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Can Biocalculus help fix the calculus image problem?

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Abstract

Calculus has an image problem in life science undergraduate studies. Students are anxious about taking it, due to tricky past experiences and because calculus as a major gateway course contributes to attrition in biology. Calculus teaching to life science students often does not convey its utility in problem solving and graphical interpretation. This problem is exacerbated by the increasing focus on big data and computational skills rather than proofs and algebraic techniques which have traditionally dominated calculus teaching. We present one solution to this problem: "Biocalculus", an approach to teaching calculus to life science students which incorporates modeling, computational tools, lab activities, authentic problem solving, interdisciplinary language, and an asset-based mindset. We outline the benefits of biocalculus as enabling students' connections to the theory of their disciplines, increased retention (particularly of students with identities often marginalized in STEM), and higher learning gains. We acknowledge Biocalculus implementation challenges such as instructor professional development and infrastructure that supports interdisciplinary initiatives. We conclude stressing the importance of the epistemological shifts required of both the Biology and Mathematics communities in order to tackle the disciplinary microaggressions that impede acknowledging the reciprocal meaning-making gains that Biocalculus can make possible for life science (and other) students.

Keywords: biocalculus, calculus, biology, life science, inclusive, modeling

1. Introduction

To understand Biocalculus education and research is to understand the space of tension between calculus requirements and biology education needs. Research into the mathematical needs of non-mathematics majors has been around for some time (Kent & Noss, 2003) with the students' disciplinary specialism often just a background to studies conducted by Mathematics Education researchers (Biza et al., 2016). For example, Klein's double discontinuity (1908/1931) between school, university, and workplace mathematics is often the focus of these studies. In this paper, we explore the multiple ways calculus instruction feeds into this discontinuity with biology. We also explore the reasons why biology students still need calculus. For example, mathematical modeling can help make connections between mathematics and the real world (Chiel et al., 2010; Jungck & Schaefer, 2011; Svoboda & Passmore, 2013; Viirman & Nardi, 2019, 2021), and calculus often is a tool through which students are expected to engage in mathematical modeling activities (Maaß, 2006). We suggest that calculus has an image problem in biology and biology education. To address this, in this paper we propose that we need to cross interdisciplinary boundaries, be willing to listen to the concerns of life science students, faculty, and practitioners, and stretch our imagination of what calculus for life science students can be.

1.1 Calculus is failing to address critical needs in biology education

Courses addressing specific mathematical skills for undergraduate biology students have been around for decades, and interest around calculus education for biology, specifically, increased at the turn of the century (Diaz Eaton et al., 2020). In the US, for example, this was primarily due to the release of several reports on biology education which alerted programs to the rising use of big data and modeling in the field This was accompanied by a call for reform in how mathematics was integrated into biology (Brewer & Smith, 2011; Edelstein-Keshet, 2005; Steen, 2005), and Marshall & Durán (2018) investigated the types of skills emerging in published life science research for the purposes of designing more relevant quantitative biology courses.

For all fields, there is a growing demand for computing, data, and modeling skills in the era of machine learning and artificial intelligence. Medical school entrance requirements are one way to track sentiment towards calculus in this rapidly changing technological landscape. The Medical College Admission Test (MCAT) in the US no longer recommends calculus and many medical schools have also dropped calculus as an entrance requirement (Emanuel, 2006). Now medical schools are allowing students to complete the requirement through computer science and statistics courses (Association of American Medical Colleges, 2025). Given also that there has been a notable increase in the enrollment of historically excluded and underrepresented students in medical schools in recent years, the need to address aforementioned interrelated issues in education by integrating calculus and biological sciences in a way that is relevant and accessible is even more pronounced (Association of American Medical Colleges, 2022).

On the surface, trading calculus for statistics was an attractive offer for undergraduate biology programs as the role of data and computation became more visible in biology research. Biology faculty familiar with traditional calculus courses, deemed the emphasis on algebraic calculation and proofs of convergence holding little connection. Therefore, links to calculus in their courses were often overlooked, whereas statistics often took a prominent role in laboratory courses (Neitzel et al., 2023). In addition, calculus acted as a "gatekeeper" or "filtering" (Biza et al., 2022) for students in biology. If students failed in calculus, they were also much more likely to not complete their major (Voigt et al., 2022). This is true even in biology, which has long been perceived as a science major with the lowest mathematics requirements compared to other programs in STEM. As Steen (2005) points out, "biology education is burdened by habits from a past where biology was seen as a safe harbour for math-averse science students" (p. 14).

Compounding this, studies of calculus show that students with marginalized gender, race and ethnicity are more likely to fail calculus (Hatfield et al., 2022). A 2015 Mathematical Association of America (MAA) study of the state of calculus instruction in the US revealed that among all Black and Hispanic students in Calculus I, 33% and 36%, respectively, were biological science majors (Bressoud et al., 2015, p. 11). A key study by Ellis et al. (2016) showed that women were 1.5 times more likely to leave STEM after taking Calculus and that, among women taking Calculus I, 43% were biological science majors (p. 12). Failing to meet the needs of women, ethnic and racial minorities and failing to meet the needs of biology students are interrelated issues which contribute to a lack of diversity in STEM overall. More recent studies on the effect of the COVID-19 pandemic has only indicated that this gap could get worse in the near future (Alabdulaziz, 2021).

In summary, despite the urgent call for more modeling, data skills and understanding, the poor reputation - due to the failure of calculus courses for Biology students to meet student, faculty, and program needs - has stifled this progress. Calculus has an image problem in the biological sciences that desperately needs to be addressed.

1.2 Biocalculus as an opportunity for better mathematics and life science education

This conversation takes place at a time when, within the field of university mathematics education, student-centered strategies - such as inquiry-based learning and active learning approaches that are designed to address attrition, promote equity and strengthen meaningful engagement with mathematical and other content - have been gaining traction (Reinholz et al., 2020). In recent years evidence of their positive impact has also been growing (Freeman et al., 2014; Laursen et al., 2014). A further observation is that the need to reform calculus courses for Biology students resonates with modeling-infused perspectives on undergraduate calculus (Carlson et al., 2010), which emphasize calculus as a tool for interpreting dynamic systems. Modeling education in mathematics has matured, and the related modeling reform movement in calculus was in its infancy when many of the key reports of the early 21st century were produced, but now is enjoying maturity at an opportune moment in calculus education (Bressoud et al., 2015). Biocalculus is an approach to teaching calculus to life science students which integrates these developments.

Biocalculus aims to show life science students how calculus relates to biology in a rigorous, yet informal, manner and through drawing examples from a range of biology topics. As an approach that propagates reformed and rigorous calculus teaching, it leans heavily on modeling-oriented reforms of calculus, providing opportunities for students to engage meaningfully in theoretical biology, programming, and data skills which are important to life scientists. In this context, the Biocalculus movement aims to transform calculus education to better prepare biology students for the modeling and data-driven world of the 21st century. In contrast to calculus as “filtering”, Biocalculus instead aspires to be about “scaffolding” (Biza et al., 2022).

Madlung et al. (2011) and Powell et al. (2012) outline the complexity of the mathematical landscape in biology. Multiple special issues have been devoted to curricular reform for biology (Comar, 2008; R. Robeva et al., 2022). The mathematical modeling education community has matured with the creation of the Society for Industrial and Applied Mathematics Education subgroup and the release of the Guidelines for Assessment and Instruction in Mathematical Modeling Education [GAIMME] report (Bliss et al., 2019). Mathematical modeling has also been studied as an important vehicle for the integration of mathematics and biology (Angra & Gardner, 2017; Bowen et al., 1999; Glazer, 2011; Harsh & Schmitt-Harsh, 2016). For example, Viirman & Nardi (2019, 2021) wrote about Biology students' navigating between mathematical and biological discourse as they work on mathematical modeling activities. Their focus is on where students draw to build the assumptions underpinning their emerging mathematical models of a biological situation (Viirman & Nardi, 2019) and how their graphing skills develop as they engage with said mathematical modeling tasks (Viirman & Nardi, 2021).

