial q_i}\frac{dq_i}{d\theta} + \frac{\partial P_j(q(\theta))}{\partial q_j}\frac{dq_j}{d\theta}, \qquad \text{for } i, j \in \{d, f\}, i \neq j.$$

An increase in θ lowers the domestic firm's output, q_d, which raises p_d. On the other hand, as θ increases, the foreign firm's output, q_f, increases which lowers p_d (since $\frac{\partial P_d(q(\theta))}{\partial q_f} < 0$). Thus the effect of an increase in θ on $p_d(\theta)$ might seem ambiguous. Similar ambiguity exists for $p_f(\theta)$ as well. The following proposition says that, in fact, the prices of both varieties, domestic as well as foreign, unambiguously increase with an increase in the degree of privatization.

PROPOSITION 3: *An increase in the level of privatization raises equilibrium prices. More formally, for all* $\theta \in (0, 1)$, $\frac{dp_i(\theta)}{d\theta} > 0$, $i = d, f$.

If prices of both varieties, domestic as well as foreign, increase with θ how could privatizing firm d even partially improve welfare? The short answer is: by cutting back firm d's losses. The next section explores the trade-off involved in the choice of the optimal level of privatization—lower losses versus higher prices (in particular higher p_f).

4. Endogenous Privatization

So far, we have assumed that the parameter θ, capturing the extent of privatization, is exogenously given. While this assumption is plausible in some circumstances (e.g., in the short run, when policy variables do not change), presumably, the choice of the degree of privatization is endogenous. To analyze endogenous privatization, we construct a stylized two-stage game. In stage 2, as in Section 3, firms d and f compete in quantities. In stage 1, the home government chooses θ to maximize national welfare.[11]

[11] In the real world, privatization occurs for a variety of reasons. For example, British Rail was privatized ostensibly to improve its profitability and performance, while British

We address two questions: First, is there any incentive for a national welfare-maximizing government to choose $\theta > 0$ and thus partially privatize the domestic firm? Second, does the *underpricing* result, that is, $p_d(\theta) < m$ continue to hold when θ is chosen optimally? As we show below, the answer is "yes" for both questions.

4.1. Welfare-Improving Partial Privatization

Recall that output, price, and profits of firm i in stage 2 equilibrium are given by $q_i(\theta)$, $p_i(\theta)(\equiv P_i(q(\theta)))$, and $\Pi_i(\theta) \equiv \pi_i(q(\theta))$, respectively. Now define

$$CS(\theta) = cs(q(\theta)),$$

$$W(\theta) = CS(\theta) + \Pi_d(\theta).$$

In stage 1, the government chooses θ to maximize national welfare, $W(\theta)$. Let $\hat{\theta}$ denote the solution (if it exists) to the maximization problem.

PROPOSITION 4: *The optimal level of privatization is strictly positive. That is,* $\hat{\theta} > 0$.

To understand Proposition 4, decompose $\frac{dW(\theta)}{d\theta}$ as follows:

$$\frac{dW(\theta)}{d\theta} = \frac{\partial w(q(\theta))}{\partial q_d}\frac{dq_d(\theta)}{d\theta} + \frac{\partial w(q(\theta))}{\partial q_f}\frac{dq_f(\theta)}{d\theta}. \tag{4}$$

Now consider an infinitesimally small increase in θ from $\theta = 0$. This increase in the extent of privatization lowers the domestic firm's output, q_d, but raises the foreign firm's output, q_f. The effect of lower q_d on welfare is second order since $\frac{\partial w(q(0))}{\partial q_d} = 0$. On the other hand, the effect of higher q_f on welfare is strictly positive and first order since $\frac{\partial w(q(\theta))}{\partial q_f} = -\frac{\partial P_f(q(\theta))}{\partial q_f} q_f(\theta) > 0$. Thus, an appropriate level of partial privatization always improves welfare in our framework.

Key to Proposition 4 is the idea that privatization allows the public firm to act as a Stackelberg leader. In that regard, this result is similar to the finding in the managerial delegation literature where the Stackelberg outcome is implemented when profit-maximizing owners assign weight to sales in a manager's objective function. However, unlike the standard Cournot setup with all profit-maximizing firms—where the Stackelberg leader wants to commit to a higher output level—here, the welfare-maximizing public firm wants

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Petroleum was privatized for abusing its monopoly position to make profits at the expense of national interest (De Fraja 1991, Vickers and Yarrow 1988, pp. 127–134). In Eastern Europe, political changes were a major factor, as much as outdated technologies. We follow the standard practice of the literature by setting welfare maximization as the objective of privatization.

to commit to a lower output so that the foreign firm can increase its output and domestic consumers can benefit. By assigning a positive weight to profit, partial privatization enables the welfare-maximizing public firm to credibly cut back its output.[12]

4.2. Underpricing

Recall from Proposition 1 that for $\theta < \theta^0$, the domestic firm's price is strictly below marginal cost. The natural question to ask is whether the same holds under an optimal level of privatization (i.e., $\theta = \hat{\theta}$). The first part of Proposition 5 gives a necessary and sufficient condition such that *underpricing* occurs under optimal levels of privatization. The second part of Proposition 5 says that the condition holds for the utility specifications that satisfy Assumptions 1–3.

PROPOSITION 5:

(i) *The domestic firm's price is strictly below marginal cost if and only if the foreign firm's price increases with an increase in the extent of privatization. More precisely,*

$$\mathrm{sgn}[p_d(\hat{\theta}) - m] = -\mathrm{sgn}\left[\frac{dp_f(\hat{\theta})}{d\theta}\right].$$

(ii) *Suppose Assumptions 1–3 hold. Then* $p_d(\hat{\theta}) - m < 0$.

Key to the underpricing result under endogenous privatization is the adverse price effect. As the level of privatization increases, the domestic firm's losses decline but the foreign price increases. Due to this adverse price effect, the national welfare-maximizing government finds it optimal to tolerate losses and choose $\theta = \hat{\theta}$, such that $p_d(\hat{\theta}) - m < 0$.

4.3. Discussion

We have made some simplifying assumptions to illustrate our results. First, note that there are no domestic profit-maximizing private firms in our base model because we wanted to highlight the open economy aspect. Second, we have assumed that the public firm's unit cost is the same as the foreign firm's unit cost. Also, the unit cost does not vary with the level of privatization. Introducing domestic firms (Subsection 4.3.1) or cost asymmetry (Subsection 4.3.2), we find that underpricing occurs for low levels of θ and privatization continues to be optimal. However, unlike Proposition 5, underpricing may

[12] We thank an anonymous associate editor for making this connection (with the Stackelberg interpretation) precise.

or may not occur under optimal partial privatization. In Subsection 4.3.3, we consider the possible ramifications of our analysis when loss-making is not allowed.

4.3.1. Domestic firms

In the presence of domestic profit-maximizing private firms, we find that, as in Proposition 1, a welfare-maximizing public firm's price is strictly below its marginal cost. Furthermore, as in Proposition 4, partial privatization continues to be optimal. However, under optimal privatization, underpricing may or may not occur. Below is a sketch of our analysis that underpins these claims.

