Sleeping Beauties in Economics: Allais, Arrow, and the 75th Anniversary of the 1952 Paris Conference on Risk

Ivan Boldyrev[a], Ivan Moscati[b], Robert Sugden[c,*]

This draft: June 23, 2026

Abstract

The conference on risk held in Paris in May 1952 was a landmark event in economic analysis, especially decision theory. Several groundbreaking ideas were first presented there: some had an immediate impact, while others remained "sleeping beauties," exerting their influence only much later. These include Allais's critique of expected utility theory and his paradox, and Arrow's approach to state-contingent consumption. This paper makes three main contributions. First, on the conference's 75th anniversary, it reconstructs the rich exchange of ideas that unfolded at the meeting. Second, it offers a systematic account of the theory of decision making under risk that Allais elaborated between 1952 and 1953, after reconsidering the various arguments presented at the conference. Third, it addresses the "sleeping beauty puzzle" associated with Allais and Arrow: why did their ideas have little influence for more than twenty-five years, and how did they rise to prominence in the late 1970s? Because the Allais puzzle is likely more relevant to readers of the *Journal of Risk and Uncertainty*, we focus on it more closely.

Keywords: Allais; Arrow; decision-making under risk; Allais paradoxes; Morlat and Massé's paradoxes; behavioral decision theory; general equilibrium theory; financial economics; contingent consumption.

JEL Codes: B21, B26, B31, C91, D53, D81, D91.

a Department of Economics and Business Economics, Nijmegen School of Management, Radboud University Nijmegen; e-mail: ivan.boldyrev@ru.nl.

b Department of Economics, University of Insubria, Varese; and Baffi Carefin, Bocconi University, Milan; e-mail: ivan.moscati@uninsubria.it.

c School of Economics, University of East Anglia, Norwich; e-mail: r.sugden@uea.ac.uk.

* We are grateful to Jean Baccelli, Pierpaolo Battigalli, Catherine Herfeld, Dorian Jullien, Massimo Marinacci, Edoardo Peruzzi, Benoit Walraevens, Carlo Zappia, and two anonymous referees for their valuable comments and helpful discussions on this paper. It also benefited from comments by participants at the 2026 ESHET–HES conference in Nice. Moscati thanks the Department of Decision Sciences at Bocconi University, Jesus College of the University of Oxford, and colleagues at these institutions for their kind hospitality while he was working on this paper. He also gratefully acknowledges financial support from the Italian Ministry for University and Research, grant Department of Excellence 2023–2027, CUP J37G22000330001.

1 Introduction

The international conference on "Foundations and Applications of the Theory of Risk in Econometrics," held in Paris in May 1952, was a landmark event in the history of decision theory. The meeting represented the high point of a debate on the descriptive and normative validity of expected utility theory (EUT) and its applications to specific areas of economic analysis, such as general equilibrium theory. Sparked by the novel version of the theory advanced by John von Neumann and Oskar Morgenstern in 1944, this debate engaged nearly all major utility theorists of the period from the late 1940s to the mid-1950s.

A number of groundbreaking ideas were first aired at the conference. These included Paul Samuelson's axiomatization of EUT, explicitly featuring the so-called independence axiom, and Leonard Jimmie Savage's extension of EUT from situations of risk, where the probabilities of uncertain events are assumed to be known, to situations of uncertainty, where this assumption is removed. While these contributions had an immediate impact on economic theory, others were "sleeping beauties," that is, they received only limited attention ("slept") for a long period and then, within a relatively short span, attracted significant attention and became highly influential ("were awakened"). Among them were Maurice Allais's critique of EUT, along with the paradox he introduced to challenge it, and Kenneth Arrow's approach to state-contingent consumption. Their ideas gained prominence only in the late 1970s, when Allais's critique and paradox became central to behavioral economics and Arrow's model served as a foundation of financial economics.[1]

In this paper we make three main contributions. First, on the occasion of the 75th anniversary of the conference, in Section 2 we provide a comprehensive reconstruction of the rich exchange of ideas that unfolded during the meeting. In doing so, we also draw attention to some neglected episodes in the history of decision theory, most notably the counterexamples to EUT introduced at the conference by Pierre Massé and Georges Morlat —two French scholars who, like Allais, had backgrounds in both economics and engineering and joined him in criticizing EUT.[2] While Allais did not present his paradox during the official sessions, but only informally to a few participants during one of the luncheons, Massé and Morlat did present their counterexamples during the conference and these exhibited all the ingredients of the Allais paradox.

[1] For general reviews of the scientific contributions of Allais (1911–2010; Nobel Prize in Economics, 1988) and Arrow (1921–2017; Nobel Prize in Economics, 1972), see Munier (1991) and Maskin (2019). On the important role of conferences and workshops in the history of modern economics in forging new ideas and disseminating them within relevant scholarly communities, see Düppe & Weintraub (2014); Cherrier & Saïdi (2021).

[2] Massé (1898–1987) worked in the electrical industry from the late 1920s, joined Électricité de France (EDF), the French state-owned electricity company, upon its foundation in 1946, and served as its deputy director from 1948 to 1959. Morlat (1924–2022) was hired at EDF by Massé in the late 1940s and spent most of his career there, becoming, in the 1960s, head of the division for statistical research. For more on Massé and Morlat, see Jallais & Pradier (2005).

Second, in Section 3 we offer a systematic account of the theory of decision making under risk that Allais elaborated after the conference, between 1952 and 1953, when he had time to reflect on and reconsider the various arguments presented by participants at the Paris meeting. Specifically, it was only after the conference that Allais formally articulated the paradox he had presented informally at the meeting, formulated a second paradox, and integrated both into his critique of EUT.

Third, in Sections 4–7 we address the "sleeping beauty puzzle" associated with Allais and Arrow: why did their ideas have little influence on economics for more than twenty-five years? And how were they taken up in the late 1970s and eventually rose to prominence in the field? Since the Allais sleeping beauty puzzle is likely to be more relevant for most readers of this Journal, as it is closely tied to major developments in the theory of decision making under risk and uncertainty, we devote comparatively more space to it. More specifically, Section 4 focuses on Allais's sleeping beauty puzzle. Section 5 illustrates how some of his ideas, and especially his paradoxes, were eventually revived and became foundational to the development of behavioral theories of decision making that deviate from EUT. Section 6 turns to Arrow and examines the genesis of his Paris paper and its reception in the 1950s and 1960s. Section 7 addresses the sleeping beauty puzzle associated with Arrow and traces how his Paris paper became a seminal paper in financial economics in the late 1970s.

In the concluding Section 8, we briefly discuss similarities and differences between these two temporally parallel sleeping beauties. We do not pretend to draw general conclusions about the reception of ideas in economics from a sample of two cases. Rather, we think it is more useful to see these cases as illustrations of two different ways in which ideas can re-emerge after long periods of neglect.

A final introductory comment on "sleeping beauties" in science is in order. There has been some research on scientific contributions that went almost unnoticed for a long time and exerted their influence only much later (see, e.g., Glänzel et al. 2003; van Raan 2004; 2015). Much of this research, however, has tended to focus on physics and other "hard sciences," to rely on bibliometric methods, and to examine more recent periods, for which bibliometric databases are more easily available. Our approach is different: in short, it is a history-of-ideas rather than a bibliometric approach. That is, our interest lies primarily in the conceptual and methodological developments internal to decision theory, as well as in the changes in the broader theoretical frameworks of economics, that made Allais' and Arrow's ideas come to be regarded as significant and influential, rather than in tracing patterns of reception and diffusion as reflected in citation data.

2 It all happened in Paris

The conference was organized by Allais together with the French mathematician Georges Darmois. Sponsored by the French Centre National de la Recherche Scientifique (CNRS) and the Econometric Society, it took place at the Institut Henri Poincaré in Paris's 5th arrondissement, from May 12 to 17, 1952.

Several prominent economists, mathematicians, and statisticians attended the meeting. From the United States came Arrow, Samuelson, Savage, Milton Friedman, and Jakob Marschak. In addition to Allais and Darmois, the French participants included Morlat and Massé along with mathematician Georges Guilbaud and economist Edmond Malinvaud. Also in attendance were the Norwegian economist Ragnar Frisch, Italian probability theorist Bruno de Finetti, and English economist George Shackle. The year after the conference, the CNRS published the proceedings in French (CNRS 1953), which include full transcriptions of the discussions following the presentations and thus allow us to form a fairly detailed picture of the discussions that unfolded during the meeting.

In this section, we begin by briefly outlining the fortunes of EUT prior to the conference; we then survey the papers presented there by Allais and Arrow, along with other key contributions by Savage and Samuelson, and summarize the discussions these works generated; finally, we consider the counterexamples to EUT presented at the conference by Morlat, Massé, and Allais.

2.1 The fortunes of EUT before May 1952

EUT was originally advanced by Daniel Bernoulli in the eighteenth century, entered economics in the 1870s, and was widely accepted until the 1910s (on the history of EUT, see Moscati 2023). In the 1920s and 1930s, however, it came under increasing criticism in the context of the ordinal turn in utility analysis. Ordinalists such as John Hicks objected that the utility function in EUT is cardinal rather than ordinal, thereby contravening their tenet that utility merely represents preference rankings. Critics also argued that EUT's representation of the utility of a lottery as a weighted sum of outcome utilities presupposes that utility is additively separable, an assumption that economists had long abandoned because it excludes substitutability and complementarity among commodities. Finally, EUT was criticized for focusing exclusively on the average of utilities, without explaining why rational agents should disregard other features of the utility distribution, most notably its variance. As a result of these objections, by the late 1930s and early 1940s EUT had few supporters, and several alternative theories were put forward, including utility-free approaches based on the mean and variance of payoffs.

The fortunes of EUT began to recover in the late 1940s, following von Neumann and Morgenstern's reformulation in *Theory of Games and Economic Behavior* (1st ed. 1944; 2nd ed., with an extended axiomatic treatment, 1947). Consistent with the ordinal turn, their version of EUT takes preferences between lotteries, rather than the utility of outcomes, as primitive and characterizes them through axioms. Among other things, this means that the acceptance of EUT as a normative theory depends on whether these axioms are regarded as identifying rational behavior. As already mentioned, from the late 1940s to the mid-1950s, von Neumann and Morgenstern's EUT was the subject of an intense debate, especially in the United States but also in Europe.

In the United States, between 1948 and May 1952, Arrow, Friedman, Marschak, Savage, and Samuelson took an active part in the discussion, both in published papers and in intense correspondence (see Moscati 2019, chapters 10–12). Although Samuelson initially opposed EUT, by early 1952 all five scholars had come to endorse it, albeit for different reasons. Samuelson, Marschak, and Savage accepted EUT as a normative theory because they regarded its underlying axioms as a compelling expression of economic rationality under risk, while Friedman and Arrow valued its relative simplicity and descriptive adequacy. The five researchers—whom Allais later labeled the American School—were more closely aligned in their interpretation of the cardinal utility function in von Neumann and Morgenstern's EUT, which they regarded as conceptually and mathematically distinct from the traditional utility function expressing preferences over riskless alternatives. Therefore, for them the acceptance of EUT did not entail a return to pre-ordinal utility analysis.

