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Abstract: Research often reports an overt discrepancy between theoretically / out-of context expressed teacher beliefs about mathematics and pedagogy and actual practice. In order to explore teacher knowledge in situation-specific contexts we have engaged mathematics teachers with classroom scenarios (Tasks) which: are hypothetical but grounded on learning and teaching issues that previous research and experience have highlighted as seminal; are likely to occur in actual practice; have purpose and utility; and, can be used both in (pre- and in-service) teacher education and research through generating access to teachers' views and intended practices. The Tasks have the following structure: reflecting upon the learning objectives within a mathematical problem (and solving it); examining a flawed (fictional) student solution; and, describing, in writing, feedback to the student. Here we draw on the written responses to one Task (which involved reflecting on solutions of $|x|+|x-1|=0$) of 53 Greek in-service mathematics teachers in order to demonstrate the range of teacher knowledge (mathematical, didactical and pedagogical) that engagement with these tasks allows us to explore.

USING TASKS
TO EXPLORE TEACHER KNOWLEDGE IN SITUATION-SPECIFIC CONTEXTS

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Explorations of teachers' beliefs and their relation to practice (see, for example, (Thompson 1992) for a review) acknowledge the overt discrepancy between theoretically and out-of context expressed teacher beliefs about mathematics and pedagogy (e.g. in interview-based studies) and actual practice. Therefore teacher knowledge is potentially better explored in situation-specific contexts. Within professional training courses focusing on situation-specificity, or what Shulman (1986, 1987) calls 'case knowledge', is far from novel: trainee lawyers are typically required to engage with problems that concern the application of the law in specific cases (thus exploring gaps in their understanding of the law as well as exploring the complexities of applying the law in real cases). Similar requirements feature also in the training of doctors and other professionals. In all of these cases the emphasis is on transforming theoretical knowledge into theoretically-informed practice. In the context of mathematics education, 'a domain of professional work that makes fundamental use of highly specialized kinds of mathematical knowledge, […] a kind of applied mathematics' (Bass 2005), this transformation – see (Watson & Mason, this volume) – has been described by concepts such as Chevallard's (1985) *transposition didactique*, Hill and Ball's (2004) *mathematical knowledge for teaching* and Shulman's (ibid) *pedagogical content knowledge*.

In the work we discuss in this paper we engaged mathematics teachers with classroom scenarios which are hypothetical but grounded on learning and teaching issues that previous research and experience have highlighted as seminal. We thus see these scenarios as likely to occur in actual practice. We perceive the type of task we present here as having both *purpose* and *utility* (in the words of Ainley and Pratt (2002) applied to teachers as learners) and we see the potential of these tasks both in terms of research and teacher education. We see these tasks as suitable for engaging both pre- and in-service mathematics teachers. We also believe that – in contrast to posing questions at a theoretical, decontextualised level – inviting teachers to respond to highly focused mathematically and pedagogically specific situations that are likely to occur in the mathematics classrooms they are (or will be) operating in can generate significant access to teachers' views and intended practices (Dawson 1999).

The mathematically / pedagogically specific situations that we invite teachers to engage with in our work are in the form of tasks which have the following structure:

Reflecting upon the learning objectives within a mathematical problem (and solving it)	Examining a flawed (fictional) student solution	Describing, in writing, feedback to the student

Task Structure

We propose that an examination of teacher responses to this type of task can support the following aims:

1. Explore teachers' subject-matter knowledge – crucially in terms of its gravitation towards certain types of mathematical thinking – and identify issues that their preparation for the

classroom needs to address (for example, in terms of distinctions such as relational and instrumental understanding (Skemp 1976), conceptual and procedural knowledge (Hiebert 1986) etc.).

2. Explore teachers' gravitation towards certain types of pedagogy and, crucially, explore how their preferences interact and are influenced by 1 (for example, in terms of constructivist principles (Freudenthal 1983) such as encouraging student participation in reconstructing initially incomplete or flawed solutions to mathematical problems).
3. Explore teachers' gravitation towards certain types of didactical practice, crucially, in the light of 1 and 2 and through the type of feedback they state they would provide to the student (for example, in terms of how they employ exemplification as a means for explanation, illustration etc. (Zaslavsky 2005)).