Extend $U(.)$ to include a third variety. Continue to make Assumptions 1 and 2. Let q_{d1}, q_{d2}, and q_f, respectively, denote the outputs of the public firm, the domestic profit-maximizing firm, and the foreign firm. Corresponding to $q \equiv (q_{d1}, q_{d2}, q_f)$, welfare is

$$w(q) \equiv U(q) - P_f(q)q_f - mq_{d1} - mq_{d2}.$$

Suppose $\theta = 0$, i.e., the public firm maximizes welfare. Rearranging the first-order condition corresponding to the public firm's welfare maximization problem, we get

$$\frac{\partial U}{\partial q_{d1}} - m - \frac{\partial P_f}{\partial q_{d1}} q_f = 0.$$

Since $\frac{\partial P_f}{\partial q_{d1}} < 0$, $\frac{\partial U}{\partial q_{d1}} < m$ must hold. Thus underpricing occurs in Cournot equilibrium. By the standard continuity argument, it follows that underpricing occurs as long θ is lower than a certain threshold.

Corresponding to a $\theta \in [0, 1]$, let $q_{d1}(\theta), q_{d2}(\theta), q_f(\theta)$ denote the Cournot equilibrium quantities. Define $q(\theta) \equiv (q_{d1}(\theta), q_{d2}(\theta), q_f(\theta))$ and $W(\theta) \equiv w(q(\theta))$. We have that

$$\frac{dW(\theta)}{d\theta} = \frac{\partial w(q(\theta))}{\partial q_{d1}} \frac{dq_{d1}(\theta)}{d\theta} + \frac{\partial w(q(\theta))}{\partial q_f} \frac{dq_f(\theta)}{d\theta} + \frac{\partial w(q(\theta))}{\partial q_{d2}} \frac{dq_{d2}(\theta)}{d\theta}.$$

To prove that partial privatization is still optimal, we need to show that $\frac{dW(0)}{d\theta} > 0$. As in the proof of Proposition 4, we can show that at $\theta = 0$, $\frac{\partial w(q(\theta))}{\partial q_{d1}} \frac{dq_{d1}(\theta)}{d\theta} + \frac{\partial w(q(\theta))}{\partial q_f} \frac{dq_f(\theta)}{d\theta} > 0$. Extending Proposition 2, we can show that $\frac{dq_{d2}(\theta)}{d\theta} > 0$, i.e., an increase in the level of privatization raises the output of the domestic profit-maximizing firm. The result then follows from noting that $\frac{\partial w(q(\theta))}{\partial q_{d2}} = \frac{\partial U(q(\theta))}{\partial q_{d2}} - m - \frac{\partial P_f(q(\theta))}{\partial q_{d2}} q_f(\theta) > 0$ since $\frac{\partial U(q(\theta))}{\partial q_{d2}} - m = p_{d2} - m > 0$ and $-\frac{\partial P_f(q(\theta))}{\partial q_{d2}} > 0$.

Finally, to see why underpricing may or may not occur, start from a θ such that $p_{d1}(\theta) - m = 0$. A slight increase in θ leads to higher output by the domestic private firm which improves welfare, but it also raises the price of the foreign variety, which lowers welfare. If the latter effect dominates, the public firm will continue to tolerate losses in equilibrium through underpricing. The opposite is true if the increase in the domestic private firm's output is the dominant effect.

4.3.2. Cost asymmetry

Suppose the foreign firm is more efficient. Its constant unit cost $m > 0$ and the domestic firm's unit cost is $m(\theta)$ where $m(0) = m_0 > m(1) = m$ and $m'(\theta) < 0$. The domestic firm's efficiency improves only with privatization. Note that the underpricing problem outlined in Proposition 1 does not depend on symmetric costs, so while the threshold value of θ below which underpricing occurs might differ depending on the degree of cost asymmetry, the basic result of underpricing will still hold for low values of θ. Privatization now improves welfare through an additional channel: cost reduction of the domestic firm. Hence, like Proposition 4, privatization, at least to some degree, is always optimal.

Whether underpricing occurs or not with optimal privatization depends on the properties of $m'(\theta)$. Starting from a θ where $p_d(\theta) - m = 0$, a decrease in θ lowers the price of foreign variety which improves welfare. At the same time, however, a decrease in θ worsens production efficiency since $m'(\theta) < 0$. If $|m'(\theta)|$ is large, the government may increase θ sufficiently and underpricing may not occur.

4.3.3. Loss-making firms

We have implicitly assumed that the losses of the public firm can be covered by taxes and transfers which do not impose any social cost. However, in reality, transfers can be quite costly, and in many cases politically not possible. Suppose the government can only implement those θ which induce non-negative profit for the domestic firm. Since $p_d(\theta^0) - m = 0$ (by Proposition 1) and $p_d(\theta) - m$ is strictly increasing in θ (by Proposition 3), profit is non-negative if and only if $\theta \in [\theta^0, 1]$. The constrained optimal level of privatization (satisfying the no-loss constraint) is given by $\theta_{\max} = \text{argmax}_{\theta \in [\theta^0, 1]} W(\theta)$, where $W(\theta)$ is as defined in Section 4.1. Since underpricing cannot occur at $\theta \geq \theta^0$, privatization is optimal if and only if $W(\theta_{max}) \geq W(0)$. For a linear differentiated duopoly, we find that $W(\theta_{max}) < W(0)$. Thus, privatization, irrespective of its scale, might lower welfare when no loss is permitted.

Would a welfare-maximizing government *ever* embark on privatization, if the (partially) privatized firm's profit has to be non-negative? The answer depends on the environment and the trade and investment policies in place.[13] As the discussion in Subsection 4.3.1 suggests, in an environment with domestic private firms, the public firm might not incur losses. While theoretically plausible, it is the lack of domestic private capabilities that often

[13] It might well be the case that for other functional forms $W(\theta_{max}) > W(0)$ holds. However, in the text we focus on other possible channels.

prompts governments (especially in developing and transition economies) to engage in production in the first place. More relevant to our analysis, at least in the short run, are other policy options that arise in an open economy setting.

Suppose the domestic government imposes a tariff $t > 0$ per unit of q_f. The domestic welfare is

$$w(q) = U(q) - P_f(q) - mq_d + tq_f.$$

The first-order condition of the welfare maximization problem—counterpart of (14)—is

$$\frac{dW(\hat{\theta})}{d\theta} = 0 \Leftrightarrow (p_d(\hat{\theta}) - m)\left(\frac{dq_d(\hat{\theta})}{d\theta}\right) - q_f(\hat{\theta})\frac{dp_f(\hat{\theta})}{d\theta} + t\frac{dq_f(\hat{\theta})}{d\theta} = 0,$$

where $\hat{\theta}$ denotes the optimal level of privatization. Given $\frac{dq_d(\hat{\theta})}{d\theta} > 0$

$$p_d(\hat{\theta}) - m < (>)0 \Leftrightarrow q_f(\hat{\theta})\frac{dp_f(\hat{\theta})}{d\theta} - t\frac{dq_f(\hat{\theta})}{d\theta} > (<)0.$$

As tariff revenues increase with q_f, and q_f increases with θ, the presence of tariff revenues prompts the government to choose higher θ than it would otherwise have chosen. In fact, for suitably high values of t, the optimal level of privatization, $\hat{\theta}(t)$, could be high enough such that $p_d(\hat{\theta}(t)) - m > 0$ holds.

In a linear differentiated duopoly,

$$q_f(\hat{\theta})\frac{dp_f(\hat{\theta})}{d\theta} - t\frac{dq_f(\hat{\theta})}{d\theta} = (q_f(\hat{\theta}) - t)\frac{dp_f(\hat{\theta})}{d\theta},$$

which is strictly negative when t is suitably large (but still not prohibitive). Thus $p_d(\hat{\theta}(t)) - m > 0$ is possible. For lower values of t (but not too low), while $p_d(\hat{\theta}(t)) - m < 0$, there exists $\theta^0(t) \neq \hat{\theta}(t)$ such that $W(\theta^0(t)) > W(0)$ and $p_d(\theta^0(t)) - m \geq 0$.[14]

Instead of imposing tariffs, the government could impose investment restrictions on the foreign firms. For example, foreign firm might be required to form a joint venture with a domestic firm. As we show below in Section 5, certain restrictions on foreign ownership yields $p_d(\hat{\theta}) - m \geq 0$.

