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Extended Planckian locus

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Abstract: The Planckian locus is a curve on a chromaticity diagram that records the color of a black body radiator for different temperatures. As temperature increases from 0 to ∞, red, orange, yellow, whitish, and bluish lights are generated, and these are broadly typical of the colors of everyday illuminations. The red for very low temperatures is on the edge of the spectral locus (it is monochromatic), but the bluest blue is in the middle of the chromaticity diagram, far from being a *pure* color. The Wien locus is parameterised by a simpler equation than the Planck locus and runs almost parallel to the Planckian locus. These two loci are so close together that a temperature conversion brings the corresponding chromaticities into an almost complete coincidence. However, the Wien locus is longer—extends more towards the short-wave part of the chromaticity diagram—than the Planck locus for an infinite color temperature. In this paper, we extend the *Planck* and *Wien* formulas to accommodate negative temperatures. The Planckian locus extends only slightly and stops in the middle of the chromaticity diagram. However, the Wien locus naturally extends all the way to intersect the spectral locus, at 360 nm. We show that the extended Wien locus is continuous: negative and positive infinite-color temperatures (the limit of the temperature as it tends to positive *and* negative ∞) converge to the same point. However, there is a substantial discontinuity at the limit of the temperature as it tends to positive *and* negative 0, evidenced by the large chromaticity difference between the violet and red ends of the Wien locus. This mathematical framework provides a firmer theoretical basis for widely used lighting indices such as correlated color temperature, thereby strengthening their practical applicability. Theoretical and practical results of this research are discussed.

1. Introduction

The Planckian locus, also known as the blackbody locus, is a curve that describes the path of the color *appearance* of a blackbody radiator as its temperature T (in Kelvin) changes. This locus is often shown in a standard chromaticity space (see Fig. 1). Here and throughout the paper, we will use (u', v') chromaticities and CIE(u', v') chromaticity diagram, as it is more perceptually relevant [1] than other choices such as CIE(x, y).

In daily life, most lights have a spectral shape that is not well described by the very smooth lights generated by Planck's equation. This is true both for natural light sources such as the sun and sky (which are not as smooth) and artificial ones like incandescent and LED lamps (which are not smooth at all). However, these non-Planckian illuminants can, in analogy to the temperature defining Planckian lights, be characterized by their correlated color temperatures (CCT). The CCT is the color temperature of the Planckian illuminant having the closest chromaticity coordinates (in terms of CIE (u, v) [1]) to those of a given illuminant [2]. Of course, reducing the spectral data to a single parameter such as CCT is inherently reductive. However, CCT is widely used in applied domains such as architecture [3], illumination engineering [4], and specifically in the most recent and modified version of calculating the Color Fidelity Index [4]. Regarding Color

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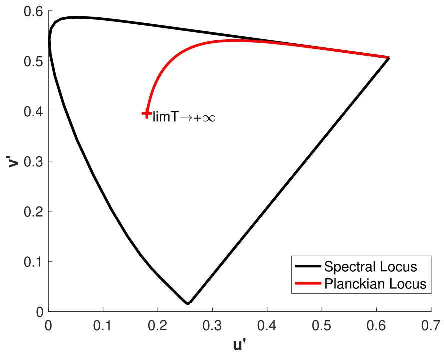
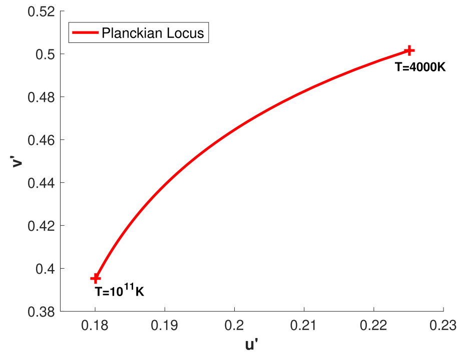


(a)



(b)

Fig. 1. (a) Planckian locus and its infinite-T point in CIE (u', v') chromaticity diagram, (b) Exact (u', v') coordinate of infinite-T of Planckian locus.

Fidelity, in general, lights that score highly tend to lie close to the Planckian locus thus ensuring that CCT is actually a tolerable single-number summary of the color of a light. Importantly, the CCT derived from the concept of color temperature (T), and in this paper we work directly with T, which underpins the CCT.

In Fig. 1, the deep red (the lowest possible temperature: $T \to 0^+$) light intersects the spectral locus. Now, as the temperature of T increases, the perceived color of a Planckian black-body radiator transits through the colors: orange, yellow, white and blue with the bluest light being defined by an infinite color temperature ($T \to \infty$). However, the meaning of high temperatures, represented by ∞ in the diagram, is not yet fully understood. So, this paper starts by deriving the Planckian spectrum at infinite temperature and this facilitates the computation of *exact* color coordinates, such as (u', v') and sRGB.

Robertson [5] reported the infinite T-chromaticity of the Planckian locus in CIE 1960 (u, v) diagram to be at $(0.18006, 0.26352)$. However, Robertson [5] did not explain how the point was derived. For all practical purposes, we might suppose that 10^{11} Kelvin is sufficiently close to $+\infty$ of the Planckian locus, so we can generate spectra and plot the corresponding (u', v') coordinate. A zoomed-in view of the Planckian $+\infty$ is shown in Fig. 1(b). Notice that we transformed this point to the CIE 1976 (u', v') chromaticity as $(0.18006, 0.39528)$ which is $(0.18006, 0.26352)$ (the same as Robertson) in (u, v). In Subsection 3.2 we will analytically redetermine the chromaticity of Planckian infinite-T and we retrieve the same chromaticity coordinates as Robertson, and our intuition that $T = 10^{11}$K was large enough for all practical purposes is shown to be correct. Potentially, we have re-invented Robertson's method and yet our derivation is the first time—to our knowledge—that this limit point has been published.

Keeping in mind the current Planckian locus, recently, the theory of Locus Filters [6] has been proposed as a unique class of transmissive filters that map any Wien–Planckian light of color temperature T_1 to another of color temperature T_2, while ensuring that the transformed spectrum remains on the Wien–Planckian locus. An important aspect of the theory is that a given *Locus Filter* (defined with reference to a given pair of lights) can be applied to any light on the Wien locus and the filtered light is also on the Wien locus. The theory has been introduced on the basis of Wien's law, not the Planckian equation. So, in this paper, we say Wien light instead of Wien-Planckian for convenience. Importantly, in [7], it was shown that we can adjust a given Planckian temperature T to T' such that the spectrum produced by Wien's equation for T' has almost the same spectral shape as the Planckian for T. That is, we can use Wien's equation to generate—to a very good approximation—all Planckian lights. Thus, although Locus Filter theory was introduced for Wien's equation, it actually applies to Planckian lights [7].