In sum the tasks offer an opportunity to explore and develop teachers' sensitivity to student difficulty and needs (Jaworski 1994) as well as an ability to provide adequate (pedagogically sensitive and mathematically precise) feedback to the student. Particularly by asking the teacher to engage with a specific (fictional yet plausible) student response that is characterised by a subtle mathematical error we can explore not only whether the teacher can identify the error but probe into its causes and grasp the didactical opportunity it offers (and the fruitful cognitive conflict it has the potential to generate). In this respect in designing these tasks we bear in mind the following:

- The mathematical content of the task concerns a topic or an issue that is known for its subtlety or for causing difficulty to students (from literature and/or previous experience).
- The fictional student response reflects this subtlety (or lack of) or difficulty and provides an opportunity for the teacher to reflect on and demonstrate the ways in which s/he would help the student achieve subtlety or overcome difficulty.
- Both mathematical content and fictional student response provide a context in which teachers' choices (mathematical, pedagogical and didactical) are allowed to surface.

In what follows we focus on one of the tasks we have used in the course of an ongoing study[1] and as part of a selection process for a Masters in Mathematics Education programme. As part of this selection process candidates sat an exam. This Task was amongst the exam questions. The 53 candidates were in-service secondary mathematics teachers: all are mathematics graduates with teaching experience that ranges from a few to many years. Most have attended in-service training of about 80 hours.

In a mathematics test students were given the problem:

"Solve the equation: $|x| + |x-1| = 0$"

a. What do you think the examiner intended by setting this problem?

b. A student responded as follows:

"It is true that

$$|x| + |x-1| = 0 \Leftrightarrow \left(|x| + |x-1|\right)^2 = 0 \Leftrightarrow x^2 + (x-1)^2 + 2|x(x-1)| = 0 \Leftrightarrow$$

$$x^2 + x^2 - 2x + 1 + 2|x(x-1)| = 0$$

Case 1: $x(x-1) \leq 0$

Then

$$2x^2 - 2x + 1 - 2x^2 + 2x = 0 \Leftrightarrow 1 = 0 \text{ Impossible.}$$

Case 2: $x(x-1) \geq 0$

Then $2x^2 - 2x + 1 + 2x^2 - 2x = 0 \Leftrightarrow 4x^2 - 4x + 1 = 0 \Leftrightarrow (2x-1)^2 = 0 \Leftrightarrow x = \frac{1}{2}$

[1] Supported by an EU ERASMUS Programme grant and by the University of Athens (ELKE). The data presented here have been translated from Greek.

Therefore the solution of the equation is $x = \frac{1}{2}$."

What comments would you make to this student with regard to this response?

The Task

We recognise that in the course of their engagement with the Task the teachers were *not* in the classroom and, for example, they had some time to think about their reaction. However we consider that the latter may allow teacher responses to be more representative of their *intentions* and to be more reflective. We see fostering the habit of this type of reflection as a significant by-product of the teachers' engagement with this type of task.

In this paper we aim to demonstrate how our rationale for these tasks is reflected in the design of this particular Task and exemplify the type of teacher knowledge that engagement with these tasks allows us to explore. We do so by drawing on teacher responses to the Task and we conclude with a brief proposition on ways in which this type of task can be integrated in teacher education programmes.

Analysis of the Task

The mathematical content of this Task – solving equations that involve the absolute value || – is central in the Year 10 Algebra syllabus of the Greek national curriculum. The notion of absolute value of a rational number appears for the first time in the Year 7 textbook[2] but teachers usually choose to introduce it a bit later at the beginning of Year 8 (in the textbook of which it appears briefly again in a recap chapter). In these textbooks the concept is introduced as follows: 'The absolute value of a positive number is the number itself, of a negative number is the opposite number and of zero is zero'. The definition of $|a|$ as *distance* on the number line is also mentioned but most applications and exercises in the Year 7-9 textbooks do not use it. In the Year 10 textbook the section on absolute value revisits the *distance* definition and emphasises that for a real number a**:**

$$|a| = \begin{cases} a, & if\ \ a \geq 0 \\ -a, & if\ \ a < 0 \end{cases}$$

Various properties of $|a|$ are then introduced and are followed by applications and exercises that include algebraic expressions, equations and inequalities involving absolute value. Previous research has highlighted difficulties that learners encounter with the concept at this stage and especially in an algebraic context (Chiarugi et al 1990).

