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Colour Corrected Smoothest Reflectance Estimation

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Abstract. The same colour response can result from many different spectral reflectances: there are many metamers. This said, there is interest in finding a simple computational method to recover a single ‘good’ metamer for a given sensor and lighting condition. One such method - smoothest reflectance estimation - returns the reflectance whose sum of squared derivatives is minimum over all possible metamers. A weakness in the smoothest reflectance methodology is that different smoothest reflectances are recovered under different lights. In this paper, we try and mitigate this problem by using a colour correction preprocessing step. Simply, we colour correct - by regression - colour responses for a given acquisition light to a fixed reference D65 illumination. We then recover the smoothest reflectance with respect to D65. Experiments show that, on a cross-validation basis, adding the colour correction step can reduce the variability in recovered smoothest reflectances by up to 50%.

1 Introduction

It is relatively easy to model how an RGB is formed given knowledge of the light spectrum illuminating the scene, the spectral sensitivities of the camera and the reflectances (of objects) that are present. However, methods are often sought to invert the image formation process and to estimate spectral quantities from RGB images. This might be estimating the light spectrum (e.g. [1]), the spectral sensitivities of the camera (e.g. [2]) or, the focus of this paper, the spectral reflectances.

Spectral reflectance estimation from RGB image is widely used. Exemplar applications include in the analysis of agricultural products such as coffee [3], bananas and papayas [4], and in the graphics domain where RGB textures are *upsampled* to spectral counterparts [5]. It has also been proposed that spectral reflectance estimation accounts for how human observers perceive colour across viewing illuminants [6].

In restricted circumstances there exists a unique metamer: there is a single mapping from RGB to reflectance. For example in [7], colour signal spectra (light multiplied by reflectance) are modelled by a 3-dimensional linear basis. It is shown that - when this 3-D assumption holds exactly - that the colour signal spectra can be, uniquely, recovered from an RGB image by a simple linear transform. Reflectance spectra can then be found by ‘dividing out’ the illuminant. In general, however, for a given known viewing illuminant and camera spectral sensitivities there are many reflectances, called *metamers*, that integrate to the same RGB.

When a single RGB is available then - together with known camera spectral sensitivities and a known measured light - there are many algorithms for recovering a single metamer. For example, given a good understanding of the underlying statistics of reflectances, the problem of reflectance estimation can be cast as a maximum likelihood estimation problem [8]. Further, recently this problem has been solved using deep learning (e.g. [9] is a leading exemplar algorithm). Here, it is assumed that several reflectances are measured simultaneously (the deepnets take a patch of RGBs as input). Interestingly, most deep nets do not recover metamers in the sense that the spectra they recover do not integrate to the RGBs from which they are estimated [10]. Though, it is straightforward to adjust networks so that metamers are



