



# Mutual fund performance and flow-performance relationship under ambiguity

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## ABSTRACT

Since the exact probability distribution of asset returns is often unknown, the type of uncertainty affecting financial assets may be better characterized as ambiguity rather than risk. Using data from the U.S. mutual fund market, we examine the relationships between mutual funds' ambiguity exposure, risk-adjusted performance, and investment flows. We introduce a novel measure of ambiguity exposure based on the smooth ambiguity model, which provides insight into how funds are priced in the presence of ambiguity. We find that risk-adjusted fund returns include a positive premium that compensates for greater ambiguity exposure in the fund's asset holdings. The flow analysis, however, suggests that fund investors pursue positive risk-adjusted returns overall, regardless of whether seemingly superior returns are driven by the ambiguity premium. This behavior indicates that fund investors are primarily attracted to performance outcomes and less concerned with whether these reflect managerial expertise or increased ambiguity exposure.

## 1. Introduction

Recent developments in decision theory have greatly enhanced our understanding of decision-making under ambiguity and its impact on financial markets. Ambiguity occurs when the exact probability distribution of outcomes is unknown to decision-makers, setting it apart from risk – another type of uncertainty – where the exact distribution is known.<sup>1</sup> Most financial assets can thus be said to exhibit ambiguity, in contrast to raffle tickets which pose risk by offering a fixed, say 1-in-10, chance of winning. In an influential study, [Klibanoff et al. \(2005\)](#) develop the smooth ambiguity model, which generalizes expected utility theory by distinguishing ambiguity aversion from traditional risk aversion. [Maccheroni et al. \(2013\)](#) further adapt this new model for portfolio analysis, developing a representation that aligns with the classic mean–variance utility function. [Mukerji et al. \(2019\)](#), later published as [Mukerji et al. \(2023\)](#), show that given this representation and heterogeneous ambiguity preferences across investors, risk-adjusted returns in the market model can be seen as an ambiguity premium, which compensates for an asset's exposure to systematic ambiguity.

Using monthly data from the U.S. mutual fund market, we examine the impact of a fund's ambiguity exposure on its risk-adjusted performance and investment flows. To measure the ambiguity exposure, we apply a metric inspired by the theoretical literature on smooth ambiguity aversion. This measure, referred to as delta, is calculated by subtracting the fund's standard market beta from its ambiguity-related beta. The latter represents the sensitivity of alternative signals regarding the fund's expected return

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<sup>1</sup> As [Etner et al. \(2012\)](#) point out, the nomenclature in decision theory can be confusing to navigate: *Uncertainty*, without any qualifier, is sometimes used as a synonym for *ambiguity* to describe a “non-probabilized” uncertainty, as opposed to *risk* which describes a “probabilized” uncertainty.

to overall market movements. Following recent studies incorporating decision theory into asset returns analysis (Barberis et al., 2016; Li et al., 2017; Gu and Yoo, 2021), our approach treats the projection of the fund's future returns as a lottery, interpreting its historical average returns across various periods as indicators that may give mixed signals about future outcomes. Typically, a fund's risk-adjusted returns – often called alpha – are attributed to the fund manager's skills, such as their ability to select stocks effectively. Our study investigates whether this alpha may be attributed to the fund's delta, which captures its exposure to a type of uncertainty that traditional risk metrics do not account for. By also examining whether fund flows decrease when delta contributes to a significant portion of alpha, we assess whether investors are drawn to overall risk-adjusted returns or specifically to those components linked to managerial skills.

We find that risk-adjusted fund returns incorporate a positive premium that compensates for greater ambiguity exposure. Funds in higher deciles of ambiguity exposure tend to outperform funds in lower deciles, in the sense that the average value of alpha tends to increase across the delta deciles. In relation to the asset pricing equilibrium predicted by the smooth ambiguity model (Mukerji et al., 2019), the significant alpha-delta correlation suggests that the market for the underlying equity assets comprises of investors with heterogeneous ambiguity preferences, and its positive sign suggests that at least some of those investors are ambiguity-averse. This positive relation between alpha and delta is robust to the use of two-way fixed effects models that explain fund performance using ambiguity exposure, fund fixed effects, and year-month fixed effects along with a standard set of observed fund characteristics. This relation also holds on a within-month basis, in the sense that funds with relatively high levels of ambiguity exposure in a particular month tend to outperform those with relatively low levels of exposure in the same month.

Our findings on the flow-performance relation, however, suggest that fund investors chase positive risk-adjusted returns as a whole, without discriminating against higher values of alpha that arise from greater ambiguity exposure. We assume that a fund's flow depends on how well it has recently performed relative to other funds in terms of alpha (Sirri and Tufano, 1998), and augment the implied model specification with a measure of its relative ambiguity exposure, based on delta, over the same period. In a two-way fixed effects analysis that controls for fund and year-month fixed effects along with observed fund characteristics, the fund flow is estimated to show a positive response to an improvement in the fund's alpha rankings, but a statistically and/or quantitatively insignificant response to a change in its delta rankings. In other words, fund investors, on average, appear to show indifference between two alternative funds which have achieved equally strong performance, even though one may have exposed itself to greater ambiguity in the process than the other. One might hypothesize that institutional investors have better resources to assess a fund's ambiguity exposure level than retail investors, but we do not find that this nexus of fund flow, performance, and ambiguity exposure differs between the two types of investors.

Since our measurement approach relies on a theoretical construct – delta – that has not been applied to the fund market previously, we also examine which funds display greater ambiguity exposure in terms of this measure. Ambiguity in the present context arises as investors are unable to form precise expectations about a fund's future performance. Our results support the construct validity of delta in the sense that this measure tends to take higher values for those funds with characteristics that can be plausibly linked to difficulties in forming such expectations. In regards to size, age, and turnover, funds with higher delta tend to have smaller asset holdings, be newer to the market, and have lower turnover, the last of which may reflect the role of active management in mitigating the effects of unexpected developments in the market. Concerning historical returns, a fund's delta tends to be higher when different moments of its return distribution send conflicting signals about its attractiveness – specifically, when a fund that appears less attractive based on mean returns looks more attractive in terms of standard deviation and skewness. Finally, with respect to the asset pricing factors in the Carhart model (Carhart, 1997), funds with higher delta tend to have lower loadings on the momentum factor. This suggests that their asset holdings are more dissimilar to momentum portfolios, making it more difficult for investors to exploit market momentum to predict their future performance.

Our study makes several contributions to existing literature. First, it complements experimental research on ambiguity attitudes in controlled environments with findings from the mutual fund market. Typically, experimental studies elicit ambiguity attitudes in laboratory (Ahn et al., 2014; Stahl, 2014; Abdellaoui et al., 2015) or lab-in-the-field (Dimmock et al., 2016) settings that link choices over artificial lotteries to earnings from the experiment. The mutual fund market serves as an ideal natural laboratory since fund investors are primarily motivated by returns (Choi and Robertson, 2020) rather than confounds like controlling company interests, with mutual funds being a major component of U.S. household portfolios.<sup>2</sup> The experimental literature shows wide variability in ambiguity attitudes, spanning the entire spectrum of ambiguity seeking, neutral, and averse behavior. In a pricing equilibrium predicted by the smooth ambiguity model, the positive alpha-delta relation that we estimated is consistent with these results. That we are able to broadly replicate the experimental results using the mutual fund data is a finding that supports the predictive validity of the smooth ambiguity model.

Second, our findings suggest that considering ambiguity may offer a unifying explanation for two seemingly unrelated anomalies in risk-adjusted financial asset returns. Frazzini and Pedersen (2014) sparked renewed interest in the *beta anomaly*, which highlights that stocks with higher market betas tend to underperform in terms of risk-adjusted returns. Bali et al. (2017) show that conventional risk factors in asset pricing models fail to fully account for macroeconomic risks, as risk-adjusted stock returns decrease with *uncertainty beta*, which measures a stock's sensitivity to macroeconomic uncertainty in their study. The measure of ambiguity exposure we adopt from the smooth ambiguity literature, delta, represents the difference between a fund's ambiguity-related beta and market beta, with the former acting as an alternative measure of a fund's sensitivity to macroeconomic uncertainty (Mukerji

<sup>2</sup> According to the 2021 Investment Company Fact Book ([https://www.ici.org/system/files/2021-05/2021\\_factbook.pdf](https://www.ici.org/system/files/2021-05/2021_factbook.pdf)), 47.4% of all U.S. households own mutual funds.

et al., 2019). We find that the positive net effect of delta on alpha occurs because the negative effect of ambiguity-related beta is outweighed by the larger negative effect of market beta. These findings suggest that the two anomalies are interconnected aspects of how ambiguity exposure influences risk-adjusted performance.

Third, the insignificant flow response to ambiguity exposure in our study aligns with two recent findings on the flow-performance relationship. Evans and Sun (2021) suggest that improved fund performance generates a positive flow response not because investors actively seek alpha *per se*, but because they use heuristic decision criteria correlated with alpha. Since our measure of ambiguity exposure, delta, is an abstract construct based on a very recent theoretical development, it is unlikely to have been incorporated into a heuristic investing strategy during our sampling period (1999–2019). Additionally, in a study of defined contribution pension plans, Christoffersen and Simutin (2017) note that fund managers may strategically increase market beta to outperform benchmark indices, as higher market beta does not appear to deter pension plans from investing in the fund. Our findings suggest that fund managers may similarly leverage increased ambiguity exposure to improve alpha.