In Europe, an important discussion on EUT took place in France between 1949 and 1952 among Allais, Darmois, and other future participants in the conference (see Jallais & Pradier 2005). Roughly speaking, a group of economists with engineering backgrounds—led by Allais and including Morlat and Massé—opposed EUT on both descriptive and normative grounds, while a group of mathematicians led by Darmois and Guilbaud favored it. In 1950, the French scholars invited Friedman to Paris to give a talk on EUT and subsequently decided to organize an international conference on the topic.

2.2 In the arena

Savage and Arrow delivered their papers on the second day of the conference. Savage (1953) presented an axiomatization of EUT in which the probabilities of uncertain events are not given, as in von Neumann and Morgenstern's model, but inferred from the decision maker's preferences over actions whose outcomes depend on those events. This is the "subjective" version of EUT that Savage would publish two years later in the *Foundations of Statistics* ([1954] 1972). Although he had completed a first draft of the book in 1951 and circulated it privately to Samuelson and other colleagues, the Paris conference was the first occasion on which he presented his model to a wider audience.

Savage advocated the axioms underlying his model only from a normative viewpoint, as "principles of conduct … that no one would want to transgress deliberately" (Savage 1953: 29; here and throughout the paper, translations from the conference proceedings are ours).

The axioms largely match those in *Foundations*, with what became the sure-thing principle in the book appearing in Paris as axiom 5. This axiom requires that the preference ranking between two actions, or, in Savage's terminology, "acts," depend only on the parts that distinguish them, and not on the parts they have in common.[3] Following the more familiar terminology of *Foundations*, we will refer to axiom 5 as the sure-thing principle.

In the discussion following Savage's presentation, Allais criticized the sure-thing principle on both descriptive and normative grounds for ignoring the "complementarity effect" between outcomes:

> It seems impossible to assume that, if I am very cautious, the psychological value I attach to a one-in-ten chance of winning 100 million would be the same whether I already hold the other nine chances out of ten or do not hold them. There is, in fact, a very strong complementarity effect that ought to be taken into account. There is nothing irrational in this effect. (CNRS 1953: 39)

In his contribution, Arrow (1953; translated into English in Arrow 1964 and as chapter 4 of Arrow 1971) extended general equilibrium theory, previously developed only for riskless situations, to settings involving risk. He introduced state-contingent commodities x_{isc}, denoting the quantity x of commodity c claimed by individual i if state of the world s occurs. He also assumed that each individual i assigns a subjective probability π_{is} to the occurrence of each state s, maximizes expected utility calculated using these probabilities, and is risk-averse. This reformulation of the general equilibrium model implied that trading the state-contingent commodities leads essentially to the same efficiency results as in the general equilibrium model with certainty. Arrow (1953: 43) further posited trade in "elementary values," now called Arrow securities, which pay one monetary unit if state s occurs and nothing otherwise. He showed that, under perfect competition, trade in these securities leads to an "optimal allocation of risk-bearing" (41), in the sense that no alternative allocation of the state-dependent—and therefore risky—commodities x_{isc} could make everyone better off. The paper thus demonstrated that endowing the economy with securities for every future state of the world would allow to achieve the same efficient allocation as the trade in state-contingent commodities.

On the third day of the conference, Allais presented two papers. In the morning, he addressed the same topic as Arrow, namely the extension of general equilibrium theory to situations involving risk (Allais 1953a). In the 1940s, in fact, general equilibrium theory had been Allais's main research area (see especially Allais 1943; 1947), and it was for his pioneering contributions to this field, and not for his paradox, that he was awarded the Nobel Prize in Economics in 1988.

3 More formally, consider two acts, A_1 and A_2, that yield the same outcome x when event E occurs, so that, under event E, x is "sure" regardless of which act the decision maker chooses. The axiom states that the preference ranking between A_1 and A_2 should not depend on what the outcome under event E is.

In the discussion following Allais's presentation, several participants, including Arrow and Allais themselves, highlighted important differences between their two approaches (CNRS 1953: 110–120). Arrow emphasized that his model allows heterogeneous beliefs among individuals, while Allais assumed that everyone shares the same beliefs. This criticism of Allais was echoed by other discussants, notably Marschak, who stressed that "in reality, financial markets are characterized by differences in individuals' estimations of the future, such as those between the *bulls* (optimists) and the *bears* (pessimists)" (110). In turn, Allais criticized Arrow's reliance on EUT and claimed that his own model could be generalized to allow for heterogeneous beliefs, modeled by "substituting subjective probabilities for objective probabilities" (110), and that its key insights would hold even within this more general setting. However, he did not provide any systematic treatment of this claim.

The afternoon session featured Allais's second paper (1953b) and Samuelson's paper (1953; English translation in Samuelson 1966). Allais addressed decision making under risk but, contrary to what one might expect, he neither introduced his famous paradox nor offered a detailed criticism of EUT. In fact, he developed those criticisms in the discussions following the presentations of other papers, and, as discussed in Section 2.4, he presented the paradox only informally at one of the conference lunches. His second paper instead represents an early attempt to outline the "foundations of a positive theory of choice involving risk" (1953b: 127). The key features of this theory will be discussed at some length in Section 3. For present purposes, it suffices to note that the theory is highly general, featuring several functions whose mathematical form is left unspecified, and that it subsumes EUT as a special but, in Allais's view, unlikely case.

Samuelson proposed an axiomatization of EUT that explicitly featured the independence axiom, which had been implicit but hidden in von Neumann and Morgenstern's axiomatization. The idea behind this axiom is essentially the same as that behind the sure-thing principle, namely that the preference ranking between uncertain options should be independent of the parts those options have in common and depend only on the parts that distinguish them. However, Samuelson's theory differs from Savage's in that it takes lotteries, in which consequences are associated with given probabilities, rather than acts, which do not explicitly feature probabilities, as the objects of preference.[4] In effect, Morlat, Allais, and other conference participants often treated the independence axiom and the sure-thing principle interchangeably, referring to violations of the former as violations of the latter.

4 More formally, the independence axiom states that, for any three lotteries L_1, L_2, and L_3, if the decision maker prefers L_1 to L_2, then she should also prefer the compound lottery $[L_1, p;\ L_3, 1-p]$, yielding L_1 with probability p and L_3 with probability $(1-p)$, to the compound lottery $[L_2, p;\ L_3, 1-p]$, yielding L_2 with probability p and L_3 with probability $(1-p)$, and this for any probability p. More on the difference between the sure-thing principle and the independence axiom in Steele & Stefánsson (2025).

Samuelson explained that he endorsed EUT only because, thanks to an exchange with Savage, he had come to regard the sure-thing principle as a requirement of rational behavior under uncertainty, and the independence axiom as a special case when the probabilities of outcomes are known (on this exchange, see Moscati 2016). Samuelson, however, remained skeptical about EUT's descriptive power: "For explaining the *actual behavior* of men on our planet," he wrote, "the Bernoulli hypothesis [i.e., EUT] appears to me of rather trifling significance" (1953: 142). He also argued that the independence axiom is unrelated to the discredited hypothesis of additively separable utilities. This is because lottery outcomes are mutually exclusive, so potential relationships of substitutability or complementarity between them are irrelevant (149–150).

The discussion following Allais's and Samuelson's presentations was the most intense of the entire conference (CNRS 1953: 150–164). Savage, Marschak, de Finetti, and Guilbaud argued, though with varying emphases, that Allais's very general theory yielded no workable formula and therefore offered no viable alternative to EUT. Allais replied that it was "better to use a formula that is more general but consistent with the facts," like his own, than "a simple formula that is manifestly inaccurate," like EUT (154).

As for Samuelson's contribution, Malinvaud (163–164) pointed out that the independence axiom is indeed hidden in von Neumann and Morgenstern's EUT, specifically in their stating the axioms over preferences between indifference classes of lotteries rather than preferences between single lotteries. He later published this comment as Malinvaud (1952).

Allais also rejected Samuelson's claim that the mutual exclusiveness of lottery outcomes renders complementarity relationships between them irrelevant. He argued that the claim is correct only *ex post*—that is, after the decision is made and a single outcome is obtained—but incorrect *ex ante*, before the choice is made. As discussed in Section 3.2, this *ex ante*/*ex post* distinction became a key element of the approach to decision making under risk that Allais elaborated after the conference.

Finally, in commenting on Samuelson, Morlat introduced the first of the two "paradoxes" we will discuss in the following section.

2.3 Morlat and Massé's paradoxes

Morlat (CNRS 1953: 156) envisioned an urn containing one hundred balls numbered from 1 to 100 and, following Savage's approach, considered choices between pairs of actions whose outcomes depend on the number on the ball drawn from the urn. Each action involves the threat of execution by guillotine, a capital punishment used in France until its abolition in 1981.

Action A_1 yields guillotine if ball 1 is drawn; 1,000 French francs (approximately \$50 today) if the ball is numbered 2–50; and 2,000 francs (approximately \$100 today) if it is numbered 51–100. Action A_2 yields guillotine if the number is 1 or 2; 1,000 francs if it is 3–50; and 1 million francs (around \$50,000 today) if it is 51–100. Note that actions A_1 and A_2 share a common outcome: 1,000 francs when the ball drawn is numbered 3–50. Morlat constructed the second pair of actions by replacing this common outcome with another common outcome, namely guillotine. Accordingly, A_3 yields guillotine if the ball is numbered 1 or 3–50; 1,000 francs if it is numbered 2; and 2,000 francs if it is 51–100. A_4 yields guillotine if the ball is 1–50 and 1 million if it is 51–100.

Savage's sure-thing principle implies that substituting one common outcome with another should not affect the preferences between the resulting actions. Therefore, if an individual prefers A_1 over A_2, she should also prefer A_3 over A_4. However, Morlat noted, this is not what many reasonable persons would do:

> I assume that many persons reputed to be reasonable will prefer A_1 to A_2 because the risk of death is half as great, but will prefer A_4 to A_3 because, in that case, the risks of death will be close (49 percent or 50 percent), whereas, if they are spared, they will be much better off in the case of option A_4. (CNRS 1953: 157)

Since in Morlat's setting the probabilities of the states of the world are known, a counterexample to the sure-thing principle also represents a counterexample to the independence axiom. However, Morlat did not make this point; instead, together with Massé, he constructed a specific and more direct counterexample to the latter.