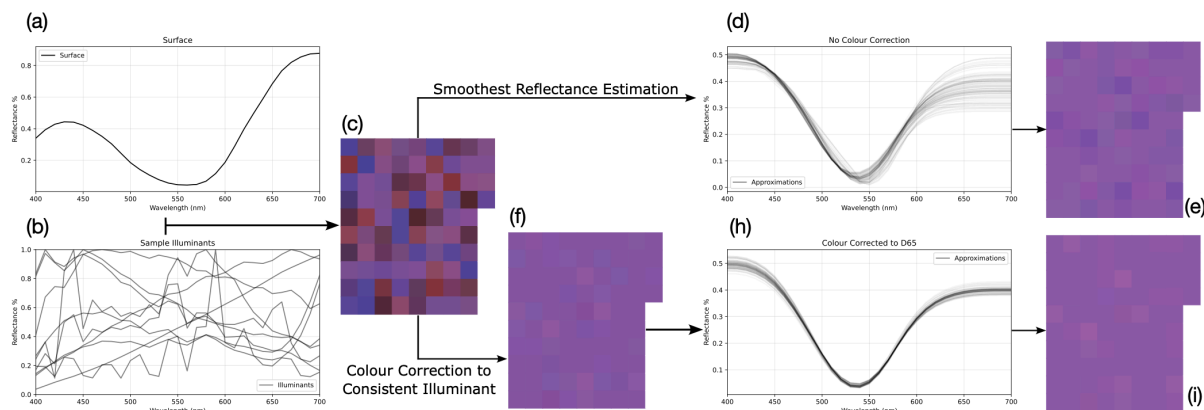


Figure 1: Given a surface distribution and set of illuminants (left), coloured pixels are produced (centre-left). Applying smoothest reflectance estimation causes significant variation in curves and pixels produced from these are inconsistent (top-right). Instead, preprocessing by colour correcting the pixels results in less variation both in produced curves and pixels (bottom-right).

always returned [10]. Unfortunately, when the statistics in the world are different from those assumed during training, the reflectance estimation - by deepnets or classical methods - can be relatively poor [11].

Thus, there is an interest in a recovery technique that is not based on a strong statistical assumption (i.e. on the statistics of a large corpus of measured reflectances). One such method - and the focus of this paper - is smoothest reflectance recovery. This method starts with the *weak* observation that reflectances tend to be smooth. This smoothness concept is codified in the language of derivatives and we say that one metamer is smoother than another if its sum of squared derivatives is less. In smoothest reflectance recovery an optimisation is solved to find the smoothest reflectance that might have induced a given RGB (given knowledge of the spectral sensitivities of the camera and the spectral power distribution of the illumination). The smoothest reflectance can be found using a simple quadratic programming formulation [12] or - in a more complex formulation - in closed-form [13].

A shortcoming of the smoothest reflectance approach is that a different smoothest reflectance is recovered under different lights. In this paper, we will attempt to mitigate this problem. Our new approach - which we call ‘colour corrected smoothest reflectance estimation’, or **CCSRE** - and a comparison to the simple smoothest reflectance methodology is shown in Figure 1. In Figure 1a, we plot a single reflectance. In 1b we show a variety of viewing illuminants (a subset of 102 [14] we will use in our experiments). In 1c - for a Canon 300D camera - we show the corresponding RGBs rendered in sRGB colour space. For each light we recover the smoothest reflectance. All 102 recovered reflectances are plotted in 1d. These 102 reflectances are then rendered under D65, Figure 1e. It is apparent that these reflectances are not metameric under D65.

In our **CCSRE** approach, we will colour correct - by mathematical means - the RGBs under a given light to corresponding colours under D65. This results in the sRGB inputs shown in 1f. In Figure 1h we show the smoothest reflectance estimates based on our colour corrected D65 RGBs. Clearly, the variance in the smoothest reflectance estimates is less than in 1d (no colour correction). Figure 1i (rendering the recovered reflectances under D65) is - as it has to be - the same as shown 1f. Clearly, the colours are closer to being metamers (under D65) than in the original smoothest reflectance workflow, Figure 1e.

To validate our method, we will train and test - on a cross-validation basis - on corpora of reflectances widely used in colour science. Our experiments will show that colour correcting to a standard light can result in a significant reduction in the variance of smoothest reflectance recovery.

2 Background

2.1 Colour Image Formation

A camera RGB, represented as a 3-vector denoted ρ - componentwise ρ_i ($i \in \{R, G, B\}$) - is a function of the spectral sensitivities of the camera, the light present in a scene and an object’s spectral reflectance:

$$\rho_i = \int_{\omega} R_i(\lambda) E(\lambda) S(\lambda) d\lambda \quad (1)$$

Here $E(\lambda)$ denotes the spectral power distribution of the viewing illuminant, $S(\lambda)$ the surface spectral reflectance and $R_i(\lambda)$ the i th spectral sensitivity of a camera. Integration is taken over the visible spectrum ω (400 to 700 nanometres).