Finally, our results suggest that identifying the sources of shared variation across alternative behavioral predictors of fund flows offers an intriguing opportunity for future research. Several recent studies (Li et al., 2017; Gu and Yoo, 2021; Artavanis and Eksi, 2024) have reported significant flow responses to variables measuring the attractiveness of a fund's historical return distribution under the maximin expected utility model (Gilboa and Schmeidler, 1989) and prospect theory (Tversky and Kahneman, 1992). Unlike delta in our study, these behavioral predictors are not formal indices of ambiguity exposure. Nevertheless, we find that they closely relate to the fund's ambiguity-related beta which, along with its market beta, is one of the two components of delta. Moreover, despite the overall flow insensitivity to delta, we observe a significant flow response to ambiguity-related beta when it is included explicitly as a distinct regressor, rather than implicitly via delta. Evans and Sun (2021) suggest that flow responses to alpha – risk-adjusted returns – arise from alpha's correlation with Morningstar ratings, which provide more accessible information to investors. Given the lack of a formal theory to compare the growing list of behavioral predictors, studying whether these predictors approximate common, accessible information for investors could offer valuable insights into investor decision-making.

## 2. Conceptual framework

Our empirical analysis is motivated by a context-free theory of decision making under ambiguity due to Klibanoff et al. (2005) and its extensions to financial asset pricing due to Maccheroni et al. (2013) and Mukerji et al. (2019). The first of these studies develops the smooth ambiguity model (also known as the KMM model after Klibanoff, Maccheroni, and Mukerji) that separates the notion of ambiguity, which refers to a type of uncertainty, from ambiguity aversion, which refers a type of attitude to uncertainty. As Ahn et al. (2014) explain graphically, this model is analytically tractable because it implies smooth indifference curves over state-dependent atomic securities, in contrast to earlier theories such as the maximin expected utility model (Gilboa and Schmeidler, 1989) that result in kinked indifference curves.

At the core, the smooth ambiguity model is a two-stage expected utility model that draws an analogy between ambiguity and risk. Let  $r$  denote an outcome of a lottery or risky prospect which is represented by the density function  $f(r|\theta)$ , where  $\theta$  is some parameter that describes the statistical distribution of  $r$ . Under classical expected utility theory, the decision maker's evaluation of this lottery is given by

$$EU_\theta = \int u(r)f(r|\theta)dr \quad (1)$$

where  $u(\cdot)$  is a utility function which is concave if the decision maker is risk-averse and convex if risk-seeking. In the smooth ambiguity model, ambiguity is related to the notion that the decision maker is uncertain about exactly what type of risk they are facing; that is, they have several candidate distributions for  $f(r|\theta)$  in mind. This “second-order uncertainty” or ambiguity can be operationalized by seeing the type of distribution, summarized in  $\theta$ , as an uncertain outcome that is distributed according to its own density function  $g(\theta|\omega)$ , where the hyper-parameter  $\omega$  describes the probability distribution of  $\theta$ . Under ambiguity, the decision maker's evaluation of the lottery is then represented by the expected utility of expected utilities

$$KMM_\omega = \int v[EU_\theta]g(\theta|\omega)d\theta = \int v\left[\int u(r)f(r|\theta)dr\right]g(\theta|\omega)d\theta \quad (2)$$

where  $v[\cdot]$  is a second-order utility function which is concave if the decision maker is ambiguity-averse and convex if ambiguity-seeking.

In a subsequent work that establishes a link between the smooth ambiguity model and the mean–variance portfolio analysis, Maccheroni et al. (2013) put the notion of ambiguity on a more concrete footing by associating the uncertain parameter  $\theta$  with the expected value of asset returns. Specifically, suppose that an investor is asked what the expected return to a financial asset is. If the investor simply sees the asset as a risky prospect, they will be able to provide a single figure, say 2.2%. If the investor perceives ambiguity, however, they will quote several candidate figures and provide a qualified response, such as that they are 70% sure that the expected return is 1% and 30% sure that it is 5%. In relation to Eq. (2), the two candidate answers 1% and 5% correspond to two possible values of  $\theta$ , and the qualifying probabilities 70% and 30% make up its probability distribution  $g(\theta|\omega)$ .

As well-known, if  $u(\cdot)$  is an exponential utility function and  $f(\cdot)$  is a normal density function, the expected utility functional in Eq. (1) coincides with the mean–variance utility specification, the workhorse objective function in portfolio analysis. Maccheroni et al. (2013) show that the preference functional of the smooth ambiguity model in Eq. (2) can be seen as a dual representation of an extended mean–variance utility specification that incorporates a penalty for uncertainty about  $\theta$ . Specifically, they show that if  $u(\cdot)$

is an exponential utility function,  $v(\cdot)$  is a power utility function, and both  $f(\cdot)$  and  $g(\cdot)$  are normal density functions, the smooth ambiguity model implies an extended utility specification

$$MV_{KMM} = MV_{EU} - a\sigma_{\mu_r}^2 = (\mu_r - b\sigma_r^2) - a\sigma_{\mu_r}^2 \quad (3)$$

where  $MV_{EU}$  is the usual mean–variance utility given the unconditional expectation ( $\mu_r$ ) and variance ( $\sigma_r^2$ ) of asset returns  $r$ ;  $\sigma_{\mu_r}^2$  is the variance of the investor's beliefs (i.e., that of their candidate answers to the question of what the expected return is); and the preference parameters  $b$  and  $a$  represent the investor's aversion to the risk measure  $\sigma_r^2$  and the ambiguity measure  $\sigma_{\mu_r}^2$ , respectively.

In a setup where investors display the extended mean–variance preferences given by Eq. (3), Mukerji et al. (2019) derive a remarkable result that relates non-zero “alpha” or risk-adjusted asset returns to the distribution of ambiguity preferences in the investor population. They show that in equilibrium, the relation between expected asset returns ( $E[r]$ ) and expected market returns ( $E[r^{mkt}]$ ) takes the form of the usual market regression

$$E[r] - r^{free} = \alpha + \beta_{mkt}(E[r^{mkt}] - r^{free}) \quad (4)$$

where  $r^{free}$  is the risk free rate, and  $\alpha$  and  $\beta_{mkt}$  denote the asset's alpha and beta, respectively. Furthermore, they show that alpha can be expressed as

$$\alpha = \kappa(E[r^{mkt}] - r^{free})(\beta_{mkt}^{UNC} - \beta_{mkt}) \quad (5)$$

where  $\kappa$  is a constant term which is a function of population heterogeneity in ambiguity aversion, and  $\beta_{mkt}^{UNC}$  is an ambiguity-related beta that measures how sensitive the beliefs about the expected return are to general moves in the market, similarly as the usual beta  $\beta_{mkt}$  measures how sensitive the asset returns *per se* are to general moves in the market. The value of  $\kappa$  is non-zero if the degree of ambiguity aversion represented by the parameter  $a$  in Eq. (3) varies across different investors, and the sign of  $\kappa$  is positive if at least some of those investors are ambiguity-averse. Thus, in a heterogeneous population that includes ambiguity-averse investors, the asset's alpha captures an ambiguity premium that increases in the beta difference,  $(\beta_{mkt}^{UNC} - \beta_{mkt})$ .

We will define “delta” as shorthand for the asset's ambiguity exposure measured by the beta difference:  $\Delta^{UNC} := (\beta_{mkt}^{UNC} - \beta_{mkt})$ . In this connection, the ambiguity-related beta,  $\beta_{mkt}^{UNC}$ , may be seen as a measure of the asset's gross ambiguity exposure whose *net* part,  $\Delta^{UNC}$ , is priced in equilibrium and the rest is not. On a more concrete footing the ambiguity-related beta may be seen as a measure of the asset's sensitivity to general macroeconomic uncertainty (Mukerji et al., 2019, pp. 32–33). We will refer to  $\beta_{mkt}$  as “market beta” and  $\beta_{mkt}^{UNC}$  as “uncertainty beta” henceforth, to better distinguish them from each other as well as the equilibrium measure of ambiguity exposure, delta.

The financial asset of interest in our analysis is the mutual fund. As summarized above, the smooth ambiguity model suggests that the fund alpha, which is often seen as a baseline measure of the fund manager's stock-picking skills, may be confounded with the ambiguity premium that compensates for the fund's ambiguity exposure in the sense of second-order uncertainty about its expected return. Our first objective is to evaluate whether a positive relation exists between the fund alpha and the fund delta: that is, whether there is evidence of a positive ambiguity premium, as one would expect in a plausible scenario where investors have heterogeneous ambiguity attitudes and ambiguity-averse investors are present in that heterogeneous population. Laboratory (Abdellaoui et al., 2015; Stahl, 2014) and field (Dimmock et al., 2016) experiments typically find heterogeneity in individual ambiguity attitudes that spans both ambiguity-seeking and ambiguity-averse behavior. Our second objective is to analyze flow-performance relationship and evaluate whether fund investors strip away the ambiguity premium from other components of alpha when they make fund allocation decisions. Both objectives require that we operationalize the empirical measurement of the uncertainty beta. To construct proxies for the uncertainty beta, we adopt a similar perspective as recent studies that model financial assets as lotteries over past returns (Barberis et al., 2016; Zhong and Wang, 2018; Gu and Yoo, 2021) and apply a rolling window regression procedure to successive 60-month periods. More detailed information is available below in Section 3.2.

### 3. Data

#### 3.1. Database and sample selection

Our mutual fund sample is selected from CRSP U.S. Survivorship-Bias-Free Mutual Fund Database, which supplies information on monthly fund returns, total net assets (TNA), fund management structures, and other fund characteristics. We focus on actively managed diversified domestic mutual funds and our sample excludes international, bond, money market, and index funds. We remove all non-U.S. funds by excluding funds with CRSP investment objective codes that do not start with “ED” (Equity Domestic).<sup>3</sup> Further, to better capture mutual funds which trade mainly in stocks, we select funds that held an average of 70% or more of their assets in common stock ( $per\_com > 70\%$ ). Finally, to mitigate small-fund bias, we exclude fund share classes with TNA less than 1 million dollars.