While the Biocalculus movement is promising, it has its own set of challenges. Some consider Biocalculus to be a watered-down calculus option that is easier to pass (Diaz Eaton & Highlander, 2017). There are also concerns in the mathematics education literature about further distancing between the what/how of mathematics for Biology and the why/when (Viirman & Nardi, 2021), a somewhat antiquated conflict within the binary of conceptual understanding and procedural fluency (Österman & and Bråting, 2019). Finally, who can teach Biocalculus well is complicated as it requires content knowledge about modeling and biology, which may be less likely for theoretical mathematicians who may be teaching a calculus course (Aikens et al., 2021). Ideally, the biology context of the calculus topics should reflect the diverse interests and career path needs of the students and this can vary extensively within biology (Neitzel et al., 2023). The good news is that there is promising research which can help dispel myths and suggest ways to overcome these challenges.

In this paper, we review current research and reform activities in Biocalculus education. We show how Biocalculus can help re-narrate beliefs about the role of calculus in biology education. We also propose that making space for a Biocalculus that challenges mainstream calculus epistemology will help ensure the place of calculus in the future of biology education.

2. From Calculus to Biocalculus

2.1 What is Calculus?

Calculus designed for the needs of life science students is typically different from a "traditional" calculus course, and as such, we emphasize this comparison. This "Traditional Calculus" is sometimes known as "Stewart's Calculus," which leans on a reference to the popular use of the calculus textbook authored by Stewart throughout calculus classrooms in the United States (US) (e.g., Stewart, 2012). The table of contents of this book (Table 1) is the dominant syllabus for the Calculus I, II, and Multivariate courses in the US.

Chapter	Title	Traditional Course (US context)
1	Functions and Limits (includes continuity)	Precalculus and/or Calculus I
2	Derivatives	Calculus I
3	Applications of Differentiation	Calculus I

4	Integrals	Calculus I* or Calculus II
5	Applications of Integration	Calculus I* or Calculus II
6	Exponential, Logarithmic, and Inverse Trigonometric Functions	Calculus II
7	Techniques of Integration	Calculus II
8	Further Applications of Integration	Calculus II
9	Differential Equations	Calculus II* or ODE course
10	Parametric Equations and Polar Coordinates	Calculus II* or Calculus III
11	Infinite Sequences and Series	Calculus II
12	Vectors and the Geometry of Space	Calculus III
13	Vector Functions	Calculus III
14	Partial Derivatives (includes multivariable functions)	Calculus III
15	Multiple Integrals	Calculus III
16	Vector Calculus	Calculus III
17	Second-Order Differential Equations	Calculus III* or ODE course

Table 1: Above are the chapter numbers in the 7th edition and their traditionally corresponding US course designations. Chapters with a * indicate that this is often considered optional content for that course. While not always included in the "calculus" umbrella, ordinary differential equations (ODE) courses are courses which might also cover some of the chapters in Stewart (2011).

Another way that calculus is defined, is through some broad statement of cohesive conceptual knowledge. One possible unifying concept of the study of calculus in Calculus I is the concept of the derivative. For example, a research study on conceptual understanding in Calculus I used the prompt:

> *"Explain what a derivative is to someone who hasn't encountered it before. Use diagrams, examples and writing to include everything you know about derivatives"* (Bisson et al., 2016, 2020).

Often the unifying concept of Calculus II in the US context is considered the accumulation of change and Calculus III is the calculus of multivariable functions. However, these boundaries are somewhat artificial and, for example, several authors in the IJRUME Special Issue edited by Ely and Jones (2023), make the case for the centrality of the concept of integral (Bajracharya et al., 2023; Dray & Manogue, 2023; Ely & Jones, 2023). For example, modeling with ordinary differential equations could then reasonably fall under Calculus I theme if the rate of change is emphasized and under Calculus II if the emphasis is on deriving analytic solutions or numerical approximations. Partial derivatives could fall under Calculus I as rate of change, but just as easily fall under Calculus III for discussion of multivariate functions. In this paper, we use the term "calculus" in the postsecondary instructional context as the mathematical study of rates of change and accumulations of rates of change encompassing broadly the conceptual ideas typically presented in a year-long sequence of Calculus I and II.

2.2 What is Biocalculus?

Biocalculus courses are quite varied. Some carry the name Calculus or Calculus for Life Science. Some are named more broadly, such as Mathematics for Life Science. Some of these are one year sequences of Calculus I and II, some are reimaginings into one semester of calculus and one semester of statistics, and some eschew calculus boundaries altogether. While the umbrella of Biocalculus is broad, overall there is an understanding that Biocalculus is more than just traditional "Stewart Calculus" with some biology examples (Akyen-Odoom et al., 2024). While adding examples of biology applications is often a step to a broader transformation for many, this is seen as a very limited change. Below we introduce nine common features which distinguish Biocalculus courses: modeling approaches, integrating data and statistics, programming and/or algebraic solvers, lab activities and active learning, Rule-of-Five, discrete modeling, big problem-driven, asset-based, and translating across disciplinary boundaries. We intersperse examples as boxed vignettes which utilize these strategies in a particular mathematical and biological context. The mathematical concepts or biological contexts introduced in a particular Biocalculus instance are often customized to cater towards the specific mathematical skills that may be demanded by particular life science majors and in the life science contexts which are most relevant (Diaz Eaton & Highlander, 2017; Ganter & Haver, 2011; Marshall & Durán, 2018), so we have endeavored to showcase a range in our boxed vignettes.

2.2.1 Modeling approaches

From a content perspective, many Biocalculus courses offer a perspective focused on the learning of concepts and motivated by the use of calculus for modeling. This is not specific to Biocalculus, but rather was championed by waves of "Calculus Reform" driven by mathematics researchers such as the Gleason-Hallett Calculus Consortium (Bressoud et al., 2015). Mumford (1997) wrote of this movement in the Notices of the American Mathematical Society (AMS):

> *"Are we teaching calculus in the hope that a small percentage of our students will catch our love of rigor, or so that most of our students will emerge with the ability to use calculus in their specialties?"* (p. 563).

Resulting calculus textbooks of the reform movement included the version authored by Hughes-Hallett, which continued to maintain a similar set of topics as Stewart, but with this more conceptual and modeling approach. Biocalculus courses have largely adopted at least this calculus for modeling framework - leveraging models to understand biology and leveraging calculus as a tool for understanding these dynamical system models (Carlson et al., 2010). Many adaptations have focused on how to make the modeling more authentic, make modeling more accessible, and rethink the traditional calculus table of contents to provide the necessary space and scaffolding. As a result, modeling techniques such as difference equations and differential equations are also introduced. This is particularly valuable because students can understand the linkage between foundational models such as the exponential and logistic equation and theory in population dynamics (Diaz Eaton & Highlander, 2017). See the Boxes integrated throughout this section for additional examples.

Box 1. Example - The logistic growth model

This is an example of a module used in Bodine et al. (2014), and utilized by many experienced Biocalculus instructors. This module is particularly effective for students who are familiar with the conceptual model of logistic growth, but can also be used to motivate the relevance of calculus for future biology courses.

Biology context: The logistic growth model is a well-known model of limited population growth, typically introduced in an introductory biology course which covers concepts in population ecology. In this context, students learn that logistic growth in populations is characterized by exponential-like growth for low population sizes. However, as the population size (*P*) reaches its intrinsic carrying capacity (*K*), intraspecific competition (competition within a species), leads to a plateau in the population growth. Some students may also be introduced to the differential equation version of the logistic growth model, though this may happen in a later course focused specifically on ecology.