This second counterexample is included in Massé and Morlat's (1953: 165) conference paper, delivered on the morning of the fourth day of the conference. It introduces an element absent from Morlat's earlier example, namely "the advantage of certainty." Here the alternatives are two pairs of lotteries, and the likelihood of outcomes is expressed directly in probability terms. Lottery L_1 pays 10 million francs (around \$500,000 today) with certainty, while L_2 yields 20 million francs (about \$1 million today) with probability $p = 0.90$ and nothing with $p = 0.10$. Lotteries L_3 and L_4 are derived from L_1 and L_2, respectively, by mixing each in the same ratio $(2{:}\,8)$ with another lottery that yields nothing with certainty (call it L_5). Thus, $L_3 = [L_1, 0.2;\ L_5, 0.8]$, yielding 10 million francs with $p = 0.20$ and nothing with $p = 0.80$, and $L_4 = [L_2, 0.2;\ L_5, 0.8]$, yielding 20 million francs with $p = 0.18$ and nothing with $p = 0.82$.

Massé and Morlat contended that "many prudent people" would prefer L_1 to L_2 because of the advantage of the certain gain offered by the former (179). But most of these same people, they argued, will also prefer L_4 to L_3:

> Most of them, we believe, will consider the prospects L_3 and L_4 as follows: a gain of 10 million with a probability of 20 percent, weighed against a gain of 20 million—i.e., double the amount—with a probability of 18 percent, that is, not very different from the former. And they will then decide, *no longer having to take into account the advantage of certainty*, L_4 preferred to L_3 (CNRS 1953: 179; italics in the original)

The preferences for L_1 and L_4, however, violate the independence axiom, which states that mixing two lotteries in the same ratio with a third lottery should not affect the preferences between the resulting lotteries.

Although Massé and Morlat presented their decision problems in the plenary sessions and discussions of the conference, and although participants such as Savage ([1954] 1972, 101) later referred to Morlat's example, their paradoxes were, and remain, little known among decision theorists. One reason for this obscurity is that, after the conference, Massé and Morlat devoted themselves primarily to their professional careers at Électricité de France (see footnote 2), leaving Allais to play the principal role in criticizing EUT and promoting the paradoxes. They also did not publish their conference paper in a journal, so it remained accessible only in the conference proceedings, which were difficult to obtain, especially outside France. Finally, after the publication of Savage's *Foundations* in 1954, Morlat (1956; 1957) revised his position and came to accept EUT as a normatively compelling theory because he now regarded Savage's postulates as "a fundamental logical achievement worthy of serving as a rule of conduct for every reasonable person" (Morlat 1957: 378; translation from the French by the authors).

2.4 Allais's paradoxes

The counterexample to EUT commonly indicated as the Allais paradox is conspicuously absent from the conference proceedings. All available evidence suggests, in fact, that Allais presented it only informally to Savage and a few other participants during one of the conference lunches (Allais 1979, 533; Jallais & Pradier 2005; Mongin 2019).

The paradox first appeared in print in 1953, in two publications. The first was a 75-page memoir in French that Allais included as an appendix to the conference proceedings (English translation in Allais [1953] 1979). Taking stock of the conference discussions, in this memoir he extended his critique of EUT and further developed the positive model of decision making introduced in his second conference paper (Allais 1953b). The second, and far more visible, publication was an article in *Econometrica*, written in French and preceded by an extended English abstract that refers to the paradox only in very general terms (Allais 1953c). This article is essentially an abridged version of the memoir, focusing on the critique of EUT rather than on Allais's positive decision model. In the acknowledgements of both publications, Allais ([1953] 1979, 132; 1953c, 503) thanked, among others, Massé and

Morlat "for their many excellent comments and suggestions." Both publications also introduced a second, similarly paradoxical decision problem.

Allais's first decision problem—that is, the Allais paradox in strict sense—was intended as a counterexample to Savage's sure-thing principle, just as Morlat's was. Unlike Morlat, however, Allais (1953c: 525–527; [1953] 1979: 314–315) framed the problem directly in terms of choice between lotteries rather than between acts *à la* Savage. The first pair consists of L_1, which pays 100 million francs (approximately, $5 million today) with certainty, and L_2, which yields 100 million francs with $p = 0.89$, 500 million francs (about $25 million today) with $p = 0.10$, and nothing with $p = 0.01$. Like Morlat's actions A_1 and A_2, Allais's lotteries L_1 and L_2 share a common outcome: 100 million francs with $p = 0.89$. Allais constructed the second pair of lotteries by replacing this common outcome with a different one, namely nothing. Accordingly, lottery L_3 yields 100 million francs with $p = 0.11$ and nothing with $p = 0.89$, and L_4 yields 500 million francs with $p = 0.10$ and nothing with $p = 0.90$.

As already observed, Savage's sure-thing principle implies that substituting one common outcome with another should not affect the preferences between the resulting lotteries. Therefore, if an individual prefers L_1 over L_2, she should also prefer L_3 over L_4. However, Allais argued, "most highly prudent people … who are regarded as rational by common opinion" (1953c: 527) prefer L_1 and L_4, and thus violate the sure-thing principle. Even Savage himself preferred L_1 and L_4 when presented with the problem in Paris (see Savage ([1954] 1972: 103). As discussed more extensively in Section 3.3, according to Allais this pattern of choice is due to the psychological "advantage of certainty" (526) generated in L_1 by the complementarity effect between the gains of 100 million francs with probabilities $p = 0.11$ and $p = 0.89$.

Notice that this paradox combines two features of the counterexamples to EUT presented by Massé and Morlat. First, Allais's replacement of one pair of common outcomes with another is also central to the paradox Morlat presented on the third day of the conference. Second, Allais's idea that outcomes can be complementary and combine to produce a certain gain, and that the certainty of the resulting combination exerts a significant influence on choice, was already part of Massé and Morlat's explanation of the paradox they presented on the fourth day. One difference between Allais's and Morlat's paradoxes is that, whereas all outcomes in Allais's example are gains, in Morlat's the most salient consequence is a dramatic loss (being guillotined). Yet, like the proponents of EUT at that time, Allais did not draw a categorical distinction between gains and losses.[5]

[5] On this point, see also Section 3.3, footnote 7.

The second paradoxical situation envisioned by Allais was not presented at the Paris conference but appeared in print the following year (Allais 1953c: 528–530; [1953] 1979: 315–317). It nevertheless seems appropriate to illustrate it here, immediately after the other three Parisian paradoxes, with which this fourth one is closely related.

Allais's second paradox is very similar to Massé and Morlat's counterexample, and, like theirs, targets Samuelson's independence axiom. As in Allais's first paradox, lottery L_1 yields 100 million francs with certainty, while in this variant L_2 yields 500 million francs with $p = 0.98$ and nothing with $p = 0.02$. As in the Massé–Morlat paradox, Allais derived lotteries L_3 and L_4 from L_1 and L_2, respectively, by mixing each with a lottery that yields nothing with certainty, in the same ratio $(1:9)$. This yields L_3, which pays 100 million francs with $p = 0.10$ and nothing with $p = 0.90$, and L_4, which pays 500 million francs with $p = 0.098$ and nothing with $p = 0.902$. Allais argued that "people regarded as perfectly rational, though prudent" (1953c: 529) prefer L_1 and L_4, thereby violating the independence axiom. According to him, this pattern of choice arises for psychological reasons analogous to those operating in the first paradox: rational but prudent individuals prefer L_1 to L_2 because of the "advantage of complementarity" (528) and because they "attach a great value to certainty" (530).

Clearly, there are strong similarities between these decision problems and those presented at the Paris conference by Massé and Morlat. Without intending to diminish Allais's centrality over many years in bringing sustained attention to violations of EUT, it seems to us that what are now known as the "Allais paradoxes" would more appropriately be called the "Massé–Morlat–Allais paradoxes"

3 Allais's general theory of behavior in the face of risk

In this section, we offer a reconstruction of Allais's considered views about the theory of choice under risk, after he had reflected on the various arguments presented at the Paris conference.[6] As noted in Section 1, it was only after the conference that Allais integrated his two paradoxes into his critique of EUT.

[6] Our primary source is the memoir that Allais included as an appendix to the conference proceedings (Allais [1953] 1979). Most of the passages we cite also appear in the more accessible *Econometrica* paper (Allais 1953c). Where this is the case, we give *Econometrica* page references but use the translations in the English edition of the memoir.

3.1 Allais's conception of rationality

Allais presents his theory both as a "positive theory of choices involving risk" (in the title of the memoir) and as an analysis of "the behavior of the rational man in the face of risk" (in the title of the *Econometrica* paper). That both descriptions are appropriate reflects Allais' understanding of "rationality."

Allais makes a crucial distinction between two different concepts of rationality. The "abstract" definition expresses the idea that rationality consists in pursuing internally consistent ends by choosing means appropriate to those ends. In the context of risk, this concept of rationality requires only that (i) a rational individual has a preference ordering over probability distributions of satisfaction, (ii) this ordering respects what is now generally known as first-order stochastic dominance, and (iii) objective probabilities, if known by the individual, "are considered" (1953c: 518). In defense of this relatively undemanding conception of rationality, Allais argues that attitudes to risk are like tastes: "they are what they are, and differ from one person to the next" (1953c: 521).

Alternatively, rationality can be defined "experimentally," by observing the behavior of individuals "who are considered rational by public opinion" (1953c: 505). Choices under risk are experimentally rational when they are those that reasonable and prudent individuals would make. It is in this sense that the behavior observed in the paradoxes is rational. Allais's "positive" theory of choice under risk is essentially a theory of experimentally rational behavior, interpreted broadly. His objection to EUT is that it uses normative axioms that rule out certain forms of such behavior.

Allais recognizes that, if considered in the abstract, the independence axiom and the sure-thing principle can be viewed as simple and obvious requirements of consistency among preferences. If behavior contrary to these requirements is regarded as rational by common opinion, isn't common opinion simply wrong? Allais's response, in the context of the first paradox, is:

> This example is a good illustration of the *pseudo-evident* character of [the sure-thing principle]. *That it is accepted by so many is due to the fact that not all its implications are perceived although far from being rational, some of them are in certain psychological situations wholly irrational*. (1953c: 527–528, italics in original)

For Allais, "pseudo-evidence" is the perception of self-evidence or "rational obviousness" (1953c: 538) that can be cued by abstract propositions of the type that rational choice theorists present as axioms. Reasonable people should not accept abstract axioms on the basis of such perceptions: they need to think about those axioms' concrete implications. This perspective on rationality is crucial to Allais's interpretation of the paradoxes. For Allais, the paradoxes do not reveal limitations of the rationality of ordinarily reasonable people; they are evidence of the unreasonableness of some of the allegedly rational requirements of EUT.