This gives us an unbalanced panel of 14,728 fund share classes, collectively observed across 246 calendar months from January 1999 through June 2019. Two considerations motivate our choice of 1999 as the starting point of the study period. First, in the CRSP Mutual Fund Database, data prior to 1999 were primarily compiled using a different and potentially more error-prone method

<sup>3</sup> The details of CRSP investment objective code can be accessed from [https://www.crsp.org/wp-content/uploads/guides/CRSP\\_US\\_Mutual\\_Funds\\_Guide\\_SAS\\_ASCII\\_R.pdf](https://www.crsp.org/wp-content/uploads/guides/CRSP_US_Mutual_Funds_Guide_SAS_ASCII_R.pdf).

based on transcriptions from printed materials. The current provision of electronic records by Lipper and Thomson Reuters dates back to only August 1998. The start of our sample period therefore corresponds to the first full calendar year for which electronically recorded data are available. Second, daily net asset value (NAV) data at the fund level are available only since September 1998. While our analysis focuses on monthly flows and returns, restricting the sample to the post-1998 period facilitates future comparisons with studies that would utilize the daily data.<sup>4</sup>

Similarly as Huang et al. (2011) and Li et al. (2017), we treat each fund share class as an individual fund. Mutual funds typically report performance for each share class separately, even when the stock holdings remain identical across various share classes of the same mutual fund. Moreover, we are interested in testing for the difference between retail and institutional shares investors, which can only be distinguished at the share class level. Since our analysis considers fund flows, treating each fund share class separately will not lead to a double-counting problem. We note, nevertheless, that our main findings below remain qualitatively unchanged when we test our hypotheses using aggregated share classes at the fund level.

### 3.2. Alpha, beta, and delta

Our analysis in Section 4 begins by examining the relationship between a fund's risk-adjusted return (alpha) and its ambiguity exposure (delta), *i.e.*, between  $\alpha$  and  $\Delta^{UNC}$ . Fund delta is defined as the difference between the fund's uncertainty beta and market beta, *i.e.*,  $\Delta^{UNC} := (\beta_{mkt}^{UNC} - \beta_{mkt})$ . As discussed in Section 2, the smooth ambiguity model predicts a positive relationship between alpha and delta when investors are heterogeneous in their attitudes towards ambiguity and some are ambiguity-averse.

We compute each fund's alphas and market betas over 60-month horizons using a rolling window regression of the fund's excess returns on pricing factors. The regression specification follows either the market model or the Carhart model (Carhart, 1997). Let  $n = 1, 2, \dots, N$  index funds and  $t = 1, 2, \dots, T$  index months, where  $N = 14,728$  and  $T = 246$ . Denote by  $\Omega_{60}[s]$  the set of 60 consecutive months ending in month  $s$ . For fund  $n$  in month  $s$ , we estimate alpha and market beta using the following regression:

$$r_{nt} - r_{nt}^{free} = \alpha_{ns} + \beta_{mkt,ns}(r_{nt}^{mkt} - r_{nt}^{free}) + \epsilon_{nt}, \quad (6)$$

where  $t \in \Omega_{60}[s]$ ,  $\epsilon_{nt}$  is an error term, and  $r_{nt}$ ,  $r_{nt}^{free}$ , and  $r_{nt}^{mkt}$  denote fund returns, risk-free rates, and market returns, respectively. The Carhart-based alpha and beta measures are similarly obtained by augmenting the model in Eq. (6) with book-to-market, size, and momentum factors from the Fama–French database.<sup>5</sup>

In the theoretical framework of Maccheroni et al. (2013) and Mukerji et al. (2019) that underpins our analysis, ambiguity arises from second-order uncertainty about a fund's expected return. Specifically, investors face multiple, potentially mixed signals about the expected return but cannot determine with certainty which one is true. They respond to this ambiguity by assigning subjective probabilities to each signal being true. The uncertainty beta relates to how sensitive investors' expectations about the true expected return are to market-wide developments.

Operationalizing uncertainty beta in empirical work necessarily requires auxiliary assumptions about the nature and dimensionality of the alternative signals and investors' second-order beliefs. While the theoretical framework does not prescribe specific guidance on these aspects, recent studies at the intersection of decision theory and empirical asset pricing offer a useful set of empirical assumptions on which we can build. Barberis et al. (2016) is the first to demonstrate an empirical relationship between expected returns and prospect theory (PT) values of stocks. To compute the PT values, the authors model each stock as a probability distribution over its possible returns, an approach that requires auxiliary assumptions about which returns are feasible and how likely each return is. They fill in this gap by assuming a discrete uniform distribution, the support of which is the stock's historical monthly returns from the preceding 60 months (five years). This time window is motivated by the observation that investment publications frequently report five-year return histories, making it arguably a plausible approximation to the information investors typically evaluate.<sup>6</sup> Building on Barberis et al. (2016), subsequent studies have applied similar modeling strategies to other asset markets. For example, Zhong and Wang (2018) examine corporate bonds and Gu and Yoo (2021) examine mutual funds; both assume a uniform distribution over the respective asset's 60-month historical returns and find that PT values help explain investor demand for those assets. Outside the PT-based studies, Li et al. (2017) analyze mutual fund flows using the maxmin expected utility framework of Gilboa and Schmeidler (1989). Noting that publicly available fund performance statistics are often reported over 12-, 36-, and 60-month horizons, they use a fund's worst historical performance over those periods as proxies for investors' worst-case expectations. These worst-performance indicators are found to be significant predictors of fund flows.

Building on this literature, we model investors as facing ambiguity about a fund's expected return in the form of five alternative signals. These signals correspond to the fund's historical average gross returns over the preceding 12-, 36-, and 60-month horizons, as well as the corresponding measures over the quarterly and semi-annual (*i.e.*, 3- and 6-month) horizons, which are similarly accessible. By construction, the five averages are calculated over overlapping time periods, with the overlapping elements receiving greater weights in shorter-horizon measures; for example, each of the fund's returns over the past three months is weighted by 1/3 in the 3-month return, compared to 1/36 in the 36-month return. We further assume that investors' second-order beliefs over these

<sup>4</sup> Perhaps the most straightforward way to evaluate the robustness of our analysis without affecting the integrity of our main sample would be to examine pre-1999 data separately. As summarized towards the end of Section 4.1, we obtain qualitatively similar findings for the pre-1999 data.

<sup>5</sup> The factors include book-to-market (HML), size (SMB), and momentum (UMD), abbreviated from High-Minus-Low, Small-Minus-Big, and Up-Minus-Down, respectively.

<sup>6</sup> The assumption of a uniform subjective distribution over possible outcomes, in the absence of known objective probabilities, is also commonly adopted by experimental studies into choice under ambiguity (*e.g.*, Ahn et al., 2014).

**Table 1**  
Summary statistics.

Variable	Definition	Obs	Mean	SD
$ALPHA_{Market}$	Market: Risk-adjusted returns (in %)	1,352,416	0.076	0.389
$DELTA_{Market}$	Market: Fund delta	1,352,416	−0.856	0.194
$BETA_{Market}^{UNC}$	Market: Fund uncertainty beta	1,352,416	0.123	0.052
$BETA_{Market}$	Market: Fund beta	1,352,416	0.980	0.220
$ALPHA_{Carhart}$	Carhart: Risk-adjusted returns (in %)	1,352,416	0.035	0.296
$DELTA_{Carhart}$	Carhart: Fund delta	1,352,416	−0.831	0.159
$BETA_{Carhart}^{UNC}$	Carhart: Fund uncertainty beta	1,352,416	0.132	0.057
$BETA_{Carhart}$	Carhart: Fund beta	1,352,416	0.963	0.177
FLOW	Growth rate (in %) of assets under management	1,352,370	4.080	1284.270
SIZE	Fund total net assets (TNA) in \$ mn.	1,352,414	785.526	4401.897
LOAD	Fund load	1,352,416	0.007	0.015
EXP	Fund expense ratio	1,352,416	0.012	0.007
TURN	Fund turnover ratio	1,352,029	0.756	2.019
AGE	Fund age in years since inception in CRSP	1,352,416	12.500	7.400

Notes: Growth rate is equal to  $[SIZE_t - SIZE_{t-1}(1 + RET_t)]/[SIZE_{t-1}(1 + RET_t)] \times 100$  where  $t$  indexes months and  $RET_t$  is the gross return between  $t - 1$  and  $t$ . The subscript *Market* (*Carhart*) indicates that the variable is derived from 60-month rolling window regressions of fund returns described in SubSection 3.2, based on the market (Carhart) model specification. In our two-way fixed effects model analyses, we winsorize the variables *FLOW*, *SIZE*, *EXP*, and *TURN* at their respective 1st and 99th percentiles; the summary statistics in this table refer to their non-winsorized values.

five signals follow a discrete uniform distribution; that is, investors assign an equal chance (namely, 1/5) to each signal being the true expected return.

Let  $\bar{r}_{nt}[\tau]$  denote the fund's average return across months  $t - \tau$  through  $t$ . Given our auxiliary assumptions, the market model measure of fund  $n$ 's uncertainty beta in month  $s$  is given by the estimate of  $\beta_{mkt,ns}^{UNC}$  from the following regression

$$\bar{r}_{nt}[\tau] - r_{nt}^{free} = \alpha_{ns}^{UNC} + \beta_{mkt,ns}^{UNC}(r_{nt}^{mkt} - r_{nt}^{free}) + v_{nt} \quad (7)$$

where  $t \in \Omega_{60}[s]$ ,  $\tau \in \{3, 6, 12, 36, 60\}$ , and  $v_{nt}$  is an error term. The possible values of  $\tau$  imply that, in each month  $t$ , the data used in Eq. (7) encompass five average returns, in contrast to Eq. (6), which includes only the return for month  $t$  itself. Consequently, the rolling window sample is five times larger in the former case. The Carhart measure of the uncertainty beta is similarly obtained from a regression specification that augments the market model in Eq. (7) with book-to-market, size, and momentum factors.<sup>7</sup>

We compute fund  $n$ 's delta in month  $s$  by taking the difference between the rolling window estimates of its uncertainty beta and market beta. That is,  $\Delta_{ns}^{UNC} = (\beta_{mkt,ns}^{UNC} - \beta_{mkt,ns})$ . Our analysis is based on a fund-level panel data set that merges the CRSP data with the collection of alphas, market betas, uncertainty betas, and deltas across all fund-month observations. Since the alpha, delta, and beta measures vary depending on whether the market model or the Carhart model is used for risk adjustment, we will study each set of measures separately in Section 4.