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Let us illustrate how Locus Filters work. In Fig. 2, a Wien light of $T_1 = 5500\text{K}$ (red cross) is filtered using a bluish Locus Filter (according to the theory [6]), producing another Wien light of 12222 K as the green circle. Both lights fall *approximately* (we say approximately as Wien and Planck diverge at high temperatures [7]) along the Planckian locus, although they are defined by Wien's equation rather than the Planck function. However, applying the same Locus Filter again generates a bluer light—the blue square—which now lies beyond the conventional Planckian locus. This chromaticity—according to Locus Filter theory—can be produce by integrating a Wien spectrum with a negative temperature of −55000K.

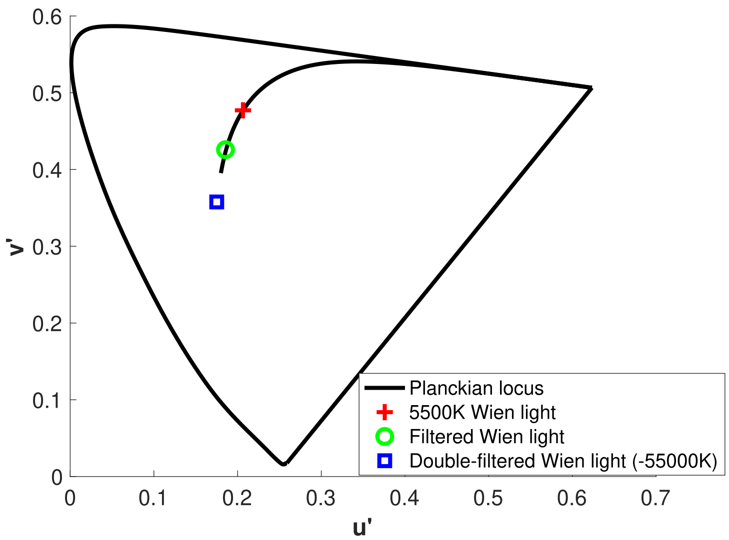

Fig. 2. Illustration of Locus Filter theory [6]. A Wien light of $T_1 = 5500\text{K}$ (red cross) is transformed by a blue Locus Filter with LFT = −10000K, yielding the green circle. Applying the same Locus Filter for the second time produces the blue square, which lies beyond the conventional Planckian locus.

We make several contributions in this paper. First, we derive (possibly rederive) the infinite–T limits for Planck and Wien loci. We will see that the Wien locus pushes further into bluer lights than possible with respect to Planck's equation. As discussed in Locus Filter theory, negative Wien's equation using negative color temperature well describes Wien lights filtered by blue filters whose chromaticities lie beyond the infinite Planckian chromaticity. We can also—as a mathematical stratagem—insert negative temperatures into Planck's equation. As our second contribution, we investigate how the admission of negative temperatures extends the Planck and Wien loci. We show that the extended Wien locus is continuous over any chromaticity diagram and intersects the spectral locus such that it starts from the monochromatic red and ends in the monochromatic violet. In comparison, the Planckian locus extends a little when negative temperatures are admitted but it does not extend to the spectral locus (limiting it's potential utility). Moreover, the Planck extension only really makes sense in the chromaticity diagram representation since negative power spectra are generated (which themselves have no physical meaning). Finally, we discuss an application for the negative-T Wien extension to describe real blue-filtered lights. This is a not-unlikely scenario and pertains to when a bluish color correction filter is applied to an already bluish light.

The remainder of the paper is organized as follows. In Section 2 we present the fundamental equations that will be used. Before presenting the main equations, we summarize in Table 1 the notation and symbols used throughout this paper for clarity. In Section 3, we investigate how negative temperatures extend both Wien and Planckian loci. The extended Wien locus,

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encompassing all temperatures except 0, is presented, and we prove its continuity in chromaticity space. In Section 4 we show some results to demonstrate the usefulness of our research. The paper ends with a brief conclusion.

Table 1. Table of symbols used in this paper.

Symbol	Description
$E_P(\lambda, T)$	Spectral radiant emittance from Planck's formula
$E(\lambda, T)$	Spectral radiant emittance from Wien's formula
T	Color temperature (K), positive or negative in this extension
$Mired$	Reciprocal color temperature (MK^{-1})
λ	Wavelength (m or nm)
c_1	Radiation constant, 3.74183×10^{-16} W·m^2
c_2	Radiation constant, 1.4388×10^{-2} m·K
k	Scalar factor that modulates spectrum intensity
T_{lf}	Locus Filter Temperature (LFT)
F_{Locus}	Locus Filter Transmission Function
u, v	CIE 1960 chromaticity coordinates
u', v'	CIE 1976 chromaticity coordinates
U	Absolute inverse of color temperature, $U = 1/T$
P_i	Color signal in the i^{th} channel
p_i	Chromaticity coordinate for the i^{th} channel
$R_i(\lambda)$	Color-matching function of the i^{th} channel
$M(\lambda), N(\lambda)$	Weighting functions in Wien/Planck locus proofs
$G(U)$	General ratio function used in continuity proofs

2. Background

For a blackbody radiator, the spectral radiant emittance E_P is calculated using Planck's formula as a function of wavelength λ and color temperature T [1] (measured in Kelvin, K):

$$E_P(\lambda, T) = kc_1\lambda^{-5}(e^{\frac{c_2}{T\lambda}} - 1)^{-1} \tag{1}$$

where c_1 and c_2 are two radiation constants equal to $3.74183 \times 10^{-16} Wm^2$ and $1.4388 \times 10^{-2} mK$, respectively. In addition, the scalar k modulates the intensity of the light spectrum.

Although color temperature T is conventionally expressed in kelvin, it could also be represented using its reciprocal $\frac{10^6}{T}$, as Judd [8] suggested that reciprocal color temperature (say Mired), expressed in reciprocal mega-kelvin (MK^{-1}), would be a more convenient parameter for general use than color temperature itself, since differences in reciprocal color temperature are proportional to the corresponding chromaticity differences. However, since the main contributions of this paper are representational—extending the locus and understanding both its intersection points with the spectral locus and between positive and negative temperature loci, as well as its continuity—we mainly stick with temperature T in kelvin. Mired units are also presented in Table 2 (our only table of results).

