Regular orbits of cyclic subgroups of the simple classical groups

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Abstract

Let G be a group contained in $Sym(\Omega)$, and let C be a cyclic subgroup of G. Then the question we ask is: for which groups $G \subset Sym(\Omega)$ does every cyclic subgroup C have a regular orbit on Ω? In this thesis we look at the simple symplectic, orthogonal and unitary groups of rank three. Let V be a vector space such that $V = F_q^n$ and let $I(V)$ be a symplectic, orthogonal or unitary group. We denote by $I(V)'$ the commutator subgroup of $I(V)$. Then the main result contained here is the following:

Let G be a group such that $I(V)' \subseteq G \subseteq I(V)$ with the conditions that $I(V) \neq O^{\pm}(2,q), O^{+}(4,q)$ and $q > 5$. Let Ω be a non-trivial, transitive G-set with kernel contained in $Z(G)$. Then C has an orbit of size $|C/(C \cap Z(G))|$ on Ω.

If C has an orbit of size $k = |C/(C \cap Z(G))|$ on Ω then there exists an $\omega \in \Omega$ such that $|C\omega| = k$. Thus if $c\omega = \omega$ for some $c \in C$ then $c \in Z(G)$. Hence an easy corollary to the main result is:

Let G be a simple symplectic, orthogonal or unitary group over a field of more than 5 elements. Then every cyclic subgroup of G has a regular orbit in every non-trivial G-set Ω.

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