Calculus context: The analytic form of the logistic growth model is:

$$P(t) = \frac{P_0 K}{(K-P_0)e^{-rt}+P_0},$$

where P_0 is the initial population size at time $t = 0$. Sometimes, this version of the model appears in the discussion of limits, as $\lim_{t\to\infty} P(t) = K$.

The first-order ordinary differential equation form is commonly written either as:

$$\text{(ii) } \frac{dP}{dt} = \frac{r}{K}P(K - P) \text{ or (i) } \frac{dP}{dt} = rP - \frac{r}{K}P^2,$$

where r is the rate of growth.

In a traditional calculus approach, this version of the model, if discussed, is typically in the context of solving separable ordinary differential equations (ODEs).

Biocalculus integration: In Biocalculus, the ideas presented in biology are connected to the ideas presented in calculus. Students would have already discussed exponential models as a very simple ODE. The version (ii) of the ODE can be leaned into to make this connection. We can interpret the ODE as positive change factors - negative change pressure in a "bathtub model" approach, or

$$\frac{dP}{dt} = P_{in} - P_{out}.$$

In this scenario, we can see that P_{in} in Equation (ii) is the exponential ODE. The P_{out} term is proportional to P^2, in other words, the growth is moderated by the interaction between members of the population. For small population sizes, the "out" term is much smaller than the "in" term, so indeed, the population grows approximately exponentially. This can also be seen clearly from the version (i) of the equation. The version (i) of the equation can also be used to show how, for values of P near K, the derivative gets close to 0, leading to the plateau effect.

In the ODE version (i), one can quickly infer the equilibrium solutions as $P^* = 0$, which is unstable, and $P^* = K$, which is stable. Equilibrium analysis helps connect the ideas of derivatives and limits, but is rarely introduced in traditional calculus experiences.

By analyzing this and other limited growth models this way, students begin to understand that these models are chosen not just for the fact that they lead to limited growth, but that the model choice implies that certain mechanisms are responsible for the type of limited growth being observed.

2.2.2 Integrating data and statistics

The use of real data is prevalent in all branches in biology, whereas in mathematics it is often the case that data is generated *in silico* from models. This makes it difficult for students and faculty to understand how mathematical theory directly supports and connects to observation and experiment (Diaz Eaton et al., 2019). Multiple studies point to data and statistics as a top most valuable skill out of their quantitative courses (Bennoun, 2022; Diaz Eaton & Highlander, 2017). Biocalculus adoptions vary in their approach to considering data. Garfinkel focuses on modeling and the resulting numerical [*in silico*] data generated by said models for the purposes of graphing solutions and comparing trends. The Bodine et al., (2014) book integrates biostatistics into the first semester and first half of the book. That is because the "Mathematics for Life Sciences" curriculum at the University of Tennessee serves as a combined one year substitute for Statistics, Calculus I and Calculus II. In one of the most integrated cases, Robeva et al. (2022) introduces statistical perspectives alongside modeling perspectives. For example, Chapter 9 examines endocrinology and period cycling from a statistical perspective, then Chapter 10 introduces a mathematical modeling perspective (Robeva et al., 2008).

2.2.3 Programming and/or algebraic solvers

Most Biocalculus revisions include Matlab/Octave (Bodine et al., 2014; Diaz Eaton & Highlander, 2017; Robeva et al., 2022), Excel/Google Spreadsheets (Diaz Eaton & Highlander, 2017), R (Bodine et al., 2014; Diaz Eaton & Highlander, 2017), Mathematica (Bodine et al., 2014), SageMath (Garfinkel et al., 2017) and/or Python (Bodine et al., 2014; Garfinkel et al., 2017; Robeva et al., 2022). Statistical software and spreadsheets can be used to connect data to mathematical models (Box 2). Computer algebra solvers are introduced to overcome barriers related to algebra and to better focus on patterns in the algebraic results - for example, illustrating why certain differentiation "shortcuts" work (e.g., Box 2). Computer programming languages are typically introduced for the purposes of numerical approximation of solutions to differential equations and integrals (e.g., Box 3). In some renditions of Biocalculus, a lab is introduced which is to support the programming aspects of the course. This feature is not unique to Biocalculus as many versions of "applied calculus" also incorporate technological tools. However, the introduction of programming languages and/or spreadsheets can also be used to fit models to data, supporting other goals above.

Box 2. Example - The Keeling Curve

In a survey of biology and environmental science faculty, feedback loops and fitting data to models were identified as the two most important topics that should appear in a calculus course (Diaz Eaton & Highlander, 2017). This module meets these needs and is particularly applicable for students who are interested in sustainability and climate change and additional details about its adaptation for and use in Biocalculus can be found in Diaz Eaton (2023).

Biology context: Feedback loops are important to understanding population dynamics and climate change because of the rapid acceleration that builds over time without limits if kept unchecked. In the context of climate change, increasing carbon dioxide concentration ([CO_2]) levels present a threat because carbon dioxide accumulation in the atmosphere traps heat. The greenhouse gas effect refers to the feedback loop caused by the increase in [CO_2].

Data for [CO_2] concentration levels has been recorded by the Mauna Loa Observatory in the Hawaiian Islands, United States since 1958 (NOAA, 2024). The exponential model is the best fit model for this data, and is referred to as the Keeling Curve (see Figure 1). The words "exponential growth" are used often colloquially to describe an observed increase that is faster than linear growth. Students seeing the exponential model as the best fit for the [CO_2] data are then alerted to the idea that [CO_2] levels could spiral out of control.

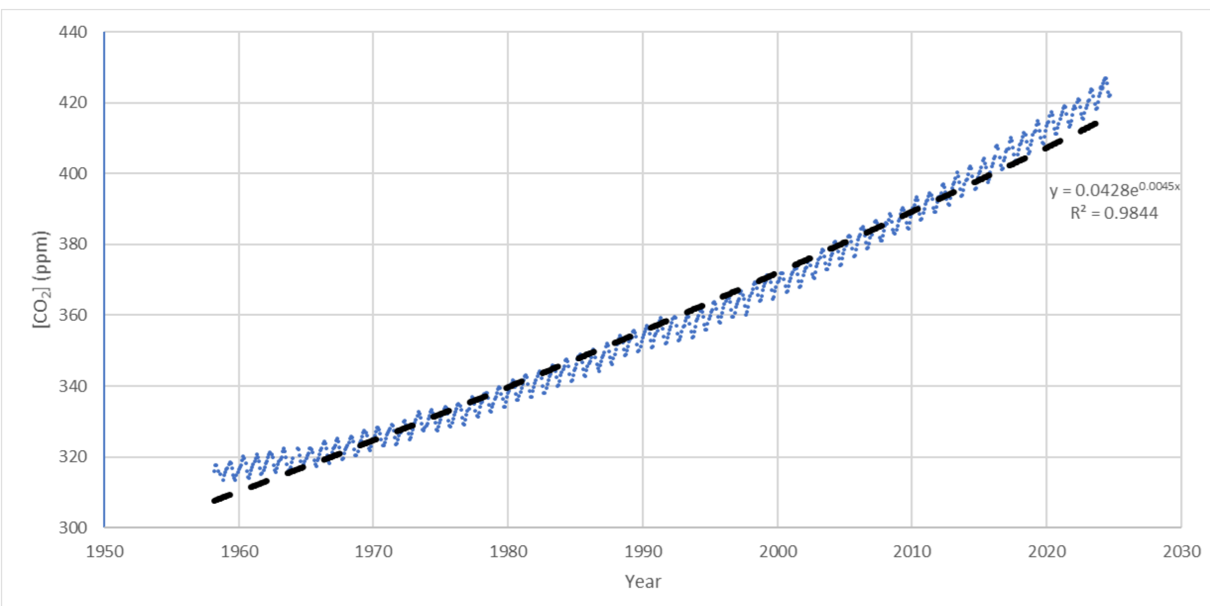

Figure 1. Carbon dioxide concentration, [CO_2], levels obtained from the Moana Loa observatory, plotted with the Keeling Curve fit in Excel with an exponential fit. The Excel file is included as Supplementary 1.