Another formula is the Wien displacement law that also describes the blackbody radiation E as [1]:

$$E(\lambda, T) = kc_1\lambda^{-5}e^{-\frac{c_2}{T\lambda}} \tag{2}$$

The Wien and Planck functions describe similar spectra for low (say <4000 K) color temperatures. However, the generated spectra become more different the higher the temperatures

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become. We show this in Fig. 3 by plotting two loci in the CIE(u', v') diagram. Notice that in Fig. 3, red text shows the temperature of the Planck locus, while blue text indicates the Wien temperature. Here, to calculate the chromaticity of radiators with different temperatures (4000 to $+\infty$K), we use Planck's and Wien's laws and refer to them as Planckian and Wien loci. The (u', v') point for 4000K is almost coincident for Wien and Planck and so is plotted as a single black point. When temperature increases, two loci diverge such that their infinite temperatures are far from each other, resulting in a visually significant difference in chromaticity [9]. In general, we see mismatch between the Planck and Wien loci for a given temperature such that each point on the Wien locus is cooler than its corresponding point on the Planck locus. However, Daneshvar et al [7], showed that a given Planckian temperature could be corrected to a Wien counterpart so that the Wien spectrum defined with the corrected temperature is a close spectral match to the desired Planckian. Let's elaborate on the details of this correction method [7] in the following subsection.

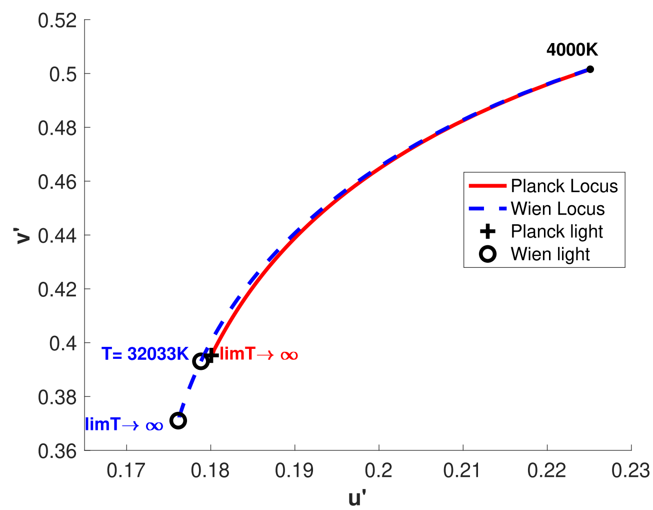

Fig. 3. Common Planckian locus compared to the Wien locus in (u', v') diagram.

2.1. Temperature correction function

To mitigate the discrepancy between Planck and Wien spectra for a given temperature, a *temperature correction function* [7] was defined that maps a Planckian temperature T to a corrected Wien temperature $T' = f(T)$ such that the resulting Wien spectrum more closely approximates the desired Planck spectrum in terms of angular error between two spectra.

The correction problem can be stated formally as:

$$E_P(\lambda, T) \approx \alpha\, E(\lambda, f(T)) \tag{3}$$

where $T' = f(T)$ denotes the corrected Wien temperature. In practice, the mapping $f(\cdot)$ can be constructed in different ways, including look-up-table, polynomial and arctangent functions [7]. Across all formulations, the corrected Wien temperature is always *lower* than the corresponding Planckian temperature ($T' < T$). Accordingly, for all practical purposes, we can use Wien's displacement formula to generate all Planckian spectra as long as a temperature conversion is carried out [7].

Let's assume that the infinite-T chromaticity of the Planck locus is effectively equal to the chromaticity at 10^{11}K (see Fig. 1 for details). This very high temperature converted, using

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Look-up-table correction function of [7], to the Wien domain, it is just 32033 K, as shown in Fig. 3.

In Fig. 4 we plot the 10^{11}K Planckian (red line) and the 32033 Wien (blue dashed line). For reference we also show the 10^{11}K Wien (blue dotted line). All 3 curves have their maximum power normalized to 1. For the same color temperature, the Wien equation always produces a 'bluer' light than Planck's equation, with relatively more power in the shorter wavelengths.

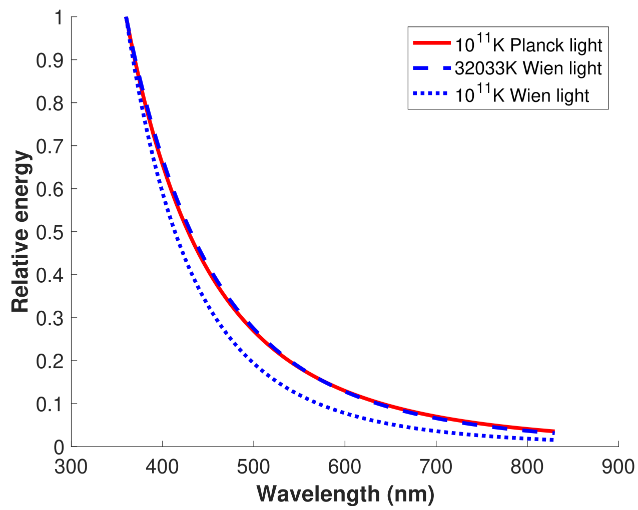

Fig. 4. The spectrum of a Planckian light (solid line) with its corrected Wien temperature (dashed line), compared to an uncorrected Wien light (dotted line).

2.2. Locus filter theory

As we mentioned in the introduction, the motivation for this paper is the theory of locus filter [6,10]. Here, we review the most fundamental equations related to our current research. In Locus Filter theory, if we have a Wien illuminant with a temperature T_1, applying the Locus Filter to that obtains another Wien illuminant with a temperature T_2. By dividing the spectra of these two Wien lights provided by Eq. (2) we obtain a filter transmittance function $F_{Locus}\left(\lambda, T_{lf}\right)$ that has the property:

$$F_{Locus}\left(\lambda, T_{lf}\right) = e^{-\frac{c_2}{T_{lf}\lambda}} \tag{4}$$

where the T_{lf} parameter is the main parameter of Locus Filter called *Locus Filter Temperature* (LFT), and is equal to:

$$T_{lf} = \frac{1}{\frac{1}{T_2} - \frac{1}{T_1}} \tag{5}$$

From Eq. (5), the *LFT* depends on two temperatures, T_1 and T_2. Now, let's apply F_{Locus} to a third light that has a temperature T_3 to make a new light spectrum as

$$E^{filt}(\lambda, T_4) = E^{light}(\lambda, T_3) F_{Locus}(\lambda, T_{lf}) \tag{6}$$

the new light E^{filt} is also lies on the Wien locus with temperature T_4 calculated as (see [6] for details):

$$T_4 = \frac{1}{\frac{1}{T_{lf}} + \frac{1}{T_3}} \tag{7}$$

Interestingly, it is immediate from Eq. (5) that LFTs can be negative. It follows that when we apply a Locus Filter with a negative temperature, T_4 could be negative depending on the value of T_3. Fortunately, Wien's law also works for negative temperatures. When T_4 in Eq. (7) is negative, the corresponding Wien light spectrum becomes bluer than the bluest Planckian light possible. An important and necessary property of the Locus Filter theory—when negative color temperatures are admissible—is that any given Locus Filter must map the entire Wien locus onto itself. These properties of locus filter theory establish a mathematical framework that is directly related to the concept of color temperature. In the following subsection, we review the correlated color temperature (CCT), a widely used index in lighting science, to highlight how our theoretical extension might provide a stronger basis for its definition.