There is a range of approaches to solving the equation in this Task. Two approaches that are ubiquitous in the Greek secondary mathematics classroom, both aiming to eliminate ||, are as follows:

- Distinguishing cases according to the sign of x and x-1. This method employs the above algebraic definition of absolute value. One requirement of this approach is that the solver needs to make sure that the solution emerging in each case is for an acceptable value of x;
- Squaring both sides of the equation. This method employs the property of ||, $|x|^2=x^2$ (itself connected to the property $x = y \Leftrightarrow x^2 = y^2$ for $xy \geq 0$). This method requires considerable facility with algebraic manipulation (e.g. expansions, employment of algebraic identities such as $(a+b)^2=a^2+2ab+b^2$ etc.).

[2] Teaching of school subjects in Greek state secondary schools typically follows a textbook distributed nationally by the Ministry of Education. The information available in this paragraph originates in the editions of the Ministry textbooks used at the time of publication for teaching the Year 7-10 mathematics syllabus.

Both approaches are generalisable and demonstrate substantial ability in algebraic manipulation. We will call these two approaches 'procedural'.

The fictional student response in Part (b) is a combination of both as the student first squares and then distinguishes cases. The student's solution also involves heavy algebraic manipulation. The subtle flaw in the student's solution is that s/he has failed to observe that in the second case ½ is not a solution (it does not satisfy $x(x-1) \geq 0$) and therefore the equation has no solutions.

Two other, also generalisable, approaches are graphical: sketching the graph of $f(x) = |x|+|x-1|$ and finding the intersections with the x-axis; and, sketching the graphs of $f(x) = |x|$ and $g(x) = -|x-1|$ and finding their common point(s), if any.

Finally there is an approach based on the observation that $|x|$ and $|x-1|$ must be simultaneously zero in order to add to zero (as they are both by definition non-negative quantities). As this is impossible the equation has no solutions. This approach relies on selecting a crucial attribute of the entity involved, the non-negativity of ||, and taking the logical step that if two non-negative quantities add to zero then both have to be zero. This approach is brief, elegant and requires a remarkable degree of conceptual understanding of || as well as the application of one subtle logical step. While it is specific to this particular equation, it does offer an opportunity for discussing equations more globally (e.g. observing that $|x|+|x-1|=0$ means $|x|=-|x-1|$ and realising the absurdity of the latter: how can something non-negative be definitely non-positive at the same time while it cannot be zero?). Also, from a didactical point of view, acknowledging this solution offers an opportunity to discuss meta-cognitive issues such as the benefits from having awareness of the multiple ways in which a problem can be approached (e.g. procedural, graphical and conceptual). For the above reasons we will call this approach 'optimum'[3].

We envisage responses to the Task as operating at least at three levels: the substantive (algebraic and logical manipulations in solving equations; conceptual understanding of absolute value), the meta-cognitive (acknowledgement of the multiple and qualitatively different ways in which an equation can be solved; optimal choice of solution) and the didactical (utilise the opportunity offered by the problem to discuss problem-solving skills such as the above mentioned elements of meta-cognitive awareness).

More specifically, and with relation to Aims 1-3, we propose that teachers' responses to the Task provide evidence for exploring the following questions:

- Can the teacher identify the underlying pedagogical aim of the equation (which we described above as involving a deeper conceptual understanding of absolute value, the application of a subtle logical step and the opportunity to discuss a range of solutions)?
- Can the teacher identify the subtle error, and its cause, in the student's response? Incorrect identification may provide evidence of issues in subject matter knowledge that need to be addressed.
- In the case of correct identification of the student's error, and its cause, how does the teacher choose to help the student cope with the error? Does the teacher simply point at the error and its cause? Does the teacher facilitate the student in identifying the error herself? What are the didactical benefits the teacher reaps from this error? The choices the teacher makes offer starting points for discussion of the rationale that underlies these choices.
- Does the teacher offer any other solutions to the student or does s/he limit feedback to a reconstruction of the student solution? This may be revealing both in mathematical and didactical terms. For example, perhaps the teacher does not see any other solution to the

[3] We note however that, again from a didactical point of view, a valid case for a preference for one of the other approaches can be made as well: it may be too confusing to bombard students at the relatively early stages of their algebraic learning with alternative solutions when they are still struggling with the standard methods.

equation (mathematical issue) or, even if s/he does, s/he does not attribute enough significance to discussing these other solutions with the student (didactical issue). In the cases s/he does discuss them with the student what is then the quality of this discussion? Is it just a fragmented citation of these solutions? Or is it a comparative, juxtaposing type of presentation that has the capacity to generate reflection on the choices one faces when solving equations?