5. Privatization and Joint Ventures

Consider a joint venture (JV hereafter) between the foreign firm and the domestic private partner who owns a fraction $\alpha \in [0, 1]$ of the JV. If $\alpha = 0$,

[14] The effect of privatization on optimal tariff in a homogenous good setting with a fixed number of firms has been discussed in Fjell and Pal (1996), Chao and Yu (2006), and Long and Stähler (2009). However, none of these papers look at optimal privatization.

the firm is completely foreign-owned while if $\alpha = 1$, the firm is effectively a second domestic firm. However, our interest lies in the intermediate case of $\alpha \in (0, 1)$. We assume that α is exogenously given.

As before, in the Cournot competition stage, the JV chooses q_f to maximize $(P_f(q) - m)q_f$. Firm d chooses q_d to maximize $(1-\theta)w(q,\alpha) + \theta\pi_d(q)$ where

$$w(q,\alpha) = cs(q) + \pi_d(q) + \alpha\pi_f(q) \equiv U(q) - P_f(q)q_f - mq_d$$
$$+\alpha(P_f(q) - m)q_f.$$

The new term above, $\alpha(P_f(q) - m)q_f$, captures the domestic partner's profits in the JV.

LEMMA 1: *Let $q_d(\theta,\alpha)$ and $q_f(\theta,\alpha)$ denote the equilibrium output of the domestic firm and the JV, respectively.*

(i) An increase in the level of privatization lowers the domestic firm's output and raises the JV's output (i.e., $\frac{dq_d(\theta,\alpha)}{d\theta} < 0$, $\frac{dq_f(\theta,\alpha)}{d\theta} > 0$).
(ii) An increase in the domestic partner's share in the JV lowers the domestic firm's output and raises the JV's output (i.e., $\frac{dq_d(\theta,\alpha)}{d\alpha} < 0$, $\frac{dq_f(\theta,\alpha)}{d\alpha} > 0$).

Consider (ii) first: the effect of an increase in α on equilibrium outputs. As α increases, the leakage from the economy declines since a larger share of the JV's profits enters into domestic welfare. As long as the domestic firm cares about welfare, i.e., as long as $\theta < 1$, this prompts the domestic firm to cut back its output. Since outputs are strategic substitutes, lower q_d, in turn leads to higher q_f. Concerning the effect of privatization on equilibrium output—stated in part (i)—see the discussion after Proposition 1. While that discussion pertains to the case $\alpha = 0$, the same intuition applies here.

Now let us turn to stage 1 where the level of privatization is chosen. Define $q(\theta,\alpha)$, $p_i(\theta,\alpha)\,(i = d, f)$, and $W(\theta,\alpha)$ as the second stage equilibrium output vector, prices, and social welfare, respectively, for any given θ and α. In stage 1, the government chooses θ to maximize $W(\theta,\alpha)$. Let $\hat{\theta}(\alpha)$ denote the solution to the government's welfare maximization problem. We find that $\hat{\theta}(\alpha) > 0$ and furthermore, underpricing occurs if α is not too high.

PROPOSITION 6:

(i) The optimal level of privatization is strictly positive irrespective of the domestic partner's share in the JV. That is, $\hat{\theta}(\alpha) > 0$ for all $\alpha \in [0, 1]$.
(ii) There exist α_1 and α_2 satisfying $0 < \alpha_1 \leq \alpha_2 < 1$ such that

- *for all $\alpha \in [0, \alpha_1)$, $p_d(\hat{\theta}(\alpha), \alpha) - m < 0$, and*
- *for all $\alpha \in (\alpha_2, 1]$, $p_d(\hat{\theta}(\alpha), \alpha) - m > 0$.*

Proposition 6 (i) says that an appropriate level of privatization of the domestic public firm improves welfare. The underlying logic is similar to that of Proposition 4.

Part (ii) of Proposition 6 is concerned with a public firm's pricing. From Proposition 5(ii) we know that for $\alpha = 0$, a public firm's price lies strictly below its marginal cost even when the level of privatization is optimally chosen. That is $p_d(\hat{\theta}(0), 0) - m < 0$. Applying the standard continuity argument, it is straightforward to show that $p_d(\hat{\theta}(\alpha), \alpha) - m < 0$ for sufficiently low α. On the other hand, if α is sufficiently large, $p_d(\hat{\theta}(\alpha), \alpha) - m > 0$. To see why, fix $\alpha = 1$ and consider

$$W(\theta, 1) \equiv U(q(\theta, 1)) - m(q_d(\theta, 1) + q_f(\theta, 1)).$$

The following first-order condition must hold at $\theta = \hat{\theta}(1)$:

$$\frac{dW(\theta, 1)}{d\theta} = (p_d(\theta, 1) - m)\frac{dq_d(\theta, 1)}{d\theta} + (p_f(\theta, 1) - m)\frac{dq_f(\theta, 1)}{d\theta} = 0. \quad (5)$$

Since the JV maximizes profit, $p_f(\hat{\theta}(1), 1) - m > 0$. By Lemma 1, $\frac{dq_f(\theta,1)}{d\theta} > 0$ and $\frac{dq_d(\theta,1)}{d\theta} < 0$. Then

$$\frac{dW(\theta, 1)}{d\theta} = 0 \Rightarrow p_d(\hat{\theta}(1), 1) - m > 0.$$

Once again, using the continuity argument (for appropriate functions) we get $p_d(\hat{\theta}(\alpha), \alpha) - m > 0$ for sufficiently large α.

The intuition is that when α is large, the JV is effectively a domestic firm. Privatizing beyond the zero profit level becomes then optimal, because the leakage from the national economy, in the form of JV's repatriated profit, is not significant.

For sharper characterization, we now turn to the quadratic utility function which has been used extensively in the differentiated oligopoly literature (see, for example, Dixit 1979, Singh and Vives 1984, and Qiu 1997)

$$U(q) = a(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + q_2^2\right) - bq_1q_2,\ b \in (0, 1). \quad (6)$$

The goods are independent if $b = 0$ and perfect substitutes if $b = 1$. The restriction that b lies strictly between 0 and 1 implies that the goods are imperfect substitutes. The degree of substitutability increases, or equivalently, the extent of product differentiation decreases, as b increases.

PROPOSITION 7: *Suppose $U(q)$ is given by* (6). *Then*

$$\hat{\theta}(\alpha) = \frac{b(1-b)}{2(2-b^2) + b - 2\alpha b(2-b)}. \quad (7)$$

The optimal level of privatization, $\hat{\theta}(\alpha)$, decreases as the foreign share in the JV, $1-\alpha$, increases. Furthermore, $p_d(.) - m < (>) 0$ as long as the foreign firm's share is strictly greater (less) than 50%.[15]

Proposition 7 hints at a possible conflict between privatization and investment liberalization. Relaxing restrictions on foreign ownership might reduce the incentives for privatization, and eventually will have to tolerate losses. This conflict, fortunately, is present only in the short run. In the long run, as we show next, with the free entry of foreign firms, the public firm will no longer have to suffer losses.

6. Free Entry

To analyze endogenous privatization with the free entry of foreign firms, we consider a stylized three-stage game. In the first stage, the government chooses the degree of privatization to maximize national welfare. In the second stage, a large number of identical potential foreign entrants exist, each of whom must decide whether or not to enter. Should a foreign firm decide to enter, it must incur a setup cost of K. Stage 3 involves Cournot competition among profit-maximizing foreign firms and the domestic firm that maximizes a weighted combination of its own profits and national welfare.