2.3. Correlated color temperature

Correlated color temperature (CCT) is typically calculated by computing the smallest chromaticity difference between the test light source and the Planckian locus in the CIE (u, v) chromaticity diagram [2]. Several different methods have been used to calculate CCT based on the Planckian locus [2,5,11–14]. All current methods for estimating CCT are applicable in a limited range of temperature, beyond their accuracy decreases. For example, the widely used McCamy method [2], which employs a cubic polynomial approximation based on CIE 1931 xy chromaticity coordinates, provides reliable results only within approximately 2856–6504 K. In part because of this range restriction, CCT remains an open research topic in color science and lighting engineering. Significantly, CCT is being investigated in research forum RF-03, under the CIE's research strategy topic "Color Quality of Light Sources Related to Perception and Preference" [15]. See also the proposal of a new definition of CCT (i.e., CIE DR 1-67 [16]). Together, RF-03 and DR 1-67 investigate how CCT is used in lighting practice, its theoretical basis, and how the definition can be usefully extended.

Our work contributes to this active area by providing a firmer mathematical foundation on which indices such as CCT might be defined and extended more consistently. In this sense, our theoretical extension is directly linked to long-term practical goals: improving the reliability and applicability of widely used lighting indices and ultimately supporting their further development in standardization and engineering practice.

In the next section, we propose admitting negative temperatures into both the Wien and Planckian equations. The admittance of negative temperatures allows Wien to generate a continuous path over the chromaticity diagram, sweeping from monochromatic red to the violet region. This new path, the extended Wien locus, describes *Locus Filtered* lights and almost all available real/commercial blue lights, and also accounts for several that are not well modeled by the conventional Planckian locus.

3. Theory: extending Wien and Planck loci

Let us begin by inserting negative temperatures into the Wien and Planck equations. The form of Wien's equation readily admits negative temperatures (see Eq. (2)). Whether T is positive or negative $e^{-\frac{c_2}{T\lambda}}$ is always all positive, so the Wien equation overall returns a all positive spectrum. However, when negative temperatures are inserted into the Planck equation, we *always* obtain negative values for the exponent (i.e., $\frac{c_2}{T\lambda}$<0). Further, when we calculate the exponential function, the largest value we can obtain is 1, $\max_{T\in(-\infty,0)} e^{\frac{c_2}{T\lambda}} = 1$. Substituting into Eq. (1), it follows that over negative temperatures $0 < e^{\frac{c_2}{T\lambda}} < 1$ which implies that Planckian spectra for negative temperatures are all be negatives (which doesn't make physical sense). So, to use negative temperatures, we modify Planck's equation:

$$E_P^m(\lambda, T) = |kc_1\lambda^{-5}(e^{\frac{c_2}{T\lambda}} - 1)^{-1}| \tag{8}$$

where $|.|$ denotes the absolute value function. In Fig. 5, we plot the relative energy of Wien and Planck light with the color temperature of -20000K . Both have their dominant energy in the shorter wavelengths. We see that the $-20000K\ E_P^m$ is bluer than the infinite Planckian, while the $-20000K$ Wien is the bluest in this figure, again indicating that Wien lights are always cooler than the corresponding Planck lights (for the same temperature T).

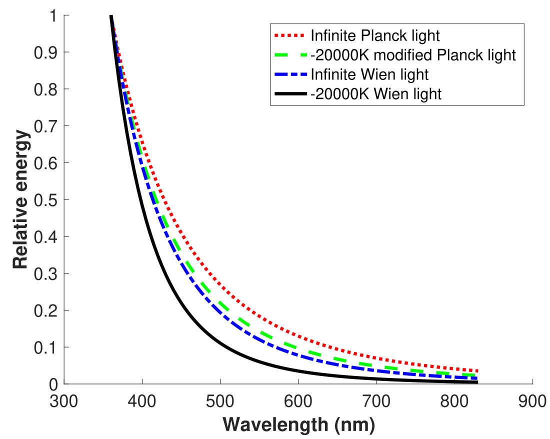

Fig. 5. Spectra at negative temperatures compared to the infinite Planck and Wien lights.

The key concern of this paper is negative color temperatures that produce extended Wien and Planck loci. In the next subsections, we will prove that both extended Wien and Planck loci start from a chromaticity point coincident with their respective $+\infty$ temperature spectrum. For Wien spectra, from $-\infty$ (which has the same chromaticity point as $+\infty$) as the temperature increases to 0^- colors become progressively bluer before eventually ending in monochromatic violet at the spectral locus. The negative temperatures for Planck start at the infinity chromaticity point (which again is the same for both $+\infty$ and $-\infty$). As temperature increases from $-\infty$ the Planckian locus extends to more bluish lights, but stops at the point corresponding to the infinite temperature for Wien's equation (which is far from the spectral locus). We also address the question of whether these extended loci are *continuous everywhere* over the chromaticity diagram. If they are, what is the intersection point of the *positive* (main locus) and *negative* (extended locus) loci?

3.1. Properties of the Wien locus

Historically, Wien's displacement law was superseded by Planck's equation, as it fails to predict blackbody spectra at long wavelengths [1]. In this paper, we do not use Wien's formula as a replacement for Planck's law in radiometry. Indeed, we use it as a mathematical foundation in chromaticity space. Unlike Planck's law, Wien's formulation admits negative temperatures and extends naturally to the spectral locus, thus enabling a continuous curve of color temperature across the entire visible domain. Furthermore, a recent study has demonstrated that using an elegant temperature correction function, Wien's law can closely approximate Planckian spectra in the visible range [7]. Thus, while its physical accuracy is limited, Wien's law provides a unique theoretical framework for the chromaticity-based extension developed in this work.

3.1.1. Wien locus continuity

By continuity, we refer to the absence of disruption in the chromaticity path that Wien's law generates. Although the Wien locus is, by this token, mostly well-ordered in T, there are a few exceptions. Indeed, while $\lim_{T\to0^+}$ and $\lim_{T\to0^-}$ make two different points (two ends of locus), $\lim_{T\to\infty^+}$ and $\lim_{T\to\infty^-}$ make the same point.

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Theorem: Wien locus is a continuous path in an arbitrary chromaticity diagram, and the chromaticity coordinate of the infinite-T of Wien locus is equal to the chromaticity of the light spectrum λ^{-5}.

Proof: Let's rewrite Wien's spectrum (Eq. (2)) in terms of absolute inverse temperature:

$$E\left(U,\lambda\right)=kc_1\lambda^{-5}e^{\frac{-c_2U}{\lambda}} \tag{9}$$

where $U=\frac{1}{T}$.