It is often useful to rewrite Equation (1) in the language of linear algebra. Here we sample the visible spectrum at 10nm intervals. Thus, all spectral quantities are represented as 31-component vectors (their respective values at the 31 sampling points). Now, the RGB sensor response can, equivalently, be written as

$$\underline{\rho} = R^T \text{diag}(\underline{e}) \underline{s} \quad (2)$$

where R is a 31×3 matrix: one spectral sensitivity per column, $\underline{e}$ and $\underline{s}$ are respectively the 31-component vector forms of light and reflectance. The operator $\text{diag}()$ places its argument along the diagonal of a diagonal matrix. Matrix transpose is denoted, here and throughout the paper, by the superscript T .

2.2 Spectral Reflectance Estimation

In many contexts, it is desirable to calculate the reflectance distribution $\underline{s}$ of a surface. Given the camera sensitivities and the illuminant are known, Equation (2) becomes a simple, underdetermined linear equation: there are typically multiple reflectances (called metamers) which will satisfy Equation (2) and a number of methods have been proposed for finding a single *good* reflectance estimation.

2.2.1 Linear Basis methods Spectral reflectances can be modelled as a linear combination of a set of basis functions [15]. The surface function is written as $S(\lambda) \approx \sum_{i=1}^3 \sigma_i S_i(\lambda)$ given $S_i(\lambda)$ basis functions and σ_i weights. The basis functions $S_i(\lambda)$ can be rewritten as the columns of an $N \times 3$ matrix S . Now, we can rewrite Equation 2 as:

$$\underline{\rho} = R^T \text{diag}(\underline{e}) S \underline{\sigma} \quad (3)$$

Adopting the notation of [15], we write $\Lambda(\underline{e}) = R^T \text{diag}(\underline{e}) S$ ($\Lambda(\underline{e})$ is called a 3×3 *lighting matrix*). We can solve for $\underline{\sigma}$:

$$\underline{\sigma} = [\Lambda(\underline{e})]^{-1} \underline{\rho} \quad (4)$$

and the recovered reflectance is equal to $S \underline{\sigma}$.

The linear basis idea is both simple and elegant. However, the dimensionality of real reflectances cannot be captured by a 3-dimensional model. Spectral reflectances are often taken to be 6 to 8 dimensional [16], or higher.

2.2.2 Smoothest Reflectance Estimation In the continuous domain, the degree of smoothness of a spectral reflectance function $S(\lambda)$ is calculated as

$$\text{smooth}(S(\lambda)) = \int_{\omega} \left(\frac{dS}{d\lambda} \right)^2 d\lambda \quad (5)$$

For sensors that are *convex* - the sensitivity at one wavelength is not a convex combination of the sensitivities at other wavelengths - van Trigt [17] showed how for a given RGB the smoothest reflectances can be recovered. However, most works that use smoothest reflectances - including our own methodology - use tools from numerical analysis. We will recapitulate this more practical formulation - as part of our own algorithm development - in Section 3.

2.3 Colour Correction

Colour correction refers either to mapping camera responses to corresponding human vision system referred values (such as XYZ or sRGB) or to ‘discounting’ the illuminant i.e. mapping colours under one light to another. It is this latter aspect of colour correction that is of interest in this paper. Linear colour correction (LCC) is the simplest method for colour correction. Here, a 3×3 matrix is used to linearly map RGBs between illuminants.

Given our discussion of Linear Basis methods of spectral recovery, a natural colour correction method follows. If we have two lights $\underline{e}^a$ and $\underline{e}^b$ - and reflectances are 3-dimensional - the RGBs are mapped between lights (a to b) according to:

$$\underline{\rho}^b = \Lambda(\underline{e}^b)[\Lambda(\underline{e}^a)]^{-1}\underline{\rho}^a \quad (6)$$

where $[\Lambda(\underline{e}^a)]^{-1}$ is 3×3 matrix.