First we will consider the Cournot competition in stage 3 where n foreign firms and the domestic firm d compete on quantities. Label these $n+1$ firms from 0 to $n+1$ such that firm 0 is the domestic firm while firms 1 to n are foreign. We will use the subscripts 0 and d interchangeably. Both pertain to the domestic firm. Once again we consider the quadratic utility specification

$$U(q, n) = a\sum_{i\in\mathcal{N}} q_i - \frac{1}{2}\sum_{i\in\mathcal{N}} q_i^2 - b\sum_{i\in\mathcal{N}}\sum_{j>i} q_i q_j, \quad b \in (0, 1), \tag{8}$$

where $q \equiv (q_0, q_1, ..., q_n)$ is the output vector, $n(\geq 1)$ is the number of foreign firms, $\mathcal{N} \equiv \{0, 1, \ldots, n\}$ is the set of firms, and $a > m$. Observe that for $n = 1$, the utility function is the same as that stated in (6).[16]

Corresponding to a given $n(\geq 1)$ and an output vector $q \equiv (q_0, q_1, ..., q_n)$, firm i's profits, consumer surplus, and welfare, denoted by $\pi_i(q, n)$, $cs(q, n)$, and $w(q, n)$, respectively, are as follows:

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[15] Note that $\hat{\theta}(\alpha) > 0$ unless the product is completely independent ($b = 0$) or homogenous ($b = 1$). In both cases, the optimality requires the public firm's price to be equal to marginal cost. This is achieved with $\theta = 0$.

[16] See p. 146 in Vives (1999) for more on this utility specification.

$$\pi_i(q, n) = (P_i(q) - m)\, q_i, \quad i \in \mathcal{N},$$

$$cs(q, n) = U(q, n) - P_0(q) q_0 - \sum_{i=1}^{n} P_i(q) q_i,$$

$$w(q, n) = cs(q, n) + \pi_d(q, n) \equiv U(q, n) - \sum_{i=1}^{n} P_i(q) q_i - m q_0.$$

In stage 3, the domestic firm chooses q_0 to maximize

$$R(q; \theta, n) \equiv (1 - \theta) w(q, n) + \theta \pi_0(q, n),$$

while each foreign firm $i \in \{1, 2, \ldots, n\}$ chooses q_i to maximize $\pi_i(q) = (P_i(q) - m) q_i$.

6.1. Cournot Equilibrium

For any given $\theta \in [0, 1]$ and $n \geq 1$, let $q(\theta, n) = \{q_i(\theta, n)\}_{i=0}^{n}$ denote the unique equilibrium output vector. Define

$$p_i(\theta, n) = P_i(q(\theta, n)),$$

$$\Pi_i(\theta, n) = \pi_i(q(\theta, n)).$$

Since $q_i(\theta, n), p_i(\theta, n)$, and $\Pi_i(\theta, n)$ are same for all $i \in \{1, \ldots, n\}$, hereafter, we denote the output, price, and profit of a foreign firm in stage 3 equilibrium as $q_f(\theta, n)$, $p_f(\theta, n)$, and $\Pi_f(\theta, n)$, respectively. Lemma 2 records the effect of (i) an increase in θ, and (ii) an increase in n on equilibrium outputs.

LEMMA 2:

(i) For a given number of foreign firms, an increase in the level of privatization raises the foreign firm's output and lowers that of the domestic firm. That is, $\frac{\partial q_f(\theta, n)}{\partial \theta} > 0$, *and* $\frac{\partial q_d(\theta, n)}{\partial \theta} < 0$.

(ii) For a given level of privatization, an increase in the number of foreign firms lowers the output for all firms. That is, $\frac{\partial q_i(\theta, n)}{\partial n} < 0$ *for* $i = d, f$.

6.2. Free Entry

Now we turn to the entry stage. Each foreign firm incurs a fixed entry cost $K > 0$ (which is not prohibitive) to enter the domestic market. Then, for a given $\theta \in [0, 1]$, the free entry number of foreign firms, denoted by $n^*(\theta)$, is the value of n that solves

$$\pi_f(\theta, n) - K = 0. \tag{9}$$

LEMMA 3: *For all* $\theta \in [0, 1]$, $n^*(\theta)$ *is unique and strictly increasing in* θ.

Equipped with Lemmas 2 and 3, we are now ready to analyze the case for privatization and the possibility of underpricing.

6.3. Endogenous Privatization and Underpricing

First, define

$$q_i^*(\theta) = q_i^*(\theta, n^*(\theta)), p_i^*(\theta) = p_i^*(\theta, n^*(\theta)), i = 1, 2, \ldots, n$$

and

$$q^*(\theta) = (q_0^*(\theta), q_1^*(\theta), q_2^*(\theta), ..., q_n^*(\theta)),$$

where $q_i^*(\theta)$ and $p_i^*(\theta)$, respectively, are firm i's output and price in stage 3 equilibrium where $n = n^*(\theta)$. Now define

$$W(\theta) \equiv w(q^*(\theta), n^*(\theta)).$$

Then, noting that $q_0^*(\theta) = q_d(\theta)$ and $q_i^*(\theta) = q_f(\theta)$ for all $i \in \{1, 2, \ldots, n\}$ we get

$$\begin{aligned} W(\theta) &= w(q^*(\theta), n^*(\theta)), \\ &= U(q^*(\theta), n^*(\theta)) - n^*(\theta)p_f^*(\theta)q_f^*(\theta) - mq_d^*(\theta). \end{aligned}$$

The government chooses θ to maximize $W(\theta)$ and let θ^* be the optimal θ. The existence of θ^* follows from appealing to the standard arguments: the continuity of $W(\theta)$ in θ and the compactness of the interval $[0, 1]$. We claim that $\theta^* > 0$.

PROPOSITION 8: *The optimal level of privatization is strictly positive, i.e.,* $\theta^* > 0$.

As in the short run case, we find that partial privatization (to an appropriate extent) improves welfare; however, the reasoning is different. An infinitesimally small increase in θ from $\theta = 0$ raises p_f which lowers welfare. In the presence of free entry, there is an additional effect: an increase in θ leads to more entry which in turn lowers p_f and raises welfare. If $U(q, n)$ is given by (8), these two effects exactly cancel each other. That is,

$$\frac{dp_f^*(\theta)}{d\theta} = \frac{dp_f(\theta, n^*(\theta))}{d\theta} = \frac{\partial p_f(\theta, n^*(\theta))}{\partial \theta} + \frac{\partial p_f(\theta, n^*(\theta))}{\partial n}\frac{dn^*(\theta)}{d\theta} = 0.$$

Nevertheless, by encouraging the entry of foreign varieties, privatization Directly benefits the consumers which raises welfare. This creates a rationale for privatization.

6.4. (No) Underpricing

While partial privatization improves welfare in the presence of foreign firms, we find that the underpricing result no longer holds. In other words, the public firm (once optimally privatized) never makes losses.

Recall that in the short run mixed duopoly case ($n = 1$), it is the adverse price effect of the foreign variety that holds back privatization and generates losses in the public firm. This argument remains valid even for arbitrary $n(\geq 1)$, as long as n is fixed.