Table 1 reports sample statistics on variables to be used. The unit of our analysis is a fund observed in a particular month of a given year. Across all observational units, the sample funds have average total net assets (TNA) of \$785.5 million and an average monthly net flow of 4.08%. They have an average total expense of 1.2% and total load of 0.68 basis points each month.

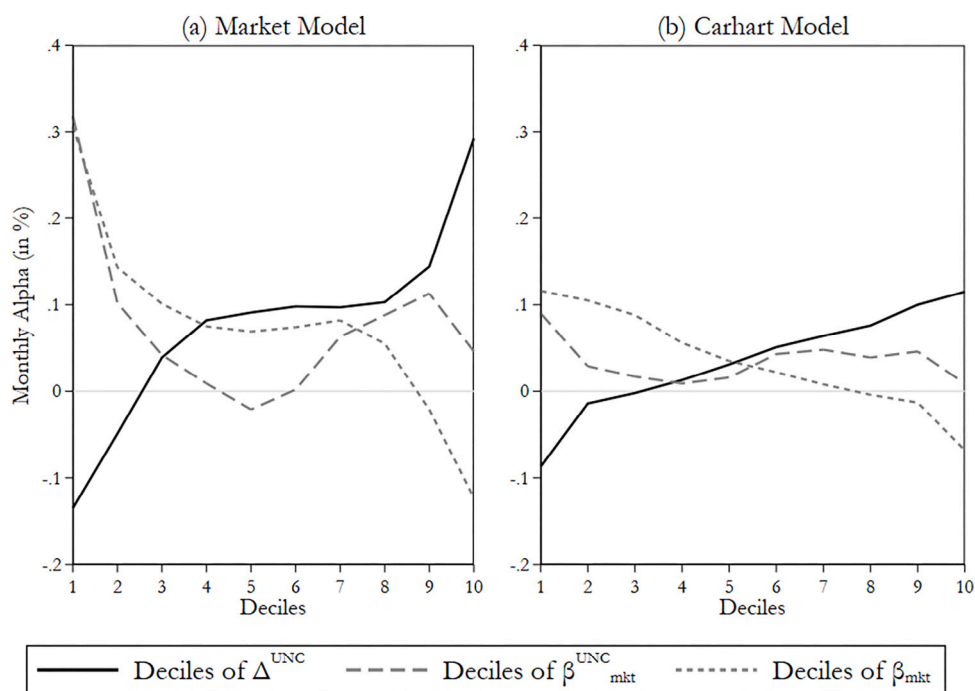
## 4. Results

### 4.1. Fund performance and ambiguity exposure

We first consider the bivariate relation between risk-adjusted returns (alpha;  $\alpha_{ns}$  in Section 3.2) and ambiguity exposure (delta;  $\Delta_{ns}^{UNC}$ ). Fig. 1 graphically summarizes the results.<sup>8</sup> The bold lines display positive slopes which are consistent with the notion that equity assets making up mutual fund holdings command positive ambiguity premiums. In each panel of the figure, we categorize fund-month observations by deciles of their delta and report the average alpha within each decile. The left panel is based on the market model estimates of alpha and delta. We observe that the average alpha increases from −0.135 percentage points (pp) in the lowest decile to the 0.292 pp in the highest decile, and their difference of 0.427 pp is statistically significant ( $p$ -value < 0.01). Moreover, the average alpha increases across consecutive deciles, except for a plateau between deciles 6 and 7 where the average alpha is equal to 0.098 pp and 0.097 pp, respectively. The right panel applies the same procedure to the Carhart model estimates of alpha and delta. The results show that the positive relation between fund performance and ambiguity exposure is robust to adjustment for the effects of book-to-market, size, and momentum factors on the fund performance. The upward trend in the average

<sup>7</sup> An alternative approach to estimating uncertainty beta is to regress the fund's excess returns on an index of market uncertainty, as speculated by Mukerji et al. (2019) and independently applied to stock returns by Bali et al. (2017). In Section 4.4, we compare our main results with those obtained using these index-based measures. We thank an anonymous reviewer for encouraging us to explore comparisons with this type of approach.

<sup>8</sup> Table A1 in the online appendix tabulates the corresponding numerical results.



**Fig. 1.** Average alpha in decile – all share classes.

Notes:  $\Delta^{UNC}$ ,  $\beta_{mkt}^{UNC}$ , and  $\beta_{mkt}$  refer to delta, uncertainty beta, and market beta measures described in Section 3.2. Each plot traces the sample average value of risk-adjusted fund returns, alpha ( $\alpha$ ), across relevant deciles. All averages and top-bottom decile differences are individually significant at the 1% level, with standard errors adjusted for two-way clustering in fund and year-month dimensions. Table A1 in the online appendix reports the corresponding numerical results. The left (right) panel uses alpha, delta and beta measures estimated from 60-month rolling window regressions of fund returns following the market (Carhart) model specification.

alpha across consecutive delta deciles is more apparent without any plateau. The top-bottom decile difference in alpha shrinks to 0.201 pp mainly due to less pronounced outperformance in the top decile (0.201 pp) but remains significant at the 1% level.

As reviewed in Section 2, the smooth ambiguity model by Mukerji et al. (2019) predicts that a financial asset's risk-adjusted returns and ambiguity exposure has a non-zero correlation if investors have heterogeneous preferences for ambiguity, and that the direction of this correlation is positive if ambiguity-averse investors are present in the heterogeneous population.<sup>9</sup> The bivariate relations identified by the bold lines in Fig. 1 support both predictions. The simultaneous presence of heterogeneous ambiguity attitudes and ambiguity aversion is consistent with the growing body of findings from experimental research that elicits individual-level ambiguity attitudes in controlled environments (Abdellaoui et al., 2015; Dimmock et al., 2016; Stahl, 2014). That we are able to replicate, in a broad sense of the term, the experimental results using the mutual fund returns data may be seen as findings supporting the predictive validity of the smooth ambiguity model and the construct validity of our empirical measurement of delta.

The results in Fig. 1 also suggest that measuring the fund's ambiguity exposure by its uncertainty beta ( $\beta_{mkt,ns}^{UNC}$ ) would have masked the positive slopes supporting the presence of positive ambiguity premiums. The dashed line in each panel plots variations in the average alphas across the uncertainty beta deciles. The top-bottom decile difference in the average alpha is negative, rather than positive, and significant ( $p$ -values < 0.01) in both panels: −0.272 pp for the market regression (left) and −0.081 pp for the Carhart regression (right). Moreover, the pattern of variations in the intermediate deciles is non-monotonic, displaying two turning points in the left (deciles 5 and 9) and three turning points (deciles 4, 8 and 9) in the right.

The fund's ambiguity exposure measure, delta, is equal to its uncertainty beta minus its market beta ( $\beta_{mkt,ns}$ ). We observe that the fund's market beta, the other leg of the bipod for delta, plays an integral role in generating the positive ambiguity premiums. The dotted lines in Fig. 1 plot variations in the average alphas across market beta deciles. In each panel, the dotted line is practically a mirror image of the bold line based on the delta deciles. We observe that the average alpha declines across consecutive deciles, except for a brief plateau which only affects the left panel; the top-bottom decile difference is equal to −0.430 pp and −0.184 pp in the left and right panels, respectively ( $p$ -values < 0.01). The importance of controlling for the fund's exposure to market risks in evaluation of the ambiguity premium is reminiscent of the analysis by Abdellaoui et al. (2011). They propose an index of ambiguity

<sup>9</sup> Conversely, if investors have homogeneous ambiguity preferences, the risk-adjusted returns and ambiguity exposure (i.e., alpha and delta) are expected to display no correlation. If investors have heterogeneous preferences because all investors are ambiguity-seeking to varied extents, a negative correlation is expected.

**Table 2**  
Fund performance and ambiguity exposure.

	ALPHA <sub>Market</sub>		ALPHA <sub>Carhart</sub>	
	(1)	(2)	(3)	(4)
DELTA	0.846*** (0.051)		0.250*** (0.046)	
BETA <sup>UNC</sup>		−1.442*** (0.162)		−0.985*** (0.122)
BETA		−0.654*** (0.048)		−0.206*** (0.047)
ln(SIZE)	0.067*** (0.003)	0.065*** (0.003)	0.049*** (0.002)	0.047*** (0.002)
LOAD	0.012 (0.071)	0.014 (0.067)	0.059 (0.057)	0.065 (0.056)
EXP × 100	−0.150*** (0.029)	−0.153*** (0.028)	−0.113*** (0.023)	−0.110*** (0.022)
TURN	−0.031*** (0.005)	−0.026*** (0.005)	−0.033*** (0.004)	−0.031*** (0.004)
Fund FE	YES	YES	YES	YES
Year-month FE	YES	YES	YES	YES
Obs	1,351,979	1,351,979	1,351,979	1,351,979

Notes: \*, \*\*, and \*\*\* indicate statistical significance at the 10%, 5% and 1% levels. Standard errors in parentheses are adjusted for two-way clustering in fund and year-month dimensions. The subscript *Market* (*Carhart*) indicates that alpha, delta, and beta measures relevant to the columns below are estimated from 60-month rolling window regressions of fund returns following the market (*Carhart*) model specification. The dependent variable is alpha in month *t* and the independent variables are delta, uncertainty beta, market beta, and other fund characteristics in the same month. Table 1 provides variable definitions. *ln(SIZE)* is the natural log of *SIZE* and *EXP* × 100 is *EXP* multiplied by 100. The model does not control for fund age (*AGE* in Table 1) explicitly because allowing for two-way fixed effects (FE) at the fund and year-month levels absorbs the effects of fund age.

aversion, which is constructed by comparing how pessimistic individuals are towards known and unknown probability distributions (i.e. risky and uncertain prospects) in an experimental setting.