3.2 *Ex ante* versus *ex post* viewpoints on uncertainty

As anticipated in Section 2.2, Allais's fundamental criticism of EUT is that it conflates *ex ante* and *ex post* viewpoints on uncertainty. Preferences between lotteries express an *ex ante* view of probability distributions over still-possible outcomes, but the "utility" of a potential outcome is intelligible only from an *ex post* viewpoint in which it has become the *actual* outcome:

> In reality, the "ex ante" standpoint is the only one that may properly be taken in considering choices involving uncertainty. *In taking an ex post stance the element specific to risk, to wit, the importance of the shape of the distribution of uncertainty, is eliminated unawares.* That this is happening [in EUT] is absolutely obvious only in extreme cases [such as those exemplified by the paradoxes], *but in fact is happening throughout*. (1953c: 538, italics in original)

Allais's paradoxes (or, as he characterizes them, "facts indicated by experience" [1953c: 524]) are intended as particularly sharp illustrations of the general significance of the difference between the *ex ante* and *ex post* viewpoints.

Explaining the general principle behind the first paradox (now generally known as the *common consequence effect*), Allais says that it is easy to create similar examples by using "extreme cases in which the advantages (or drawbacks) of complementarity may be particularly strong" (1953c: 536). Complementarity effects are particularly likely when a combination of uncertain outcomes creates a certainty. Explaining the second paradox (now generally known as the *common ratio effect*), Allais refers again to "complementarity relations" and adds a footnote drawing a parallel with complementarity between consumption goods (1953c: 528 and note 33). The paradoxes illustrate how, as viewed by a reasonable person, there can be *ex ante* complementarity between outcomes that are mutually exclusive *ex post*. Allais's own theory of choice under risk takes the *ex ante* standpoint.

3.3 Allais's "positive" theory

Allais's methodological stance was very different from that of the proponents of EUT. It was important for Samuelson and Savage that their normative axioms were formulated as properties of preferences that were in principle observable in choice behavior. As noted in Section 2.1, Arrow and Friedman valued EUT as a simple, tractable theory that produced generally reliable predictions. In contrast, Allais maintained that a scientific theory should include *all* factors that are believed to have bearing on the phenomenon to be explained, even if, when the theory is first proposed, the scientist does not know how to calculate their effects.

Allais took what in 1952 would have been seen as the old-fashioned approach of effectively treating cardinal *psychological value* or *satisfaction* (Allais treats these terms as synonyms) as a primitive for his theory. Measures of satisfaction represent the *ex post* viewpoint. The *ex ante* viewpoint is that of choosing between different probability distributions of satisfaction.

Allais's theory is a theory of *prospects*, defined as probability distributions over outcomes that have monetary values.[7] In effect, it is a generalization of the Bernoullian form of EUT used in early neoclassical economics. The key features of the theory are encapsulated in four factors "which must necessarily be taken into account, even in a first approximation, by every theory of risk" (1953c: 504). Elements I and III are properties of Bernoullian EUT; Elements II and IV are not.

Element I is "the distinction between monetary and psychological values" (504), i.e., the non-linearity of the function that assigns satisfaction levels to monetary outcomes. Allais assumes diminishing marginal satisfaction over the domain of the function, but local linearity for small gains (1953c: 507–508, 524–525).

Element II is "the distortion of objective probabilities and the appearance of subjective probabilities" (504). Allais's conception of probability is frequentist; subjective probability is interpreted as "psychologically appraised frequency" ([1953] 1979: 42). The subjective probability judgements of reasonable people may differ from objective probabilities because of cognitive limitations, optimism or pessimism (1953c: 508).

Element III is "the mathematical expectation of psychological values." Allais treats this measure, in which satisfactions are weighted by subjective probabilities, as a first approximation to a true *ex ante* evaluation of a prospect (1953c: 508–509).

Element IV is "the dispersion (variance) as well as general properties of the form of the probability distribution of psychological values." For Allais, this is the factor "that is specific to the theory of risk" (1953c: 504). Dispersion in the distribution of psychological value is crucial because of the local linearity of the satisfaction function; attitudes to dispersion explain risk aversion for small gains. As viewed by Allais, Samuelson's and Savage's axioms conflate the *ex post* concept of diminishing marginal satisfaction (Element I) with the *ex ante* concept of risk aversion (Element IV). By implication, the properties of complementarity and certainty that induce the paradoxes are among the other "general properties" of Element IV.

4 Allais's sleeping beauty puzzle

This section addresses the "sleeping beauty" puzzle associated with Allais: namely, why his critique of EUT, and particularly the two paradoxes, had little influence on economics for more than twenty-five years, from the early 1950s to the late 1970s.

---

7 Allais usually refers to prospects as distributions of "gains," but in the memoir he makes clear that the monetary value of each outcome is the sum of the individual's "initial capital" C and a "gain" g that can be positive or negative. Treating C as "the origin of monetary values" is merely a convenient normalization ([1953] 1979: 38–39).

4.1 Why was Allais's contribution neglected for so long?

One obvious explanation—ignorance—can be ruled out, for three reasons. First, the leading decision theorists of the time attended the Paris conference and were therefore aware of Allais's ideas. Second, these ideas were published in *Econometrica*, the discipline's premier journal, although with the handicap that the article was written in French and that the significance of the paradoxes was not brought out in the English summary. Third, and most importantly, Savage discussed the first Allais paradox at length in his seminal *Foundations of Statistics* ([1954] 1972: 101–103).

However, Savage's response to the paradox gave other economists and decision theorists apparent justifications for ignoring it. Savage reformulated Allais's decision problem, making explicit the violation of the sure-thing principle that had been implicit in the preferences he expressed in Paris. Still viewing the principle as normatively valid and acknowledging that he still felt "an intuitive attraction to those preferences," he concluded that his initial preferences were mistaken and revised them. In an article published in *Econometrica* shortly after the Paris conference, Samuelson (1952: 678) gave a similar but blunter response: "[I]t is anything but a casual thing to have and know one's complete ordering or set of indifference curves; and if you ask a casual question, you must expect to get a casual answer."

As noted in Section 2.2, neither Savage nor Samuelson claimed that EUT produced reliable predictions of actual behavior. But, even if (contrary to Allais's criticism of "pseudo-evidence") their defenses of the *normative* status of the EUT axioms were accepted, the paradoxes posed obvious challenges for economics as an empirical science. If the paradoxes were a product of normal human psychology, would that psychology induce significant violations of EUT in real economic contexts, and if so, what form would those violations take? Could EUT be generalized to accommodate these violations while retaining its overall predictive power? These questions surely belonged on the agenda of a discipline that was using EUT for explanation and prediction. The puzzle is why they lay dormant for so long.

A possible explanation is that, although Allais presented the paradoxes as illustrations of behavioral tendencies that were inconsistent with EUT, he did not offer a tractable theory that explained and generalized those particular tendencies. But that leaves open the question of why economists did not see the value of developing such a theory. One might also ask how far such a theory, had it been produced in the 1950s, would actually have been used.

There is a significant parallel here with an article published in the same year as the Paris conference by Harry Markowitz, a Chicago economist who would later become one of the founders of financial economics (see Section 7). Like Allais, Markowitz (1952a) criticized Friedman and Savage's (1948) EUT-based explanation of individuals' simultaneous purchase of insurance policies and lottery tickets.[8] He instead explained this phenomenon by developing an elegant model of reference-dependent utility that was consistent with recent psychological experiments on gambling behavior (Mosteller & Nogee 1951). Although the theory was published in *Journal of Political Economy*, it remained a sleeping beauty until it was used by Daniel Kahneman and Amos Tversky (1979) as a major component of their prospect theory. It seems that, as viewed by most economists in the 1950s and 1960s, there was no need to investigate or explain behavior that violated EUT.

4.2 Why was Allais's contribution eventually recognized?

We suggest that the trajectory of Allais's influence on economics reflected the changing role of rational choice theory in the discipline and the cumulative effect of experimental evidence.

Although economics had used the rational "Economic Man" as a theoretical device from at least the 1870s, formal axiomatic rational choice theory remained a niche activity in the early 1950s. Initially, rational choice concepts were mainly used to provide microfoundations for theories of aggregate behavior in fields such as consumer demand, capital asset pricing and Keynesian macroeconomics. There was widespread acceptance of Friedman's (1953) methodological argument that theories should be judged by their predictive power, not the realism of their assumptions. Isolated anomalies, observed outside the intended domain of economic theories, could be tolerated. For the same reason, experimental investigations of individual behavior were generally viewed as no more than curiosities.

However, over the 1950s, 1960s, and 1970s, there was a continuous expansion of the domain of rational choice theory. Increasingly, that theory was used in contexts in which its implications could not be read off as predictions about observable aggregate behavior—for example, in the theory of public goods, cost–benefit analysis, normative social choice theory, and formal game theory. In such applications, rationality is not merely an "as if" assumption. Increasing awareness of paradoxes in the concept of rational choice, such as the backward induction paradox and the refinement problem (e.g., Selten 1975 1978) encouraged reconsideration of the foundations of the theory. Indeed, from the 1970s until at least the end of the century, foundational work on rational choice theory was one of the most prestigious sub-fields of economics.

In parallel with these theoretical developments, there was a gradual accumulation of experimental and survey evidence of systematic violations of EUT—not only the Allais paradoxes and related problems of decision making under risk (e.g., MacCrimmon 1968), but also the Ellsberg paradox concerning decisions under uncertainty (Ellsberg 1961), the phenomenon of preference reversal (Lichtenstein & Slovic 1971), and the disparity between

[8] Allais ([1953] 1979: 86–87) discusses simultaneous insuring and gambling as an additional "fact based on experience" that reveals violations of EUT by reasonable individuals.

willingness to pay and willingness to accept (e.g., Bishop & Heberlein 1979).[9] The choice problems presented by Allais, Massé and Morlat had initially been conceived as thought experiments, but when their more extreme consequences were replaced by more manageable gains and losses, they could be recognized as elegant experimental designs for controlled tests of the independence and sure-thing axioms.

5 How (some of) Allais's ideas were eventually taken up

5.1 The descriptive dimension

The late 1970s and early 1980s were a tipping point, with a burst of publications of theories that deviated from EUT by allowing patterns of behavior that generalized the Allais paradoxes.

For many of these publications, a key insight was that the paradoxes (along with the coexistence of insuring and gambling) could be rationalized by generalizing EUT in ways that allowed the utilities of more "outlying" outcomes in a probability distribution to be weighted more heavily. One of the earliest such theories, proposed by Ole Hagen (1969 1979), developed Allais's Element IV (see Section 3.3) by making the *ex ante* utility of a lottery a function of the mean, variance, and skewness of the distribution of psychological satisfaction.[10] Other things equal, positive skew (as in low-stake high-prize gambles) was assumed to be desirable. David Bell (1985) and Graham Loomes and Sugden (1986) proposed theories similar to Hagen's but with the psychological rationale of "disappointment aversion." Many other theories followed EUT in not using the concept of psychological satisfaction, but weakened the independence axiom in a variety of ways that allowed greater weight to be given to more outlying outcomes (e.g., Chew & MacCrimmon 1979; Machina 1982; Fishburn 1983).