More commonly, a colour correction matrix is found by regression. Respectively, let P^a and P^b denote $3 \times N$ matrices of corresponding RGBs measured for the same reflectances under two lights. The 3×3 matrix $M^{a,b}$ taking responses between the pair of lights (from light a to b) as found by least-squares regression Equation (7).

$$M^{a,b} = P^b[P^a]^T(P^a[P^a]^T)^{-1} = P^b[P^a]^+ \quad (7)$$

where $[P^a]^+$ denotes the Moore-Penrose inverse. It follows $M^{a,b}P^a \approx P^b$

A simple way to improve the accuracy of regression is to *expand* the RGBs to higher dimensional counterparts. One example could be to use a second degree polynomial expansion. Here $[r \ g \ b]^T$ is mapped to $[r \ g \ b \ r^2 \ g^2 \ b^2 \ rg \ gb \ rb]^T$ and $M^{a,b}$ would be a 3×9 map. In this paper, in addition to the LCC, we will consider polynomial [18] and root-polynomial [19] colour corrections, respectively labelled PCC and RPCC.

3 Colour Corrected Smoothest Reflectance Estimation (CCSRE)

We will develop our CCSRE method for the discrete representation of spectra (spectra represented at 31 wavelength samples). We begin by recapitulating the *smooth()* - Equation (5) - measure of reflectance in the discrete domain. The discrete derivative is written as a matrix operator in [13, 20] as:

$$\nabla = \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & 0 & -1 & 1 \\ 0 & \cdots & 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (8)$$

And the sum of squared derivatives is written as

$$smooth(\underline{s}) = \|\nabla \underline{s}\|^2 = \underline{s}^T \nabla^T \nabla \underline{s} \quad (9)$$

We can now write the optimisation statement for conventional smoothest reflectance recovery [13]:

$$\underset{\underline{s}}{\operatorname{argmin}} \|\nabla \underline{s}\|^2 \text{ s.t. } R^T \operatorname{diag}(\underline{e}) \underline{s} = \underline{\rho} \ \& \ 0 \leq \underline{s} \leq \underline{1} \quad (10)$$

where respectively $\underline{0}$ and $\underline{1}$ denote 31-vectors of 0s and 1s (we need to explicitly bound the reflectance to lie between 0 and 1). Equation 10 is a quadratic objective with linear equality and inequality constraints. As such it can be efficiently solved - with the global minimum guaranteed to be found - by quadratic programming [21].

Perhaps counter-intuitively, it is simple to show that as we change the viewing illuminant the smoothest reflectance also changes. That is for a given *real reflectance* the smoothest metamers that induces the same RGB (as the actual reflectance) under different lights varies. This is a result of the formulation of the problem and the ornery nature of metamerism. Returning to panel (d) of Figure 1 the variation in recovered smoothest reflectances can be large.

Let us now set forth our Colour Corrected Spectral Reflectance Estimation, CCSRE, method to try and find smooth metamers and to mitigate this large variation in smoothest reflectance recovery. Our idea is a simple one. Suppose we can - by mathematical means - transform the RGB measured under a given viewing illuminant $\underline{e}^v$ to a second canonical (or standard) viewing illuminant $\underline{e}^c$. Clearly, if we could solve this problem *exactly* then we would always recover the same smoothest reflectance. However, metamerism dictates no exact solution is possible as there will always be pairs of reflectances that look the same under one light but different under another. And yet, finding the best colour correction transform has been the source of much research and year-by-year the efficacy of correction transforms has improved [19, 22, 23, 24, 25].