Calculus context: Exponential functions, $y = y_0 e^{kx}$, are typically relegated to a "pre-calculus" course in the US, though many calculus texts, including Stewart (2012, p32), have versions that include exponential and trigonometric functions as the first chapter. In Stewart and other traditional textbooks, software expertise and datasets are not integrated, so students are not asked to perform model fitting or model selection. Occasionally, the exponential function invoked to ask students what the [C02] levels in parts per trillion (ppm) are predicted to be in 2030. In Stewart, derivatives of exponential functions are not discussed until Chapter 6. Here, the early treatment focuses on the derivation of the derivative, but in Section 6.5 eventually introduces the autonomous ordinary differential equation that characterizes the exponential function as the simplest feedback loop:

$$\frac{dy}{dx} = ky.$$

Note that because the discussion of exponential function growth is delayed until after integration, it is likely that a Calculus I course following the Stewart textbook would not touch the two calculus topics deemed most important to biology faculty.

Biocalculus integration: In the first semester of Biocalculus offered by Diaz Eaton (Diaz Eaton, 2025), the motivation for students to learn derivatives is to better understand and describe how things change. This idea is incredibly important to all natural systems. This change is explored at first through discrete models. For example, adding two individuals to a population each year ($x_{n+1} = x_n + 2$ or $\Delta x_n = 2$) is a constant rate of change, and the resulting arithmetic model is similar to a linear continuous function. Reproduction, which increases population size by a fixed 5% each year ($x_{n+1} = 1.05x_n$ or $\Delta x_n = 0.05x_n$), results in a yearly change proportional to the current population size, also known as the geometric model. Here, students are introduced to spreadsheets and/or computer programming to explore calculations and visualizations of these models.

To understand change in continuous and compelling contexts such as global climate change and ocean acidification, we need additional ways to talk about change. The same computational skills can be used to introduce numerical approximation methods in addition to algebraic methods to calculate instantaneous change at a point, i.e.

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

As students move from derivatives at a point to determining functions which describe the change at any point, visual representations are sketched at a few points using (*x,* approximate slope at *x*) to visually estimate what these derivatives might look like, providing some intuition.

It is at this point that students are presented with the Mauna Loa data and its context. Student teams are asked to use numerical methods to help understand how fast [CO2] levels have changed (and may continue to change) over time. First, they will need to estimate the function that describes this data, by fitting curves - and students are encouraged to try a few. The best fit exponential function they derive forms the basis for the rest of their investigation, which is to numerically approximate the derivative at each year. For this, it is a bit easier to implement using the Δx form of the instantaneous rate of change:

$$\lim_{\Delta x \to 0} \frac{f(a+\Delta x)-f(a)}{\Delta x}.$$

For example, each row an x value with each column representing a Δx of 1, 0.1, 0.001, etc. Students are likely to observe that the rate of change is similar to that of the original function - both begin to climb rapidly. Students can even fit the exponential function to one of their derivative estimates.

As students begin writing up their results from this lab, it is natural to start thinking about generalizations. After exploring algebraic derivations of derivatives for simple polynomial or power functions (e.g., $y = mx + b$, $y = x^2$), one can present a list of derivatives for other common functions (e.g., $y = \sin x$ and $y = e^x$) and look to visual or numerical methods to confirm the accuracy. This allows for a more rapid introduction of the derivative of the natural exponential. As soon as the chain rule is introduced, students are given the general form for the natural exponential, $y = y_0 e^{kx}$, and are stepped through an in-class activity to get them to derive the feedback loop equation $\frac{dy}{dx} = ky$. Students may even notice the similarity to the discrete geometric model.

The take-home idea is that we use exponential models not just because they increase rapidly, but because more specifically the rate of change is proportional to current size. This removes the black box of why the "ugly" exponential function with the "made up number" e is so common in living systems. This helps students relate a mathematical concept to an existing biological concept with which they are already familiar.

2.2.4 Lab activities and active learning

Labs for introductory science courses are extremely common. For many science majors, this is the most exciting part of the course and can bring science to life (Diaz Eaton et al., 2019). In some renditions of Biocalculus, a lab is introduced which is to support the programming aspects of the course. This can be an in-class lab over one or more classes or a dedicated lab section. Labs can be computational, as in the Keeling Curve lab described in Box 2. However, some programs use labs as an opportunity to engage in experiential perspectives on the models and concepts discussed (Diaz Eaton et al., 2019). For example, Diaz Eaton has brought Calculus II students to a nearby natural area to sketch land topography and draw cross-sections and slopes. This is the first activity in an introduction to multivariate functions and partial derivatives. Students then move back into the classroom to study topographical maps of the local National Park. Labs are one, but not the only way, in which Biocalculus courses engage students heavily in active learning. Numerous studies have suggested that active learning approaches can improve conceptual understanding and improve student retention (Freeman et al., 2014; Laursen et al., 2014).

2.2.5 Rule-of-Five

The Rule-of-Three and Rule-of-Four are well known in calculus communities for connecting representations of calculus ideas in textbooks across *verbal, numerical, algebraic*, and *visual* representations. The sequence of activities in Box 2 illustrates how important it is to introduce these multiple representations for increased understanding of the concept of derivative (Silver & Charles, 1989), and more specifically the relationship between exponential growth and feedback loops. However, as seen in the discussion of laboratory and field activities, the experiential experience is of utmost importance to biology students. Diaz Eaton et al. (2019) expanded the Rule-of-Four to a Rule-of-Five, distinguishing many Biocalculus classrooms by adding *experiential* models to the list of model representations that appear in life science education and Biocalculus classrooms (Table 2). The addition of experiential, while newer to mainstream calculus and modeling education, was already established in mathematics education research in grade schools and adopted in precalculus (Simundza, 2006). Another advantage of integrating the Rule-of-Five framework is that it can help students understand how calculus informs biology theory in a manner complementary, but different to the way that statistics informs theory (investigating a hypothesis via algebraic methods versus data methods, respectively).

Model Representation	**Examples**
Experiential	Animations, simulations, physical models, experiments, observations
Verbal	Hypotheses, predictions, qualitative data, descriptions of data trends
Numerical	Simulated data, quantitative experimental data, quantitative observational data
Symbolic	Equations, state variables, parameters
Visual	Graphs, schematics
Modeling Activities	**Description**
Reality-to-model	Moving from observations of reality to an abstracted model, either as an initial step in developing a model or as part of a model revision.
Between representations	Moving from one model representation to another representation of the same model.
Comparing models	Comparing different models to each other or to reality, for example model selection and model validation.

Table 2. Model representations and model activities as described in Diaz Eaton et al. (2019).

2.2.6 Discrete modeling

Several versions of Biocalculus include discrete models. While sequences and series are commonly a part of the "traditional" Calculus II, discrete difference equations are also included. Diaz Eaton & Highlander (2017) point out that difference equations play an important role in wildlife management models, making them particularly important to some subdisciplines of life science. As described in Box 3, discrete models are often included at the beginning of

Biocalculus courses to help develop skills for students to discuss change. Introducing discrete difference equations at the start of Biocalculus can also create a more even playing field between students who are new to calculus and students who have taken a calculus course in high school. Difference equations are new to almost all students, provide an easy on-ramp towards modeling for students to start asking questions about equilibrium and stability, which students might not have access to otherwise until a course in differential equations, can be explored easily with tools as accessible as spreadsheets. Discrete sequences are built, convergence is explored but with the lens of equilibria and stability, and then notions of measuring change are developed (Diaz Eaton, 2023; Robeva et al., 2022). This leads into conversations about continuity. While this order may be newer for the introductory calculus curriculum, many undergraduate real analysis courses already use a progression from sequences to develop notions of continuity as that change gets small. The Bodine, Gross and Lenhart (2014) text introduces discrete before continuous, but in its implementation appears towards the end of the first semester of Biocalculus after statistics instead of the beginning of the second semester before derivatives. Robeva et al. (2008) covers other discrete models beyond 1-dimensional linear models (see for example Box 3). Finally, as programming skills are becoming more common in biology, understanding the relationship between continuous models and their discretized analogs becomes more important.