Now, we calculate the color signal of a Wien light, E, as:

$$P_i=\int_\omega R_i(\lambda)E(U,\lambda)d\lambda \tag{10}$$

where $R_i(\lambda)$ is the response function of the ith sensor for m-dimensional vision system and integration is taken over the interval ω.

A chromaticity is a color projected into a $m-1$ dimensional space such that color signal normalizes to the amount of color vector $\underline{P}$.

$$p_i(U)=\frac{P_i}{\sum_{j=1}^{m}P_j} \tag{11}$$

By substituting Eq. (9) and Eq. (10) in Eq. (11), we have

$$p_i(U)=\frac{\int_\omega R_i(\lambda)c_1\lambda^{-5}e^{\frac{-c_2U}{\lambda}}d\lambda}{\int_\omega\left[\sum_{j=1}^{m}R_j(\lambda)\right]c_1\lambda^{-5}e^{\frac{-c_2U}{\lambda}}d\lambda} \tag{12}$$

Now, we are interested in finding $\lim_{T\to\infty^\pm}p_i$, and this statement is simply equal to find $\lim_{U\to0^\pm}p_i$. Let's assume that color-matching functions—defined as $R(\lambda)$—of any arbitrary kind of observer are bounded positive functions of visible wavelength over a compact domain that does not include $\lambda=0$, where Wien shows troublesome behavior (i.e., infinity). Therefore, it is sufficient for continuity arguments that troublesome points (i.e., U=0) be investigated only for the *General ratio* of Eq. (12). Thus, we define G as the general ratio of Eq. (12):

$$G(U)=\frac{\int_\omega M(\lambda)e^{\frac{-c_2U}{\lambda}}d\lambda}{\int_\omega N(\lambda)e^{\frac{-c_2U}{\lambda}}d\lambda} \tag{13}$$

where $M(\lambda)$ and $N(\lambda)$ are positive weighting functions over the *visible* range. The factor λ^{-5} has been folded into $M(\lambda)$ and $N(\lambda)$ functions to make it clear that we expect no trouble from wavelength. Note that $M(\lambda)$ will be different for each chromaticity coordinate. To evaluate the limit of Eq. (13), the important part to consider is the exponential, *exp*, in Wien's equation. Remembering we are using U (absolute inverse temperature), the infinite-temperature limit can be conveniently evaluated in the U domain, at U=0. There is no pathology, such as an indeterminate form or directionality in the limit at U=0, so direct evaluation at U=0 suffices. Thus, let's determine the limit of Eq. (9) at U=0, so we have:

$$\begin{aligned}\lim_{U\to0^\pm}e^{\frac{-c_2U}{\lambda}}&=1\\ \lim_{U\to0^\pm}E\left(U,\lambda\right)&=kc_1\lambda^{-5}\end{aligned} \tag{14}$$

Now, using the above limit, we calculate the limit of general ratio (i.e., Eq. (13)) for $U\to0^+$ as

$$\lim_{U\to0^+}G(U)=\frac{\int_\omega M(\lambda)d\lambda}{\int_\omega N(\lambda)d\lambda} \tag{15}$$

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Also the limit of general ratio for $U \to 0^-$ is equal to

$$\lim_{U\to 0^-} G(U) = \frac{\int_\omega M(\lambda)d\lambda}{\int_\omega N(\lambda)d\lambda} \tag{16}$$

Finally, for every chromaticity space, we have:

$$\lim_{U\to 0^+} p_i = \lim_{U\to 0^-} p_i \tag{17}$$

Based on our first assumption, this equation is equal to

$$\lim_{T\to +\infty} p_i = \lim_{T\to -\infty} p_i \tag{18}$$

As both limits are equal, we can almost say that the extended-Wien locus is continuous in U everywhere in the domain of $(-\infty, +\infty)$ over the chromaticity space. The exception is the infinite limit of U, which is the absolute zero of T. In that limit, there is a substantial discontinuity between the $+T$ and $-T$ sides of $T = 0$. In chromaticity space, that gap comprises the two ends of the extended-Wien locus.

Now we know that the Wien locus is continuous at the location of infinitely positive and negative color temperatures, let us determine the corresponding chromaticity coordinate.

As we calculated the limit of the ratio in Eq. (15) and Eq. (16), now we write the special ratios, at $U = 0$ that are the chromaticity coordinates of the infinite temperature limit of the Wien blackbody. Now, rather than use the M and N functions, we rewrite the ratio in terms of the color matching functions:

$$\lim_{T\to\infty^\pm} p_i = \frac{\int_\omega R_i(\lambda)\,\lambda^{-5}d\lambda}{\int_\omega [\sum_{j=1}^m R_j(\lambda)\,\lambda^{-5}]\,d\lambda} \tag{19}$$

So, the chromaticity of infinite temperature Wien light is equal to the chromaticity of light spectrum—defined by $\underline{\omega} = [\lambda_1, \lambda_2, \ldots, \lambda_n]$, a $1 \times n$-vector in the visible range—to the power of -5. Notice that this theorem holds true in any system, even a non-trichromatic one. To compute Eq. (19) numerically, we used the CIE 1931 color-matching functions (2° observer) as the spectral sensitivities $R_i(\lambda)$ over the 360–830 nm range, in 1 nm wavelength steps [17]. Evaluating the integral, we find that the chromaticity point corresponding to the infinite Wien color temperature is $(0.17613,\ 0.37104)$ in the CIE (u', v') diagram. Notice that the CIE 1931 color-matching functions are used only as a representative example. The extended Wien locus that we propose is continuous across all types of observers. This means that if alternative functions such as the CIE 1964 or cone fundamentals are used, the extension still holds and the general statements remain unchanged.

3.1.2. Intersection of the Wien and Spectral loci

An interesting property of the Wien locus is that it *appears* to meet the two ends of the spectral locus. Curiously, where it meets is governed by the lowest and highest wavelength where (here) the color matching functions have a non-zero response in one color channel. In this paper we integrated from 360 to 830 nanometers. So, these points on the spectral locus delimit the extended Wien locus. If we had integrated only from (say) 361 to 379 then it would be at these wavelengths where the extended locus would end. Let us prove this *end points of the integration interval property*.