We now turn to further analyses of the relation between fund performance and ambiguity exposure using panel data methods. We estimate two-way fixed effects models of risk-adjusted fund performance that account for date (i.e., year-month) fixed effects alongside the effects of any time-invariant fund characteristic. Each model also controls for the effects of observed fund characteristics (fund size, load, expense ratio, and turnover ratio) as defined in Table 1. Fund age is omitted from the list of explicit control variables because it is perfectly collinear with the date fixed effects, implying that our two-way fixed effects specifications control for the effects of fund age implicitly.

The two-way fixed effects estimates in Table 2 lend further support to our inferences about positive ambiguity premiums based on the bivariate analyses. The first two columns report the results that utilize the market model measures of alpha, delta, uncertainty beta, and market beta. Column 1 is the two-way fixed effects model that regresses alpha on delta and other fund characteristics. The coefficient of 0.846 on delta is significant at any conventional level ( $p$ -value < 0.01). Its positive sign suggests that a fund's risk-adjusted return (alpha) increases in its ambiguity exposure (delta), a finding that is consistent with Fig. 1. The sample standard deviations (SDs) of alpha and delta are equal to 0.389% and 0.194, respectively. Thus, the coefficient's magnitude implies that a one SD increase in delta leads to an increase in alpha that is equal to  $0.846 \times 0.194 / 0.389 = 0.422$  SDs.

In the second column of Table 2, we re-estimate the model in the first column after replacing delta with its two components, uncertainty beta and market beta. The coefficients on both types of betas have negative signs ( $p$ -value < 0.01), which are also consistent with general trends observed in Fig. 1. Specifically, we find that a one SD increase in the fund's uncertainty beta (market beta) is associated with a 0.193 SD (0.370 SD) decrease in the fund's alpha.<sup>10</sup> In sum, these market model results suggest that the positive ambiguity premium, captured by the positive effect of delta on alpha, is generated as the negative effect of market beta dominates that of uncertainty beta. We continue to draw similar inferences when we use the Carhart model measures of alpha, delta, uncertainty beta, and market beta. The last two columns of Table 2 report the relevant results. In column 3, we find that greater ambiguity exposure (delta) is associated with higher risk-adjusted fund returns (alpha) after controlling for the two-way fixed effects and observed fund characteristics. Recall that delta is derived by subtracting market beta from uncertainty beta; in column 4, we find that both components of delta have negative and significant effects on alpha, suggesting that the positive net effect of delta arises from the dominating negative effect of market beta.

Overall, the results in Table 2 suggest that one may see two separate findings in the recent literature as two sides of the same coin, which is the ambiguity premium (i.e., the effect of delta on alpha) predicted by the smooth ambiguity model. First, Bali et al. (2017) show that risk-adjusted stock returns decrease in stocks' exposure to macroeconomic uncertainty. This finding refers

<sup>10</sup> These results have been derived by combining the coefficient estimates with the descriptive statistics in Table 1, in an analogous manner as how the implied effect of a one SD increase in delta (namely a 0.422 SD change in alpha) has been derived.

**Table 3**  
Rankings of fund performance and ambiguity exposure.

	RK:ALPHA <sub>Market</sub>		RK:ALPHA <sub>Carhart</sub>	
	(1)	(2)	(3)	(4)
RK:DELTA	0.319*** (0.019)		0.155*** (0.018)	
RK:BETA <sup>UNC</sup>		−0.208*** (0.016)		−0.130*** (0.012)
RK:BETA		−0.249*** (0.021)		−0.149*** (0.021)
ln(SIZE)	0.055*** (0.002)	0.052*** (0.002)	0.053*** (0.002)	0.050*** (0.002)
LOAD	0.011 (0.056)	0.006 (0.053)	0.016 (0.056)	0.026 (0.055)
EXP × 100	−0.061*** (0.015)	−0.069*** (0.014)	−0.069*** (0.014)	−0.067*** (0.014)
TURN	−0.027*** (0.003)	−0.023*** (0.003)	−0.033*** (0.003)	−0.031*** (0.003)
Fund FE	YES	YES	YES	YES
Year-month FE	YES	YES	YES	YES
Obs	1,351,979	1,351,979	1,351,979	1,351,979

Notes: \*, \*\*, and \*\*\* indicate statistical significance at the 10%, 5% and 1% levels. Standard errors in parentheses are adjusted for two-way clustering in fund and year-month dimensions. The subscript *Market* (*Carhart*) indicates that alpha, delta, and beta measures relevant to the columns below are estimated from 60-month rolling window regressions of fund returns following the market (*Carhart*) model specification. The prefix *RK* indicates that the variable is equal to a within-month percentile ranking of the suffixed variable, expressed in fractional units so that it is equal to unity for the fund with the largest value of the suffixed variable in a given month. The dependent variable is the percentile ranking of alpha in month *t* and the independent variables are three percentile rankings (namely those of delta, uncertainty beta, and market beta) and other fund characteristics in the same month. [Table 1](#) provides variable definitions. *ln(SIZE)* is the natural log of *SIZE* and *EXP* × 100 is *EXP* multiplied by 100. The model does not control for fund age (*AGE* in [Table 1](#)) explicitly because allowing for two-way fixed effects (FE) at the fund and year-month levels absorbs the effects of fund age.

to the negative effect of “uncertainty beta” on stock-level alpha, where “uncertainty beta” in their study is obtained by regressing excess stock returns on a publicly available macroeconomic uncertainty index. A common thread across their uncertainty beta and our uncertainty beta, which approximates how sensitive expectations about future fund returns are to general moves in the equity market, is that both parameters are intended as measures of systematic uncertainty affecting asset returns over and above what is reflected in the traditional market beta ([Mukerji et al., 2019](#)). The negative effect of uncertainty beta on fund performance in our analysis supports the conclusion by [Bali et al. \(2017\)](#) that market beta does not fully capture the priced aspects of systematic uncertainty.

Second, a study by [Frazzini and Pedersen \(2014\)](#) has generated a renewed interest in the “beta anomaly” in stock returns, referring to a negative association between market betas and risk-adjusted returns at the stock level. The authors show that the beta anomaly exists in U.S and 20 international stock markets, and study its relation to leverage constraints that investors face. On the one hand, the negative effect of market beta on fund performance that we have found can be seen as another instance of the beta anomaly. On the other hand, seeing the market beta through the lens of the smooth ambiguity model, *i.e.*, as a component of delta that measures the fund’s ambiguity exposure, allows us to gain an insight that normalizes this beta anomaly as a reflection of a positive ambiguity premium.

The analysis so far has been atemporal in the sense that we have focused on the relation between fund performance and ambiguity exposure measured in levels. To examine whether the positive relation also exists on a month-by-month basis, we evaluate whether a positive relation exists between the within-month rankings of funds in terms of ambiguity exposure and their within-month performance rankings. We express a fund’s percentile performance ranking in month *t* in fractional units so that it is equal to 1 for the fund with the largest alpha and  $1/N_t$  for the fund with the smallest alpha, where  $N_t$  refers to the total number of funds observed in month *t*. A fund’s within-month rankings in terms of delta, market beta, and uncertainty beta are similarly derived in fractional units.

In [Table 3](#), we re-estimate each model specification in [Table 2](#) after replacing alpha, delta, uncertainty beta, and market beta with the fund’s within-month percentile rankings in those measures. Since we observe at least 1719 funds per month, the value of each ranking indicator is virtually equal to zero ( $1/N_t \leq 1/1719$ ) for the worst-performing fund in each month. Therefore, the coefficient on the delta ranking is practically equal to the change in the fund’s performance ranking in response to a maximal increase (*i.e.*, from the least to the most) in its ambiguity exposure ranking. The coefficients on the market beta and uncertainty beta rankings can be similarly interpreted as the effects that maximal increases in the respective ranking indicators have on the fund performance ranking.

In the results based on the market model measures, we observe that a maximal increase in the ambiguity exposure (delta) ranking is associated with a 0.319 increase in the performance ranking (column 1). In comparison, maximal increases in the uncertainty beta ranking and the market beta ranking lead to decreases in the performance ranking by 0.208 and 0.249 (column 2), respectively.

Separately controlling for the two beta ranking measures also result in negative coefficient estimates.<sup>11</sup> The last two columns show that replacing the market model measures with the Carhart model measures lead to qualitatively similar findings: A maximal increase in the fund's delta ranking leads to a 0.155 increase in its alpha ranking (column 3), whereas maximal increases in its ambiguity and beta rankings respectively lead to 0.130 and 0.149 decreases in its alpha ranking (column 4). To sum up, the ranking-based results reproduce all directional relationships that we have reported in Fig. 1 and Table 2, and agree with the results above that a positive ambiguity premium is generated as the negative effect of market beta dominates that of uncertainty beta.

Considering that the composition of the investor population, as well as their ambiguity preferences, may change over time, the relationship between fund performance and ambiguity exposure may also vary over time. In the online appendix (Appendix A), we present additional findings on the robustness of our results across different sampling periods. Specifically, we divide our main study period (1999–2019) into successive, non-overlapping five-year subsamples: 1999–2003, 2004–2008, 2009–2013, and 2014–2019.<sup>12</sup> Additionally, we analyze two five-year samples preceding our main study period: 1989–1993 and 1994–1998.<sup>13</sup>

For the market model measures of alpha and delta, between which the smooth ambiguity model of Mukerji et al. (2019) explicitly predicts a positive relation, we find the predicted relation across all six estimation samples, regardless of whether we employ the levels or percentile rankings (Table A3).<sup>14</sup> Underlying theory itself does not make any prediction about the Carhart model measures, but our main findings indicated that a positive relation existed there too. Similarly, we observe positive relations in the Carhart model measures in all but one of the six estimation samples (Table A4), the exception being 2009–13. This period coincides with the aftermath of the global financial crisis starting in late 2008, during which returns to other pricing factors, such as return to momentum, are also known to have experienced reversals (Daniel and Moskowitz, 2016). Nevertheless, the positive relation between alpha and delta shows resilience in the sense that it re-emerges in the subsequent subsample covering 2014–2019.