2.2.7 Big problem-driven

Biocalculus is more than just some homework problems at the end of a section. Big questions or "wicked problems" in life, health, and environmental science can be the source of synthesis projects or motivating the introduction of new mathematical ideas. Garfinkel has an associated project with each chapter. Diaz Eaton (2023) discusses the use of starting the unit on derivatives with the context of rising carbon dioxide levels and coral reef health. The project to be completed at the conclusion of this chapter asks students to fit a curve of carbon dioxide levels to the Mauna Loa observatory historical data set, which finds the best fit model, namely a function describing exponential growth. This is indicative of a feedback loop, described by a simple differential equation. Then, students numerically estimate the rate of change. However, this context is used as motivation to engage students early, rather than just to add context to a project at the end of the unit. Research from Aikens (2020) points to the importance of including these authentic problems. Robeva begins each chapter with a key biological context: Chapter 5 is entitled "Risk Analysis of Blood Glucose Data" (2008). Bodine et al. (2014), Garfinkel et al. (2017), and Robeva et al. (2008) focus on the modeling and build programming skills so that students have the time and skills to computationally explore models which exhibit oscillatory and/or chaotic behavior. Oscillations are related to many biological functions: having exposure to these ideas is highly valued in a Biocalculus experience (Bennoun, 2022). Garfinkel et al. (2017) include a project which relates chaotic behavior to the study of cardiac rhythms and supply chains. The biological contexts chosen for these motivating problems are ideally chosen based on the interests and majors of the students. Diaz Eaton and Highlander (2017) discuss this extensively in the curriculum design as Diaz Eaton's wildlife and environmental science students had differing interests from Highlander's primarily pre-medical students. In addition to what appears in textbooks, there is an extensive amount of scholarship devoted to creating such tailored syllabi and in-depth examples for particular institutional and student contexts (Ledder, 2013; Stoner & Joyner, 2022). Many resources for teaching Biocalculus can be also found as Open Educational Resources (OER), which are free for instructors to download, adapt and use (Diaz Eaton et al., 2022). For example, the Calculus group on QUBESHub (https://qubeshub.org/community/groups/calculus/) has example syllabi, and other resources. There are also many more, found by searching the full database of OER for Calculus and related topics (Quantitative Undergraduate Biology Education & Synthesis [QUBES] Project, 2025).

Box 3. Example - Drug dosing

A significant portion of life science students are students interested in health professions, for example nursing, dentistry, physical therapy, veterinary school, and medical school. Some texts particularly cater to these students, such as Robeva et al. (2008). Here, we discuss an example from this text which uses exponential decay and discrete models to model how drugs are metabolized (Robeva et al., 2008). Although still only in Chapter One, at this point, students have already been exposed to the differential equation for exponential growth.

Biology context: Understanding the implications of drug dosing within a calculus perspective allows future health care providers to accurately assess how drugs behave within the human body over time, pharmacokinetics (Ernstmeyer & Christman, 2023). By utilizing a calculus framework, these future health professionals can analyze rates of change, which again is important for understanding how the human body behaves in response to administered drugs as well as the concentration of the drug and time dependent implications. Calculus models that emphasize dose optimization and intervals using differential equations aid in ensuring safe and efficient drug concentration levels in the bloodstream - therefore, determining the therapeutic amounts of drugs and preventing drug toxicity.

Calculus context: A dosage, *C,* of a drug is delivered at intervals of time, *T,* and the amount of drug remaining in the body after the *n*th interval can be described as:

$$R_n = C\left[\sum_{k=1}^{n}\left(e^{-Tr}\right)^k\right]$$

This drug dosing example utilizes an understanding of discrete time models (to describe the drug intake), exponential functions (exponential decay to describe drug metabolization). The resulting equilibrium is discussed (where the "steady state" is periodic) and visualized through computer simulation. These are all considered important mathematical topics to biologists. While exponential growth models are associated with population growth (see Box 2), exponential decay is often more associated with ideas like Carbon-dating, which are useful also for scientists. However, it would be unusual for a traditional calculus class to consider a continuous and discrete combination model. Periodic behavior as a type of stability is also not commonly described in traditional calculus texts. However, recall that periodicity and nonlinear systems have a key importance to those studying life science.

Biocalculus integration: Robeva et al. (2008) prompts students to ask themselves:

> "Why do you need to take two acetaminophen [paracetamol] tablets every 4-6 hours when you have a headache? Why is there a warning label that cautions you not to take any more than four doses in a given 24-hour period? And why does your head start to ache again after four hours when the warning suggests you really ought to wait six hours before you take the next dose?"

The above leverages student curiosity about the natural world. In addition, it has the underlying message that understanding drug dosing is not just a medical student or mathematics student interest, it also has personal implications. Other examples mathematically similar to drug dosing may include alcohol metabolism, or snow accumulation with some percentage melting between snowfall.

In this activity, students are introduced to different types of absorption methods for drugs (oral, intravenous, intramuscular, transdermal, and inhalation). Some of these would result in an nearly instantaneous uptake, motivating the need for a discrete modeling approach. Then students discuss how drugs are eliminated. Instead of it being instantaneous, there may be transport required, there will be some metabolized into the body, and there will be some excreted. In this context, students are reminded about exponential models as a way we can perhaps model this elimination. If $C(t)$ is the concentration in the bloodstream at time, t, then the rate of elimination could be modeled as proportional to the current concentration, *i.e.*,

$$\frac{dC(t)}{dt} = -\, rC(t).$$

The half-life of a drug, $\tau = \ln(2)/r$, can also be discussed in this context, relating this to carbon isotope half-life which many students may have seen in chemistry, physics or precalculus. Here, the half-life of a drug means the amount of time it takes for the body to eliminate half of the drug in the system. Students may notice that it is not a constant amount in the half-life or specific time interval that is eliminated, but rather half of whatever exists. Once you are in an equilibrium pattern, the drug level to maintain should be the same, but what about trying to reach and establish that equilibrium level?

Students are then brought through some calculations to imagine the amount in the body after one dose,

$$R_1 = Ce^{-rT}$$

then two doses,

$$R_2 = (R_1 + C)e^{-rT} = C(e^{-rT})^2 + Ce^{-rT}$$

and can try to generalize a pattern based on this. This formula generalization uses ideas of geometric series to leverage a closed form solution. That closed-form solution can then be used to explore the limit.

Before or after developing a closed form solution, students can be introduced to computer programming options to develop a numerical simulation for their model (see Figure 2). This would allow students to explore the effect of changing initial conditions, *r, T,* or a number of other ideas, leveraging that natural curiosity, and providing an opportunity for them to notice patterns.