For a given calibration set of N reflectance spectra let us denote the corresponding RGBs under the canonical light as P^c . The corresponding RGBs for the viewing illuminant of interest is denoted P^v . Both these matrices are $3 \times N$. We admit polynomial and root-polynomial expansions of the data. We define an expansion function $\Psi^{alg,order}$ where $alg \in \{p, r\}$ (polynomial or root-polynomial expansions) and $order \in \{0, 1, 2, 3, 4, 5, \dots\}$ is the order of the expansion. As an example, a second order root-polynomial expansion is written as [19]:

$$\Psi^{r,2}(\rho) = [r \ g \ b \ \sqrt{rg} \ \sqrt{rb} \ \sqrt{gb}]^T \quad (11)$$

See [19] for a general formal statement of n th order polynomial and root-polynomial expansions.

We write the expansion of the matrix of RGBs under the viewing illuminant as $\Psi^{r,2}(P^v)$. For higher order expansions the expanded data matrices can become rank deficient. Thus, we need to regularise the colour correction transform. We define the best regularised colour correction matrix as

$$M^{v,c} = P^c [\Psi^{alg,order}(P^v)]^\times \quad (12)$$

where we define $A^\times$ as the regularised pseudo inverse of A defined as

$$A^\times = A[A^T A + \alpha \mathcal{I}]^{-1} \quad (13)$$

Here α is a user defined quantity controlling the degree of regularisation. If $\alpha = 0$ then $A^\times = A^+$. As formulated $A^\times$ is an instantiation of Tikhonov regularisation [26]. In this paper α is defined to be $0.000001 \times \text{trace}(A^T A)$

We now write our algorithm for smoothest reflectance recovery. The input to the algorithm is the observed RGB, $\underline{\rho}^v$ under a known viewing illuminant $\underline{e}^v$ together with knowledge of the spectral sensitivities of the camera, R .

Algorithm 1 Colour Corrected Smoothest Reflectance Estimation (CCSRE)

- 1: Preprocessing. For a given calibration set of reflectance spectra, viewing and canonical lights ($\underline{e}^v$ and $\underline{e}^c$), alg , $order$ and α calculate $M^{v,c}$ (according to Equations (12) and (13)).
 - 2: Estimate RGB under canonical light: $\underline{\hat{\rho}}^c = M^{v,c} \underline{\rho}^v$
 - 3: Attempt to solve CCSRE quadratic program: $\text{argmin}_{\underline{s}} \|\nabla \underline{s}\|^2$ s.t. $R^T \text{diag}(\underline{e}^c) \underline{s} = \underline{\hat{\rho}}^c$ & $0 \leq \underline{s} \leq 1$
 - 4: **if** solution exists **then** return $\underline{s}$
 - 5: **else** Solve for simple smoothest reflectance: $\text{argmin}_{\underline{s}} \|\nabla \underline{s}\|^2$ s.t. $R^T \text{diag}(\underline{e}^v) \underline{s} = \underline{\rho}^v$ & $0 \leq \underline{s} \leq 1$
 - 6: **end if**
-

In conventional smoothest reflectance estimation, for a given viewing illuminant there *must* be a physically realisable reflectance that generated the RGB (the actual reflectance is a viable solution). However, once we introduce a colour correction step it is possible that the estimated RGB under the canonical illuminant does not correspond to any physically realisable reflectance. This is why in step 3 of the algorithm we write ‘Attempt to solve’. On rare occasions ($< 1\%$) there is no solution after colour correction. In this case we revert to the conventional smoothest reflectance algorithm, step 5.

4 Results and Discussion

Now we wish to systematically quantify the degree to which a colour correction step might lead to more consistent reflectance recovery across different lights. For our camera we used the spectral sensitivities of a Canon 300D camera [27] and, for viewing illuminants, we used the 102 lights from [14]. The target of our colour correction step is the camera responses under CIE standard D65 [28]. In our experiments we use 4 reflectance datasets

- **MUN**: 462 reflectances from the Munsell colour chips dataset were used [29], these contain reflectances of colour chips from Munsell Book of Colour, Matte Edition.
- **OBJ**: 170 reflectances from assorted objects were used [30]. These include natural objects like rocks and plants, as well as man-made ones like fabrics.
- **DUP**: 120 reflectances from the DuPont paint chips dataset [30], these contain reflectances from the DuPont Colour Sampler.