A variant of this approach retained the idea of expected-utility maximization but with non-linear weighting of probabilities (e.g., Handa 1977; Kahneman & Tversky 1979; Quiggin 1982; Tversky & Kahneman 1992). One implication of using probability weighting to explain deviations from EUT is that deviations are predicted to occur even when gains and losses are very small relative to an individual's wealth—an effect that was one of Allais's "facts indicated by experience" (1953c: 524–525) and that has been confirmed in many

[9] The Ellsberg paradox is a violation of the sure-thing principle that occurs when some combinations of events have well-defined objective probabilities but the probabilities of other combinations are "ambiguous." Preference reversals occur when a decision maker chooses lottery L_1 over lottery L_2 in a binary decision problem, but attaches a higher monetary value to L_2 when valuations of the two lotteries are elicited independently. The willingness-to-pay/willingness-to-accept disparity is the finding that individuals typically require more money to give up a good they possess than they would pay to acquire it.

[10] Together with Allais, Hagen organized the first "Foundations of Utility and Risk" (FUR) conference in Oslo in 1982. From then on, biennial FUR conferences have served as an important platform for the presentation, discussion and dissemination of decision models that supersede EUT.

experiments. Kahneman and Tversky's *prospect theory* combined a probability-weighting mechanism with Markowitz's (1952b) mechanism of reference-dependent utility. Their 1979 paper was particularly influential because it reported an array of experimental evidence of violations of EUT, including Allais's two paradoxes. The publication of the paper in *Econometrica* signaled the economics profession's recognition of theoretical and experimental studies of decision-making behavior as complementary and mutually informative.

For all the contributions described in the two preceding paragraphs, explaining the Allais paradoxes was a fundamental motivation. Other theorists shared this motivation but proposed fundamentally different explanations of the paradoxes. Regret theory (Bell 1982; Loomes & Sugden 1982; 1987) explained them as results of non-linearity in attitudes to "regret" and its opposite ("rejoicing"). Ariel Rubinstein (1988) attributed the paradoxes to heuristics that lead people to treat similar magnitudes, whether probabilities or outcomes, as if they were equal—a hypothesis later generalized by Loomes (2010). Interestingly, Massé and Morlat used a similar concept of "not very different from" in explaining their paradox (see Section 2.3 above).

Undoubtedly, the Allais paradoxes were central to the early development of experimental and behavioral economics. In the sense described by Sugden (2005), they are *exhibits* that an empirical science needs to explain. Precisely because "good" exhibits produce strong effects and are robust to variations in experimental implementation, there is reason to suspect that they are located at the intersection of more than one causal mechanism. It should be no surprise that many of the different explanations that were proposed in the 1970s and 1980s, including Allais's, have initiated productive lines of enquiry that continue today.

5.2 The normative dimension

As explained in Section 3.1, it was fundamental to Allais's critique of EUT that the normativity of the EUT axioms was merely "pseudo-evident." In the late 1960s and early 1970s, as part of the growing interest in experimental evidence about the adequacy of rational choice theory, there was some experimental investigation of how far people who had been exposed to explanations of the sure-thing principle exhibited Allais paradox behavior.[11] Building on earlier work by Kenneth MacCrimmon (1968), Paul Slovic and Tversky (1974) and Herbert Moskowitz (1974) reported experiments in which each participant first faced a pair of Allais paradox problems and reported their decisions. In Moskowitz's experiment, they were then shown two opposing arguments, one giving Savage's argument for the sure-thing principle, the other giving Allais's argument against it. In Slovic and Tversky's experiment, they were shown whichever of the arguments was contrary to their choice.

[11] This strand of research, and its relationship to Allais's critique of EUT, is discussed by Mongin (2019).

Finally, they were given the opportunity to change their decisions. In both experiments the sample sizes were very small, but the common finding was compatible with Allais's position: most participants did not revise their decisions in either direction, and revisions towards conformity with EUT were no more frequent than revisions in the opposite direction.

In the two decades after the "awakening" of Allais's critique of EUT, experimental economists' investigations of the paradoxes were almost entirely focused on the descriptive adequacy or otherwise of EUT. However, the question of whether the EUT axioms are principles of rationality has become more significant as economists have tried to reconcile welfare economics with behavioral findings. Many contributors to behavioral economics maintain that, although the axioms are frequently violated by real decision makers, they remain requirements of rationality. For example, Kahneman and Tversky (1979: 277) argued that the deviations from EUT explained by prospect theory lead to "normatively unacceptable consequences" that are normally corrected by decision makers who become aware of them. Behavioral economists commonly treat observed violations of EUT as "errors" or "biases" relative to (what are assumed to be) individuals' latent welfare-relevant preferences, and try to correct for these effects in normative analysis (e.g., Bleichrodt et al. 2001).

The research question originally posed by Slovic, Tversky and Moskowitz has recently been revisited in incentivized experiments. Steven Humphrey and Nadia-Yasmine Kruse (2024) report a replication of Slovic and Tversky's experiment that produced similar results. Kirby Nielsen and John Rehbeck (2022) investigate how far individuals want their decisions to satisfy various rational-choice axioms. Although they summarize their results as showing that "many individuals want to follow canonical axioms and revise their choices to be consistent with the axioms" (Nielsen & Rehbeck 2022: 2327), their evidence about the independence axiom suggests a more nuanced interpretation. 83 per cent of participants in the experiment reported willingness to delegate decisions to an algorithm that implemented the independence axiom, effectively endorsing what Allais (1953c: 538) called the "rational obviousness" of the axiom. However, in 75 per cent of actual decisions by participants who had endorsed the independence axiom, that axiom was violated. These participants were then shown their inconsistency and given the opportunity to revise their decision and/or "de-select" the axiom. In 34 per cent of these cases, participants maintained the axiom and revised their choices to satisfy it. In 16 per cent of cases, they maintained their choices and de-selected the axiom. In 47 per cent of cases, they made neither revision—they were content to live with the inconsistency (Nielsen & Rehbeck 2022, Table 1). This evidence suggests that, for many people and in line with Allais's claim about "pseudo-evidence", there is a disconnect between perceiving an abstract principle of consistent choice as rational and perceiving actual choices that contravene that principle as unreasonable.

Obviously, Allais's critique of the normative claims of EUT is not a sleeping beauty in the strict sense of an idea that achieves prominence only after a long period of neglect. Yet neither has it been shown to be clearly mistaken. We therefore do not exclude the possibility that it may eventually prove to be one.

6 Arrow's model: genealogy and general significance

As noted in Section 2.2, Arrow's motivation for writing the paper for the Paris conference was very much parallel to that of Allais: the idea was to introduce uncertainty into general equilibrium theory. In doing this, Arrow drew on various traditions.

To begin with, the idea of "dated" commodities goes back to John Hicks. In his *Value and Capital* ([1939] 1946), Hicks considered "dating" economic parameters as a distinct feature of economic dynamics. He further distinguished a "Spot Economy" with no forward trading and no prior coordination, and a "Futures Economy," in which, "for a considerable period ahead, everything [is] fixed up in advance" and "all goods [a]re bought and sold forward" (136). This imaginary device of a Futures Economy is prefiguring Arrow's idea of a complete market, in which a relevant security exists for any conceivable contingency. In a later recollection, Arrow (2009: 6) summarized how he perceived Hicks' idea:

> Decisions on commodity consumption and production today are made jointly with consumption and production in the future. Therefore, simply put a time subscript on commodities; use the static formalism with the enlarged commodity space.

Importantly, Arrow's approach meant that the future-dependent nature of economic decisions had to be made endogenous to the model. This feature characterized his full-fledged general equilibrium model with uncertainty, whereas was absent in Hicks (on this point, see Duffie & Sonnenschein 1989).

More immediate "technical" inspiration came from the theory of statistical decisions initiated by Abraham Wald (1950). In an exposition of Wald's book, Savage (1951: 55) defined this theory as dealing with "deciding on a reasonable course of action on the basis of incomplete information," as opposed to merely making an argument on this basis, which would be the theory of statistical inference. Savage refers to the unknown facts about the future consequences of the actions "as states of the world, or simply states" (56). Thus, to refer to Arrow's (1983: 47) own later reconstruction, "a decision under uncertainty could be regarded as a function of states of nature." Arrow (47) further recollected:

> In trying to incorporate uncertainty into general equilibrium theory, I was led by the Wald–Savage viewpoint to consider an elementary decision as one that took a unit value for one state of nature and zero elsewhere; thus all general decisions could be regarded as bundles of elementary decisions.[12]

In a later interview, Arrow (2006) brought together the various inspirational elements behind his Paris paper:

> I knew of Hicks's device of dating commodities, and suddenly somewhere, like the fall of '51, it finally came to my mind that the way to think about it is that a consumption vector could be thought of as consumption at a particular state of the world, just as you had consumption at a particular time. Consumption of a particular commodity at a particular state of the world is analogous to Hicks's consumption at a particular point of time. Then it struck me that ordinary securities are that sort of thing. You don't write them out that way, but your payoff … is a function of the state of the world.

Arrow's model is often praised as a brilliant generalization (Duffie & Sonnenschein 1989; Rubinstein 2006). In this more general case, any economy becomes not a collection of commodities but a collection of *assets*. The standard existence and optimality results were simply extended to the case of state-contingent commodities.

It is this formal equivalence of certainty and uncertainty that mostly interested Arrow's co-author, and a co-creator of modern general equilibrium theory, Gérard Debreu (1959; 1960).[13] If the economy can be represented as a large decision tree, at the origin of which all decisions are taken and the contracts agreed beforehand, moving along this tree does not require any notion of probability: one gets a *certain* amount of commodities on each of the "branches." In *Theory of Value* (1959), Debreu's contribution consisted in a more elaborate and comprehensive introduction of uncertainty into a general equilibrium model (with production and in a multiperiod setting) that had not yet been fully articulated in Arrow's 1952 paper. At the same time, while developing the model in all its generality, Debreu was less interested in the implications of introducing the securities trading in this economy for the optimal allocation of risk. Rather, he reformulated the very definition of "uncertain" commodity by directly including in it its physical characteristics, its location and what Debreu calls an "event"—a combination of contingent circumstances that also implies the date of delivery and captures the uncertainty. He then demonstrated more explicitly how the relevant equilibrium and efficiency results are derived for the case of uncertainty.

[12] This literature clearly belonged to Arrow's conceptual toolbox at the time: He authored an influential overview (Arrow 1951) of the "Alternative approaches to the theory of choice in risk-taking situations", which featured Wald's and Savage's work.

[13] Note that their joint contribution to general equilibrium theory—the famous existence proof (Arrow & Debreu 1954)—did not include the case of uncertainty.