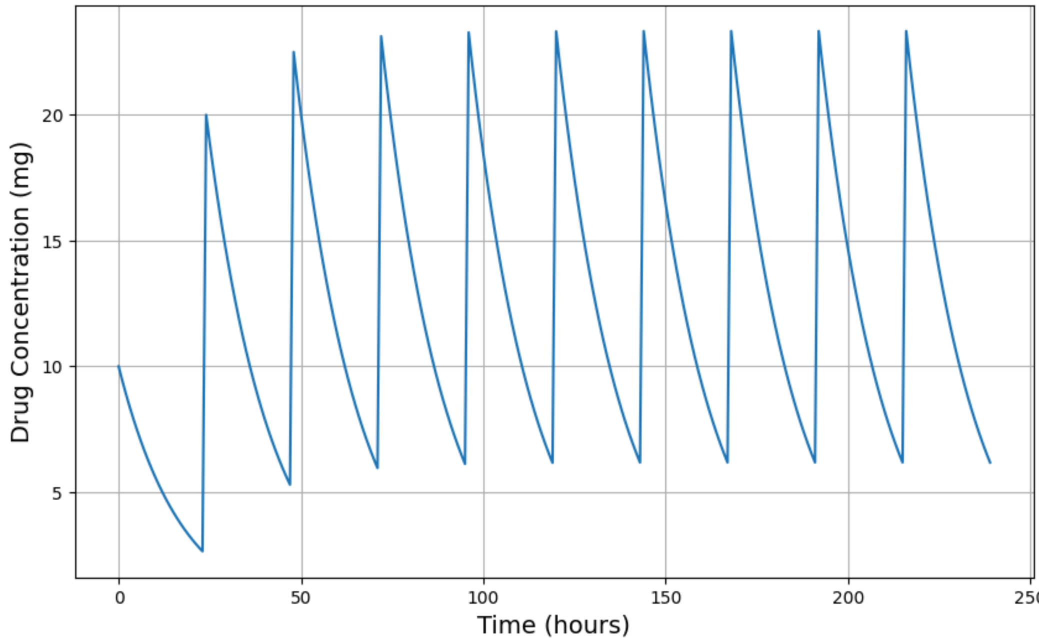

Figure 2. Drug concentration over time, for a drug with a half-life of 12 hours, taken in a 10 mg dose every 24 hours. The simulation was performed in Google Colab using Python, and the file is provided as Supplement 2.

2.2.8 Asset-based

Being authentic-problem driven is more than just engaging students with questions that matter to them. Mathematicians typically rely on foundational mathematics knowledge to abstract into pattern and show support for advanced ideas (Tall, 1991; 2013). With this narrow perspective of foundational skill ability, life science students are often perceived as coming to calculus with deficits in what they need to know. For example, Diaz Eaton & Highlander (2017) noted that biology students performed significantly worse on the first exam in Calculus I covering primarily precalculus review skills. However, life science students come with a vast knowledge of expertise in their own fields, which can also be leveraged to help them understand advanced mathematical concepts. For example, life science students often hear about exponential and logistic growth. They can sketch both the population size curve and verbally explain the rate of change even without the graph. Using their existing knowledge to build skills sketching graphs of derivatives is an asset-based approach to calculus instruction for life and environmental science students (see Box 1 and 2). Likewise, geology and environmental studies students' familiarity with transects and topographic maps can be leveraged to teach multivariate surfaces. Asset-based approaches are part of broader theories of inclusive instructional strategies (Yosso, 2005). Students come with a *conocimiento*, a body of knowledge that encompasses a broad range of experiences entangled with social identity (Diaz Eaton, 2023; Gutiérrez et al., 2024). As all students in Biocalculus share some interest, enthusiasm, and knowledge of life science, this aspect of their social identity may be more effectively leveraged in a Biocalculus course (Diaz Eaton, 2023; Silver & Charles, 1989). Also, many life science issues are broader community or individual issues, such as understanding climate change (Box 2) and personal health (Box 3), which may appeal to other identities and interests. Diaz Eaton (2023) discusses in depth how the carbon dioxide example in Box 2 evolved over time to become more relevant and authentic, as well as to engage students in questions about power and privilege as a newly proposed observatory at Mauna Kea was met with resistance from Native Hawaiian protectors (Garcia Chua et al., 2022).

2.2.9 Translating across disciplinary boundaries

Language to describe calculus concepts may vary across disciplinary boundaries (Redish & Kuo, 2015). For example, "rate of change," "rate equation," or "velocity" are used throughout their introductory science courses, but derivative notation and the word "derivative" is unlikely to appear. Biocalculus courses focus on making these connections clear (e.g., Box 1) and helping students understand the different languages used in their worlds to describe the same underlying concepts. Many life science students take statistics courses or are introduced to statistics in early laboratory courses. Connecting finite integrals to area under the curve calculations for probability distributions helps students better understand statistical tables, critical values, and calculators. Introducing students to the Rule-of-Five for models and modeling can help students understand how their calculus theory is part of a broader set of scientific approaches used to understand complex phenomena (Diaz Eaton et al., 2019). Through metacognitive reflection exercises, students can be asked to reflect on how topics discussed in class relate to their other classes, their research interests, or their passions (Diaz Eaton & Highlander, 2017). Independent choice projects can further help. For example, Diaz Eaton assigns as a final project that students choose an article in their major discipline to class which uses an idea from calculus, and then summarize and present that article to the class. Students value these experiences, but also professors in their major disciplines value calculus instruction more when students develop the ability to translate ideas from calculus to their classes. Developing a deeper translation fluency among mathematics instructors is an open area of research and a top challenge in building capacity for teaching Biocalculus. Neitzel et al., (2023) used a database of calculus ideas used in a Marine Biology course after the instructor sat in on Calculus I and found that quantitative language seemed more developed in the lecture content closest to the instructor's own research area. This suggests that offering differentiated opportunities in various subject areas to learn and integrate mathematics and biology together may be a successful tactic.

In summary, Biocalculus challenges the epistemological values of traditional calculus courses. Some use the language of rigor to describe adherence to a particular established textbook, though in research on epistemology, qualitative research suggests they actually value problem solving, which Biocalculus does (Stockton, 2010). Biocalculus also adheres to best practices in calculus education. Even the latest guide by the MAA Committee on the Undergraduate Program in Mathematics (CUPM) of what mathematics education researchers and respected mathematics educators say should be in the calculus classroom matches more with forms of Biocalculus rather than many publisher-backed popular texts (Bressoud et al., 2015). Biocalculus educators should be considered innovative frontliners pushing best practices in calculus education.

3. The Benefits of a Biocalculus approach

3.1 Usefulness, Values and Language

For Biocalculus to stay relevant in the curriculum, life science program faculty must see what it helps their students do and understand. When life and environmental faculty were initially surveyed using calculus language such as "derivative", the results for usefulness were much lower than when they were queried using the language of calculus often used by biologists, such as "rates of change" (Diaz Eaton & Highlander, 2017). When courses were changed to reflect the cultural values of surveyed faculty, these faculty responded positively - to both the process which garnered buy-in and collaboration as well as to what their students were able to do as a result.

Qualitative research comparing pre-post student responses indicated that after a Biocalculus course, students could articulate specific examples of the role of calculus in biology, for example models and modeling, making predictions, explaining patterns, evaluating change, and forming hypotheses (Aikens et al., 2021). O'Leary et al. (2021) noted that enrollment in Biocalculus was associated with increased grades in science courses where traditional calculus was not. Students vote with their enrollment as well, with many programs noting increased enrollment in Biocalculus over the traditional (Diaz Eaton & Highlander, 2017; O'Leary et al., 2021). One marine biology program re-added Biocalculus to their list of program requirements after initial elimination and multiple programs highlighted it in program reviews (Diaz Eaton & Highlander, 2017). Mathematics departments that rely on calculus as a primary source of ultimately funding faculty lines, should see Biocalculus as an opportunity to partner.

Students also picked up on how Biocalculus courses were relevant and applicable - citing it as a top factor in what helped them learn (Aikens et al., 2021). Their responses indicated that mathematics was more accessible and that they developed a sense of agency around their ability to use mathematics. Interestingly, they also talked about how their professors cared. Discussion of care is often associated with student evaluations of women professors (Sprague & Massoni, 2005). However, Aikens et al. (2021) suggested that translating calculus - tailoring courses and speaking in their language - may be also contributing to students' perception of instructor care. Disciplinary language can be a powerful tool academics use to indicate their epistemological stance and values, and can be used as a "disciplinary microaggression," adding to feelings of not belonging (Diaz Eaton et al., 2019). This could be an avenue for further research. However it points to the importance of seeing our students not just as knowledge containers, but as people whose identities and presence crave to be acknowledged and valued.