The CIE Standard 15:2004 [18] on standard colorimetric observers recommends that the CIE tristimulus values of a color stimulus be obtained by multiplying at each wavelength the value of the color stimulus function by that of each of the CIE color-matching functions and integrating each set of products over the wavelength range corresponding to the entire visible spectrum, 360 nm to 830 nm. The integration is calculated as a numerical summation at wavelength intervals, $\Delta\lambda$, equal to 1 nm (the fundamental colorimetric tables are the 1 nm tables in CIE standards, and all rigorous calculations should use these 1 nm tables) [18]. Therefore, according to the CIE methodology, we simply change the integral to summation in Eq. (10), obtaining the color signal (here, let's say CIE-XYZ response to a standard illumination) in the discrete domain as:

$$P_i = \sum_{\lambda} R_i(\lambda)E(U,\lambda)\Delta\lambda \tag{20}$$

Now, in this domain, let's consider wavelengths, λ and λ', which are two wavelengths from the integer interval (from 360 nm to 830 nm in 1 nm steps), and assume that the illuminant has power only at one of these two wavelengths. We are interested in finding what happens at a single wavelength of light in the context of the Wien locus and the negative Wien locus. To do this, we calculate the limit of the ratio $\frac{E(U,\lambda)}{E(U,\lambda')}$ expressed in terms of the discrete Delta function, to show that the two ends of the Wien locus intersect with the spectrum locus.

To achieve this, we must demonstrate that as the temperature T approaches zero from the *positive side* (i.e., $U = 1/T \rightarrow +\infty$), the ratio of the spectral energy at a *longer wavelength* to that at a *shorter one* becomes unbounded (say infinity). That is, for $\lambda > \lambda'$:

$$\lim_{U\to\infty^+} \frac{E(U,\lambda)}{E(U,\lambda')} = \lim_{U\to\infty^+} \frac{kc_1\lambda^{-5}e^{\frac{-c_2U}{\lambda}}}{kc_1\lambda^{-5}e^{\frac{-c_2U}{\lambda'}}} = \lim_{U\to\infty^+} e^{\frac{(\lambda-\lambda')c_2U}{\lambda\lambda'}} \tag{21}$$

The result of above limit depends on λ and λ'. Since $\lambda > \lambda'$, then Eq. (21) is equal to:

$$\lim_{T\to 0^+} \frac{E(T,\lambda)}{E(T,\lambda')} = +\infty \tag{22}$$

This confirms that when $T \rightarrow 0^+$, longer wavelengths dominate. This means that the Wien spectrum at longer wavelengths always dominates compared to shorter wavelengths.

We also find the limit of the aforementioned ratio as T approaches 0 from the negative side (i.e., for the violet color), we have:

$$\lim_{U\to\infty^-} \frac{E(U,\lambda)}{E(U,\lambda')} = \lim_{U\to\infty^-} \frac{kc_1\lambda^{-5}e^{\frac{-c_2U}{\lambda}}}{kc_1\lambda^{-5}e^{\frac{-c_2U}{\lambda'}}} = \lim_{U\to\infty^-} e^{\frac{(\lambda-\lambda')c_2U}{\lambda\lambda'}} \tag{23}$$

To ensure that the ratio in Eq. (23) tends to $+\infty$, the exponent must again grow positively without bound. This occurs if $\lambda < \lambda'$, and so:

$$\lim_{T\to 0^-} \frac{E(T,\lambda)}{E(T,\lambda')} = +\infty \tag{24}$$

Hence, as $T \rightarrow 0^-$, it is the *shortest wavelength* that dominates. Finally, from Eqs. (22) and (24), it follows that the spectrum *behaves* like a Dirac delta function in the limit as $T \rightarrow 0$. Only the lower and upper wavelength bounds (e.g., 360 nm and 830 nm) matter in the computation of chromaticities (since the responses here are infinitely larger than the contribution of all other wavelengths in the limit).

3.2. Properties of the Planckian locus

Following the same procedure as with Wien, we apply negative temperatures to Planck's law and explore the resulting behavior of the Planckian locus. As $-ve$ temperature Planckians result in negative power spectra, we could use the modified Planck Eq. (8), which solves the negative power problem by taking absolute values. However, if we are examining properties of the Planckian locus (and a putative extension) we can also use Planck's equation directly (as in this case the numerator and denominator in the chromaticity calculation are both negative—for negative temperatures—and so the negative cancels).

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3.2.1. Properties of the Planckian locus

In the literature related to illuminant color, the Planckian locus end chromaticity point—for bluish colored lights—is for the infinite color temperature. Here, we demonstrate that, interestingly, this end-point actually corresponds to both $-\infty$ and $+\infty$. Unlike Wien, however, admitting $-ve$ temperatures does not extend the modified Planckian locus to the short-wave side of the spectral locus though it does admit some bluer spectra but only so far as those described by positive temperatures in the context of Wien's Equation.

Theorem. The Planckian locus is continuous as the temperature transitions from $-\infty$ to $+\infty$ where the limit spectrum in both cases is equal to λ^{-4}.

Proof: In order to find the chromaticity of infinite (positive and negative) temperature light, we replace Wien's formula in Eq. (12) with Planck's equation.

$$p_i = \frac{\int_\omega R_i(\lambda)c_1\lambda^{-5}(e^{\frac{c_2U}{\lambda}}-1)^{-1}d\lambda}{\int_\omega [\sum_{j=1}^m R_j(\lambda)]\, c_1\lambda^{-5}(e^{\frac{c_2U}{\lambda}}-1)^{-1}d\lambda} \tag{25}$$

As in the Wien proof, we can further simplify Eq. (25) by writing the general ratio, G as:

$$G(U) = \frac{\int_\omega M(\lambda)(e^{\frac{c_2U}{\lambda}}-1)^{-1}d\lambda}{\int_\omega N(\lambda)(e^{\frac{c_2U}{\lambda}}-1)^{-1}d\lambda} \tag{26}$$

where $M(\lambda)$ and $N(\lambda)$ are bounded positive weighting functions over the *visible* range. But equation is not as simple as Wien limit. Thus, one way to simplify this equation is replacing *exp* function by its Maclaurin series expansion (we can use Maclaurin and not Taylor as the point of interest is $U = 0$). Lets write the Maclaurin expansion of *exp* function as

$$e^q = 1 + q + \frac{q^2}{2!} + \frac{q^3}{3!} + \cdots \tag{27}$$

where $q = \frac{c_2U}{\lambda}$, also the zero'th-order term is 1, and the first-order term q dominates and remains (as $U \longrightarrow 0$, as temperature increases). Thus, using Eq. (27) near $U = 0$, we can write

$$\begin{aligned} e^q &\approx 1 + q \\ e^{\frac{c_2U}{\lambda}} &\approx 1 + \frac{c_2U}{\lambda} \end{aligned} \tag{28}$$

It follows by substituting the Maclaurin series

$$\lim_{U\to 0^\pm}(e^{\frac{c_2U}{\lambda}}-1)^{-1} = \frac{\lambda}{c_2U} \tag{29}$$

Now, remembering that $U = \frac{1}{T}$ and substituting in Eq. (1) (where we ignore the scaling term i.e. $k = 1$)

$$\lim_{U\to 0^\pm} E_P(U,\lambda) = \frac{c_1\lambda^{-4}}{c_2U} \tag{30}$$

Now, we calculate the chromaticity coordinates at the infinite-temperature limit of the Planck black-body equation:

$$\lim_{T\to\infty^\pm} p_i = \frac{\int_\omega R_i(\lambda)\,\lambda^{-4}d\lambda}{\int_\omega [\sum_{j=1}^m R_j(\lambda)\,\lambda^{-4}]\,d\lambda} \tag{31}$$

The final result is similar to the final Wien expression (Eq. (19)), except the Wien exponent -5 becomes -4 in the Planck case. Accordingly, the chromaticity (in any system, even a non-trichromatic one) of the infinite-temperature Planck radiator is the chromaticity of λ^{-4}.