In the free entry scenario, as θ increases, more foreign firms enter the domestic market. Entry creates downward pressure on foreign prices, which exactly offsets the adverse price effect (described in the previous paragraph) when $U(q, n)$ is given by (8). If the foreign price does not increase with privatization, it is no longer optimal to tolerate losses. Thus $p_d^*(\theta^*) - m \geq 0$. In fact, the inequality is strict, since starting from $\theta = \theta'$ where $p_d(\theta') - m = 0$, an infinitesimally small increase in θ generates a first-order welfare gain by increasing the number of foreign varieties.

PROPOSITION 9: *With the free entry of foreign firms, the domestic firm's price is strictly above marginal cost under the optimal level of privatization. That is,* $p_d^*(\theta^*) - m > 0$.

7. Concluding Remarks

We examined privatization and the financial health of a public sector firm in an open economy setting. We found that partial privatization of the public firm to an appropriate extent always improves welfare in a differentiated mixed oligopoly with foreign firms. In the short run with a fixed number of firms, the public firm makes losses in a sufficiently open economy. To be more precise, even if the government can choose a level of privatization such that the public firm earns strictly positive profit, it will not choose to do so. Our analysis highlights a possible short-run conflict between trade/investment liberalization and the privatization of public sector enterprises. Partial privatization continues to be optimal in the long run with free entry of foreign firms. However, in the long run, under optimal privatization, the public firm makes positive profit because its price lies strictly above the constant marginal cost.

Finally, note that, following the literature on mixed oligopoly, we have assumed that privatization does not affect the public firm's efficiency. Clearly, this assumption does not accord well with the existing evidence. Megginson and Netter (2001) argue that one of the primary goals of privatization is to improve efficiency.[17] However, we shut off the efficiency

[17] See Bennett and Maw (2000) for an insightful analysis on the relationship between efficiency-enhancing investment, market structure, and privatization.

channel to stress that privatization can improve welfare even if it does not improve the efficiency of the erstwhile public firm. If efficiency improves, then welfare improvement becomes more likely under privatization both in the short run and the long run.

Appendix

The Appendix contains the proofs of Lemmas 1–3 and Propositions 1–9.

Proof of Proposition 1: Simplifying (2) and then rearranging gives

$$p_d(\theta) - m = -\theta q_d(\theta)\frac{\partial P_d(q(\theta))}{\partial q_d} + (1-\theta)\frac{\partial P_f(q(\theta))}{\partial q_d} q_f(\theta). \quad \text{(A1)}$$

Since $\frac{\partial P_i(q(\theta))}{\partial q_j} < 0$, for $i, j \in \{d, f\}$ we have that

$$p_d(0) - m = \frac{\partial P_f(q(0))}{\partial q_d} q_f(0) < 0, \quad p_d(1) - m = -q_d(1)\frac{\partial P_d(q(1))}{\partial q_d} > 0,$$

and consequently

$$\Pi_d(0) = (p_d(0) - m)q_d(0) < 0, \quad \Pi_d(1) = (p_d(1) - m)q_d(1) > 0.$$

Then the result follows from noting that $p_d(\theta)$ and $\Pi_d(\theta)$ are continuous in θ. ■

Proof of Proposition 2: Totally differentiating (2) and (3) with respect to θ and then solving for $\frac{dq_d(\theta)}{d\theta}$ and $\frac{dq_f(\theta)}{d\theta}$, we get

$$\frac{dq_d(\theta)}{d\theta} = -\frac{\left[q_d(\theta)\frac{\partial P_d(q(\theta))}{\partial q_d} + q_f(\theta)\frac{\partial P_f(q(\theta))}{\partial q_d}\right]\frac{\partial^2 \pi_f(q(\theta))}{\partial q_f^2}}{\Delta}, \quad \text{(A2)}$$

$$\frac{dq_f(\theta)}{d\theta} = \frac{\left[q_d(\theta)\frac{\partial P_d(q(\theta))}{\partial q_d} + q_f(\theta)\frac{\partial P_f(q(\theta))}{\partial q_d}\right]\frac{\partial^2 \pi_f(q(\theta))}{\partial q_f \partial q_d}}{\Delta}. \quad \text{(A3)}$$

The following second-order conditions must hold at $q = q(\theta)$: $\frac{\partial^2 \pi_f(q(\theta))}{\partial q_f^2} < 0$, and $\Delta = (\frac{\partial^2 R(q(\theta),\theta)}{\partial q_d^2})(\frac{\partial^2 \pi_f(q(\theta))}{\partial q_f^2}) - (\frac{\partial^2 R(q(\theta),\theta)}{\partial q_f \partial q_d})(\frac{\partial^2 \pi_f(q(\theta))}{\partial q_f \partial q_d}) > 0$. Then the result follows from noting that $q_i(\theta) > 0$, $\frac{\partial P_i(q(\theta))}{\partial q_j} < 0$, $i, j = d, f$, and $\frac{\partial^2 \pi_f(q(\theta))}{\partial q_f \partial q_d} < 0$ (Assumption 3). ■

Claim 1: $\frac{\partial^2 U(q)}{\partial q_d \partial q_f}\frac{\partial^2 \pi_f(q)}{\partial q_f^2} - \frac{\partial^2 U(q)}{\partial q_f^2}\frac{\partial^2 \pi_f(q)}{\partial q_d \partial q_f} > 0$.

Proof: Using $\frac{\partial^2 U(q)}{\partial q_d \partial q_f} = \frac{\partial P_d(q)}{\partial q_f}$, $\frac{\partial^2 U(q)}{\partial q_f^2} = \frac{\partial P_f(q)}{\partial q_f}$, $\frac{\partial^2 \pi_f(q)}{\partial q_d \partial q_f} = 2\frac{\partial P_f(q)}{\partial q_f} + q_f\frac{\partial^2 P_f(q)}{\partial {q_f}^2}$, $\frac{\partial^2 \pi_f(q)}{\partial q_d \partial q_f} = 2\frac{\partial P_f(q)}{\partial q_f} + q_f\frac{\partial^2 P_f(q)}{\partial {q_f}^2}$ we find that $\frac{\partial^2 U(q)}{\partial q_d \partial q_f}\frac{\partial^2 \pi_f(q)}{\partial q_f^2} -$

$\frac{\partial^2 U(q)}{\partial q_f^2}\frac{\partial^2 \pi_f(q)}{\partial q_d \partial q_f} = \frac{\partial P_d(q)}{\partial q_f}\frac{\partial P_f(q)}{\partial q_f} + q_f\left[\frac{\partial P_d(q)}{\partial q_f}\frac{\partial^2 P_f(q)}{\partial q_f^2} - \frac{\partial P_f(q)}{\partial q_f}\frac{\partial^2 P_f(q)}{\partial q_d \partial q_f}\right]$. The result then follows from applying Assumptions 1 and 2. ■

Proof of Proposition 3: Differentiating $p_i(\theta) \equiv P_i(q(\theta))$, $i = d, f$, with respect to θ gives

$$\frac{dp_i(\theta)}{d\theta} = \frac{\partial P_i(q(\theta))}{\partial q_d}\frac{dq_d}{d\theta} + \frac{\partial P_i(q(\theta))}{\partial q_f}\frac{dq_f}{d\theta}.$$