3.2 Inclusion and Exclusion in Calculus drives Biology (and STEM)

Calculus attrition is a problem for all STEM programs (Ellis et al., 2016). If you do not pass calculus, there is a trickle down effect to the rest of the major. One measure of attrition is DFW rate, which compiles course grades below a 70% (D or F) or course withdrawal (W). The DFW percentage indicates how many students will need to repeat the course if it is needed as a prerequisite or program requirement (Diaz Eaton & Highlander, 2017). Diaz Eaton & Highlander (2017) reported Biocalculus as reducing DWF rates by half or more, from 25-30% DFW rates to 5-10%. These findings indicate that Biocalculus has the potential to reposition calculus as a gateway to STEM instead of a gatekeeper. When mathematics programs are slow to respond to this research and life science program needs, life science departments have hired the expertise to teach it within their departments (Diaz Eaton & Highlander, 2017) - a practice already common for biostatistics.

Biocalculus is not just a "watered-down" version of calculus. This myth assumes that the reason for closing gaps in DWF is that students are held to a mathematically lower standard. This is a fundamental misunderstanding about what Biocalculus is. In fact, most of the Biocalculus courses described in literature are quite the opposite: they are designed to meet the needs of biology programs and allow students to successfully transition into calculus II (Robeva et al., 2022). This is often because, even if differentiated Biocalculus classes may be economically feasible at the Calculus I level due to high enrollment, not all institutions can offer a differentiated Calculus II. Success in later Calc II courses has been demonstrated by both Comar (2008) and in (Uhl & Holdener, 2013). Diaz Eaton & Highlander used common exams between calculus and Biocalculus sections to demonstrate "rigor" (2017). Students in the regular calculus statistically out performed students enrolled in Biocalculus on the common precalculus exam at the beginning of the semester. By halfway through the semester, traditional calculus students outperformed Biocalculus students in the differentiation exam on average, but the difference was no longer statistically significant. By the end of the term, the Biocalculus students were outperforming the traditional calculus students. This signal was not statistically significant, but even no statistical difference between classes, successfully dispels this myth. The findings support the idea that Biocalculus can build a better scaffold for students, rather than serve a filter (Biza et al., 2022).

We also know that success in calculus is un-equitably related to various social characteristics such as socioeconomic status, gender, first-generation to college status, race, and ethnicity. Biocalculus studies are only now starting to disaggregate student data. A more recent study by O'Leary et al. (2021) suggested a meaningful, but not statistically significant impact on grades of low SES, first gen, and underrepresented minority students. An open avenue for research is to see whether Biocalculus is making an impact on disproportionately affected students. For example, Diaz Eaton (2023) suggested that epistemological violence and disciplinary microaggressions are not neutral, *i.e.* our scientific constructs are not independent of our social constructs. In addition, several of the course reforms which Biocalculus employs are also considered inclusive pedagogical practices, *e.g.,* asset-based perspectives. The Biocalculus community needs to think about employing a more intersectional analysis of student identity as their disciplinary identities grow and use this analysis to discuss how race, gender and class identities introduce multiple, intersecting, and potentially multiplicative axes of oppression (Crenshaw, 1991).

There are also opportunities to analyze the effect of specific instructional strategies. For example, some courses with a systems perspective and programming employ "just-in-time" calculus instruction. Does the relevance and new focus of the models and systems perspective outweigh social characteristics related to the likelihood of prior exposure to calculus and programming? Are students who have already taken calculus or programming at their high school given an advantage that is difficult to overcome. Could students' diverse cultural wealth be more valued and leveraged by certain choices of the authentic problems which drive instructional units?

Biocalculus approaches to teaching are by definition more drawn towards learning through active participation, which are associated with lower attrition (Freeman et al., 2014; Laursen et al., 2014). This will help the ongoing struggles in mathematics departments move away from chalk and talk approaches (Cevikbas & Kaiser, 2023). In this way, Biocalculus offers a blueprint for approaches to inclusion and access issues that can be useful in mathematics departments which increasingly are learning to address these issues.

4. Beyond epistemological violence and disciplinary microaggressions: future directions for a Biocalculus approach

> "Like all people, we perceive the version of reality that our culture communicates. Like others having or living in more than one culture, we get multiple, often opposing messages. The coming together of two self-consistent but habitually incomparable frames of reference causes un choque, a cultural collision." - **Anzaldúa (1987, p. 3)**

4.1 Challenges

Why has the Biocalculus movement not become more universal? Resistance to change may be due to deeply-embedded institutional and cultural barriers, the lack of intercommunity involvement (e.g. of mathematics educators in course development) and the fact that mathematicians teaching calculus are not necessarily trained as applied mathematicians and modelers; they may feel that they are asked to contribute to teaching this new kind of course and feel unprepared to do so (Bressoud, 2020; Schoenfeld, 2013). A flawed narrative about what Biocalculus is (a fundamental redesign and translation) and what it is not (watered down calculus, the traditional text with a few biology examples) may also be a key obstacle.

Some departments in the US hire specialized faculty with research areas in mathematical biology to teach Biocalculus courses. The BIO2010 report (National Research Council, 2003) and the Vision and Change report (AAAS, 2011), both foundational to setting postsecondary biology education agendas in the US, helped spur an infusion of grants and initiatives to create curricula (Diaz Eaton et al., 2020). These resources can help provide support for instructors and students to make connections. However, those without understanding of the deeper Biocalculus philosophy may still struggle. Some models for peer support and resource sharing have been particularly fruitful; QUBESHub pairs open educational resources with professional development to support instructors in the process of implementation (Akman et al., 2020; Gutiérrez et al., 2024).

However, even reform calculus uptake is still in progress (Bressoud, 2021), without the complications of adding biology. As we move to a modeling and computational framework for calculus, is it still calculus as we have come to know it? While originally the push to incorporate more programming and data into calculus was because of the specific needs of biologists contributing to bioinformatics and ecoinformatics, these skills have become increasingly called for in the newest big data and artificial intelligence push. We have to confront the relevance of the current curriculum to meet these broad challenges and needs in its foundational service courses. We also need to confront how our abstractness and neutrality may not be leaving room for aspects of calculus that could be relevant and humanizing (Diaz Eaton, 2023).

4.2 Opportunities

Instead of asking why we do not change, we might ask: what are we trying to preserve by continuing to leave calculus as is? We have the opportunity and call to enhance our mission as mathematics educators to help improve the transfer of knowledge across disciplines, contribute our techniques to science to create a broader understanding, and care for our students as people with identities that matter and as people who shape our future. (Gutiérrez et al., 2024) define *conocimiento* in the context of teaching mathematics as a form of relational knowing that intertwines mathematics, pedagogies, students, and politics. A *conocimiento* lens "brings the margins to the center by recognizing political issues cannot be separated from other dimensions or added on, as if politics are not already present in mathematics teaching and learning" (Gutiérrez et al., 2024, p.755).

There are still many avenues open for researchers in Biocalculus to understand what helps create successful Biocalculus experiences, particularly among students who are often left behind in traditional calculus instruction. We propose Biocalculus as a unique setting to explore the teaching of social justice, environmental awareness, sustainability and biodiversity topics, opening new avenues for educational research on these topics. As more modeling and meaning-making versions of calculus have been introduced, there are new research questions available about how to assess this type of applied and meaning-making calculus understanding across these multiple instances. There are new assessment alternatives for calculus that could be applied across many calculus contexts. Finally, as Biocalculus moves to be more relevant in an age of big data and computing, there are epistemological and pedagogical shifts for mathematics and biology educators alike that are probable as well as necessary.