Our discussion thus far has focused on the contemporaneous relationship between fund performance and ambiguity exposure, which is the focus of the equilibrium analysis in Mukerji et al. (2019). A related empirical question – arguably more relevant from an investor's perspective – is whether a fund's ambiguity exposure can serve as a leading indicator of its future performance. Since our measure of ambiguity exposure, delta, is constructed from average returns across overlapping horizons within 60-month windows, one may expect it to exhibit positive autocorrelation at the fund level.<sup>15</sup> Combined with the positive contemporaneous relationship between alpha and delta, this suggests that the existence of a predictive relationship is a plausible hypothesis.

Under both the market (Table A5) and Carhart model specifications (Table A6), our results remain *quantitatively* similar when the level and ranking measures of delta in Tables 3 and 4 are replaced with their 12-month lags. This suggests that *greater* ambiguity exposure today predicts *greater* risk-adjusted returns one year ahead, regardless of whether *greater* is defined in absolute terms or relative to other funds in the same month. While a positive directional relationship is also observed when using the 36- or 60-month lags in place of the 12-month lags, the corresponding coefficients are smaller relative to the 12-month specifications. Moreover, in the case of the market model measures, once we partial out the predictive power of the 12-month lag by jointly including all three lags in the same regression, the coefficients on the 36- and 60-month lags lose statistical significance. Overall, this exploratory evidence suggests that our ambiguity exposure measure has potential as a leading indicator of fund performance over a relatively short horizon, although its merit compared to other possible combinations of past return measures requires further scrutiny.

#### 4.2. Flow-performance relationship and ambiguity premium

The results so far suggest that risk-adjusted fund performance incorporates an ambiguity premium as the smooth ambiguity model of Mukerji et al. (2019) predicts for a heterogeneous population with ambiguity-averse investors. We next turn to the implications of this finding for the flow-performance relationship. The empirical literature on mutual fund flows shows that past performance is a significant predictor of future fund flow. A recent study by Choi and Robertson (2020) offers behavioral insight into this empirical regularity; in a large-scale primary survey, they find that about a half of actively managed fund investors in the U.S. believe that the past performance is a reflection of the manager's stock picking skills.

Do fund investors discriminate between the ambiguity premium, which compensates for greater ambiguity exposure, and the remaining components of the fund's risk-adjusted performance, which includes returns to managerial skills? We use two-way fixed effects models of fund flow response that control for fund and date fixed effects to study the relation between fund flow in month  $t$

<sup>11</sup> The results are available upon request.

<sup>12</sup> The first three subsamples each cover 60 months, running from January of the start year through December of the end year, while the last subsample spans 66 months, from January 2014 through June 2019. Excluding the six months in 2019 from the final subsample does not materially affect our results.

<sup>13</sup> As discussed in Section 3.1, the CRSP Mutual Fund Database applied a different data collection method prior to August 1998. Each pre-1999 sample covers 60 months, running from January of the start year through December of the end year. Aggregating the two samples do not affect our finding on the positive alpha-delta relation. This relation is also robust to further expanding the aggregated sample to cover January 1979 through December 1998, that is, all pre-1999 data at our disposal after running 60-month rolling window regressions to obtain the alpha and delta measures. The results for these aggregated 1989–1998 and 1979–1998 samples are available upon request.

<sup>14</sup> All but one of the 16 coefficients – two for each five-year estimation sample, one based on the level and the other on the ranking measure – are significant at the 1%; the only exception concerns the ranking-based relation for the 1989–1993 sample where we have considerably a smaller number of observations than others due to the smaller number of mutual funds having been on the market then. In the case of the Carhart results to be summarized next, all 16 coefficients are significant at the 1% level.

<sup>15</sup> Indeed, a bivariate regression of the market model measure of delta on its 12-month lag yields an  $R^2$  of 0.82 and a slope coefficient of 0.90. The corresponding results are 0.54 and 0.72 for its 36-month lag, and 0.32 and 0.53 for its 60-month lag. In all three cases, the slope coefficients are significantly greater than zero ( $p$ -values  $< 0.01$ ). Bivariate regressions using the Carhart measures of delta display a similar pattern of positive autocorrelation.

and fund performance (alpha) and ambiguity exposure (delta) as at month  $t-1$ . Fund flow may be affected by the general movement of assets in and out of the mutual fund market, as well as the reallocation of the existing assets in the mutual fund market across different funds. The date fixed effects absorb macroeconomic shocks that drive the former type of movement, leaving the latter reallocation decision as the main driver of flow responses in our analysis. We thus follow Sirri and Tufano (1998) and Li et al. (2017) by measuring the past performance of each fund relative to that of others. Specifically, we employ a within-horizon percentile ranking of the fund's alpha, where the horizon refers to a 12-month, 36-month, or 60-month period ending in month  $t-1$ .<sup>16</sup> We similarly evaluate the corresponding within-horizon percentile ranking of the fund's ambiguity exposure, delta. Our models also control for the standard deviation of the fund's gross returns during the same period, along with the fund's observed characteristics (namely fund size, load, expense ratio, and turnover ratio) as at month  $t-1$ .

The results in Table 4 suggest that fund investors are practically indifferent between two funds with varied levels of ambiguity exposure, as long as those funds display equally good performance. Put another way, fund investors chase good performance, regardless of whether that good performance is driven by compensation for increased ambiguity exposure. The model specification in the first column employs the fund's 12-month percentile rankings in terms of the market model measures of alpha and beta. As we measure fund flow in percentages and percentile rankings in fractional units, the coefficient of 2.093 ( $p$ -value < 0.01) on the fund's 12-month alpha ranking implies that a maximal increase in the fund's 12-month performance ranking (*i.e.* from worst to best) leads to a positive flow response of 2.093 pp. This is about two-thirds of the mean of absolute fund flow in our sample (2.909%). The effect of a maximal increase in the fund's delta or ambiguity exposure ranking (*i.e.* from least to most exposed) is negligible in comparison ( $-0.037$  pp) and insignificant ( $p$ -value = 0.730), suggesting that fund investors neither penalize nor reward funds for increased ambiguity exposure. The third column applies the same 12-month specification apart from that it employs the Carhart measures of alpha and beta. Our findings remain qualitatively similar: A maximal increase in the fund's 12-month performance ranking generates a positive flow of 2.363 pp ( $p$ -value < 0.01), whereas a maximal increase in the fund's 12-month ambiguity exposure ranking generates a much smaller flow of 0.182 pp ( $p$ -value = 0.060).

We obtain similar findings in the remaining columns of Table 4 that employ the percentile rankings over 36-month or 60-month horizons. In column 2 (column 3), the results based on the market model measures of alpha and delta show that a maximal increase in the 36-month (60-month) performance ranking generates a positive flow of 1.156 pp (0.737 pp), whereas that in the ambiguity exposure ranking generates a relatively small flow response of 0.040 pp ( $-0.019$  pp). The latter flow response is statistically insignificant with a  $p$ -value of 0.740 (0.890). Column 5 (column 6) reports the corresponding results based on the Carhart model measures. We find that a maximal increase in the 36-month (60-month) performance ranking generates a positive flow of 1.432 pp (0.884 pp), which is significant at the 1% level. In comparison, a maximal increase in the ambiguity exposure ranking generates a relatively small flow response of 0.185 pp (0.176 pp), which is insignificant at the 5% (10%) level.

Our findings on statistically and practically insignificant responses to the fund's ambiguity exposure can be related to two recent empirical studies, albeit neither study is concerned with ambiguity exposure directly. First, Evans and Sun (2021) find that the positive relation between fund flow and alpha exists not because fund investors actually use alpha to evaluate fund performance, but because they apply heuristic decision rules which are correlated with alpha, for example by using Morningstar ratings to evaluate fund performance. In light of their findings, the insensitivity of fund flow to the fund's ambiguity exposure can be seen as a reflection of the fact that those heuristic decision rules do not discriminate between alpha and its component delta. Since the notion of ambiguity exposure expressed in delta is a new theoretical construct inspired by the smooth ambiguity model of Mukerji et al. (2019), the existence of substantial correlation between delta and prevailing decision heuristics can be expected to be quite fortuitous. Second, in an analysis of mutual fund flow attributable to the fund allocation decisions by defined contribution (DC) pension plans, Christoffersen and Simutin (2017) conclude that mutual fund managers may be tempted to hold high market beta stocks to beat performance benchmarks used by DC pension plans as there is no negative flow response to an increase in the fund's market beta. Our findings on the positive ambiguity premium in Section 4.1 and the insensitivity of fund flow to ambiguity exposure are consistent with their findings that the well-documented beta anomaly in fund performance does not have positive or negative implications for fund flow.

In the online appendix (Appendix B), we examine whether our findings on fund performance and flow-performance relationships remain robust when retail and institutional funds are analyzed separately.<sup>17</sup> The ambiguity premium in fund performance is a reflection of the ambiguity premium generated in the equity market: The mutual funds in our sample hold equity assets, and it is those assets that command ambiguity premiums which show up in the funds' alphas. Since which type of fund investor is more sophisticated has no bearing on equity investors' ambiguity attitudes, one should expect to find positive and significant ambiguity premiums in fund performance regardless of whether one looks at retail or institutional funds. By comparison, whether fund investors discriminate between two equally well performing funds with varied levels of ambiguity exposure is a behavioral response which depends potentially on their level of sophistication.

We continue to find a positive relationship between alpha and delta, regardless of whether we focus on retail or institutional funds. Our flow analysis also finds no differences between retail and institutional funds that would alter our earlier conclusions: In each subsample, the flow response to a maximal increase in the delta ranking remains below 0.200 pp and is insignificant at the 5% level.