- Does the teacher attempt to engage the student meta-cognitively? For example does s/he attempt to foster the idea that prior to engaging with the application of a standard procedure it is worth exploring whether other, potentially simpler approaches are possible in this particular case?

In what follows we provide evidence of the types of teacher knowledge that engagement with the Task allowed us to explore through reference to the analysis[4] of the 53 scripts/written teacher responses.

Exploring teacher knowledge: analysis and examples from the data

In sum for each script we produced an Analytical Summary, a narrative of approximately 100-200 words, in which we described the script's contents and evaluated the teacher's response in accordance with the above list of questions. Further scrutiny of the scripts and the Analytical Summaries led to a three-dimensional taxonomy and all scripts were characterised in terms of the *mathematical (M), didactical (D)* and *pedagogical (P)* issues they raised (five, nine and five categories respectively). Here we exemplify from the first and the second of these[5]. Numbers in brackets indicate number of teacher responses allocated to the category.

The summary lists for the *Didactical* and the *Mathematical Issues* are as follows[6]:

Mathematical Issues Emerging from Teacher Responses to the Task

1. Regards flaw of the student response to be the squaring in the first line (13).
2. Regards that, in Case 1, $x(x\text{-}1)\leq 0$ needs to be solved – this is unnecessary (7).
3. Does not see the 'optimum' solution (6).
4. Makes technical mathematical mistakes, e.g. in algebraic manipulations (8).
5. Does not identify any flaw in the student's response (1).

Didactical Issues Emerging from Teacher Responses to the Task

1. Does not reconstruct the student response and proceeds directly to presentation of solution (3).
2. In reconstructing the student's solution, in particular in order to reject ½ as an acceptable solution, proposes the use of standard procedural methods that are unnecessarily convoluted – e.g. solving

[4] See (Biza, Nardi & Zachariades 2006) for more details on this analysis.

[5] See (Biza, Nardi & Zachariades ibid) for an example of our analysis *across* M, D and P: there we focused on about a fifth of the scripts that demonstrated pedagogical sensitivity but were constrained mathematically (at the substantive or meta-cognitive level) and we examined how these constraints may divert teachers from materialising their good pedagogical intentions (for example, how insistence on the routine methods (D2 and D3) may have diverted the teachers from thinking about and/or suggesting to the student the 'optimum' solution (D4)). We identified three types of constraints: insistence on standard procedural methods; inappropriate contextualisation of otherwise commendable pedagogical practices; and, inadequate reflection on student thinking.

[6] Under *P* we listed: presenting solution without encouraging participation/discussion (21); no attempt to 'psychoanalyse' the student response (10); lacking meta-cognitive reflection (21); mere identification and correction of the mathematical flaw of the response (13); and, drawing hasty, largely unfounded and clichéd generalisations about the student's ability (3). Overall we hesitated to draw more definitive inferences from *P*: most *P* categories highlight the *absence* of a certain reference in the teachers' response – for example, to encouraging student participation. We feel that we cannot infer a teacher's conscious choice against student participation from this non-reference in their script (even though many responses started with 'I would tell…'). To draw such an inference we would need further and more solid evidence, e.g. from observing their classroom practice or interviewing. On the other hand we feel more confident in acknowledging commendable pedagogical intent in scripts where there are overt references to encouraging student participation (some responses did start off with 'I would encourage the student to…' or 'I would ask him to…').

inequalities, graphing the intervals in which x needs to belong etc. – instead of simply substituting x with ½ in the inequality and seeing it needs to be rejected. (21).

3. Presents a standard procedural method other than the student's (2).
4. Despite identifying the 'optimum' solution in part (a), does not refer to it in part (b) – either at all (14) or faintly (3).
5. Describes an overly general and theoretical pedagogical approach (7): offers substantively and meta-cognitively rich propositions but does not embed them in the specific situation set in the question (5); alludes to constructivist ideas such as encouraging the student's own reconstruction of the solution but in fact simply lets the student unguided and possibly lost (1); offers limited feedback based on superficial generalisations on student's ability (1).
6. Appears to aim at the use of commendable pedagogical practices, such as exemplifying, but employs them unsuccessfully – e.g. proposes examples that are incorrect, miss the point or are potentially misleading (3).
7. Uses mathematical (terms, symbolism) or ordinary language problematically (8).
8. Focuses excessively on insubstantial, trivial aspects of the question (2).
9. Uses mathematical formalism in an over-the-top and potentially misleading way (1).