For our story, it is significant that despite the fact that the general equilibrium model with uncertainty is often, and rightly, associated with both names, Debreu's results in this area were not perceived as superseding those by Arrow. In the economic theory literature from the late 1960s through the 1980s, Arrow (1953) and Debreu (1959) were lumped together as Arrow–Debreu and used as a framework (see the references in footnote 14). The reason is that it was Arrow and Debreu (1954) that contained the first general equilibrium existence and efficiency proof within a full-fledged model, while its extension to the case of uncertainty appeared in print a year before. In finance, it was this earlier Paris paper by Arrow that eventually attracted most attention (Duffie & Sonnenschein 1989; Duffie 1992).

It is important to emphasize the fundamental link of Arrow's approach to *economics*: the problem it solves is the *optimal allocation* of risk-bearing—that is, contingent commodities—done on the financial markets, while uncertainty can be "priced" (in fact, "state prices" measure the value of a dollar in a particular state of the world).

Indeed, independently of the reception history in finance we will come to in Section 7, these features shaped the subsequent fortunes of Arrow's paper. Arrow's theory supplies a normative benchmark—the idea of "complete markets." This normative dimension, similar to the status of the EUT in the 1950s, in fact became crucial for the subsequent research on risk-bearing. As Arrow (1996) himself explained, once the benchmark of completeness was introduced, it was natural to study real-world deviations from it.[14]

One important case of market incompleteness is information asymmetry that Arrow explored in his pathbreaking article "Uncertainty and the welfare economics of medical care" (1963). There, the recognition of the real-world market incompleteness—one has troubles conditioning securities on events that are only privately observed—led not only to the exploration of specific institutional arrangements in healthcare markets, but also to the new developments in the economics of information.

7 Arrow and finance: sleeping beauty puzzle

As already noted, despite all the insights it offered, Arrow's approach did not have an immediate impact on financial economics in the 1950s, and its acceptance in finance in the 1960s remained quite limited. How could we explain that work later hailed as "Arrow's most influential contribution to equilibrium theory" (Duffie & Sonnenschein 1989: 584), and even as "the most important paper ever written for financial economics" (Rubinstein 2006: 107),

[14] Indeed, beyond the field of asset pricing, the paper laid the groundwork for a broader abstract theory exploring the assumption that the set of securities necessary to allocate risks is incomplete, namely the theory of incomplete markets. Representative contributions include Diamond (1967); Radner (1968; 1972); Hart (1975); Duffie & Shafer (1985); Geanakoplos & Polemarchakis (1986).

initially remain "comparatively unfamiliar" (Hirshleifer 1965: 523)?[15] We argue that it was the institutional and intellectual history of academic finance that mattered.

During this period, financial economics was emerging as a field, shaped largely by individuals closely interacting with Arrow, such as Markowitz, Franco Modigliani, and Samuelson (another Paris conferee).[16] Thus, as for Allais, it was not the case that the profession was completely unaware of Arrow's model. But its full-fledged integration into finance took time.

Despite major conceptual advances, the field of finance, as it was becoming more rigorous and was beginning to understand itself as *financial economics*, was still in search of coherent and parsimonious theoretical foundations. In his Nobel memorial speech, Markowitz (1990: 286) recounted that:

> When I defended my dissertation as a student in the Economics Department of the University of Chicago, Professor Milton Friedman argued that portfolio theory was not Economics, and that they could not award me a Ph.D. degree in Economics for a dissertation which was not in Economics. I assume that he was only half serious, since they did award me the degree without long debate. As to the merits of his arguments, at this point I am quite willing to concede: at the time I defended my dissertation, portfolio theory was not part of Economics. But now it is.

Being initially much more applied and focused on institutional details of financial transactions and the corporate world, finance developed as a theoretical discipline during the 1950s and 1960s. Milestones of this development included portfolio theory, analysis of optimal capital structure, and the exploration of arbitrage and market efficiency. At the same time, in the 1950s and 1960s, Arrow's model essentially remained part of general equilibrium literature (Kirtchik & Boldyrev 2024). Its republication in English in 1964 coincided with the discussions and changes within the finance profession in mid-1960s, in terms of both research and teaching: by embracing economics, American finance reinvented its theoretical foundations (Jovanovic 2008); by gradually shifting attention from corporations to markets over the 1960s and 1970s, US business schools became the strongholds of the new academic orientation tying them more closely to—and increasing their influence in—mainstream economic theory. This orientation clearly included finance, with financial *economics* becoming a core discipline (Fourcade & Khurana 2013; 2017). It is in this new context that Arrow's model could gain new prominence.

[15] The histories of financial economics are either entirely silent on Arrow's approach (Bernstein 1992; Harrison 1997; Miller 1998; MacKenzie 2006; Jovanovic 2008) or, if they do discuss it (Rubinstein 2006; Mehrling 2016; Brine & Poovey 2017; Boldyrev 2021), they do not address the "sleeping beauty" question.

[16] Samuelson, who was six years older than Arrow, had already become a leading theorist and a crucial reading by the time Arrow began his career (see Arrow 2009). Indeed, in the year of the Paris conference Samuelson was serving as president of the Econometric Society. His contribution to finance, however, is a later development (Samuelson 1965). Markowitz and Modigliani both worked at the Cowles Commission, an institution Arrow was also affiliated with. Modigliani even commented on Arrow's earlier work in welfare economics in the late 1940s.

As the archival evidence suggests, the publication itself (Arrow 1964) was initiated by Peter Newman (1928–2001), at that time a professor of economics at the University of Michigan.[17] Newman was not a finance scholar, but he had deep interest in general equilibrium theory and its history (see, e.g., Newman 1961) and, generally, a broad perspective on the economics as a whole, something that later led him to become a co-editor of the *New Palgrave Dictionary of Economics*.

At the same time, Arrow did *not* position himself as a finance theorist and this did not happen even after his work was fully recognized as a classic contribution to financial economics. It is not presented as such in his 1972 Nobel lecture (Arrow 1974) and played only a minor role in the contemporaneous assessment of Arrow's work (see, e.g., von Weizsäcker 1972). When in 1989, Newman, then the co-editor of the *Palgrave Dictionary of Money and Finance*, invited Arrow to join the advisory board of the Dictionary, Arrow initially declined the invitation arguing that he had never written on the subject.[18] This remarkable mismatch highlights the perception of Arrow's fundamental contribution as a "sleeping beauty," not immediately recognized as a foundational work even by its progenitor.

So, how could Arrow become a classic? Apart from institutional dynamics discussed above, the key answer is provided by the intellectual history of the field. The framework suggested in the 1952 paper ultimately provided the unifying structure that brought together different strands of financial economics. If we wish to locate more precisely how this happened, we need to single out the work of UCLA economist Jack Hirshleifer (1964; 1965; 1966).[19] Hirshleifer was the first to connect Arrow's ideas to the Modigliani–Miller (1958) theorem in corporate finance—which states the irrelevance of capital structure to a firm's market value—and to compare Arrow's approach to the mean–variance portfolio selection models of Markowitz (1952b) and Tobin (1958).

In fact, the linear structure employed by Arrow and the analysis of "spanning" (roughly, the idea that, in equilibrium, a set of Arrow securities can provide the same possibilities for state-contingent consumption as a complete set of state-contingent commodities) led Robert Merton (1982) to treat his model as one of portfolio selection—alongside those of Markowitz (1959) and Tobin (1958) (see also Bodie 2020). In particular, in Arrow's model each security's return can be replicated by a portfolio of Arrow securities. Markowitz himself (1990: 281) later commented that in his work, he sought:

[17] Arrow to Newman, May 11, 1989 (Kenneth Arrow papers, Box 39).

[18] Newman eventually won the argument by claiming that Arrow's 1952 Paris paper on securities and risk-bearing "is one of the foundation stones of the entire subject" (Kenneth Arrow papers, Box 39). In fact, Arrow committed for another reason, but this is not relevant for our narrative.

[19] In the early 1960s, Hirshleifer belonged to the small community of mainstream economists interested in finance (Harrison 1997). At UCLA, he formally supervised William Sharpe, who was developing what later came to be called Capital Asset Pricing Model (CAPM), although Sharpe's principal intellectual mentor was Markowitz, who was then working at RAND (see Sharpe 2004).

> A set of rules which investors can follow [...]—at least investors with sufficient computational resources. Thus, we prefer an approximate method which is computationally feasible to a precise one which cannot be computed. I believe that this is the point at which Kenneth Arrow's work on the economics of uncertainty diverges from mine. He sought a precise and general solution. I sought as good an approximation as could be implemented. I believe that both lines of inquiry are valuable.

Hirshleifer demonstrated the advantage of the general equilibrium formulation over the mean–variance approach—the former being more explicit in envisioning what he called a "choice-theoretic structure"—and showed how one could think of the Modigliani–Miller result in such a way that "financing via alternative debt–equity ratios [would] then represent different possible ways of distributing this wealth [i.e., market value] over time-state claims" (Hirshleifer 1966: 267).

Through the works of Hirshleifer (1965; 1966), Arrow's model gained broader recognition in the academic finance community. However, *financial economists* at that time still focused primarily on models and theories that allowed empirical verification—like the Capital Asset Pricing Model (Sharpe 1964) and the efficient market hypothesis (Fama 1965)—and paid less attention to the broader and most general theoretical framework of complete markets as an ideal benchmark. It was only somewhat later in the 1970s, in the wake of the Black–Scholes options pricing theory (Black & Scholes 1973), that the generality of this framework became more significant. At that time, another key finance scholar, Stephen Ross (1976) and others explicitly related contingent claims analysis to option pricing (of course, options themselves are contingent claims). Arrow's approach—specifically, the idea that any security can be valued as a sum of its payoffs in different states, weighted by state prices—played a pivotal role in Michael Harrison and David Kreps's (1979) reformulation of the Black–Scholes option-pricing model. While the dynamic hedging strategy underlying arbitrage-free asset pricing theory is a way to "complete" the market, Arrow's work was the foundation of this more general approach to constructing new assets from the existing ones.[20]

Importantly, while not active himself, Arrow was closely following these developments. Thus, as he wrote to Princeton economist Richard Quandt, he immediately recognized that Harrison and Kreps's (1979) work "showed the full power of arbitrage conditions and demonstrated precisely the extent to which rationally foreseen securities prices could or could not replace the full set of contingent markets."[21]

---

[20] On the connection of the "no-arbitrage condition" with the existence of state prices for each asset serving as linear weights for its returns in the respective "states of the world" see Varian 1987.

[21] Arrow to Richard Quandt, 26 October 1987 (Kenneth Arrow papers, Box 9). From other letters it is clear that Arrow knew of Harrison and Kreps's paper prior to its publication (see, Arrow to Hansen, 7 November 1979, Kenneth Arrow papers, Box 9).