4.3 Pedagogical and Epistemological Shifts

At the heart of the "What is Biocalculus" section in our paper are nine approaches that make a strong case for its pedagogical potency. For each, we dive into one specific example (of a particular problem in biology modeled through particular calculus techniques) and/or we present evidence of specific learning outcomes for students. We see student learning as making new narratives: about biology; about calculus; about the purpose and utility of calculus in biology; about their own interest in, and appreciation for, this purpose and utility; about their self-efficacy as users of calculus in biology and so on.

Also encoded are the necessary pedagogical and epistemological shifts that the use of these approaches entails. For example, we aimed that our vignettes have mathematical and biological specificity and we see many places where *all* (not life science only) students can benefit from approaches that

- connect hitherto sparsely-connected mathematical topics (such as limits and derivatives in the "logistic growth" example) and

- provide opportunities to see a tangible example that showcases the differences between mathematical objects (such as exponential and logistic growth in the "logistic growth" example)

4.3.1 Cultivating Deep Mathematical Connections Across Disciplines

The need to see connections between mathematical topics - appreciate what the distinctive features of a mathematical object are - is present in the learning experiences of *all* mathematical learners. And therein lies an epistemological and pedagogical shift that considering Biocalculus approaches may imply for Mathematics educators mainly trained as mathematicians: "carrying capacity", say, is not a mere realization of the abstract mathematical object of "limit" that has relevance and significance to life science students only. What Biocalculus approaches teach us is that, through engaging in an activity that involves, say, the concept of "carrying capacity", calculus learners and users across disciplines - including in Mathematics itself - may learn something about the mathematical object of "limit" they would not learn otherwise. This idea has been explored extensively by those working at the interface of physics and mathematics education (e.g. Redish & Kuo, 2015).

Interdisciplinary learning communities can help instructors practise this reciprocity of learning benefits (Soares et al., 2024). We see in Biocalculus approaches an opportunity for making "fuel" narratives about mathematics - meaningful engagement with a variety of mathematical, and other, "realisations" of a mathematical object as opposed to "fossil" narratives about mathematics - externally imposed, and often abstract, reverence for a mathematical object, yet alienation from its meaning, utility and purpose (Nardi, 2023).

The Biocalculus approaches we make a case for in this paper aim to humanize calculus for life science students via "translating dialects" (where, for example, a conversation about limits becomes a conversation about carrying capacity). The mathematical modeling intervention study in Viirman and Nardi (2019, 2021) is an example that lends additional support for this claim. In that study, the participating students often talked about whether what they were being asked to do was mathematics or biology and, at least at the start, they felt more comfortable when they perceived their task as biological rather than mathematical. As the intervention progressed, the boundaries - and their anxiety or concern about these boundaries - started to fade.

4.3.2 Leveraging the Breadth of Calculus

Another set of boundaries that our paper navigates across is between an exclusive focus on Calculus, while simultaneously defining Biocalculus, as including mathematical modeling, discrete mathematics, and data science. We take issue with "applicationism", a Stewart-like canon narrative about calculus that marvels at its beauty, elegance and powerful abstraction and where applications are just a cheerleading periphery for Mathematics celebrated as the centre. We see taking a pluralist and centrist view in a typically polarised landscape as potentially more productive than a somewhat obsolete *bras de fer* between "applicationism" and a less precious narrative about the rigour and abstraction of mathematics in which mathematics is a useful instrument for advancing work in the disciplines. A more centrist view celebrates beauty and elegance and so on in the way mathematics helps make meaning in applications. The examples in Boxes 1-3 illustrate that applications can be an excellent opportunity to introduce higher order mathematical thinking skills.

Instead of trimming what we know as calculus down to a few theorems, we play in the ways that thinking about change has itself changed. We compare and contrast continuous and discrete approaches and, in so doing, aim to help students understand why it was so groundbreaking to consider what happens when the width of intervals goes to zero. Introducing mechanics of finding derivatives helps us understand the approaches of mathematicians - often very different from the approaches of empiricists, but this appreciation is earned because we have also centered and embraced the power and ubiquity of simple ordinary differential equations in biology. We invite students to play in these spaces with us, ask why, explore ideas with technology, even collect data - moving our classrooms to spaces of active and curious inquiry where students become mathematicians for a moment. By seeing the spaces between the "canon" calculus and biology, not as a cultural *choque* (Anzaldúa, 1987), but instead as a space of connections that can lead to collaborative innovation, we hope to foster a new generation of scientific possibilities.

4.4 Connecting to Current Conversations in the Field

The Biocalculus approaches highlighted in our paper may face epistemological and institutional barriers to change in calculus. There are forces that hold the "canon" sacred. At the Calculus conferences (Dreyfus et al., 2023) that inspired this Special Issue, we sensed some defensiveness on this from some attendees from the mathematics education research community, where many researchers from the "other disciplines" communities seemed less committed to the "canon" and therefore more comfortable questioning it. But there is space now for this conversation to be had. For example, in 2024 at the INDRUM (International Network for Didactic Research in University Mathematics) conference (Florensa et al., 2024), a gathering mostly of mathematics educators, the panel discussion highlighted assumptions that merit questioning about the what and how of Calculus in other disciplines; our paper does some of that questioning for the life sciences.

The panel session was entitled "Mathematics and other disciplines: epistemological issues and their impact on teaching practices at tertiary level". The panelists (chaired by Laura Branchetti, its members were Frank Feudel, Felix Ho, Ricardo Karam and Noemí Ruiz Munzón) had diverse areas of expertise and zoomed in well-known challenges in university mathematics education such as that the perception that mathematics courses at university are of little use for succeeding in subsequent disciplinary courses, as well as in the practice of professional fields. In tandem with our scepticism about "applicationism" (see earlier comment), the panelists shared examples of how professionals in different fields have their own aims and values, necessitating a departure from an "applicationist" and instrumental approach to mathematics courses—an approach predicated on the "illusion of prerequisites" (ibid, p. 50), viewing mathematics as merely a toolbox. Their examples offered designs of interdisciplinary teaching and learning activities, as well as disciplinary teaching that aimed at better integration with other subjects. They spoke of the need for multiple perspectives in a respectful and inclusive environment and stressed several aspects of the disciplines such as reasoning, modeling, and the relationship between the disciplinary knowledge as well as societal and professional priorities.

We see the Biocalculus proposition in our paper as aligned with the efforts described, and examples offered, by the INDRUM2024 panelists - as well as the authors of another IJRUME Special Issue the papers of which resonate greatly with ours (Biza et al., 2022). Biza et al. (2022) use the “filtering” (calculus as a filter, a cause for diminished access and dropping out) / “scaffolding” (calculus taught in ways that allow students to scaffold their appreciation for, and confidence in, mathematical methods, reasoning and abstraction) metaphor to problematise the canonical version of mathematics typically mathematicians have been trying to bring on curricula and students in other disciplines while appreciating some elements of this “preciousness”. They make the case that there is intrinsic tension between the scaffolding and filtering institutional roles of calculus and alert the mathematics education research community to the need for interdisciplinary research that resolves this tension. If the inter-community goodwill evidenced in the INDRUM2024 panel discussion - and the works reported in the *Calculus at the intersection of institutions, disciplines and communities* IJRUME Special Issue (Biza et al., 2022) - is anything to go by, a convergence towards calculus teaching that is empowered by its meaningfulness in other disciplinary contexts (as we hope our paper shows in the disciplinary context of life sciences) may not be that far fetched an aspiration.

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Supplementary Information

S1: Excel file containing the Mauna Loa Observatory carbon dioxide data and the generated figure of the exponential fit.

S2: Python notebook downloaded from Google Colab which contains the code and the resulting figure for the drug dosing model. Much of the code was derived using Colab's built-in Gemini

AI feature, making it easy also for students to generate on their own. This .ipbyn used this generated code as a foundation, but co-authors reformatted and reorganized it for clarity.