Substituting the expressions for $\frac{dq_d(\theta)}{d\theta}$ and $\frac{dq_f(\theta)}{d\theta}$ from (A2) and (A3) in above and noting that $\frac{\partial P_i(q(\theta))}{\partial q_j} \equiv \frac{\partial^2 U(q)}{\partial q_i \partial q_j}$, $\frac{\partial P_i(q(\theta))}{\partial q_i} = \frac{\partial^2 U(q)}{\partial q_i^2}$ we get

$$\frac{dp_d(\theta)}{d\theta} = -\frac{q_d(\theta)\frac{\partial P_d(q(\theta))}{\partial q_d} + q_f(\theta)\frac{\partial P_f(q(\theta))}{\partial q_d}}{\Delta}\left(\frac{\partial^2 U(q)}{\partial q_d^2}\frac{\partial^2 \pi_f}{\partial q_f^2} - \frac{\partial^2 U(q)}{\partial q_d \partial q_f}\frac{\partial^2 \pi_f}{\partial q_d \partial q_f}\right),$$

$$\frac{dp_f(\theta)}{d\theta} = -\frac{q_d(\theta)\frac{\partial P_d(q(\theta))}{\partial q_d} + q_f(\theta)\frac{\partial P_f(q(\theta))}{\partial q_d}}{\Delta}\left(\frac{\partial^2 U(q)}{\partial q_d \partial q_f}\frac{\partial^2 \pi_f}{\partial q_f^2} - \frac{\partial^2 U(q)}{\partial q_f^2}\frac{\partial^2 \pi_f}{\partial q_d \partial q_f}\right).$$

From the proof of Proposition 2 we know that $q_d(\theta)\frac{\partial P_d(q(\theta))}{\partial q_d} + q_f(\theta)\frac{\partial P_f(q(\theta))}{\partial q_f} > 0$ and $\Delta > 0$. Furthermore, by claim 1, $\frac{\partial^2 U(q)}{\partial q_d \partial q_f}\frac{\partial^2 \pi_f(q)}{\partial q_f^2} - \frac{\partial^2 U(q)}{\partial q_f^2}\frac{\partial^2 \pi_f(q)}{\partial q_d \partial q_f} > 0$. Then it follows that $\frac{dp_f(\theta)}{d\theta} > 0$. Since $|\frac{\partial^2 U(q)}{\partial q_i^2}| > |\frac{\partial^2 U(q)}{\partial q_d \partial q_f}|$ and $\frac{\partial^2 \pi_f}{\partial q_f^2} < 0$, $\frac{\partial^2 \pi_f}{\partial q_d \partial q_f} < 0$ we have that

$$\frac{\partial^2 U(q)}{\partial q_d^2}\frac{\partial^2 \pi_f}{\partial q_f^2} - \frac{\partial^2 U(q)}{\partial q_d \partial q_f}\frac{\partial^2 \pi_f}{\partial q_d \partial q_f} > \frac{\partial^2 U(q)}{\partial q_d \partial q_f}\frac{\partial^2 \pi_f}{\partial q_f^2} - \frac{\partial^2 U(q)}{\partial q_f^2}\frac{\partial^2 \pi_f}{\partial q_d \partial q_f} > 0,$$

which in turn implies that $\frac{dp_f(\theta)}{d\theta} > 0$. ■

Proof of Proposition 4: Existence of $\hat{\theta}$ immediately follows from noting that $W(\theta)$ is continuous in θ and θ lies in a compact interval $[0, 1]$. To prove $\hat{\theta} > 0$ it suffices to show that $\frac{dW(0)}{d\theta} > 0$, that is, $\frac{dW(\theta)}{d\theta} > 0$ at $\theta = 0$. We have that

$$\frac{dW(\theta)}{d\theta} = \frac{\partial w(q(\theta))}{\partial q_d}\frac{dq_d(\theta)}{d\theta} + \frac{\partial w(q(\theta))}{\partial q_f}\frac{dq_f(\theta)}{d\theta}. \tag{A4}$$

Applying envelope theorem gives (a) $\frac{\partial w(q(0))}{\partial q_d} = \frac{\partial R(q(0), 0)}{\partial q_d} = 0$. By Proposition 2, (b) $\frac{dq_f}{d\theta} > 0$. Finally, we have that (c) $\frac{\partial w(q(\theta))}{\partial q_f} = \frac{\partial U(q(\theta))}{\partial q_f} - p_f(\theta) - \frac{\partial P_f(q(\theta))}{\partial q_f}q_f(\theta) > 0$ since $\frac{\partial U(q(\theta))}{\partial q_f} \equiv p_f$, and $\frac{\partial P_f(q)}{\partial q_f} < 0$. Applying (a)–(c) gives $\frac{dW(0)}{d\theta} > 0$. ■

Proof of Proposition 5:

(i) Substitute $\frac{\partial w(q(\theta))}{\partial q_d} = p_d(\theta) - m - \frac{\partial P_f(q(\theta))}{\partial q_d} q_f(\theta)$ and $\frac{\partial w(q(\theta))}{\partial q_f} = -\frac{\partial P_f(q(\theta))}{\partial q_f} q_f(\theta)$ in (4). Then using the fact that $\frac{dW(\hat{\theta})}{d\theta} = 0$, we get

$$(p_d(\hat{\theta}) - m)\left(\frac{dq_d(\hat{\theta})}{d\theta}\right) - q_f(\hat{\theta})\frac{dp_f(\hat{\theta})}{d\theta} = 0, \tag{A5}$$

where $\frac{dp_i(\theta)}{d\theta} = \frac{\partial P_i(q(\theta))}{\partial q_i}\frac{dq_i}{d\theta} + \frac{\partial P_j(q(\theta))}{\partial q_j}\frac{dq_j}{d\theta}$. The result follows from observing (A5) and noting that $\frac{dq_d(\hat{\theta})}{d\theta} < 0$ (Proposition 2) and $q_f(\hat{\theta}) > 0$.

(ii) If Assumptions 1–3 hold, $\frac{dp_f(\theta)}{d\theta} > 0$ (by Proposition 3). Then the claim follows from applying part (i).

■

Proof of Lemma 1: Let the second stage Cournot equilibrium output vector be denoted as $q(\theta, \alpha) = (q_d(\theta, \alpha), q_f(\theta, \alpha))$. Let $q_i(\theta, \alpha) > 0$ for $i = d, f$. It satisfies the following equation and (3):

$$\begin{aligned}\frac{\partial R(q(\theta,\alpha),\theta,\alpha)}{\partial q_d} &= P_d(q(\theta,\alpha)) - m - (1-\theta)q_f(\theta,\alpha)\frac{\partial P_f(q(\theta,\alpha))}{\partial q_d}\\ &\quad + \theta q_d(\theta,\alpha)\frac{\partial P_d(q(\theta,\alpha))}{\partial q_d} + \alpha q_f(\theta,\alpha)\frac{\partial P_f(q(\theta,\alpha))}{\partial q_d} = 0.\end{aligned} \tag{A6}$$

Totally differentiating (A6) and (3) with respect to θ and then solving for $\frac{dq_d(\theta,\alpha)}{d\theta}$ and $\frac{dq_f(\theta,\alpha)}{d\theta}$ we obtain

$$\frac{dq_d(\theta,\alpha)}{d\theta} = -\frac{\left[q_d(\theta,\alpha)\frac{\partial P_d(q(\theta,\alpha))}{\partial q_d} + q_f(\theta,\alpha)\frac{\partial P_f(q(\theta,\alpha))}{\partial q_d}\right]\frac{\partial^2 \pi_f(q(\theta,\alpha))}{\partial q_f^2}}{\tilde{\Delta}}, \tag{A7}$$

$$\frac{dq_f(\theta,\alpha)}{d\theta} = \frac{\left[q_d(\theta,\alpha)\frac{\partial P_d(q(\theta,\alpha))}{\partial q_d} + q_f(\theta,\alpha)\frac{\partial P_f(q(\theta,\alpha))}{\partial q_d}\right]\frac{\partial^2 \pi_f(q(\theta,\alpha))}{\partial q_f \partial q_d}}{\tilde{\Delta}}, \tag{A8}$$

where $\tilde{\Delta} = \left(\frac{\partial^2 R(q(\theta,\alpha),\theta,\alpha)}{\partial q_d^2}\right)\left(\frac{\partial^2 \pi_f(q(\theta,\alpha))}{\partial q_f^2}\right) - \left(\frac{\partial^2 R(q(\theta,\alpha),\theta,\alpha)}{\partial q_f \partial q_d}\right)\left(\frac{\partial^2 \pi_f(q(\theta,\alpha))}{\partial q_f \partial q_d}\right)$.