So, when was the "sleeping beauty" awakened? We argue that it happened in the late 1970s. There is a striking contrast to be observed that supports this claim. First, if we compare authoritative appraisals of Arrow's work, we see that Darrell Duffie and Hugo Sonnenschein (1989), unlike Christian von Weizsäcker (1972), devoted substantial attention to restating Arrow's Paris contribution and its role for financial economics. Second, while Fama and Miller's (1972) textbook, arguably the most influential manual in finance at the time, barely mentions Arrow's approach, the important textbooks of the new generation—Huang & Litzenberger (1988) and Duffie (1992)—are almost entirely based on it. We thus see that as different bits and pieces of finance got progressively reformulated in a general equilibrium framework, Arrow's approach gained traction during the 1970s and, by the 1980s, it had become part of the standard theoretical discourse in finance.

Some further historical details also deserve to be mentioned. One could argue—following Merton (1998) and Donald MacKenzie (2006)—that developments in the financial sector, also happening in the 1970s and coinciding with the increasing importance of financial sector for the economy as a whole (Mehrling 2016), played a significant role, too. Arrow's model suggested that securities span future contingencies, thus indicating a genuinely economic mechanism for risk allocation. While the complete markets model preceded the institutional innovations that led to the creation of derivatives markets, once these markets emerged, along with the relevant asset pricing theories and techniques, the scope of Arrow's original insights became more visible. As Merton (1998: 341) commented,

> The "pure" securities developed by Kenneth J. Arrow (1953) … were nowhere to be found in the real world until the broad development of the options and derivative-security markets. It is now routine for financial engineers to disaggregate the cash flows of various securities into their elemental Arrow-security component parts and then to reaggregate them to create securities with new patterns of cash flows.

All in all, we can now better appreciate the role of Arrow's contribution and the patterns of its delayed reception. While in the 1950s and 1960s, financial economics developed in a much more piecemeal and uncoordinated way, toward the end of the 1970s, armed with a full-scale general equilibrium construction and inspired by the development in financial engineering, it could be much more easily reformulated in the framework suggested in Paris in 1952. It is along the lines drawn by Arrow 75 years ago that finance actually began to understand itself as part of economic theory.

8 Conclusion

On the occasion of the 75th anniversary of the 1952 Paris conference on risk, our paper has offered a comprehensive reconstruction of this landmark event in the history of economic analysis and has addressed the "sleeping beauty puzzles" associated with the ideas that Allais and Arrow presented at the meeting.

Certain aspects are common to the two cases. In the 1950s, the papers of Allais and Arrow were published in their entirety only in French; both were known for decades, yet their implications were fully developed only in the late 1970s; and in both instances, wider recognition came only after being associated with new intellectual frameworks, namely behavioral economics and financial economics.

However, important aspects make Allais's and Arrow's cases very different. In the 1950s, the direction of travel in economic theory was towards parsimony, generality and mathematical abstraction. Notwithstanding the depth of Arrow's economic intuition and his genuine interest in the realities of the economic world, he was universally recognized as a leading figure in this methodological shift. In contrast, Allais was an outspoken critic of stylized theoretical models and what he called "unduly abstract mathematical formulations" ([1953] 1979: 37).

The core idea in Arrow's Paris paper was that, by modelling markets as exchanges of state-contingent claims, existing theories of general equilibrium could be generalized to take account of uncertainty, consistently with EUT but without any explicit reference to probability. This was exactly the type of advance that was most highly valued by the economic theorists of the time. It became part of the framework associated with Arrow and Debreu's 1954 analysis of general equilibrium—an analysis that was immediately recognized as a ground-breaking contribution to economics. Arrow's sleeping beauty was a particular application of his Paris model to financial markets. Its awakening was the eventual recognition that an abstract theory based on rational choice could serve as an organizing principle of financial economics—a previously unsystematic area of applied economics.

In contrast, Allais's Paris contribution was a critique of current developments in the theory of rational choice. The general theory he proposed was an adaptation and extension of a Bernoullian approach that used cardinal psychological satisfaction as a primitive notion and was regarded as outdated by the theorists of the 1950s. However, his (and Massé and Morlat's) paradoxes opened up the possibility that an empirical theory of decision making might need to learn from experiments of the kind used by psychologists, rather than simply applying a priori principles of normative rationality. Their awakening was the eventual recognition that economics could not be divorced from psychology.

Summing up this comparison between the two cases, we suggest that the gradual acceptance of Arrow's ideas in the developing field of financial economics was a victory for rational choice theory, whereas the gradual acceptance of Allais's ideas in the developing field of behavioral economics was a defeat.

References

Allais, M. (1943). *À la recherche d'une discipline économique*. Ateliers Industria.

Allais, M. (1947). *Économie et intérêt*. Imprimerie Nationale.

Allais, M. (1953a). Généralisation des théories de l'équilibre économique général et du rendement social au cas du risque. In CNRS 1953, pp. 81–109.

Allais, M. (1953b). Fondements d'une théorie positive des choix comportant un risque. In CNRS 1953, pp. 27–140.

Allais, M. (1953c). Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'École Americaine. *Econometrica*, 21(4), 503–546.

Allais, M. ([1953] 1979). The foundations of a positive theory of choice involving risk and a criticism of the postulates and axioms of the American School. In M. Allais & O. Hagen (Eds.), pp. 27–145.

Allais, M. (1979). The so-called Allais paradox and rational decisions under uncertainty. In M. Allais & O. Hagen (Eds.), pp. 437–681.

Allais, M. & Hagen. O. (Eds.) (1979). *Expected utility hypotheses and the Allais paradox*. Dordrecht: Reidel.

Arrow, K.J. No date. Papers. David M. Rubenstein Rare Book & Manuscript Library, Duke University.

Arrow, K.J. (1951). Alternative approaches to the theory of choice in risk-taking situations. *Econometrica*, 19(4), 404–437.

Arrow, K.J. (1953). Le rôle des valeurs boursières pour la répartition la meilleure des risques. In CNRS 1953, pp. 41–48.

Arrow, K.J. (1963). Uncertainty and the welfare economics of medical care. *American Economic Review*, 53(5), 941–973.

Arrow, K.J. (1964). The role of securities in the optimal allocation of risk bearing. *Review of Economic Studies*, 31(2), 91–96.

Arrow, K.J. (1971). *Essays in the theory of risk-bearing*. North-Holland.

Arrow, K.J. (1974). General economic equilibrium: Purpose, analytic techniques, collective choice. *American Economic Review*, 64(3), 253–272.

Arrow, K.J. (1983). *Collected papers*, Vol. 2. Harvard University Press.

Arrow, K.J. (1996) The theory of risk-bearing: Small and great risks. *Journal of Risk and Uncertainty*, 12 (2–3), 103–111.

Arrow, K.J. (2006). Interview by Darrell Duffie, April 6. Series *Masters of finance*, American Finance Association. Retrieved October 15, 2025, available at https://afajof.org/masters-of-finance-videos.

Arrow, K.J. (2009). Some developments in economic theory since 1940: An eyewitness account. *Annual Review of Economics*, 1, 1–16.

Arrow, K.J. & Debreu, G. (1954). Existence of an equilibrium for a competitive economy. *Econometrica*, 22(3), 265–290.

Bell, D.E. (1982). Regret in decision making under uncertainty. *Operations Research*, 30(5), 961–981.

Bell, D.E. (1985). Disappointment in decision making under uncertainty. *Operations Research*, 33(1), 1–27.

Bernstein, P.L. (1992). *Capital ideas: The improbable origins of modern Wall Street*. The Free Press.

Bishop, R.C. & Heberlein, T. (1979). Measuring values of extramarket goods: Are indirect measures biased? *American Journal of Agricultural Economics*, 61(5), 926–930.

Black, F. & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3), 637–654.

Bleichrodt, H., Pinto, J.L., & Wakker, P.P. (2001). Making descriptive use of prospect theory to improve the prescriptive use of expected utility. *Management Science*, 47(11), 1498–1514.

Bodie, Z. (2020). Robert C. Merton and the science of finance. *Annual Review of Financial Economics*, 12, 19–38.

Boldyrev, I. (2021). The ontology of uncertainty in finance: The normative legacy of general equilibrium. *Topoi: An International Review of Philosophy*, 40 (4), 725–731.

Brine, K.R. & Poovey, M. (2017). *Finance in America*: *An unfinished story*. The University of Chicago Press.

Cherrier, B. & Saïdi, A. (2021). Back to front: The role of seminars, conferences and workshops in the history of economics. *Revue d'économie politique*, 131(4), 609–635.

Chew, S.H. & MacCrimmon, K.R. (1979). Alpha-nu choice theory: A generalization of expected utility theory. Working Paper 669, University of British Columbia.

CNRS 1953. *Économétrie*. Actes du colloque "Fondements et applications de la théorie du risque en économétrie." Centre National de la Recherche Scientifique.

Debreu, G (1959). *Theory of value*. Yale University Press.

Debreu, G. (1960). Une économique de l'incertain. *Économie appliquée*, 13(1), 111–116.

Diamond, P. (1967). The role of a stock market in a general equilibrium model with technological uncertainty. *American Economic Review,* 57(4), 759–776.

Duffie, D. (1992). *Dynamic asset pricing theory*. Princeton University Press.

Duffie, D. & Shafer W. (1985). Equilibrium in incomplete markets: I: A basic model of generic existence, *Journal of Mathematical Economics*, 14(3), 285–300.

Duffie, D. & Sonnenschein, H. (1989). Arrow and general equilibrium theory. *Journal of Economic Literature*, 27(2), 565–598.

Düppe, T. & Weintraub, E.R. (2014). Siting the new economic science: The Cowles Commission's Activity Analysis Conference of June 1949. *Science in Context*, 27(3), 453–483.

Ellsberg, D. (1961). Risk, ambiguity, and the Savage axioms. *Quarterly Journal of Economics*, 75(4), 643–669.

Fama, E.F. (1965) The behavior of stock-market prices. *Journal of Business*, 38(1), 34–105.

Fama, E.F. & Miller, M.H. (1972). *The theory of finance*. Holt.

Fishburn, P.C. (1983). Transitive measurable utility. *Journal of Economic Theory*, 31(2), 293–317.

Fourcade, M., & Khurana, R. (2013). From social control to financial economics: The linked ecologies of economics and business in twentieth century America. *Theory and Society*, 42(2), 121–159.

Fourcade, M., & Khurana, R. (2017). The social trajectory of a finance professor and the common sense of capital. *History of Political Economy*, 49 (2), 347–381.

Friedman, M. (1953). The methodology of positive economics. In M. Friedman, *Essays in positive economics*, pp. 3–43. Chicago: The University of Chicago Press.

Friedman, M. & Savage, L.J. (1948). The utility analysis of choices involving risk. *Journal of Political Economy*, 56(4), 279–304.