When we evaluate this integral we find that the infinite color temperature chromaticity point of Planckian locus is $(0.18006, 0.39528)$ in CIE (u', v') diagram. Notice that we only show five decimal numbers to demonstrate that our derivation recovers the same chromaticity as Robertson [5]. It is entirely possible that Robertson followed a similar line of reasoning to ourselves. In this case, we are happy we can make his derivation available to the community.

3.2.2. Convergence point of Planckian and Wien loci

In this section, we show why the Planckian formula, Eq. (1), doesn't reach the short-wave end of the spectrum locus from a mathematical point of view. Through this, we determine to what extent the original Planckian locus can be extended, helping to identify the chromaticity of $\lim_{T\to 0^-}$ for the Planckian locus. In so doing, we show that the current Planckian locus can be slightly extended.

Theorem: The chromaticity coordinate of the $0^- K$ of Planckian locus is equal to the chromaticity of the light spectrum λ^{-5}.

Proof: Here, we simply need to determine the limit of Eq. (26) when $U \to \infty^-$. To solve this equation, we should find the limit of Planck's law in Eq. (1) when $U \to \infty^-$ as

$$\begin{aligned} \lim_{U\to\infty^-} (e^{\frac{c_2 U}{\lambda}} - 1)^{-1} &= -1 \\ \lim_{U\to\infty^-} E_P(U, \lambda) &= -c_1 \lambda^{-5} \end{aligned} \tag{32}$$

Now lets substitute Eq. (32) in Eq. (26) as

$$\lim_{U\to\infty^-} G(U) = \frac{\int_\omega M(\lambda) d\lambda}{\int_\omega N(\lambda) d\lambda} \tag{33}$$

Let's convert to the temperature domain as $T \to 0^-$:

$$\lim_{T\to 0^-} p_i = \frac{\int_\omega R_i(\lambda)\, \lambda^{-5} d\lambda}{\int_\omega [\sum_{j=1}^m R_j(\lambda)\, \lambda^{-5}]\, d\lambda} \tag{34}$$

Now, we see that what we derived here as Eq. (34) is equal to Eq. (19) (chromaticity of Wien at ∞). This statement confirms that Wien and Planck, after diverging at high temperatures, converge again at the same point on the chromaticity diagram. Interestingly, for Planck, this coordinate corresponds to a Planckian light with $T \to 0^-$, while for Wien, it corresponds to a Wien light with $T \to \pm\infty$.

4. Theoretical and experimental results

4.1. Extended Wien and Planck loci

Figure 6(a) shows the extended Planck and Wien loci in the CIE(u', v') chromaticity space. Based on our proof, we determine the chromaticity coordinates of both positive and negative infinite temperature (both Wien and Planck loci) in the 2-dimensional CIE (u', v') diagram. We see that the positive and negative **Planckian** loci connect at the same (u', v') point $(0.18006, 0.39528)$, shown as a cross. While positive and negative **Wien** loci connects at the same (u', v') point $(0.17613, 0.37104)$, shown as a circle.

While both loci continuously start from the right at monochromatic red, the Wien locus ends on the left with monochromatic violet, whereas the Planckian locus ends in the middle of the chromaticity diagram with a non-saturated blue. Indeed, Wien locus from right to left the temperature monotonically increases from $\lim_{T\to 0^+}$ to $\lim_{T\to\infty^+}$. Also, in the extended Wien locus, from left to right the temperature decreases from $\lim_{T\to 0^-}$ to $\lim_{T\to\infty^-}$ to reach the original

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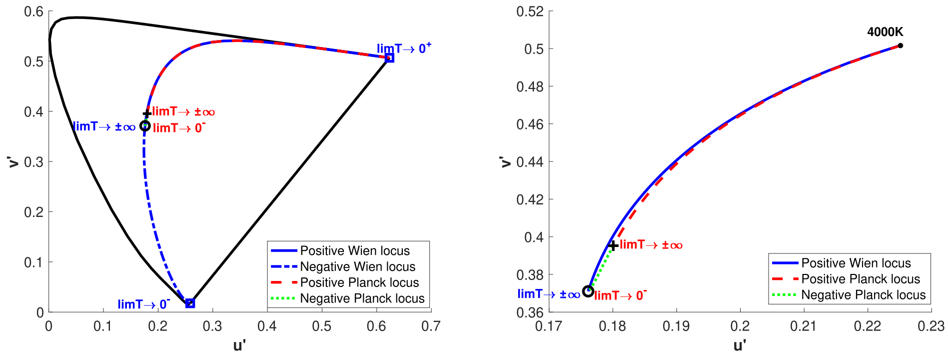

(a) (b)

Fig. 6. (a) Wien and Planck loci with their extension, (b) Zoomed-in, convergence point of Planck and Wien loci in chromaticity diagram (red text is for Planck and blue text is for Wien temperatures).

Wien locus at infinity. Regarding the chromaticities for $T \longrightarrow 0^+$ and $T \longrightarrow 0^-$, these are indicated as points respectively at 830 and 360 nanometers. We proved that as the temperature decreases towards 0 the limits must be the chromaticities recorded for Delta functions at the max and min wavelengths over which the spectral sensitivities of the visual system are defined.

In Table 2, we determine the CIE (u', v') chromaticity coordinates with 5-decimal point accuracy for both the Planck and Wien loci at the four *troublesome* color temperatures (T), which are crucial in describing the behavior of these loci. These temperatures include 0^+, 0^-, ∞^+, and ∞^-, corresponding to the positive and negative sides of the extreme points. In preparing this table, we considered the wavelength range from 360 nm to 830 nm, with increments of 1 nm.

Table 2. CIE (u', v') chromaticity coordinates for Planck and Wien loci at *troublesome* temperatures T or Mired.