By the second-order conditions we must have, at $q = q(\theta, \alpha)$, $\frac{\partial^2 \pi_f(q(\theta,\alpha))}{\partial q_f^2} < 0$, and $\tilde{\Delta} > 0$.

Similarly, totally differentiating (A6) and (3) with respect to α and solving for $\frac{dq_d(\theta,\alpha)}{d\alpha}$ and $\frac{dq_f(\theta,\alpha)}{d\alpha}$ we get

$$\frac{dq_d(\theta,\alpha)}{d\alpha} = -\frac{q_f(\theta,\alpha)\frac{\partial P_f(q(\theta,\alpha))}{\partial q_d}\frac{\partial^2 \pi_f(q(\theta,\alpha))}{\partial q_f^2}}{\tilde{\Delta}}, \tag{A9}$$

$$\frac{dq_f(\theta,\alpha)}{d\alpha} = \frac{q_f(\theta,\alpha)\frac{\partial P_f(q(\theta,\alpha))}{\partial q_d}\frac{\partial^2 \pi_f(q(\theta,\alpha))}{\partial q_f \partial q_d}}{\tilde{\Delta}}. \tag{A10}$$

Then the results of both (i) and (ii) follow from the facts that $q_i(\theta,\alpha) > 0$, $\frac{\partial P_i(q(\theta,\alpha))}{\partial q_j} < 0$, $i, j = d, f$, $\frac{\partial^2 \pi_f(q(\theta,\alpha))}{\partial q_f^2} < 0$ and $\frac{\partial^2 \pi_f(q(\theta,\alpha))}{\partial q_f \partial q_d} < 0$ (Assumption 3). ■

Proof of Proposition 6:

(i) The proof of $\hat{\theta}(\alpha) > 0$ is similar to the proof of Proposition 4. Given any $\alpha \in [0, 1]$, we write

$$\frac{dW(\theta,\alpha)}{d\theta} = \frac{\partial w(q(\theta,\alpha),\alpha)}{\partial q_d}\frac{dq_d(\theta,\alpha)}{d\theta} + \frac{\partial w(q(\theta,\alpha),\alpha)}{\partial q_f}\frac{dq_f(\theta,\alpha)}{d\theta}.$$

For any $\alpha \in [0, 1]$, we get by the envelope theorem, at $\theta = 0$, $\frac{\partial w(q(0,\alpha),\alpha)}{\partial q_d} = \frac{\partial R(q(0,\alpha),0,\alpha)}{\partial q_d} = 0$, and, for any θ, $\frac{\partial w(q(\theta,\alpha),\alpha)}{\partial q_f} = -q_f(\theta,\alpha)\frac{\partial P_f(q(\theta,\alpha))}{\partial q_f} + \alpha\frac{\partial \pi_f(q(\theta,\alpha))}{\partial q_f} = -q_f(\theta,\alpha)\frac{\partial P_f(q(\theta,\alpha))}{\partial q_f}$ due to (3). Thus,

$$\frac{dW(0,\alpha)}{d\theta} = -q_f(0,\alpha)\frac{\partial P_f(q(0,\alpha))}{\partial q_f}\frac{dq_f(0,\alpha)}{d\theta}.$$

By Lemma 1

(ii) $\frac{dq_f(0,\alpha)}{d\theta} > 0$. Hence $\frac{dW(0,\alpha)}{d\theta} > 0$. Therefore, $\hat{\theta}(\alpha) > 0$ for all $\alpha \in [0, 1]$.

■

Proof of Proposition 7: First note that optimal θ is obtained by setting the expression in (4) equal to zero, which then simplifies to

$$\begin{aligned}\frac{dW(\theta,\alpha)}{d\theta} &= [p_d(\hat{\theta}(\alpha),\alpha) - m]\frac{dq_d(\hat{\theta}(\alpha),\alpha)}{d\theta} \\ &\quad -(1-\alpha)q_f(\hat{\theta}(\alpha),\alpha)\frac{dP_f(\hat{\theta}(\alpha),\alpha)}{d\theta} \\ &\quad +\alpha(P_f(\hat{\theta}(\alpha),\alpha) - m)\frac{dq_f(\hat{\theta}(\alpha),\alpha)}{d\theta} = 0. \end{aligned} \tag{A11}$$

Now assume that $U(q)$ is given by (6). The second stage Cournot equilibrium outputs of the two firms will be

$$q_d(\theta,\alpha) = \frac{(a-m)\{2 - b(\theta + \alpha(1-\theta))\}}{2(1+\theta) - b^2\{\theta + \alpha(1-\theta)\}},$$

$$q_f(\theta,\alpha) = \frac{(a-m)(1+\theta-b)}{2(1+\theta) - b^2\{\theta + \alpha(1-\theta)\}}.$$

Further, we derive the following expressions:

$$[p_d(\theta,\alpha) - m] = (a-m)\frac{[\theta\{(2-b^2) - \alpha b(1-b)\} - b(1-b)(1-\alpha)]}{2(1+\theta) - b^2\{\theta + \alpha(1-\theta)\}},$$

$$\frac{dq_d(\theta,\alpha)}{d\theta} = -2(a-m)\frac{\{2+b(1-b) - \alpha b(2-b)\}}{[2(1+\theta) - b^2\{\theta+\alpha(1-\theta)\}]^2},$$

$$\frac{dp_f(\theta,\alpha)}{d\theta} = b(a-m)\frac{\{2+b(1-b) - \alpha b(2-b)\}}{[2(1+\theta) - b^2\{\theta+\alpha(1-\theta)\}]^2},$$

$$\frac{dq_f(\theta,\alpha)}{d\theta} = b(a-m)\frac{\{2+b(1-b) - \alpha b(2-b)\}}{[2(1+\theta) - b^2\{\theta+\alpha(1-\theta)\}]^2}.$$

As can be seen $\frac{dp_f(\theta,\alpha)}{d\theta} = \frac{dq_f(\theta,\alpha)}{d\theta} = -\frac{b}{2}\frac{dq_d(\theta,\alpha)}{d\theta}$, and also $p_f(\theta,\alpha) - m = q_f(\theta,\alpha)$. Utilizing these facts we can rewrite (5) as

$$\frac{dW(\theta,\alpha)}{d\theta} = \frac{dq_d(\hat{\theta}(\alpha,\alpha)}{d\theta}\left[p_d(\hat{\theta}(\alpha),\alpha) - m + \frac{b}{2}q_f(\hat{\theta}(\alpha),\alpha)(1-2\alpha)\right] = 0.$$

Given that $\frac{dq_d(\theta)}{d\theta} < 0$ (by Lemma 1 (i)), we must have

$$p_d(\hat{\theta}(\alpha),\alpha) - m + \frac{b}{2}q_f(\hat{\theta}(\alpha),\alpha)(1-2\alpha) = 0. \tag{A12}$$

Explicitly solving (A12) we get $\hat{\theta}(\alpha)$ as shown in (7).