Geanakoplos, J. & Polemarchakis, H. (1986). Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete. In W. Heller, R. Starr, & D Starret (Eds.), *Uncertainty, information and communication: Essays in honor of Kenneth J. Arrow*, Vol. 3, pp. 65–95. Cambridge University Press.

Glänzel, W., Schlemmer, B., & Thijs, B. (2003) Better late than never? On the chance to become highly cited only beyond the standard bibliometric time horizon. *Scientometrics*, 58, 571–586.

Hagen, O. (1969). Separation of cardinal utility and specific utility of risk in theory of choices under uncertainty. *Statsøkonomisk Tidsskrift*, 3, 81–107.

Hagen, O. (1979). Towards a positive theory of preferences under risk. In M. Allais & O. Hagen (Eds.), pp. 271–302.

Handa, J. (1977). Risk, probabilities, and a new theory of cardinal utility. *Journal of Political Economy*, 85(1), 97–122.

Harrison, P. (1997). A history of an intellectual arbitrage: The evolution of financial economics. *History of Political Economy*, 29, Annual supplement, 172–188.

Harrison, J.M. & Kreps, D. (1979). Martingales and arbitrage in multiperiod securities markets. *Journal of Economic Theory*, 20(3), 381–408.

Hart, O. (1975): On the optimality of equilibrium when the market structure is incomplete, *Journal of Economic Theory*, 11(3), 418–443.

Hicks, J.R. ([1939] 1946). *Value and capital*. Clarendon Press.

Hirshleifer, J. (1964). Efficient allocation of capital in an uncertain world. *American Economic Review*, 54(3), 77–85.

Hirshleifer, J. (1965). Investment decision under uncertainty: Choice-theoretic approaches. *Quarterly Journal of Economics*, 79(4), 510–536.

Hirshleifer, J. (1966). Investment decision under uncertainty: Applications of the state-preference approach. *Quarterly Journal of Economics*, 80(2), 252–277.

Huang, C. & Litzenberger, R.H. (1988). *Foundations for financial economics*. North Holland.

Humphrey, S.J. & Kruse, N.-Y. (2024). Who accepts Savage's axiom now? *Theory and Decision*, 96, 1–17.

Jallais, S. & Pradier, P.-C. (2005). The Allais paradox and its immediate consequences for expected utility theory. In P. Fontaine & R. Leonard (Eds.), *The experiment in the history of economics*, pp. 25–49. Routledge.

Jovanovic, F. (2008). The construction of the canonical history of financial economics. *History of Political Economy* 40(2): 213–242.

Kahneman, D. & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, 47(2), 263–291.

Kirtchik, O. & Boldyrev, I. (2024). "Rise and fall" of the Walrasian program in economics: A social and intellectual dynamics of the general equilibrium theory, *Journal of the History of Economic Thought,* 46(1), 1–26.

Lichtenstein, S. & Slovic, P. (1971). Reversals of preference between bids and choices in gambling decisions. *Journal of Experimental Psychology*, 89(1), 46–55.

Loomes, G. (2010). Modeling choice and valuation in decision experiments. *Psychological Review*, 117(3), 902–924.

Loomes, G. & Sugden, R. (1982). Regret theory: An alternative theory of rational choice under uncertainty. *Economic Journal*, 92(368), 805–824.

Loomes, G. & Sugden, R. (1986). Disappointment and dynamic consistency in choice under uncertainty. *Review of Economic Studies*, 53(2), 271–282.

Loomes, G. & Sugden, R. (1987). Some implications of a more general form of regret theory. *Journal of Economic Theory*, 41(2), 270–287.

MacCrimmon, K.R. (1968). Descriptive and normative implications of the decision-theory postulates. In K. Borch and & J. Mossin (Eds.), *Risk and uncertainty*, pp. 3–32. Macmillan.

Machina, M. (1982). 'Expected utility' analysis without the independence axiom. *Econometrica*, 50(2), 277–323.

MacKenzie, D. (2006). *An Engine, not a camera: How financial models shape markets*. MIT Press.

Malinvaud, E. (1952). Note on von Neumann–Morgenstern's strong independence axiom. *Econometrica*, 20(4), 679.

Markowitz, H. (1952a). The utility of wealth. *Journal of Political Economy*, 60(2), 151–158.

Markowitz, H. (1952b). Portfolio selection. *Journal of Finance*, 7(1), 77–91.

Markowitz, H. (1959). *Portfolio selection*. Wiley.

Markowitz, H. (1990). Foundations of portfolio theory, Nobel Prize Lecture. Retrieved December 4, 2025, from https://www.nobelprize.org/uploads/2018/06/markowitz-lecture.pdf.

Marschak, J. (1950). Rational behavior, uncertain prospects, and measurable utility. *Econometrica,* 18(2), 111–141.

Maskin, E.S. (2019). The economics of Kenneth J. Arrow: A selective review. *Annual Review Economics*, 11, 1–26.

Massé, P. & Morlat, G. (1953). Sur le classement économique des perspectives aléatoires. In CNRS 1953, pp. 165–193.

Mehrling, P. (2016). Financial economics. In H. Kurz & G. Faccarello (Eds.), *Handbook of the History of Economic Analysis*, Vol. 3, pp. 198–207. Edward Elgar.

Merton, R.C. (1982). On the microeconomic theory of investment under uncertainty. In K. Arrow & M. Intrilligator (Eds.), *Handbook of mathematical economics*, Vol. 2, pp. 601–669. Elsevier.

Merton, R.C. (1998). Applications of option-pricing theory: Twenty-five years later. *The American Economic Review*, 88(3), 323–349.

Miller, M.H. (1998). The history of finance: An eyewitness account. In J.M. Stern & D.H. Chew, Jr. (Eds.), *The revolution in corporate finance*, pp. 4–9. Blackwell.

Modigliani, F. & Miller M.H. (1958). The cost of capital, corporation finance and the theory of investment. *American Economic Review*, 48(3), 261–297.

Mongin, P. (2019). The Allais paradox: What it became, what it really was, what it now suggests to us. *Economics and Philosophy*, 35(3), 423–459.

Morlat, G. (1956). De l'usage du calcul des probabilités en matière économique. *Revue d'Économie Politique*, 66, 889–907.

Morlat, G. (1957). Sur la théorie des choix aléatoires. *Revue d'Économie Politique*, 67, 378–380.

Moscati, I. (2016). How economists came to accept expected utility theory: The case of Samuelson and Savage. *Journal of Economic Perspectives*, 30(2), 219–236.

Moscati, I. (2019). *Measuring utility: From the marginal revolution to behavioral economics*. Oxford University Press.

Moscati, I. (2023). *The history and methodology of expected utility*. Cambridge University Press.

Moskowitz, H. (1974). Effects of problem representation and feedback on rational behavior in Allais and Morlat-type problems. *Decision Sciences*, 5(2), 225–241.

Mosteller, F. & Nogee, P. (1951). An experimental measurement of utility. *Journal of Political Economy*, 59(5), 371–404.

Munier, B.R. (1991). Nobel laureate: The many other Allais paradoxes. *Journal of Economic Perspectives*, 5(2), 179–199.

Newman, P. (1961). Approaches to stability analysis. *Economica*, 28(109), 12–29.

Nielsen, K. & Rehbeck, J. (2022). When choices are mistakes. *American Economic Review*, 112(7), 2237–2268.

Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior and Organization*, 3(4), 323–343.

Radner, R. (1968). Competitive equilibrium under uncertainty. *Econometrica*, 36(1), 31–58.

Radner, R. (1972) Existence of equilibrium of plans, prices and price expectations in a sequence of markets. *Econometrica*, 40(2), 289–303.

Ross, S.A. (1976). Options and efficiency. *Quarterly Journal of Economics*, 90(1), 75–89.

Rubinstein, A. (1988). Similarity and decision making under risk: Is there a utility theory resolution to the Allais paradox? *Journal of Economic Theory*, 46(1), 145–153.

Rubinstein, M. (2006). *A history of the theory of investments*. Wiley.

Samuelson, P.A. (1952). Probability, utility, and the independence axiom. *Econometrica*, 20(4), 670–678.

Samuelson, P.A. (1953). Utilité, préférence et probabilité. In CNRS 1953, pp. 141–150.

Samuelson, P.A. (1965). Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review*, 6(2), 41–49.

Samuelson, P.A. (1966). Utility, preference, and probability. In J.E. Stiglitz (Ed.), *The collected scientific papers of Paul A. Samuelson*, Vol. 1, pp. 127–136. MIT Press.

Savage, L.J. (1951). The theory of statistical decision. *Journal of the American Statistical Association*, 46(253), 55–67.

Savage, L.J. (1953). Une axiomatisation de comportement raisonnable face à l'incertitude. In CNRS 1953, pp. 29–33.

Savage, L.J. ([1954] 1972). *The foundations of statistics*. Wiley.

Sharpe, W.F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 19(3), 425–442.

Sharpe, W.F. (2004). Interview by Steve Buser, undated. Series *Masters of finance*, American Finance Association. Retrieved May 15, 2026, available at https://afajof.org/masters-of-finance-videos.

Selten, R. (1975). Re-examination of the perfectness concept for equilibrium points in extensive games. *International Journal of Game Theory*, 4, 25–55.

Selten, R. (1978). The chain store paradox. *Theory and Decision*, 9, 127–159.

Slovic, P. & Tversky A. (1974). Who accepts Savage's axiom? *Behavioral Science*, 19(6), 368–378.

Steele, K. & Stefánsson, H. Orri. (2025). Decision theory. In E.N. Zalta & U. Nodelman (Eds.), *The Stanford encyclopedia of philosophy* (Fall 2025 edition), available at https://plato.stanford.edu/archives/fall2025/entries/decision-theory.

Sugden, R. (2005). Experiments as exhibits and experiments as tests. *Journal of Economic Methodology*, 12(2), 291–302.

Tobin, J. (1958). Liquidity preference as behavior towards risk. *Review of Economic Studies*, 25(2), 65–86.

Tversky, A. & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5, 297–323.

van Raan, A.F.J. (2004) Sleeping beauties in science. *Scientometrics* 59, 467–472.

van Raan, A.F.J. (2015). Dormitory of physical and engineering sciences: Sleeping beauties may be sleeping innovations. *PLoS ONE*, 10(10), e0139786.

Varian, H.R. (1987). The arbitrage principle in financial economics. *Journal of Economic Perspectives*, 1(2), 55–72.

von Neumann, J. & Morgenstern, O. ([1944] 1947) *Theory of games and economic behavior*. Princeton: Princeton University Press.

von Weizsäcker, C.C. (1972). Kenneth Arrow's contribution to economics. *Swedish Journal of Economics*, 74(4), 488–502.

Wald, A. (1950). *Statistical decision functions*. Wiley.