Mired (MK^{-1})	T (K)	Planck		Wien	
		u'	v'	u'	v'
∞^+	0^+	0.62337	0.50650	0.62337	0.50650
∞^-	0^-	0.17613	0.37104	0.25890	0.01757
0^+	∞^+	0.18006	0.39528	0.17613	0.37104
0^-	∞^-	0.18006	0.39528	0.17613	0.37104

We see that the Wien locus shows two distinct points for 0^+ and 0^-, with coordinates of (0.62337, 0.50650) for 0^+ and (0.25890, 0.01757) for 0^-, indicating a discontinuity in $T = 0$. Also, the Planckian locus has discontinuity in $T = 0$. However, for both loci, the chromaticity coordinates at the positive and negative infinity points are identical, indicating the continuity of the loci in CIE (u', v') diagram.

4.1.1. Convergence of the Wien and Planck loci

We previously noted that the Planck and Wien loci are close at low temperatures; however, as the temperature increases, they diverge [7]. The extent of this divergence has, to our knowledge, not been well defined anywhere. In this paper, we quantify the magnitude of divergence and prove that two loci ultimately meet at a common point (where positive and negative temperatures are considered for Planck and only positive temperatures are used for Wien). In Fig. 6(b), we observe that, despite the divergence at high temperatures, they reconverge at the (u', v') point (0.17613, 0.37104), which corresponds to the infinity of the Wien locus.

4.2. Blue filtered Granada daylights

In this section, we run an experiment to show how the extended Wien locus can describe real filtered lights that the Planckian locus cannot. Here, we use the Granada daylights (the entire dataset contains 99 daylights) [19], and the convex hull of these lights is shown in red in Fig. 7.

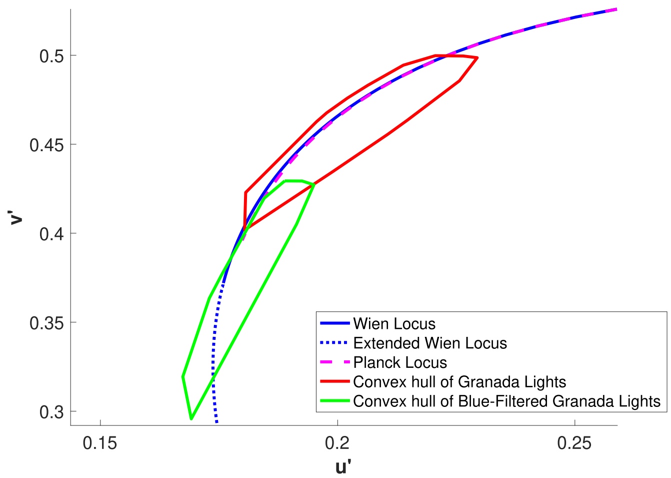

Fig. 7. The convex hull of Granada lights and Kodak-blue filtered Granada lights in CIE (u', v') chromaticity diagram.

In measurement, film and photographic applications it is common to place colored filters in front of lights (to make them warmer or cooler in color), for example Wratten filters that modify light in various ways are widely used in photography and scientific research. These filters are categorized on the basis of their effects on the acquired image and are identified using a coding system, where each filter is assigned a unique code.

Let us now consider how the Granada chromaticities shift when a 'cooling' color filter is applied on the whole dataset. We use a *blue* Kodak Wratten filter, specifically the 80A, which belongs to the color-conversion category. It is well known that color correction filters enable significant adjustments to the color temperature of an illuminant, shifting it toward a cooler (as in the case of the 80*A*) or a warmer tone. We apply the 80*A* filter to the daylight illuminants in the Granada dataset, and the convex hull of the filtered lights is shown in green in Fig. 7. We see that the hull exceeds the Planckian locus, indicating the need for another approach, such as our extended Wien locus, to represent it more accurately.

4.3. Describing commercial lamps

As a real case study, we selected the spectral power distribution (SPD) of a blue lamp from a dataset of spectra of commercial lamps [20,21]. This data set includes various types of real lights such as LED, Metal Halide, and Neon lights. Figure 8(a) shows the relative energy of a blue Neon lamp in the range 360–830 nm with a 1 nm step; its peak indicates that it is a blue light. The chromaticity of this lamp is shown as a red square on the CIE (u', v') diagram in Fig. 8(b). It is clear that this point lies outside the range of current CCT estimation methods, while being very close to the extended Wien locus. To elaborate, we identified the closest Wien light to this lamp by minimizing $D_{u'v'}$. The closest Wien light, shown as the green circle in Fig. 8(b), has $D_{u'v'} = 0.0189$ to the Neon lamp, whereas the infinite point of the Planckian locus, given by $\lim_{T\to\infty^+}$, has $D_{u'v'} = 0.2700$, indicating a noticeable color difference [9]. According to our calculation this light has a CCT of −2814 K.

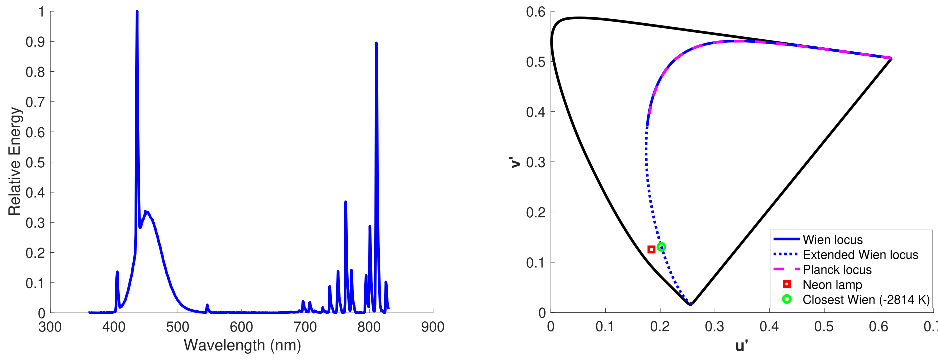

Fig. 8. (a) Relative energy of a blue Neon lamp [21], (b) CIE (u', v') chromaticity of blue Neon lamp.

5. Conclusion

The Planckian locus 'stops' for infinite color temperature. The infinite temperature Planckian light spectrum, though bluish, is far from being either the bluest blue commercially available or theoretically admissible lights. The Wien locus—which runs (almost) coincident to the Planckian locus—only accounts for slightly bluer lights than those allowed by Planck's equation. In this paper we developed the *extended* Wien locus which can account for almost all typical lights from monochromatic red, through oranges, yellows, whites, blues to monochromatic violet. A key notion in our work is the idea of negative color temperature. The equation defining the Wien displacement law naturally admits positive and negative temperatures in a way that concomitantly extends the locus. The extended locus allows us to describe many more commercially available lights by their correlated color temperatures than is currently possible and, moreover, allows us to always determine the correlated color temperature of a light that is filtered by a bluish correction filter; beyond this, our extended locus is a natural requirement of Locus Filter Theory [6,10].

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Data availability. No data were generated or analyzed in the presented research.

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