<sup>16</sup> For each horizon  $h \in \{12, 36, 60\}$  ending in month  $t-1$ , the percentile is expressed fractional units; it is equal to  $1/N_{h(t-1)}$  for the fund with the smallest (*i.e.*, the worst) alpha and unity for the fund with the largest (*i.e.*, the best) alpha, where  $N_{h(t-1)}$  refers to the total number of funds observed for that horizon.

<sup>17</sup> A fund is classified as institutional if at least one of the following three criteria is met: (1) the underlying share class is identified as institutional by the Institutional Fund Flag item in the CRSP database, (2) the share name contains the word "institutional", and (3) the share name ends with the suffix "I" or "Y". The remaining funds are classified as retail.

**Table 4**  
Fund flow, performance ranking, and ambiguity exposure ranking.

	FLOW					
	Market			Carhart		
	(1)	(2)	(3)	(4)	(5)	(6)
RK:ALPHA <sub>12</sub>	2.093*** (0.075)			2.363*** (0.071)		
RK:ALPHA <sub>36</sub>		1.156*** (0.078)			1.432*** (0.072)	
RK:ALPHA <sub>60</sub>			0.737*** (0.085)			0.884*** (0.082)
RK:DELTA <sub>12</sub>	−0.037 (0.108)			0.182* (0.097)		
RK:DELTA <sub>36</sub>		0.040 (0.122)			0.185* (0.098)	
RK:DELTA <sub>60</sub>			−0.019 (0.135)			0.176 (0.108)
VOLATILITY <sub>12</sub>	−0.137*** (0.020)			−0.153*** (0.020)		
VOLATILITY <sub>36</sub>		−0.191*** (0.021)			−0.205*** (0.022)	
VOLATILITY <sub>60</sub>			−0.223*** (0.023)			−0.235*** (0.023)
ln(SIZE)	−0.462*** (0.026)	−0.407*** (0.027)	−0.375*** (0.027)	−0.476*** (0.026)	−0.428*** (0.027)	−0.386*** (0.027)
LOAD	−1.012* (0.585)	−1.002* (0.587)	−1.005* (0.586)	−1.000* (0.578)	−0.972* (0.580)	−0.970* (0.582)
EXP × 100	−0.015 (0.124)	−0.034 (0.126)	−0.081 (0.126)	0.011 (0.123)	0.008 (0.124)	−0.057 (0.125)
TURN	0.084*** (0.031)	0.058* (0.031)	0.048 (0.032)	0.103*** (0.031)	0.064** (0.031)	0.047 (0.032)
Fund FE	YES	YES	YES	YES	YES	YES
Year-month FE	YES	YES	YES	YES	YES	YES
Obs	1,320,718	1,320,886	1,320,916	1,320,718	1,320,886	1,320,916

Notes: \*, \*\*, and \*\*\* indicate statistical significance at the 10%, 5% and 1% levels. Standard errors in parentheses are adjusted for two-way clustering in fund and year-month dimensions. The heading *Market* (*Carhart*) indicates that alpha and delta measures relevant to the columns below are estimated from 60-month rolling window regressions of fund returns following the market (Carhart) model specification. *RK : ALPHA<sub>h</sub>* measures within-month percentile rankings of funds in terms of their average alpha values over the preceding *h* months; it is expressed in fractional units and equal to unity for the fund with the largest *h*-month average alpha as at a given month. *RK : DELTA<sub>h</sub>* is similarly defined with respect to delta. *VOLATILITY<sub>h</sub>* is the standard deviation of gross fund returns over the preceding *h* months. The dependent variable is fund flow in month *t* and the independent variables are one-month lags of alpha rankings, delta rankings, volatility in gross returns, and other fund characteristics. Table 1 provides variable definitions. *ln(SIZE)* is the natural log of *SIZE* and *EXP* × 100 is *EXP* multiplied by 100. The model does not control for fund age (*AGE* in Table 1) explicitly because allowing for two-way fixed effects (FE) at the fund and year-month levels absorbs the effects of fund age.

#### 4.3. Informational content of delta

We next consider results pertaining to the informational content of our ambiguity exposure measure, delta. We begin by examining whether a specific set of observed fund attributes characterizes those funds that tend to exhibit greater ambiguity exposure, as measured by delta. We then compare delta to alternative behavioral factors that have been employed by recent studies to incorporate non-standard preferences into the empirical analyses of fund performance and flows. Our regression models continue to account for two-way fixed effects in fund and date (i.e., year-month) dimensions, and we study the results for the whole sample as well as the retail and institutional fund subsamples. As with earlier, our findings remain qualitatively unchanged regardless of whether alpha and delta are derived from the market model or the Carhart model. For brevity, we only report the Carhart-based estimates and focus on addressing those aspects of our findings that are robust across all three estimation samples.

In Table 5, the dependent variable of each model is either a fund's delta itself or a fund's within-month percentile ranking in terms of delta. Our objective is to examine how these two representations of delta co-vary with three different classes of fund characteristics. Broadly speaking, the results support the construct validity of delta. As Eqs. (2) and (3) show, ambiguity in the smooth ambiguity model is concerned with difficulties in forming precise expectations about the fund's future performance. We find that delta changes in plausible directions with those characteristics that can be intuitively linked to such difficulties.

The top panel of Table 5 reports the relation between delta and directly observed fund characteristics that have been used as control variables in our earlier analyses: namely, fund size, load, expense ratio, and turnover ratio. The results suggest that funds with smaller asset holdings are exposed to a greater degree of ambiguity. For the whole sample and the institutional fund sample, we also find that funds which trade their holdings less frequently (i.e., funds with smaller turnover) display greater ambiguity exposure, a result which may reflect the role of active management in responding to unexpected developments in the market.

**Table 5**  
Ambiguity exposure and fund profiles.

	All		Retail		Institutional	
	DELTA (1)	RK:DELTA (2)	DELTA (3)	RK:DELTA (4)	DELTA (5)	RK:DELTA (6)
<i>A. Fund Characteristics</i>						
ln(SIZE)	−0.005*** (0.001)	−0.007*** (0.001)	−0.006*** (0.001)	−0.008*** (0.002)	−0.002** (0.001)	−0.003 (0.002)
LOAD	0.027 (0.025)	0.027 (0.046)	0.026 (0.026)	0.008 (0.045)	0.308* (0.180)	1.058* (0.568)
EXP × 100	0.001 (0.005)	−0.003 (0.011)	−0.003 (0.006)	−0.010 (0.012)	0.010 (0.009)	0.000 (0.023)
TURN	−0.003** (0.002)	−0.007** (0.003)	−0.001 (0.002)	−0.002 (0.003)	−0.010*** (0.003)	−0.024*** (0.005)
<i>B. Fund Factor Loadings</i>						
$\beta_{SMB}$	−0.145*** (0.012)	0.042** (0.018)	0.018 (0.011)	0.046** (0.019)	−0.004 (0.014)	0.011 (0.027)
$\beta_{HML}$	0.065*** (0.011)	0.038*** (0.012)	−0.027*** (0.008)	0.038*** (0.013)	−0.019 (0.012)	0.049*** (0.019)
$\beta_{UMD}$	−0.081*** (0.016)	−0.289*** (0.015)	−0.273*** (0.017)	−0.273*** (0.015)	−0.260*** (0.022)	−0.348*** (0.027)
<i>C. Moments of Fund Gross Returns</i>						
Mean	−0.010*** (0.002)	−0.019*** (0.003)	−0.011*** (0.002)	−0.018*** (0.003)	−0.008*** (0.001)	−0.020*** (0.003)
Std Dev	−0.024*** (0.002)	−0.052*** (0.003)	−0.027*** (0.002)	−0.057*** (0.004)	−0.016*** (0.001)	−0.039*** (0.003)
Skewness	0.008*** (0.003)	0.024*** (0.005)	0.011*** (0.003)	0.029*** (0.006)	0.003 (0.003)	0.012** (0.006)
Kurtosis	−0.000 (0.001)	−0.001 (0.002)	0.002 (0.001)	0.005** (0.002)	−0.004*** (0.001)	−0.011*** (0.003)
Fund FE	YES	YES	YES	YES	YES	YES
Year-month FE	YES	YES	YES	YES	YES	YES
N	1,351,979	1,351,979	871,014	871,014	446,307	446,307

Notes: \*, \*\*, and \*\*\* indicate statistical significance at the 10%, 5% and 1% levels. Standard errors in parentheses are adjusted for two-way clustering in fund and year-month dimensions. The heading *ALL* (*Retail* and *Institutional*) indicates that the results in columns below use observations on all (retail and institutional) funds. The dependent variables *DELTA* and *RK:DELTA* are ambiguity exposure measures based on the Carhart model; notes to Table 2 and Table 3 provide further information. The independent variables *Fund Characteristics* are also as defined in the same notes; *Fund Factor Loadings* refer to the coefficients on the size (SMB), book-to-market (HML), and momentum (UMD) factors in the underlying Carhart model; and *Moments of Fund Gross Returns* in each month have been calculated using observations on the preceding 60 months.

Common intuition suggests that it is harder to form precise expectations about the future performance of newer funds, making fund age another characteristic of potential interest. In the online appendix, we report one-way fixed effects models (Table A2) which only account for fund fixed effects. Omitting date fixed effects enables us to estimate the coefficient on fund age explicitly, albeit at the expense of robustness to date-specific macroeconomic effects. In all three estimation samples, the results suggest that newer funds indeed display greater ambiguity exposure.

The middle panel of Table 5 reports the relation between delta and risk-adjusted returns to Carhart pricing factors. To this end, we regress a fund's delta or delta ranking on its three Carhart factor loadings – that is, the coefficients on size (HML), book-to-market (SMB), and momentum (UMD) factors from rolling window regressions described in Section 3.2 – which excludes market beta that already forms part of delta. The results for the first two factors vary with estimation samples and dependent variables used, but in all cases we find that funds with higher deltas tend to be less sensitive to the momentum factor. The latter finding suggests that the asset holdings of high delta funds tend to be dissimilar to the composition of momentum portfolios, thereby making it difficult to exploit market momentum to form expectations about their future performance.