That $\hat{\theta}(\alpha)$ is increasing in α is evident from (7), and therefore $\hat{\theta}(\alpha)$ is decreasing in $(1-\alpha)$. Further, from (A12) given $q_f(\hat{\theta}(\alpha),\alpha) > 0$, it follows that $p_d(\hat{\theta}(\alpha),\alpha) < (>) m$ if and only if $\alpha < (>) 1/2$ (or $1-\alpha > (<) 1/2$). ■

Proof of Lemma 2: If $U(q, n)$ is given by (8), routine calculations show that

$$q_d(\theta, n) = \frac{(a-m)\,[2+b(n-1) - \theta bn]}{(1+\theta)\,[2+b(n-1)] - \theta b^2 n},$$

$$p_d(\theta, n) - m = \frac{(a-m)[\{2+b(n-1) - b^2 n\}\theta - b(1-b)n]}{(1+\theta)\,[2+b(n-1)] - \theta b^2 n};$$

$$q_f(\theta, n) = \frac{(a-m)\,[1+\theta-b]}{(1+\theta)\,[2+b(n-1)] - \theta b^2 n},$$

$$p_f(\theta, n) - m = \frac{(a-m)\,[1+\theta-b]}{(1+\theta)\,[2+b(n-1)] - \theta b^2 n} = q_f(\theta, n);$$

$$\pi_f(\theta, n) = (p_f(\theta, n) - m)q_f(\theta, n) = (q_f(\theta, n))^2.$$

Partially differentiating $q_d(\theta, n)$ and $q_f(\theta, n)$ with respect to the arguments in their respective domains gives

$$\frac{\partial q_d(\theta, n)}{\partial\theta} = \frac{-(a-m)\{[(1+\theta)(2-b) + (1+\theta(1-b)]nb + [(2-b) + (1-\theta)nb][(2-b) + (1-b)nb]\}}{\left[(1+\theta)(2+b(n-1)) - \theta b^2 n\right]^2},$$

$$\frac{\partial q_f(\theta, n)}{\partial \theta} = \frac{(a-m)[(2-b)b + nb^2(1+(1+\theta)(1-b))]}{[(1+\theta)(2+b(n-1)) - \theta b^2 n]^2},$$

$$\frac{\partial q_d(\theta, n)}{\partial n} = \frac{-(a-m)\,[(2-b)(1+\theta-b)\theta b]}{[(1+\theta)(2+b(n-1)) - \theta b^2 n]^2},$$

$$\frac{\partial q_f(\theta, n)}{\partial n} = \frac{-(a-m)\,[(1+\theta-b)(1+\theta b(1-b))]}{[(1+\theta)(2+b(n-1)) - \theta b^2 n]^2}.$$

From the above four conditions the result follows. ■

Proof of Lemma 3: Since $\pi_f(\theta, n) \equiv (q_f(\theta, n))^2$ and $\frac{\partial q_f(\theta,n)}{\partial n} < 0$ we have that $\frac{\partial \pi_f(\theta,n)}{\partial n} = 2q_f(\theta, n)\frac{\partial q_f(\theta,n)}{\partial n} < 0$. This in turn implies that there is a unique value of n, say $n*(\theta)$, that solves $\pi_f(\theta, n) - K = 0$. Totally differentiating $\pi_f(\theta, n^*(\theta)) = K$ and simplifying we get $\frac{dn^*(\theta)}{d\theta} = -\frac{\frac{\partial q_f(\theta,n)}{\partial\theta}}{\frac{\partial q_f(\theta,n)}{\partial n}}$. By Lemma 2, $\frac{\partial q_f(\theta,n)}{\partial\theta} > 0$ and $\frac{\partial q_f(\theta,n)}{\partial n} < 0$. Hence $\frac{dn^*(\theta)}{d\theta} > 0$. ■

Proof of Proposition 8: Differentiating $W(\theta)$ with respect to θ we find that

$$\frac{dW(\theta)}{d\theta} = \frac{\partial w(q^*(\theta), n^*(\theta))}{\partial q_d}\frac{dq_d^*(\theta)}{d\theta} + \sum_{i=1}^{n}\frac{\partial w(q^*(\theta), n^*(\theta))}{\partial q_i}\frac{dq_i^*(\theta)}{d\theta}$$
$$+ \frac{\partial w(q^*(\theta), n^*(\theta))}{\partial n}\frac{dn^*(\theta)}{d\theta}. \tag{A13}$$

To prove the claim it suffices to show that $\frac{dW(0)}{d\theta} < 0$. By envelope theorem, $\frac{\partial w(q^*(0),n^*(0))}{\partial q_d} = 0$. Also, for all $i \in \{1, 2, \ldots, n\}$, $\frac{dq_i^*(\theta)}{d\theta} = 0$, since from (19) and (9) it follows that $q_f(\theta, n^*(\theta)) = \sqrt{\pi_f(\theta, n^*(\theta))} = \sqrt{K}$. By Lemma 3, $\frac{dn^*(\theta)}{d\theta} > 0$. Thus

$$\operatorname{sgn}\left[\frac{dW(0)}{d\theta}\right] = \operatorname{sgn}\left[\frac{\partial w(q^*(0), n^*(0))}{\partial n}\right],$$
$$= \operatorname{sgn}\left[\frac{\partial U(q^*(0), n^*(0))}{\partial n} - p_f^*(0)q_f^*(0)\right].$$

The result follows from noting that $\frac{\partial U(q^*(0),n^*(0))}{\partial n} - p_f^*(0)q_f^*(0) = \frac{(1-b)q_f^*(0)^2}{2} > 0$. ■

Proof of Proposition 9: Differentiating $W(\theta) \equiv U(q^*(\theta), n^*(\theta)) - n^*(\theta)p_f^*(\theta)q_f^*(\theta) - mq_d^*(\theta)$ with respect to θ and simplifying we have that

$$\frac{dW(\theta)}{d\theta} = (p_d^*(\theta) - m)\frac{dq_d^*(\theta)}{d\theta} - n^*(\theta)q_f^*(\theta)\frac{dp_f^*(\theta)}{d\theta} + \left[\frac{\partial U(q^*(\theta), n^*(\theta))}{\partial n} - p_f^*(\theta)q_f^*(\theta)\right]\frac{dn^*(\theta)}{d\theta}. \qquad \text{(A14)}$$

While proving Proposition 8 we have shown that $q_f^*(\theta) = \sqrt{K}$. Then using the zero profits condition in (9), we get $p_f^*(\theta) = m + \frac{K}{q_f^*(\theta)} = m + \sqrt{K}$ which implies that $\frac{dp_f^*(\theta)}{d\theta} = 0$. By Lemma 3, $\frac{dn^*(\theta)}{d\theta} > 0$. Applying Lemma 2 and Lemma 3 gives

$$\frac{dq_d^*(\theta)}{d\theta} = \frac{\partial q_d(\theta, n^*(\theta))}{\partial\theta} + \frac{\partial q_d^*(\theta, n^*(\theta))}{\partial n}\frac{dn^*(\theta)}{d\theta} < 0.$$

Then, since $\frac{dW(\theta^*)}{d\theta} = 0$, we have that

$$\operatorname{sgn}[p_d^*(\theta) - m] = \operatorname{sgn}\left[\frac{\partial U(q^*(\theta^*), n^*(\theta^*))}{\partial n} - p_f^*(\theta^*)q_f^*(\theta^*)\right].$$

The result follows from noting that $\frac{\partial U(q^*(\theta^*), n^*(\theta^*))}{\partial n} - p_f^*(\theta^*)q_f^*(\theta^*) = \frac{(1-b)q_f^*(\theta^*)^2}{2} > 0$. ■

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