- **NAT**: 404 reflectances from a dataset of natural surfaces, such as plants [31].

We will use these reflectance sets on cross-validation basis. That is each of the 4 sets will be used as a testing data set. The other 3 datasets are used for training (for our CCSRE algorithm finding the mapping from RGBs viewed under a viewing illuminant to counterparts with respect to D65).

Let us now quantify the variability of simple smoothest reflectance recovery. Since we have 102 viewing illuminants, for each spectra we will recover 102 smoothest reflectances (either using the original simple smoothest reflectance estimation - SSRE - algorithm or our CCSRE method). For the j th reflectance in our dataset let the i th recovered spectrum be denoted as $\underline{s}_{i,j}$ ($i = 1, 2, \dots, 102$, we recover the spectrum for each of 102 viewing lights). Per surface we have the mean of all recovered reflectance spectra is denoted $\underline{\mu}_i$. We calculate a percentage spectral error that measures the departure from the mean. If this % were 0 for all lights then this would mean that the same reflectance is recovered under all viewing illuminants.

$$\%Error = \frac{\|\underline{s}_{i,j} - \underline{\mu}_i\|}{\|\underline{\mu}_i\|} \quad (14)$$

As we have 102 lights and many reflectances in a test set we calculate the average %Error. Results are reported in Table 1. In the first row of the table we show the results for Simple Smoothest Reflectance Recovery, [20]. The column labelled **MUN** indicates that the Munsell dataset is used for testing. We see that the %Error for SSRE is 2.289%. Results are also shown for the 3 other reflectance sets.

		MUN	OBJ	DUP	NAT
	SSRE	2.289%	2.307%	2.887%	4.009%
	LCC	1.635%	1.551%*	2.833%*	2.655%*
PCC	2nd	1.830%	1.491%	2.376%	2.353%*
	3rd	1.550%	1.438%*	1.632%	2.022%*
	4th	1.629%	1.416%	1.537%	1.926%
RPCC	2nd	1.153%	1.489%	1.536%	2.525%
	3rd	1.293%	1.397%	1.223%	2.513%
	4th	1.333%	1.411%	1.593%*	2.588%

Table 1: %Error for each dataset using each of the CCSRE methods and SSRE. For each dataset the best method is in bold. * No solution existed for some reconstructions in these test, requiring SSRE to be used. In all cases, less than 1% were unable to be reconstructed.

We now calculate the performance of our CCSRE method. Here, when the testing data is **MUN**, the colour correction transform to D65 is computed using a training reflectance set (comprising the 3 other reflectance data sets not used for testing). When RGBs are corrected by a simple linear transform (LCC) the percentage error reduces to 1.635%. Correction transforms are also calculated for 2nd, 3rd, and 4th order polynomial (PCC) and root-polynomial (RPCC) expansion. For the **MUN** testing reflectance set the 2nd order RPCC provides the smoothest reflectance recovery that is most consistent. The % departure from the mean is only 1.153% (about half as much as SSRE).

Results are also shown for the other reflectance testing sets and in all cases there is a variant of CCSRE that delivers a 50% reduction in recovery error. Per reflectance set the lowest percentage spectral error (measuring the distance from the mean and, so, the consistency in the recover) is shown in bold.

5 Conclusion

Smoothest reflectance estimation recovers the smoothest metamer for a given measurement condition (viewing illuminant and camera). A weakness of this method is that different smoothest reflectances are recovered for different lights and the variability of the recovered reflectance can be large.

In this paper, we colour correct the RGBs recorded under a viewing illuminant to a canonical light and then recover the smoothest reflectance with respect to this canonical illumination. Experiments - on a cross-validation basis - demonstrated that this simple correction steps leads to a reduction in the variability of the recovered reflectance spectra by 50%.

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