A significant number of teachers (13) identified squaring as the source of the student's error. 'Raising to the power of 2', says one, 'we increase the number of solutions, we thus introduce new solutions'. This is correct in many cases as $x^2 = y^2$ does not imply that $x = y$ for all real numbers x and y. However in our case it is not the source of the problem as the equation is of the $f(x)=0$ kind and $x = 0 \Leftrightarrow x^2 = 0$ for every real number x. Squaring is therefore not the source of the student's error.

Three teachers identified the source of the student's error in the fact that the student allowed $x(x-1)=0$ to be examined in both the cases s/he distinguished. One teacher writes:

> 'when we distinguish cases we do not allow these cases to have values of x in common (for example the student in our case let $x(x-1)=0$ belong to both cases'.

Only four teachers attempted to highlight the problem to the student by offering simple examples. Some of these examples suggest that the teachers perceive the flaw in the student response to be the squaring in the first line (M1). For example one teacher asks the student whether the equation $x=1$ is equivalent to $x^2=1$. Another teacher asks: 'would I be allowed to square |x|+1=0?'. In these cases the teachers followed an incorrect diagnosis of the student response with a didactically commendable but mathematically flawed employment of examples.

Six teachers also do not 'see' the 'optimum' solution and seventeen teachers, even though they hint at it in Part (a), do not refer to it at all or faintly in Part (b). One issue worthy of further exploration is what determines this absence in Part (b). Furthermore, of those who refer to the 'optimum' solution in Part (b) only one juxtaposes it to the 'procedural' solution offered by the student: 'it requires significantly fewer manipulations and the result is easier to check', this teacher writes.

Twenty-one teachers point out that, in order to find out that ½ needs to be rejected as a solution to the equation, the student 'should have solved the inequality in the second case s/he distinguished'. Some even say s/he 'should have done so in both cases'. But there are far simpler ways for finding out that ½ needs to be rejected: e.g. by substituting ½ in the initial equation or by observing the limiting condition in one of the cases.

Eight teachers use mathematical terms in their writing with insufficient precision, for example 'positive number' as meaning 'non-negative number' etc. whereas eight teachers make technical mathematical mistakes (e.g. in algebraic manipulations). There is also one teacher who does not identify any flaw in the student's response.

Finally *none* of the teachers mentioned a graphical solution. We conjecture that this may emanate from the fact that graphical approaches are not usual in the Year 10 Algebra course. However this remains a potentially interesting revelation as it may reflect a certain unease of the participating teachers with graphical approaches. It may also merely reflect the fact that the chapter on the graphical representation of $|x|$ appears in the Greek textbook *after* the chapter on solving this type of equation. The fact that it emerged in the course of the teachers' engagement with the Task points at the capacity of this type of task to allow such tendencies to surface. We were to some extent surprised by this and by other occurrences in the data (e.g. the substantial number of teachers who identified the flaw in the student response to be the squaring in the first line). We welcome this element of *surprise* and credit the Task's design for allowing such surprise to occur.

Other observations that emerged in the course of our use of the Task (and others of the same type) with regard to its capacity to reveal crucial aspects of teacher knowledge include: the need for a *diversity of tasks* (as different kinds of insight into teacher knowledge emerged from teachers' engagement with different tasks); and, the need for evidence of teacher knowledge from a *diversity of sources* (e.g. observation of actual classroom practice, group discussions, interviews etc.). In resonance with the section *Ways of Working in Teacher Education* in (Watson & Mason, this volume) we conclude with a brief proposition of how these tasks can be employed in a teacher education context in productive ways.

Using this type of task in the context of teacher education

We propose that these tasks can be employed in a teacher education context as follows: as tools for the identification and exploration of mathematically, didactically and pedagogically specific issues regarding teacher knowledge (that purely theoretical questions on pedagogy or mathematics could not have identified); and, as triggers for teacher reflection on these issues. We also note that engagement with these tasks can function as a preliminary, preparatory, smoother transitory phase for pre-service teachers prior to their exposure to real classroom situations.

Overall we envisage these tasks as offering opportunities for preparing teachers to enter the classroom with a heightened ability for reflective practice (Schön 1987). These opportunities may be in the form of workshops in which teachers engage with these tasks and reflect/discuss their and others' responses to these tasks; or, in the form of following up engagement with these tasks with classroom trials of the mathematical problems in question (and juxtaposing the students' actual responses with the responses discussed during task activity). In sum we see this type of task as part of a preparatory environment that raises and develops teacher awareness (Mason 1998).

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