Finally, the bottom panel of Table 5 reports the relation between delta and moments of fund gross returns. A fund's moments in each month are calculated over the same 60-month period as the rolling window regression used in deriving its delta. The results show that delta tends to be higher during those periods when the mean return is smaller; the standard deviation is smaller; and the returns are more positively skewed. Economic analyses typically assume or find that decision makers seek a greater mean, a smaller standard deviation, and a greater positive skewness (Kimball, 1990; Ebert and Wiesen, 2011; Trautmann and van de Kuilen, 2018). Our finding suggests that the fund displays greater ambiguity exposure during those periods when its mean return and higher order moments make up conflicting signals about its desirability.

#### 4.4. Comparisons to other behavioral proxies

In the online appendix (Appendix C), we examine how delta compares with alternative behavioral factors inspired by other models of non-standard preferences, such as the maxmin expected utility model (Gilboa and Schmeidler, 1989) and prospect theory (Tversky and Kahneman, 1992). We find that these alternative behavioral factors, used in previous studies on fund flows, tend to align closely with uncertainty beta ( $\beta_{mkt}^{UNC}$  in our notation), which is one component of delta. This connection may help explain why all theory-inspired behavioral factors act as significant predictors of fund flows.

As detailed in Section 3.2, the construction of the ambiguity exposure measure, delta, in our analysis relies on a specific set of auxiliary assumptions. While our assumptions are comparable to those adopted by recent studies linking behavioral theory to empirical finance (Barberis et al., 2016; Li et al., 2017; Gu and Yoo, 2021), underlying theory ultimately provides limited guidance on how to operationalize the delta measure, leaving room for other implementation choices.

In the online appendix (Appendix A), we present results based on two alternative measures of ambiguity exposure. These apply markedly different approaches from ours to obtain a fund's uncertainty beta, from which its market beta is subtracted to derive delta. The first approach builds on the speculative suggestion by Mukerji et al. (2019, p.32) that, in empirical work, a "short cut approach to proxy" uncertainty beta may be to use the slope coefficient from a bivariate regression similar to Eq. (4), by regressing a fund's excess returns on an index of macroeconomic uncertainty – such as the one developed by Jurado et al. (2015) – rather than on market excess returns. The second approach is similar to the empirical strategy of Bali et al. (2017), who introduce their own version of uncertainty beta independently of the theoretical framework of Mukerji et al. (2019), which underpins our study. Although this version is also defined as the slope coefficient on the Jurado et al. index in a regression model of fund excess returns, it differs from the first approach in that the regression model follows a multivariate specification that controls for market excess returns and other pricing factors when estimating uncertainty beta.<sup>18</sup>

For the market model implementation of the first approach, we directly estimate a bivariate regression of fund excess returns on the Jurado et al. (2015) index of macroeconomic uncertainty.<sup>19</sup> For the Carhart model implementation, we extend this bivariate model by including controls for the three Carhart pricing factors (namely book-to-market, size, and momentum). We implement the second approach by extending the standard market and Carhart model specifications to incorporate the same index as an additional regressor.<sup>20</sup>

Our finding of a positive alpha-delta relationship is robust across the two alternative measures, regardless of whether the market model or the Carhart model is used (Table A7 in the appendix). The strength of this relationship, as reflected in the magnitude of the coefficient on delta, is also comparable, particularly in results based on percentile rankings of fund performance and ambiguity exposure that correspond to our main Table 3. We draw broadly similar, though more nuanced, conclusions about the relationship between fund flows and ambiguity exposure (Table A8). Consistent with our main flow analysis based on Table 4, we find that investors primarily chase funds with attractive risk-adjusted returns and do not penalize those with greater ambiguity exposure in terms of delta. Moreover, rather than observing an insignificant flow response to delta, we find significantly positive effects. Although this pattern was already present for the Carhart-based delta in our main analysis, it now appears across both the market model and Carhart model measures, with the magnitude of the positive effect also larger. Nevertheless, the positive effect of delta remains modest relative to that of risk-adjusted returns, alpha.<sup>21</sup>

As reported in Table 1, with our main measurement approach, the average of delta is  $-0.856$  using the market model and  $-0.831$  using the Carhart model. In comparison, the first and second alternative approaches yield averages of  $-1.058$  and  $-0.963$ , respectively, under the market model; and  $-1.063$  and  $-0.958$  under the Carhart model. All three sets of measures thus agree that individuals are less sensitive to systematic uncertainty than systematic risk, the latter being the focus of empirical asset pricing models outside the ambiguity aversion framework.<sup>22</sup>

What may explain the relative insensitivity of returns to systematic uncertainty compared to systematic risk? Interestingly, our results echo well-documented findings in experimental literature which are known as "a-insensitivity" – short for ambiguity-generated likelihood insensitivity – referring to the phenomenon that individual choice behavior shows insufficient responsiveness

<sup>18</sup> Strictly speaking, fund excess returns is a misnomer in the context of Bali et al. (2017). Their regression model concerns stock excess returns, as their analysis is focused on equity assets.

<sup>19</sup> The Jurado et al. (2015) index is available for download from the personal website of Sydney Ludvigson, a co-author of that study: <https://www.sydneyludvigson.com/macro-and-financial-uncertainty-indexes>. The index aims to capture uncertainty about macroeconomic conditions over next one-, three-, or twelve-month horizons. While the results in our appendix are based on the one-month ahead index, our findings are robust to the choice of horizon. As also reported by Bali et al. (2017), the indices across different horizons are highly correlated; during our main study period, for example, any pair of the three exhibits a correlation of at least 0.977.

<sup>20</sup> Put another way, we include market excess returns as an additional regressor in the bivariate and multivariate specifications used in the first approach.

<sup>21</sup> Specifically, regardless of the alternative measures used, the new results suggest that the fund most exposed to ambiguity is expected to attract 0.5 to 0.6 pp more investment flows than the fund least exposed; the corresponding estimate in our main Carhart-based regression is 0.2 pp. Nevertheless, even this larger effect remains modest compared to the flow difference between the best- and worst-performing funds in terms of alpha, which is around 2 pp or more. The study by Kutzner et al. (2017) offers useful intuition for why investors may exhibit ambiguity-seeking behavior in this context: When their investment objective is to earn above-average returns, a more ambiguous asset may be more appealing than a less ambiguous asset with a known limit on its upside potential, as the former does not preclude the chance of earning very high returns.

<sup>22</sup> Besides the similar averages, the three types of measures show high degrees of positive correlation. Under the market model, our measure of delta is strongly correlated with both the first (0.89) and second (0.93) alternatives, which in turn are highly correlated with each other (0.95). Under the Carhart model, the correlation coefficients are only slightly smaller: 0.84 between ours and the first alternative, 0.91 between ours and the second one, and also 0.91 between the two alternatives.

to the likelihood of ambiguous outcomes (Abdellaoui et al., 2011; Dimmock et al., 2016; Baillon et al., 2018; l'Haridon et al., 2018).<sup>23</sup> The smooth ambiguity model that has inspired our analysis is agnostic about the sign of the ambiguity exposure measure, delta, and does not provide a direct explanation for a-insensitivity. Baillon et al. (2025) develop a newer model of decision-making known as source theory, which is capable of explaining a-insensitivity, albeit this model is yet to be applied to obtain a prediction concerning risk-adjusted returns and ambiguity exposure. The empirical sign of the ambiguity exposure measure found in our mutual fund analysis, together with mounting evidence of a-insensitivity in experimental studies, suggests that formally exploring the asset pricing implications of source theory could be a promising avenue for future research.

## 5. Conclusions

In mutual fund literature, risk-adjusted performance – commonly referred to as alpha – is often linked to managers' stock-picking skills, with investors known to pursue funds showing positive alphas. However, recent advancements in asset pricing theory suggest that risk-adjusted returns also reflect an ambiguity premium. Using monthly data from the U.S. mutual fund market, we assess whether fund alpha includes compensation for ambiguity exposure in the fund's holdings. We also examine whether fund flows respond differently when more of a given alpha is driven by this ambiguity premium.

Our findings indicate that risk-adjusted returns indeed incorporate a positive ambiguity premium. A fund's alpha tends to rise with increased ambiguity exposure, and funds with higher ambiguity exposure in a given month tend to outperform those with lower exposure. However, we find that investors generally chase positive alphas without distinguishing whether these returns come from genuine managerial skills or greater ambiguity exposure. This behavior holds true for both institutional and retail investors, despite institutional investors might be better equipped to assess ambiguity exposure. Our measure of ambiguity exposure, derived from the smooth ambiguity model, shows several plausible associations with observed fund characteristics.

Our study complements experimental research on ambiguity aversion by demonstrating that the ambiguity premium, as predicted by decision theory, exists in real financial markets. Additionally, our findings suggest that two unrelated anomalies in the asset pricing literature – the beta anomaly and a premium for sensitivity to macroeconomic uncertainty – may both be part of the broader concept of the ambiguity premium. Finally, the insensitivity of fund flows to ambiguity exposure suggests that investors may be relying on heuristic decision-making tied to fund alphas, rather than actively pursuing alpha *per se*.

## CRedit authorship contribution statement

**Ariel Gu:** Writing – review & editing, Writing – original draft, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Hong Il Yoo:** Writing – review & editing, Writing – original draft, Software, Methodology, Investigation, Formal analysis, Conceptualization.

## Declaration of competing interest

Author do not have any conflict of interest to declare. Author confirm that this research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

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## Appendix A. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.jempfin.2025.101655>.

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<sup>23</sup> For instance, a given percentage change in the probability of an outcome may induce more behavioral responses when it concerns an objective probability (*i.e.*, risk) than a subjective probability assigned to an ambiguous outcome. Dimmock et al. (2016, p.1368) develop a formal and intuitive model that captures this type of behavior by employing a device known the ambiguity function.

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